Class : 11

Register Number

SECOND REVISION EXAMINATION - 2024

Time Allowed : 3.00 Hours]

MATHEMATICS

[Max. Marks : 90

PART - I

I. **Answer the following:** 20x1=20

1. If n [(AxB) ∩ (AxC)] = 8 and n(B∩C) = 2 then n(A) is

   a) 6 b) 4 c) 8 d) 16

2. The range of $\frac{1}{1-2\sin x}$

   a) $(-\infty, -1)\ (^1/_3, \infty)$ b) $(-1, {}^1/_3)$ c) $[-1, {}^1/_3]$ d) $(-, -1)\ [^1/_3, \infty)$

3. The value of $\log_3 {}^1/_{81}$ is

   a) -2 b) -8 c) -4 d) -9

4. Cos 1° + Cos 2° + ... + Cos 179° -----------

   a) 0 b) 1 c) -1 d) 89

5. If tan α and tan β are the roots of the equation $x^2+ax+b=0$ then the value of $\frac{+\sin(\alpha+\beta)}{\sin\alpha \sin\beta}$ is

   a) $^b/_a$ b) $^a/_b$ c) $^{-a}/_b$ d) $^{-a}/_b$

6. The no of sides of the regular polygon have 44 diagonals are

   a) 4 b) 40 c) 11 d) 22

7. $n\text{-}1C_r + (n-1)C_{r-1}$ is

   a) $n+1C_r$ b) $n\text{-}1C_r$ c) $n\text{-}1C_r$ d) nC_r

8. The coefficient of x^6 in the expansion of $(2+2x)^{10}$ is

   a) $10C_6 x 3^4 x 2^6$ b) $10C_6\ 2^6$ c) $10C_6\ 2^{10}$ d) $10C_6$

9. The nth term of the sequence 1, 2, 4, 7, 11 .... is

   a) n^3+3n^2+2n b) n^3-3n^2+2n c) $\frac{n(n+1)(n+2)}{3}$ d) $\frac{n^2-1n+2}{2}$

10. The length of the normal drawn from the origin to the line having slope 2 is $\sqrt{5}$. Then the equation of the line is

    a) $x-2y = \sqrt{5}$ b) $2x-y= \sqrt{5}$ c) $2x+y=5$ d) $x-2y-5=0$

11. If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and $(A+B)^2 = A^2 + B^2$ then the value of a and b are

    a) a=4; b=1 b) a=1; b=4 c) a=0; b=4 d) a=1; b=4

12. The value of the determinant $A = \begin{vmatrix} 0 & a & -b \\ -a & 0 & c \\ b & -c & 0 \end{vmatrix}$ is

    a) -2abc b) abc c) 0 d) $a^2+b^2+c^2$

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13. If a, b, c are collinear then which of the following is true?

a) $\bar{a} = \bar{b} + \bar{c}$ b) $2\bar{a} = \bar{b} + \bar{c}$ c) $\bar{b} = \bar{c} + \bar{a}$ d) $4\bar{a} + \bar{b} + \bar{c} = 0$

14. Sum of squares of disection cosines of $\bar{r}$ is --------

a) 1 b) 0 c) 2 d) -1

15. The value of $\lim_{x\to 0} \frac{xe^x - \sin x}{x}$ is

a) 1 b) 2 c) 3 d) 0

16. Differentiation of 2^x is

a) $2^x \log x$ b) $2^x \log 2$ c) log 2 d) log x

17. If $y = f(x^2+2)$ and $f'(3) = 5$ then the value of dy/dx at x=1 is --------

a) 5 b) 25 c) 15 d) 10

18. $\int \sin^3 x\, dx$ = ---------

a) $\frac{-3}{4}\cos x - \frac{\cos 3x}{12} + c$ b) $\frac{3}{4}\cos x + \frac{\cos 3x}{12} + c$

