Class : 12

Register Number

SECOND REVISION EXAMINATION - 2024

MATHEMATICS

Time Allowed : 3.00 Hours] [Max. Marks : 90

PART - I

20 x 1 = 20

1. Answer all the questions by choosing the correct answer from the given 4 alternatives
2. Write question number, correct option and corresponding answer
3. Each question carries 1 mark

1. If $A\begin{bmatrix}1 & -2\\1 & 4\end{bmatrix}=\begin{bmatrix}6 & 0\\0 & 6\end{bmatrix}$, then A =

(1) $\begin{bmatrix}1 & -2\\1 & 4\end{bmatrix}$ (2) $\begin{bmatrix}1 & 2\\-1 & 4\end{bmatrix}$ (3) $\begin{bmatrix}4 & 2\\-1 & 1\end{bmatrix}$ (4) $\begin{bmatrix}4 & -1\\2 & 1\end{bmatrix}$

2. If $0 \le \theta \le \pi$ and the system of equations $x + (\sin\theta)y - (\cos\theta)z = 0$, $(\cos\theta)x - y + z = 0$, $(\sin\theta)x + y - z = 0$ has a non-trivial solution then θ is

(1) $\frac{2\pi}{3}$ (2) $\frac{3\pi}{4}$ (3) $\frac{5\pi}{6}$ (4) $\frac{\pi}{4}$

3. The principal argument of $(\sin 40° + i\cos 40°)^5$ is

(1) -110° (2) -70° (3) 70° (4) 110°

4. The value of $\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{10}$ is

(1) $cis\frac{2\pi}{3}$ (2) $cis\frac{4\pi}{3}$ (3) $-cis\frac{2\pi}{3}$ (4) $-cis\frac{4\pi}{3}$

5. If $x^3 + 12x^2 + 10ax + 1999$ definitely has a positive zero, if and only if

(1) $a \ge 0$ (2) $a > 0$ (3) $a < 0$ (4) $a \le 0$

6. If $\cot^{-1}(\sqrt{\sin\alpha}) + \tan^{-1}(\sqrt{\sin\alpha}) = u$, then cos 2u is equal to

(1) $\tan^2\alpha$ (2) 0 (3) -1 (4) $\tan 2\alpha$

7. If the normals of the parabola $y^2 = 4x$ drawn at the end points of its latus rectum are tangents to the circle $(x-3)^2 + (y+2)^2 = r^2$, then the value of r^2 is

(1) 2 (2) 3 (3) 1 (4) 4

8. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar, non-zero vectors such that $[\vec{a}, \vec{b}, \vec{c}] = 3$, then $\{[\vec{a}\times\vec{b}, \vec{b}\times\vec{c}, \vec{c}\times\vec{a}]\}^2$ is equal to

(1) 81 (2) 9 (3) 27 (4) 18

9. The maximum slope of the tangent to the curve $y = e^x \sin x$, $x \in [0, 2\pi]$ is at

(1) $x = \frac{\pi}{4}$ (2) $x = \frac{\pi}{2}$ (3) $x = \pi$ (4) $x = \frac{3\pi}{2}$

10. If $u(x,y) = e^{x^2+y^2}$, then $\frac{\partial u}{\partial x}$ is equal to

(1) $e^{x^2+y^2}$ (2) $2xu$ (3) x^2u (4) y^2u

11. Linear approximation for $g(x) = \cos x$ at $x = \frac{\pi}{2}$ is

(1) $x + \frac{\pi}{2}$ (2) $-x + \frac{\pi}{2}$ (3) $x - \frac{\pi}{2}$ (4) $-x - \frac{\pi}{2}$

12. The value of $\int_0^{\pi} \sin^4 x \, dx$ is

(1) $\frac{3\pi}{10}$ (2) $\frac{3\pi}{8}$ (3) $\frac{3\pi}{4}$ (4) $\frac{3\pi}{2}$

13. If $\sin x$ is the integrating factor of the linear differential equation $\frac{dy}{dx} + Py = Q$, then P is

(1) $\log \sin x$ (2) $\cos x$ (3) $\tan x$ (4) $\cot x$

14. If the function $f(x) = \frac{1}{12}$ for $a < x < b$, represents a probability density function of a continuous random variable X, then which of the following cannot be the value of a and b?

