V12M

Virudhunagar District

Common Second Revision Exam - Feb 2024

Standard - 12

MATHEMATICS

Time: 3.00 Hours | Maximum Marks: 90

Part - I

Choose the correct answer: **20×1=20**

1) If $A = \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ x & \frac{3}{5} \end{bmatrix}$ and $A^T = A^{-1}$, then the value of x is

a) $-\frac{4}{5}$ b) $-\frac{3}{5}$ c) $\frac{3}{5}$ d) $\frac{4}{5}$

2) If $\omega \neq 1$ is a cube root of unity and $(1+\omega)^7 = A+B\omega$, then (A, B) equals

a) (1, 0) b) (−1, 1) c) (0, 1) d) (1, 1)

3) According to the rational root theorem, which number is not possible rational zero of $4x^7+2x^4-10x^3-5$?

a) −1 b) $\frac{5}{4}$ c) $\frac{4}{5}$ d) 5

4) $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right)$ is equal to

a) $\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$ b) $\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)$ c) $\frac{1}{2}\tan^{-1}\left(\frac{3}{5}\right)$ d) $\tan^{-1}\left(\frac{1}{2}\right)$

5) If the coordinates at one end of a diameter of the circle $x^2+y^2-8x-4y+c=0$ are (11, 2) the coordinates of the other end are

a) (−5, 2) b) (2, −5) c) (5, −2) d) (−2, 5)

6) If the direction cosines of a line are $\frac{1}{c}, \frac{1}{c}, \frac{1}{c}$ then

a) $c = \pm 3$ b) $c = \pm\sqrt{3}$ c) $c > 0$ d) $0 < c < 1$

7) The number given by the mean value theorem for the function $\frac{1}{x}$, $x \in [1, 9]$ is

a) 2 b) 2.5 c) 3 d) 3.5

8) If $v(x, y) = \log(e^x+e^y)$, then $\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y}$ is equal to

a) e^x+e^y b) $\frac{1}{e^x+e^y}$ c) 2 d) 1

9) The volume of solid of revolution of the region bounded by $y^2 = x(a-x)$ about x-axis is

a) πa^3 b) $\frac{\pi a^3}{4}$ c) $\frac{\pi a^3}{5}$ d) $\frac{\pi a^3}{6}$

10) The general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$ is

a) $xy = k$ b) $y = k\log x$ c) $y = kx$ d) $\log y = kx$

11) If $f(x) = \begin{cases} 2x, & 0 \le x \le a \\ 0, & \text{otherwise} \end{cases}$ is a probability density function of a random variable, then the value of a is

a) 1 b) 2 c) 3 d) 4

12) If a compound statement involves 7 simple statements, then the number of rows in the truth table is

a) 2^6 b) 7^2 c) 2^7 d) 7^7

13) Which of the following curves is concave down?

a) $y = -x^2$ b) $y = x^2$ c) $y = e^x$ d) $y = x^2+2x-3$

14) The area of the region bunded by the graph of $y = \sin x$ and $y = \cos x$ between $x = 0$ and $x = \pi/4$ is

a) $\sqrt{2}$ b) $\sqrt{2}-1$ c) $2\sqrt{2}-2$ d) $2\sqrt{2}+2$

15) Integrating factor of the differential equation $\frac{dy}{dx} + \frac{1}{x\log x} y = \frac{2}{x^2}$ is

a) e^x b) $\log x$ c) $\frac{1}{x}$ d) e^{-x}

16) Area of the greatest rectangle that can be inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2}$ is

a) 2ab b) ab c) $\sqrt{ab}$ d) a/b

17) Sum of the n roots of n^{th} roots of unity is

a) 1 b) −1 c) 0 d) n

18) If $\cot^{-1}2$ and $\cot^{-1}3$ are two angles of a triangle, then the third angle is

a) $\pi/4$ b) $3\pi/4$ c) $\pi/6$ d) $\pi/3$

19) If $\vec{a} = \vec{i} + \vec{j} + \vec{k}$, $\vec{b} = \vec{i} + \vec{j}$, $\vec{c} = \vec{i}$ and $(\vec{a}\times\vec{b})\times\vec{c} = \lambda\vec{a} + \mu\vec{b}$, then the value of $\lambda+\mu$ is

a) 0 b) 1 c) 6 d) 3

20) If adj A = $\begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$, adj B = $\begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}$ then adj (AB) is

a) $\begin{bmatrix} -7 & -1 \\ 7 & -9 \end{bmatrix}$ b) $\begin{bmatrix} -6 & 5 \\ -2 & -10 \end{bmatrix}$ c) $\begin{bmatrix} -7 & 7 \\ -1 & -9 \end{bmatrix}$ d) $\begin{bmatrix} -6 & -2 \\ 5 & -10 \end{bmatrix}$

Part - II

Answer any Seven questions. Question No. 30 is compulsory. 7×2=14

21) If A is a non-singular matrix of add order, prove that |adj A| is positive.