c) $\frac{-3}{4}$ d) $\frac{-3}{4}\sin x + \sin 3x + c$

19. Value of $\int \frac{dx}{3x-2}$ is

a) $\frac{-1}{3}\log|(3x-2)| + c$ b) $\log|(3x-2)|$

c) $\frac{1}{3}\log|(3x-2)| + c$ d) $\frac{(3x-2)^2}{6} + c$

20. A and B are two independent events. If P(A) = 0.35 and $P(A\cup B) = 0.6$, then P(B) is

a) $5/13$ b) $1/13$ c) $4/13$ d) $7/13$

PART - B

II. Answer any Seven questions. Question No. 30 is compulsory. **7x2=14**

21. Find the number of subsets of A if $A = \{x : x=4n+1, 2\le n\le 5, n\in N\}$

22. Solve: $\frac{1}{5}|10x-2| < 1$.

23. If A+B = 45° then prove that (1+tanA) (1+tanB) = 2.

24. If $\frac{1}{8!} + \frac{1}{9!} = \frac{n}{10!}$ then find the value of n.

25. Show that the lines are 3x+2y+9=0 and 12x+8y-15=0 are parallel.

26. If $A = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$ then find A^4.

27. If $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + 5\hat{j} - 2\hat{k}$ then find $\vec{a} \times \vec{b}$.

28. Differentiate the following (i) $y = e^{-mx}$ (ii) y = sinx + cos x.

29. Integrate the following (i) $\frac{x^{24}}{x^{25}}$ (ii) $\frac{1}{\cos^2 x}$

30. A year is selected at random. What is the probability that (i) it contains non leap year 53 sundays (ii) It is a leap year contains 53 sundays.

PART - C

III Answer any Seven questions. Question No. 40 is compulsory. **7x3=21**

31. Find the domain of $\frac{1}{1 - 2\sin x}$

32. Prove $\log a + \log a^2 + \dots\dots\dots + \log a^n = \frac{n(n+1)}{2} \log a$.

33. Integrate the following with respect to x $\int \frac{\sin x + \cos x}{\sin x - \cos x} dx$

34. If D and E are the midpoints of the sides AB and AC of a triangle ABC, prove that $\overline{BE} + \overline{DC} = \frac{3}{2} BC$.

35. If the area of triangle with vertices (-3,0) (3,9) (0,k) is 9 then find the value of k.

36. Find the derivative of ; $y = x^{\cos x}$

37. If $^nP_r = 720$ and $^nC_r = 120$ find n, r.

38. If A and B are two events such that $P(A \cup B) = 0.7$ $P(A \cap B) = 0.2$ and $P(B) = 0.5$ then show that A and B are independent.

39. If $\tan\theta = \frac{3}{4}$ and θ lies in the third quadrant, then find the value of sec θ.

40. If $y = \sin^{-1}x$ then find y^{11}.

PART - D

IV Answer all the questions. **7x5=35**

41. a) In a survey of 5000 persons in a town, it was found that 45% of the persons know language A, 25% knowledge B, 10% know language C, 5% know language A and B, 4% know language B and C and 4% know language C and A/ If 3% of the persons know all the three languages find persons know all the three languages, find the number of persons who knows only language A.

(OR)

b) Evaluate the following limits: $\lim_{x \to 0} \frac{\sqrt{1+\sin x} - \sqrt{1-\sin x}}{\tan x}$

42. a) Find all values of x that satisfies the inequality $\frac{2x-3}{(x-2)(x-4)} < 0$

(OR)

b) Integrate the following with respect to x $\frac{5x-2}{2+2x+x^2}$

43. a) Prove that $\dfrac{\text{Cot}(180^\circ+\theta)\ \text{Sin}(90^\circ-\theta)\ \text{Cos}(-\theta)}{\text{Sin}(270^\circ+\theta)\ \tan(-\theta)\ \text{Cosec}(360^\circ+\theta)} = \text{Cos}^2\theta\ \text{Cot}\theta$

(OR)

b) Show that the equation $2x^2-xy-3y^2-6x+19y-20=0$ represents a pair of intersecting lines. Show further that the angle between them is $\tan^{-1}(5)$.

44. a) By the principle of mathematical induction, prove that for all integers $n \geq 1$.

$$\frac{1}{1.2}+\frac{1}{2.3}+\ldots\ldots+\frac{1}{n(n+1)}=\frac{n}{n+1}$$

(OR)

b) Prove that $\begin{vmatrix} b+c & a-c & a-b \\ b-c & c+a & b-a \\ c-b & c-a & a+b \end{vmatrix} = 8abc.$

45. a) If $y = e^{\tan^{-1}x}$ then show that $(1+x^2)y'' + (2x-1)y' = 0$.

(OR)

b) Prove that the points whose position vectors $2\hat{i}+4\hat{j}+3\hat{k}$, $4\hat{i}+\hat{j}+9\hat{k}$ and $10\hat{i}-\hat{j}+6\hat{k}$ form a right angled triangles.

46. a) Prove that $\lim\limits_{\theta\to 0}\dfrac{\text{Sin}\theta}{\theta} = 1$

(OR)

b) State and prove Napier's formula.

47. a) If $f : R \to R$ is defined by $f(x) = 2x-3$ prove that f is a bijection and find its inverse.

(OR)

b) For what value of k does the equation $2x^2 + 2kxy + 2y^2 + 11x - 5y + 2 = 0$ represents a pair of straight lines.

