Class : 12

SECOND REVISION EXAMINATION - 2024

MATHEMATICS

Time Allowed : 3.00 Hours] [Max. Marks : 90

PART – A (Answer All the questions) 20 X 1=20

1. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ and $A(adj\,A) = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$, then $k =$
   [1]0 [2]$\sin\theta$ [3]$\cos\theta$ [4]1
2. . Subtraction is not a binary operation in
   [1]$\mathbb{R}$ [2]$\mathbb{Z}$ [3] $\mathbb{N}$ [4] $\mathbb{Q}$
3. If $\omega \neq 1$ is a cubic root of unity and $(1+\omega)^7 = A + B\omega$, then (A, B) equals
   [1]$(1,0)$ [2]$(-1\ 1)$ [3]$(0,1)$ [4]$(1,1)$
4. The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct point if
   [1]$15 < m < 65$ [2]$35 < m < 85$ [3]$-85 < m < -35$ [4]$-35 < m < 15$
5. Let A be a non singular square matrix of order 3 X 3 then the value of $|adj\,A|$ is equal to
   [1] $|A|$ [2] $|A|^2$ [3]$|A|^3$ [4]$3|A|$
6. A zero of $x^3 + 64$ is
   (1)0 [2]4 [3]$4i$ [4]-4
7. $\sin^{-1}(\cos x) = \frac{\pi}{2} - x$ is valid for
   [1]$-\pi \le x \le 0$ [2]$0 \le x \le \pi$ [3]$-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ [4]$-\frac{\pi}{4} \le x \le \frac{3\pi}{4}$
8. The value of the limit $\lim_{x \to 0}\left(\cot x - \frac{1}{x}\right)$ is
   [1]0 [2]1 [3]2 [4]∞
9. If the point (3,2) lies on the parabola $y^2=4ax$ then the length of the latus rectum is
   [1]$\frac{1}{3}$ [2]$-\frac{1}{3}$ [3]$\frac{4}{3}$ [4]$-\frac{4}{3}$
10. If α, β, γ the roots of the equation $2x^3 - 5x + 1 = 0$ then the sum of the roots is equal to
    [1] 0 [2] $\frac{5}{2}$ [3]$-\frac{5}{2}$ [4] -1
11. The value of $\int_0^{\frac{\pi}{6}} \cos^3 3x\, dx$ is
    [1]$\frac{2}{3}$ [2]$\frac{2}{9}$ [3]$\frac{1}{9}$ [4]$\frac{1}{3}$
12. The angle between the lines $\frac{x-2}{3} = \frac{y+1}{-2}, z = 2$ and $\frac{x-1}{1} = \frac{2y+3}{3} = \frac{z+5}{2}$ is
    [1]$\frac{\pi}{6}$ [2]$\frac{\pi}{4}$ [3]$\frac{\pi}{3}$ [4]$\frac{\pi}{2}$
13. The integrating factor of the differential equation $\frac{dy}{dx} + P(x)y = Q(x)$ is x, then $P(x)$
    [1]x [2]$\frac{x^2}{2}$ [3]$\frac{1}{x}$ [4]$\frac{1}{x^2}$
14. The value of $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left(\frac{2x^7 - 3x^5 + 7x^3 - x + 1}{\cos^2 x}\right) dx$ is
    [1]4 [2]3 [3]2 [4]0
15. Find the point on the curve $6y = x^3 + 2$ at which y-coordinate changes 8 times as fast as x coordinate is
    [1]$(4, 11)$ [2]$(4, -11)$ [3]$(-4, 11)$ [4]$(-4, -11)$

16. Let X have a Bernoulli distribution with mean 0.4, then the variance of $(2X-3)$ is
[1]0.24 [2]0.48 [3]0.6 [4]0.96

17. If Var$(2x+4) = 8$ then σ^2 =
[1]2 [2]3 [3]1 [4]4

18. The differential equation of the family of curves $y = Ae^{x} + Be^{-x}$, where A and B are arbitary constants is
[1]$\frac{d^2y}{dx^2} + y = 0$ [2]$\frac{d^2y}{dx^2} - y = 0$ [3]$\frac{dy}{dx} + y = 0$, [4]$\frac{dy}{dx} - y = 0$

19. The equation of the plane with intercepts 2,3,4 on x,y,and z axis is
[1] $6x+4y+3z=12$ [2] $2x+3y+4z=1$ [3] $3x+4y+6z=12$ [4] $4x+3y+2z=1$

20. The percentage error of fifth root of 31 is approximately how many times the percentage error in 31?
[1]$\frac{1}{31}$ [2]$\frac{1}{5}$ [3]5 [4]31

PART –B Answer Any Seven Question. Question No. 30 is compulsory 7 X 2 = 14

21. Solve the following system of linear equations, using matrix inversion method:
$2x + y = 4$, $x - 2y = -3$.

22. Find the equation of the ellipse with foci $(\pm 2, 0)$, vertices $(\pm 3, 0)$.

23. Suppose a discrete random variable can only take the values 0, 1, and 2. The probability mass function is defined by $f(x) = \begin{cases} \frac{x^2+1}{k} & for\ x = 0,1,2 \\ 0 & other\ wise \end{cases}$ Find the value of k

24. Find the square root of $6 - 8i$

25. A stone is dropped into a pond causing ripples in the form of concentric circles. The radius r of the outer ripple is increasing at a constant rate at 2 cm per second. When the radius is 5 cm find the rate of changing of the total area of the disturbed water?

