26-02-2024

Time: 3.00 Hours

Standard - 12

MATHEMATICS

Part - I

Maximum Marks: 90

Answer all the questions: Choose the most suitable answer from the given four alternatives and write the option code and corresponding answer: **20×1=20**

1) The augumented matrix of a system of linear equations is $\begin{pmatrix} 1 & 2 & 7 & 3 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & \lambda-7 & \mu+5 \end{pmatrix}$.

   The system has infinitely many solutions if

   a) $\lambda = 7, \mu \neq -5$ b) $\lambda = -7, \mu = 5$ c) $\lambda \neq 7, \mu \neq -5$ d) $\lambda = 7, \mu = -5$

2) The value of $\sum_{n=1}^{13} (i^n + i^{n-1})$ is

   a) $1+i$ b) i c) 1 d) 0

3) If the rank of the matrix $\begin{pmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ -1 & 0 & \lambda \end{pmatrix}$ is 2 then λ is

   a) 1   b) 2   c) 3   d) any real number

4) The value of $\left(\dfrac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{10}$ is

   a) $\text{cis } 2\dfrac{\pi}{3}$ b) $\text{cis } 4\dfrac{\pi}{3}$ c) $-\text{cis } 2\dfrac{\pi}{3}$ d) $-\text{cis } 4\dfrac{\pi}{3}$

5) If α, β, and γ are the zeros of x^3+px^2+qx+r then $\sum \dfrac{1}{\alpha}$ is

   a) $\dfrac{-q}{r}$ b) $\dfrac{-p}{r}$ c) $\dfrac{q}{r}$ d) $\dfrac{-q}{p}$

6) If $\sin^{-1}\dfrac{x}{5} + \text{cosec}^{-1}\dfrac{5}{4} = \dfrac{\pi}{2}$ then the value of x is

   a) 4   b) 5   c) 2   d) 3

7) The eccentricity of the ellipse $(x-3)^2+(y-4)^2 = \dfrac{y^2}{9}$ is

   a) $\dfrac{\sqrt{3}}{2}$ b) $\dfrac{1}{3}$ c) $\dfrac{1}{3\sqrt{2}}$ d) $\dfrac{1}{\sqrt{3}}$

8) $16x^2-3y^2-32x-12y-44 = 0$ represents

   a) an ellipse   b) a circle   c) a parabola   d) a hyperbola

9) If the direction cosines of a line are $\dfrac{1}{c}, \dfrac{1}{c}, \dfrac{1}{c}$ then

   a) $c = \pm 3$ b) $c = \pm\sqrt{3}$ c) $c > 0$ d) $0 < c < 1$

10) If the magnitude of moment about the point $\hat{j}+\hat{k}$ of a force $\hat{i} + a\hat{j} - \hat{k}$ acting through the point $\hat{i} + \hat{j}$ is $\sqrt{8}$ then the value of a is

   a) 1   b) 2   c) 3   d) 4

11) The slope of a line normal to the curve $f(x) = 2\cos 4x$ at $x = \dfrac{\pi}{12}$ is

   a) $-4\sqrt{3}$ b) -4 c) $\dfrac{\sqrt{3}}{12}$ d) $4\sqrt{3}$

12) The number given by the Rolle's theorem for the function $x^3 - 3x^2$, $x \in [0, 3]$ is

a) 1   b) $\sqrt{2}$ c) $\frac{3}{2}$ d) 2

13) If $w(x, y) = x^y$, $x > 0$ then $\frac{\partial w}{\partial x}$ is

a) $x^y \log x$ b) $y \log x$ c) yx^{y-1} d) $x \log x$

14) The value of $\int_{-\pi/2}^{\pi/2} \sin^2 x \cos x \, dx$ is

a) $\frac{3}{2}$ b) $\frac{1}{2}$ c) 0   d) $\frac{2}{3}$

15) Integrating factor of the differential equation $\frac{dy}{dx} = \frac{x+y+1}{x+1}$ is

a) $\frac{1}{x+1}$ b) $x+1$ c) $\frac{1}{\sqrt{x+1}}$ d) $\sqrt{x+1}$

16) The volume of the solid obtained by revolving $\frac{x^2}{9} + \frac{y^2}{16} = 1$ about the minor axis is

a) 48π b) 64π c) 32π d) 128π

17) The order and degree of the differential equation are $\frac{d^2y}{dx^2} + x = \sqrt{y + \frac{dy}{dx}}$

a) 2, 1   b) 1, 2   c) 2, $\frac{1}{2}$ d) 2, 2

18) The $P(X = 0) = 1 - P(X = 1)$. If $E(X) = 3Var(X)$ then $P(X = 0)$ is

a) $\frac{2}{3}$ b) $\frac{2}{5}$ c) $\frac{1}{5}$ d) $\frac{1}{3}$

19) If $f(x) = \begin{cases} 2x & 0 \le x \le a \\ 0 & \text{otherwise} \end{cases}$ is a probability density function of a random variable then the value of a is

a) 1   b) 2   c) 3   d) 4

20) Substraction is not a binary opration in

a) R   b) Z   c) N   d) Q

Part - II

Answer any Seven questions. Question No. 30 is compulsory.   7×2=14

21) Prove that $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$ is orthogonal

22) If $|z| = 2$ show that $3 \le |z+3+4i| \le 7$

23) Is $\cos^{-1}(-x) = \pi - \cos^{-1}(x)$ true? Justify your answer.