22) Find the modulus and principal argument of $2+i2\sqrt{3}$

23) Show that if p, q, r are rtional the roots of the equation $x^2-2px+p^2-q^2+2qr-r^2 = 0$ are rational.

24) If $\cot^{-1}\left(\frac{1}{7}\right) = \theta$ then find the value of $\cos\theta$.

25) If $\vec{a}, \vec{b}, \vec{c}$ are three vectors, then prove that $[\vec{a}+\vec{c},\ \vec{a}+\vec{b},\ \vec{a}+\vec{b}+\vec{c}] = [\vec{a}\ \vec{b}\ \vec{c}]$

26) Write the Maclaurin series expansion of e^x.

27) If $g(x) = x^2+\sin x$, then find dg.

28) Evaluate $\int_0^3 (3x^2-4x+5)\,dx$

29) Solve: $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$

30) Write the truth table for $\neg(p \wedge \neg q)$

Part - III

Answer any Seven questions. Question No. 40 is compulsory. **7×3=21**

31) In a competitive examination, one mark is awarded for every correct answer while $\frac{1}{4}$ mark is deducted for every wrong answer. A student answered 100 questions and got 80 marks. How many questions did he answer correctly?

32) If z_1, z_2 and z_3 are complex numbers such that

$|z_1| = |z_2| = |z_3| = |z_1 + z_2 + z_3| = 1$, find the value of $\left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right|$.

33) Obtain the condition that the roots of $x^3+px^2+9x+r = 0$ are in A.P.

34) Find the equation of the circle with centre (2, 3) and passing through the intersection of the lines $3x-2y-1 = 0$ and $4x+y-27 = 0$

35) If the vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar, then prove that the vectors $\vec{a}+\vec{b}$, $\vec{b}+\vec{c}$, $\vec{c}+\vec{a}$ are also coplanar.

36) Use the linear approximation to find approximate value of $\sqrt[3]{26}$

37) Evaluate: $\int_0^1 \frac{2x}{1+x^2} dx$

38) Suppose a person deposits Rs.10,000 in a bank account at the rate of 5% per annum compounded continuously. How much money will be in his bank account 18 months later?

39) Verify the (i) closure property (ii) commutative property (iii) associative property (iv) existence of identity (v) existence of inverse for the arithmetic operation + on z.

40) Two fair coins are tossed simultaneously. Find the probability mass function for number of heads occured.

Part - IV

Answer all questions: **7×5=35**

41) a] If $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ then prove that $A^{-1} = \frac{1}{2}(A^2 - 3I)$

(OR)

b] If $z = x+iy$ and $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$, show that $x^2+y^2 = 1$

42) a] Form the equation whose roots are the squares of the roots of the cubic equation $x^3+ax^2+bx+c = 0$

(OR)

b] If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z = \pi$ and $0 < x, y, z < 1$, show that $x^2+y^2+z^2+2xyz = 1$

43) a] A bridge has a parabolic arch that is 10 m high in the centre and 30 m wide at the bottom. Find the height of the arch 6 m from the centre on either sides.

(OR)

b] Find the parametric vector eqn and cartesian equation of the plane passing through the point (1, 1, −1) and perpendicular to the planes $x+2y+3z-7=0$ and $2x-3y+4z=0$

44) a] If the curves $ax^2+by^2=1$ and $cx^2+dy^2=1$ intersect each other orthogonally then, show that $\frac{1}{a}-\frac{1}{b}=\frac{1}{c}-\frac{1}{d}$

(OR)

b] Find the equation of tangent and normal to the ellipse $x^2+4y^2=32$ at $\theta=\pi/2$

45) a] If $\omega(x, y, z)=\frac{1}{\sqrt{x^2+y^2+z^2}}$, $(x, y, z)\neq(0, 0, 0)$, then prove that $\frac{\partial^2\omega}{\partial x^2}+\frac{\partial^2\omega}{\partial y^2}+\frac{\partial^2\omega}{\partial z^2}=0$

(OR)

b] Prove that $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$ using vector method.

46) a] Find the area of the region bounded between the parabolas $y^2=4x$ and $x^2=4y$

(OR)

b] Solve: $\frac{dy}{dx}+2y\cot x=3x^2\operatorname{cosec}^2 x$

47) a] Find the constant C such that the function $f(x)=\begin{cases}Cx^2 & 1<x<4\\ 0 & \text{otherwise}\end{cases}$ is a density function, and compute (i) $P(1.5<x<3.5)$ (ii) $P(X\leq 2)$ (iii) $P(3<x)$

(OR)

b] Prove that $p\to(\neg q, \vee r)\equiv\neg p\vee(\neg q\vee r)$ using truth table.

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