26. If α, β, and γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, find the value of $\sum \frac{1}{\beta\gamma}$ in terms of the coefficients.

27. Evaluate $\int_0^3 (3x^2 - 4x + 5)\, dx$

28. Show that $p \to q$ and $q \to p$ are not equivalent

29. show that the vectors $\hat{i} + 2\hat{j} - 3\hat{k}, 2\hat{i} - \hat{j} + 2\hat{k}$ and $3\hat{i} + \hat{j} - \hat{k}$ are coplanar

30. *If the function* $f(x) = x^3 - 3x$ then find the value of df at $x = 2$ and $dx = 0.1$

PART – C Answer Any Seven Question. Question No. 40 is compulsory 7 X 3 = 21

31. A concrete bridge is designed as a parabolic arch. the road over bridge is $40m$ long and the maximum height of the arch is 15 m. Write the equation of the parabolic each.

32. If $|z| = 1$, show that $2 \le |z^2 - 3| \le 4$.

33. If X is the random variable with distribution function $F(x)$ given by,
$F(x) = \begin{cases} 0, & x < 0 \\ x, & x \le x < 1 \\ 1, & 1 \le x \end{cases}$ then find (i) the probability density function $f(x)$
(ii) $P(0.2 \le X \le 0.7)$

34. Show that $y = mx + \frac{6}{m}, m \neq 0$ is a solution of the differential equation $xy' + 7\frac{1}{y'} = 0$

35. Find the torque of the resultant of the three forces represented by $-3\hat{i} + 6\hat{j} - 3\hat{k}$, $4\hat{i} - 10\hat{j} + 12\hat{k}$ and $4\hat{i} + 7\hat{j}$ acting at the point with position vector $8\hat{i} - 6\hat{j} - 4\hat{k}$, about the point with position vector $18\hat{i} + 3\hat{j} - 9\hat{k}$.

36. Evaluate $\lim\limits_{x \to \infty} \frac{x}{\log x}$

37. The radius of a circular plate is measured as 12.65 cm instead of the actual length 12.5 cm. find the following in calculating the area of the circular plate: (i) Absolute error (ii) Relative error (iii) Percentage error

38. Solve the following equation $\sin^2 x - 5\sin x + 4 = 0$

39. Let $*$ be defined on $\mathbb{R}$ by $(a * b) = a + b - ab - 2$. Is $*$ binary on $\mathbb{R}$? If so, find $5 * \left(\frac{2}{5}\right)$

40. Find the value of $\int_0^{\frac{\pi}{2}} \sin^5 x \cos^4 x \, dx$

PART – D (Answer All the Questions) 7 X5=35

41. a) Prove by vector method that the perpendiculars (attitudes) from the vertices to the opposite sides of a triangle are concurrent.

(OR)

b) Find the intervals of monotonicity and local extrema of the function $f(x) = \frac{x}{1+x^2}$

42. a) A multiple choice examination has ten questions, each question has four distractors with exactly one correct answer. Suppose a student answers by guessing and if X denotes the number of correct answers, find (i) binomial distribution (ii) probability that the student will get seven correct answers (iii) the probability of getting at least one correct answer.

(OR)

b) Solve : $(1 + x^3)\frac{dy}{dx} + 6x^2 y = 1 + x^2$

43. a) Investigate the values of λ and μ the system of linear equations $2x + 3y + 5z = 9$, $7x + 3y - 5z = 8$, $2x + 3y + \lambda z = \mu$, have (i)no solution (ii)a unique solution (iii)an infinite number of solutions

(OR)

b) A search light has a parabolic reflector (has a cross section that forms a 'bowl'). The parabolic bowl is 40*cm* wide from rim to rim and 30*cm* deep. The bulb is located at the focus. (1)What is the equation of the parabola used for reflector? (2) How far from the vertex is the bulb to be placed so that the maximum distance covered?

44. a) A ladder 17 metre long is leaning against the wall. The base of the ladder is pulled away from the wall at a rate of 5 m/s. When the base of the ladder is 8 metres from the wall.
(i) How fast is the top of the ladder moving down the wall?
(ii) At what rate, the area of the triangle formed by the ladder, wall, and the floor, is changing?.

(OR)

b) For of the function $f(x,y) = \tan^{-1}\left(\frac{x}{y}\right)$ find the f_x, f_y and show that $f_{xy} = f_{yx}$.

45. a) If $z = x + iy$ is a complex number such that $Im\left(\frac{2z+1}{iz+1}\right) = 0$, Find the locus of z.

(OR)

b) Find the non-parametric form of vector equation, and Cartesian equations of the plane $\vec{r} = (6\hat{i} - \hat{j} + \hat{k}) + s(-\hat{i} + 2\hat{j} + \hat{k}) + t(-5\hat{i} - 4\hat{j} - 5\hat{k})$

46. a) Find the area of the region bounded between the parabola $x^2 = y$ and the curve $y = |x|$.

(OR)

b) Let $M = \left\{\begin{pmatrix} x & x \\ x & x \end{pmatrix} : x \in R - \{0\}\right\}$ and let $*$ be the matrix multiplication. Determine whether M is closed under $*$. If so, examine the commutative and associative existence of identity, existence of inverse properties satisfied by $*$ on M.

47. a) If the normal at the point 't_1' on the parabola $y^2 = 4ax$ meets the parabola again at the point 't_2', then prove that $t_2 = -\left(t_1 + \frac{2}{t_1}\right)$.

(OR)

b) Find the value of $\tan^{-1}(-1) + \cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$