ST. JOSEPH'S HR. SEC. SCHOOL - CHENGALPATTU

class : 11 SECOND REVISION EXAMINATION - 2024

MATHEMATICS - KEY

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9.2.24

PART-A

I

1	b	4
2		MA
3	c	-4
4	a	0
5	c	$-a/b$
6	c	11
7	d	nC_r
8	c	$10C_6 2^{20}$
9	d	$\frac{n^2-n+2}{2}$
10		MA
11	b	$a=1, b=4$
12	c	0
13	b	$2a=b+c$
14	a	1
15	d	0
16	b	$2^x \log 2$
17	d	10
18		MA
19	c	$\frac{1}{3}\log\|(3x-2)\|+c$
20	a	$5/13$

PART-B

II

21. $A=\{9,13,17,21\}$, $n(A)=4$

$n(P(A))=2^4=16$

22. $-5<10x-2<5 \Rightarrow \frac{-3}{10}<x<7/10$

solution set $\left[\frac{-3}{10}, \frac{7}{10}\right]$

23. $\tan(A+B)=\tan 45^\circ$

$\tan A+\tan B = 1-\tan A\tan B$

$\tan A+\tan B+\tan A\tan B=1$

$(1+\tan A)(1+\tan B)=2$

24. $\frac{1}{8!}\left[1+\frac{1}{9}\right]=\frac{n}{8!\times 9\times 10}$

$\frac{10}{9}=\frac{n}{9\times 10} \Rightarrow n=100$

25. $\frac{a_1}{a_2}=\frac{b_1}{b_2} \Rightarrow \frac{3}{12}=\frac{2}{8} \Rightarrow \frac{1}{4}=\frac{1}{4}$

26. $A^2=\begin{bmatrix}1 & 2a\\ 0 & 1\end{bmatrix} \Rightarrow A^4=\begin{bmatrix}1 & 4a\\ 0 & 1\end{bmatrix}$

27. $\vec{a}\times\vec{b}=-17\hat{i}+13\hat{j}+7\hat{k}$

$|\vec{a}\times\vec{b}|=\sqrt{507}$

28. (i) $y'=-me^{-mx}$

(ii) $y'=\cos x-\sin x$

29. (i) $\int\frac{1}{x}dx=\log|x|+c$

(ii) $\int\sec^2 x\,dx=\tan x+c$

30. (i) $P(A)=\frac{n(A)}{n(S)}=\frac{1}{7}$

(ii) $P(B)=\frac{n(B)}{n(S)}=\frac{2}{7}$

PART-C

III

31. $1-2\sin x=0 \Rightarrow \sin x=\frac{1}{2} \Rightarrow \sin x=\sin\pi/6$

$x=n\pi+(-1)^n\pi/6, n\in Z$

$R-\{n\pi+(-1)^n\pi/6\}, n\in Z$

32. $=\log a+2\log a+3\log a+\cdots+n\log a$

$=\log a(1+2+3+\cdots+n)$

$=\frac{n(n+1)}{2}\log a$

33. $u=\sin x-\cos x$, $du=(\cos x-\sin x)dx$

$I=\int\frac{du}{u}=\log|u|+c \Rightarrow I=\log|\sin x-\cos x|+c$

34. $\vec{OA}=\vec{a}$, $\vec{OB}=\vec{b}$, $\vec{OC}=\vec{c}$, $\vec{OD}=\frac{\vec{a}+\vec{b}}{2}$, $\vec{OE}=\frac{\vec{a}+\vec{c}}{2}$

$\vec{BE}+\vec{DC}=\vec{BD}+\vec{DE}+\vec{DE}+\vec{EC}$

$=\frac{\vec{BA}}{2}+2\vec{DE}+\frac{\vec{AC}}{2}=\frac{3}{2}(c-b)=\frac{3}{2}\vec{BC}$

35. $\frac{1}{2}\begin{vmatrix}-3 & 0 & 1\\ 3 & 0 & 1\\ 0 & k & 1\end{vmatrix}=9 \Rightarrow 3|k|=9 \Rightarrow$ $\boxed{k=\pm 3}$

36. $\log y=\log x^{\cos x} \Rightarrow \log y=\cos x\log x$

$\frac{1}{y}\frac{dy}{dx}=\frac{1}{x}\cos x-\sin x\log x$

$\frac{dy}{dx}=x^{\cos x}\left[\frac{\cos x}{x}-\sin x\log x\right]$

37. $120=\frac{720}{r!} \Rightarrow r!=6! \Rightarrow$ $\boxed{r=3}$, $\boxed{n=6}$