(1) 0 and 12 (2) 5 and 17 (3) 7 and 19 (4) 16 and 24

15. The operation $*$ defined by $a * b = \frac{ab}{7}$ is not a binary operation on

(1) Q' (2) Z (3) R (4) C

16. The equation of tangent at (1,2) to the circle $x^2 + y^2 = 5$ is

(1) $x + y = 3$ (2) $x + 2y = 3$ (3) $x - y = 5$ (4) $x - 2y = 5$

17. The angle between the planes $2x + y - z = 9$ and $x + 2y + z = 7$ is

(1) $\cos^{-1}(5/6)$ (2) $\cos^{-1}(5/36)$ (3) $\cos^{-1}(1/2)$ (4) $\cos^{-1}(1/12)$

18. $\lim_{x \to 0} \frac{a^x - b^x}{c^x - d^x}$ is

(1) ∞ (2) 0 (3) $\log \frac{ab}{cd}$ (4) $\frac{\log\left(\frac{a}{b}\right)}{\log\left(\frac{c}{d}\right)}$

19. If $u = y^x$ then $\frac{\partial u}{\partial y} =$

(1) xy^{x-1} (2) yx^{y-1} (3) 0 (4) 1

20. $\int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx =$

(1) $\frac{\pi}{2}$ (2) 0 (3) $\frac{\pi}{4}$ (4) π

PART - II

1. Answer any 7 questions $7 \times 2 = 14$
2. Each question carries 2 marks
3. Question number 30 is compulsory

21. Solve the following system of linear equations by matrix inversion method: $2x + 5y = -2$, $x + 2y = -3$

22. Write $\frac{3+4i}{5-12i}$ in the $x + iy$ form, hence find its real and imaginary parts.

23. Find the equation of the parabola in each of the cases given below: focus (4,0) and directrix $x = -4$.

24. Find the acute angle between the straight lines $\frac{x-4}{2} = \frac{y}{1} = \frac{z+1}{-2}$ and $\frac{x-1}{4} = \frac{y+1}{-4} = \frac{z-2}{2}$ and state whether they are parallel or perpendicular.

[illegible] Maclaurin's series upto 4 non-zero terms for $-1 < x \leq 1$

26. Find an approximate value of $\int_1^{1.5} x^2 dx$ by applying the right-end rule with the partition {1.1, 1.2, 1.3, 1.4, 1.5}

27. Solve $\frac{dy}{dx} + 2y = e^{-x}$

28. Using binomial distribution find the mean and variance of X for the following experiments. A fair coin is tossed 100 times, and X denote the number of heads.

29. Let $A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$ be any three boolean matrices. Find $(A \vee B) \wedge C$

30. Find the modulus of [illegible]

PART - III

1. Answer any 7 questions
2. Each question carries 3 marks
3. Question number 40 is compulsory

7x3=21

31. Verify the property $(A^T)^{-1} = (A^{-1})^T$ with $A = \begin{bmatrix} 2 & 9 \\ 1 & 7 \end{bmatrix}$

32. Obtain the Cartesian form of the locus of $z = x + iy$ in each of the following cases: $[Re(iz)]^2 = 3$

33. Solve the equation $2x^3 + 11x^2 - 9x - 18 = 0$

34. Prove that $\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24} = \tan^{-1}\frac{1}{2}$

35. For any four vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ we have $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a}, \vec{b}, \vec{d}]\vec{c} - [\vec{a}, \vec{b}, \vec{c}]\vec{d} = [\vec{a}, \vec{c}, \vec{d}]\vec{b} - [\vec{b}, \vec{c}, \vec{d}]\vec{a}$

36. Suppose that for a function $f(x)$, $f'(x) \leq 1$ for all $1 \leq x \leq 4$. Show that $f(4) - f(1) \leq 3$.

37. A circular plate expands uniformly under the influence of heat. If it's radius increases from 10.5 cm to 10.75 cm, then find an approximate change in the area and the approximate percentage change in the area.