24) Prove that the general equation of the circle whose diameter is the line segment joining the points $(-4, -2)$ & $(-1, -1)$ is $x^2+y^2+5x+3y+6 = 0$

25) If the vectors $a\hat{i} + a\hat{j} + c\hat{k}, \hat{i} + \hat{k}$ & $c\hat{i} + c\hat{j} + b\hat{k}$ are coplanar. Prove that c is the geometric mean of a & b.

26) Evaluate: $\underset{x \to \infty}{Lt} \ e^{-x}\sqrt{x}$

27) Verify associative property under the binary operation * defined by $a*b = a^b$ $\forall \ a, b \in N$

28) An egg of a particular bird is very nearly spherical. If the radius to the inside of the shell is 5 mm & radius to the outside of the shell is 5.3 mm. Find the volume of the shell approximately.

29) A random variable X has the following probability mass function

X	1	2	3	4	5	6
f(x)	k	2k	6k	5k	6k	10k

find (i) k   (ii) $P(X \le 4)$

30) Evaluate: $\int_0^{\pi/2} \dfrac{\sin x}{1 + \cos^2 x} \, dx$

Part - III
Answer any Seven questions. Question No. 40 is compulsory.   7×3=21

31) Find the rank of the matrix $\begin{pmatrix} 4 & -2 & 6 & 8 \\ 1 & 1 & -3 & -1 \\ 15 & -3 & 9 & 21 \end{pmatrix}$

32) Show that the equation $z^3 + 2\bar{z} = 0$ has five solutions.

33) Prove that the length of the latus rectum of the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ is $\dfrac{2b^2}{a}$

34) Examine the concavity for the function $f(x) = x^4 - 4x^3$

35) Find the mean of the distribution $f(x) = \begin{cases} 3e^{-3x} & 0 < x < \infty \\ 0 & elsewhere \end{cases}$

36) If the two lines $\dfrac{x-1}{2} = \dfrac{y+1}{3} = \dfrac{z-1}{4}$ and $\dfrac{x-3}{1} = \dfrac{y-m}{2} = z$ intersect at a point find the value of m.

37) Form the differential equation of $y = e^{3x}(C \cos 2x + D \sin 2x)$ where C & D are arbitrary constants.

38) Construct the truth table for $\neg(p \wedge \neg q)$

39) Evaluate: $\int_0^1 x^3 e^{-2x} \, dx$

40) Find the domain of $f(x) = \sin^{-1}\sqrt{x-1}$

Part - IV
Answer all questions: 7×5=35

41) a] By using Gaussian elimination method balance the chemical reaction equation $C_2H_6 + O_2 \to H_2O + CO_2$

(OR)

b] In a binomial distribution consisting of 5 independent trials, the probability of 1 & 2 successes are 0.4096 and 0.2048 respectively. Find the mean and variance of the random variable.

42) a] If $u = \sec^{-1}\left(\dfrac{x^3 - y^3}{x + y}\right)$ show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 2\cot u$

(OR)

b] Using integration find the area of the region bounded by triangle ABC whose vertices A, B & C are $(-1, 1)$ $(3, 2)$ & $(0, 5)$ respectively.

43) a] If z_1, z_2 & z_3 are three complex numbers such that $|z_1| = 1$, $|z_2| = 2$, $|z_3| = 3$ & $|z_1 + z_2 + z_3| = 1$ show that $|9z_1z_2 + 4z_1z_3 + z_2z_3| = 6$

(OR)

b] If D is the midpoint of the side BC of a triangle ABC, show by vector method that $\left|\overline{AB}\right|^2 + \left|\overline{AC}\right|^2 = 2\left(\left|\overline{AD}\right|^2 + \left|\overline{BD}\right|^2\right)$ *Sivakumar. M, Sri Ram Matric HSS Vallam-627809 Tenkasi Dist,*

44) a] Solve $\cot^{-1}x - \cot^{-1}(x+2) = \dfrac{\pi}{12}$, $x > 0$.

(OR)

b] Solve $(x^2 + y^2)dy = xy\,dx$. It is given that $y(1) = 1$ & $y(x_0) = e$, find the value of x_0.

45) a] Prove that among all the rectangles of the given area square has the least perimeter.

(OR)

b] Solve: $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$.

46) a] A rod of length 1.2 m moves with its ends always touching the coordinate axes. The locus of a point P on the rod which is 0.3 m, from the end in contact with x axis is an ellipse. Find the eccentricity.

(OR)

b] Verfify (i) closure property (ii) commutative property (iii) associative property, (iv) existence of identi by & (v) existence of inverse for the operation X_{11} on a subset $A = \{1, 3, 4, 5, 9\}$ of the set of remainders $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

47) a] The growth of a population is proportional to the number present. If the population of a colony doubles in 50 years, in how many years will the population become triple?

(OR)

b] Find the parametric form of vector equation and Cartesian equation of the plane containing the line $\vec{r} = \left(\hat{i} - \hat{j} + 3\hat{k}\right) + t\left(2\hat{i} - \hat{j} + 4\hat{k}\right)$ and perpendicular to plane $\vec{r}\cdot\left(\hat{i} + 2\hat{j} + \hat{k}\right) = 8$

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