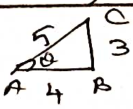
38. $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$0.7=P(A)+0.5-0.2 \Rightarrow P(A)=0.4$

$P(A)\cdot P(B)=0.4\times 0.5=0.2=P(A\cap B)$

39. $AC^2=4^2+3^2 \Rightarrow AC^2=25$

$\boxed{AC=5}$



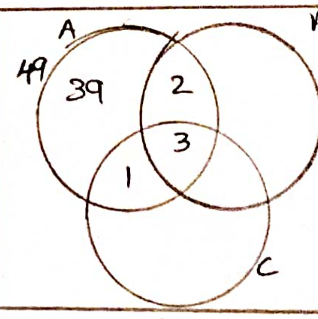
$\tan\theta=3/5 \Rightarrow \sec\theta=-\frac{5}{4}$ [II - Q]

40. $y'=\frac{1}{\sqrt{1-x^2}}=(1-x^2)^{-1/2}$

$y''=\frac{-(-2x)}{2(1-x^2)^{3/2}}=\frac{x}{(1-x^2)^{3/2}}$

PART - D

IV 41 a.



$n(A\cap B'\cap C') = 5000 \times \frac{39}{100}$

$= 1950$

b. $\lim_{x\to 0} \frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{\tan x} \times \frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}+\sqrt{1-\sin x}}$

$= \lim_{x\to 0} \frac{(1+\sin x)(1-\sin x)}{\tan x(\sqrt{1+\sin x}+\sqrt{1-\sin x})}$

$= \lim_{x\to 0} \frac{2\sin x\cos x}{\sin x(\sqrt{1+\sin x}+\sqrt{1-\sin x})} = \frac{2}{2} = 1$

42 a. critical no. $x = \frac{3}{2}, 2, 4$ — $-\infty \quad 3/2 \quad 2 \quad 4 \quad \infty$

possible Intervals $(-\infty, \frac{3}{2})(\frac{3}{2}, 2)(2,4)(4,\infty)$

$(-\infty, \frac{3}{2})$	$-$	$-$	$-$	$\ominus$
$(\frac{3}{2}, 2)$	$+$	$-$	$-$	$+$
$(2,4)$	$+$	$+$	$-$	$\ominus$
$(4,\infty)$	$+$	$+$	$+$	$+$

$\frac{2x-3}{(x-2)(x-4)} < 0$ in $(-\infty, 3/2)$ and $(2,4)$

b. $5x-2 = A\frac{d}{dx}(x^2+2x+2)+B$

$A = 5/2, \quad B = -7$

$I = \int \frac{\frac{5}{2}(2x+2)}{x^2+2x+2}dx \int \frac{7}{x^2+2x+2}dx$

$I_1 = \frac{5}{2}\log|x^2+2x+2|$

$I_2 = \int \frac{1}{(x+1)^2+1^2}dx = \tan^{-1}\left(\frac{x+1}{1}\right)$

$I = \frac{5}{2}\log|x^2+2x+2| - 7\tan^{-1}(x+1)+C$

43 a. LHS $= \frac{\cot\theta.\cos\theta.\cos\theta}{(-\cos\theta)(-\tan\theta)(\mathrm{cosec}\,\theta)} = \frac{\frac{\cos\theta}{\sin\theta}.\cos\theta.\cos\theta}{\cos\theta.\frac{\sin\theta}{\cos\theta}.\frac{1}{\sin\theta}}$

$= \frac{\cos^3\theta}{\sin\theta} = \cos^2\theta \frac{\cos\theta}{\sin\theta}$

b. $(2x^2-xy-3y^2-6x+19y-20) = (2x-3y+l)(x+y+m)$

$l = 4, m = -5 \Rightarrow y = \frac{14}{5}, x = \frac{11}{5}$

$\tan\theta = \pm\frac{2\sqrt{h^2-ab}}{a+b} = \frac{2\sqrt{(-\frac{1}{2})^2-2(-3)}}{2-3} \Rightarrow \theta = \tan^{-1}(5)$

44 a. $P(n) = \frac{1}{1\cdot2}+\frac{1}{2\cdot3}+\frac{1}{3\cdot4}+\cdots+\frac{1}{n(n+1)} = \frac{n}{n+1}$

$P(1) = \frac{1}{2} \Rightarrow P(1)$ is true

$P(k) = \frac{1}{1\cdot2}+\frac{1}{2\cdot3}+\frac{1}{3\cdot4}+\cdots+\frac{1}{k(k+1)} = \frac{k}{k+1}$