38. Find the volume of the solid formed by revolving the region bounded by the parabola $y = x^2$, x-axis, ordinates $x = 0$ and $x = 1$ about the x-axis.

39. Verify whether the following compound propositions are tautologies or contradictions or contingency: $((p \to q) \wedge (q \to r)) \to (p \to r)$

40. The probability density function of the random variable X is given by $f(x) = \begin{cases} 16xe^{-4x} & \text{for } x > 0 \\ 0 & \text{for } x \leq 0 \end{cases}$ find the mean and variance of X.

PART - IV

1. Answer all the questions
2. Each question carries 5 marks

7x5

41. a) Find the value of k for which the equations $kx - 2y + z = 1$, $x - 2ky + z = -2$, $x - 2y + kz = 1$ have (i) no solution (ii) unique solution (iii) infinitely many solution

(OR)

b) Prove that the ellipse $x^2 + 4y^2 = 8$ and the hyperbola $x^2 - 2y^2 = 4$ intersect orthogonally.

42. a) If $2\cos\alpha = x + \frac{1}{x}$ and $2\cos\beta = y + \frac{1}{y}$ show that (i) $xy - \frac{1}{xy} = 2i\sin(\alpha+\beta)$ (ii) $\frac{x^m}{y^n} - \frac{y^n}{x^m} = 2i\sin(m\alpha - n\beta)$

(OR)

b) If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$. Show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$.

43. a) Solve the equation $(2x-3)(6x-1)(3x-2)(x-12) - 7 = 0$

(OR)

b) Evaluate the following integrals using properties of integration : $\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{1}{1+\sqrt{\tan x}}dx$

44. a) Find the parametric form of vector equation, and Cartesian equations of the plane passing through the points (2,2,1), (9,3,6

and perpendicular to the plane $2x + 6y + 6z = 9$

(OR)

b) Find the population of a city at any time t, given that the rate of increase of population is proportional to the population at that instant and that in a period of 40 years the population increased from 3,00,000 to 4,00,00

45. a) A tunnel through a mountain for a four lane highway is to have a elliptical opening. The total width of the highwa (not the opening) is to be 16m, and the height at the edge of the road must be sufficient for a truck 4m high to clea if the highest point of the opening is to be 5m approximately. How wide must the opening be?

(OR)

b) A random variable X has the following probability mass function

x	1	2	3	4	5	6
f(x)	K	2k	6k	5k	6k	10k

Find (i) $P(2 < X < 6)$ (ii) $P(2 \le X < 5)$ (iii) $P(X \le 4)$ (iv) $P(3 < X$

46. a) Prove by vector method that $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$.

(OR)

b) (i) Let $M = \left\{\begin{pmatrix} x & x \\ x & x \end{pmatrix} : x \in R - \{0\}\right\}$ and let $\cdot$ be the matrix multiplication. Determine Whether M is closed under $\cdot$ so, examine the commutative and associative properties satisfied by $\cdot$ on M.

(ii) Let $M = \left\{\begin{pmatrix} x & x \\ x & x \end{pmatrix} : x \in R - \{0\}\right\}$ and let $\cdot$ be the matrix multiplication. Determine whether M is closed under $\cdot$ so, examine the existence of identity, existence of inverse properties for the operation $\cdot$ on M.

47. a) Solve $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$

(OR)

b) Solve : $2x^2\left(\frac{dy}{dx}\right) - 2xy + y^2 = 0, y(e) = e$