$P(k+1) = P(k)+\frac{1}{(k+1)(k+2)} = \frac{(k+1)}{(k+2)} \Rightarrow P(k), P(k+1)$ true

b. $\Delta = \begin{vmatrix} b+c & a-c & a-b \\ b-c & c+a & b-a \\ c-b & c-a & a+b \end{vmatrix} \Rightarrow a=0 \Rightarrow \Delta = \begin{vmatrix} b+c & -c & -b \\ b-c & c & b \\ c-b & c & b \end{vmatrix} = 0$

a, b and c are factors of Δ, $m = 3-3 = 0$

$a=1, b=1, c=1 \quad \Delta = \begin{vmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{vmatrix} = k(1)(1)(1)$

$\boxed{k=8} \Rightarrow \Delta = 8abc$

45 a. $y' = \tan^{-1}x\left(\frac{1}{1+x^2}\right) \Rightarrow y' = \frac{y}{1+x^2} \Rightarrow y'(1+x^2) = y$

$y'(2x)+(1+x^2)y'' = y'$

$(1+x^2)y''+(2x-1)y' = 0$

b. $\vec{OA} = 2\hat{i}+4\hat{j}+3\hat{k}, \ \vec{OB} = 4\hat{i}+\hat{j}-9\hat{k}, \ \vec{OC} = 10\hat{i}-\hat{j}+6\hat{k}$

$\vec{AB} = 2\hat{i}-3\hat{j}+6\hat{k} \Rightarrow |\vec{AB}| = 7$

$\vec{BC} = 6\hat{i}-2\hat{j}-3\hat{k} \Rightarrow |\vec{BC}| = 7$

$\vec{CA} = -8\hat{i}+5\hat{j}-3\hat{k} \Rightarrow |\vec{CA}| = \sqrt{98}$

$CA^2 = BC^2+AB^2 \Rightarrow 98 = 49+49$

46 a. area property $\frac{\tan\theta}{2} \geq \frac{\theta}{2} \geq \frac{\sin\theta}{2}$

$\frac{1}{\cos\theta} \geq \frac{\theta}{\sin\theta} \geq 1 \Rightarrow \cos\theta \leq \frac{\sin\theta}{\theta} \leq 1$

$\cos(-\theta) = \cos\theta$ and $\frac{\sin(-\theta)}{-\theta} = \frac{\sin\theta}{\theta}, \ (-\pi/2, \pi/2)$

$\lim_{\theta\to 0}\cos\theta = 1, \ \lim_{\theta\to 0} 1 = 1$ (app. sandwich Theorem)

$\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$

b. ΔABC, (i) $\tan\left(\frac{A-B}{2}\right) = \frac{a-b}{a+b}\cot C/2$

$\frac{a-b}{a+b}\cot\frac{C}{2} = \frac{2R\sin A - 2R\sin B}{2R\sin A + 2R\sin B}\cot C/2$

$= \frac{2\cos\frac{A+B}{2}\sin\frac{A-B}{2}}{2\sin\frac{A+B}{2}\cos\frac{A-B}{2}}\cot C/2$

$= \cot(90-C/2)\tan\frac{A-B}{2}\cot C/2$

$= \tan\frac{C}{2}\tan\frac{A-B}{2}\cot C/2 = \tan\frac{A-B}{2}$

ll^{ly} we can prove the other two results.

47 a. $y = 2x-3, \ x = \frac{y+3}{2}, \ g(y) = \frac{y+3}{2}$

$(g\circ f)(x) = \frac{(2x-3)+3}{2} = x \Rightarrow g\circ f = I_x$

$(f\circ g)(x) = 2\left(\frac{y+3}{2}\right)-3 = y \Rightarrow f\circ g = I_y$

$\Rightarrow$ f and g are bijections and Inverse to each other. $f^{-1}(y) = \frac{y+3}{2} \Rightarrow f^{-1}(x) = \frac{x+3}{2}$

b. $a = 12 \quad 2h = 2k \quad b = 2 \quad g = \frac{11}{2}, f = \frac{-5}{2}, c = -5$

$h = k$

$abc+2fgh-af^2-bg^2-ch^2 = 0$

$(12)(2)(2)+(-5)(\frac{1}{2})k-12\left(\frac{25}{4}\right)-2\left(\frac{121}{4}\right)-2(k^2) = 0$

$48-\frac{55k}{2}-\frac{300}{4}-\frac{242}{4}-2k^2 = 0$

$192-110k-300-242-8k^2 = 0$

$4k^2+55k+175 = 0$

$(k+5)(4k+35) = 0$

$k = -5, \ k = -\frac{35}{4}$