



Standard - 12
MATHEMATICS

Time Allowed: 3.00 Hours

Maximum Marks: 90

PART - A

Choose the correct answer:

20×1=20

- 1) If $A^T A^{-1}$ is symmetric then $A^2 =$ _____
a) A^T b) $(A^{-1})^2$ c) $(A^T)^2$ d) A^{-1}
- 2) If A is a non-singular square matrix of order n then
a) $|\text{adj } A| = |A|^{n-1}$ b) $(\text{adj } A)^T = \text{adj } A$
c) $|\text{adj } A| = |A|$ d) $\text{adj } (\lambda A) = \lambda \text{adj } A$
- 3) If z is a non zero complex number, such that $2iz^2 = \bar{z}$ then $|z|$ is
a) $\frac{1}{2}$ b) 1 c) 2 d) 3
- 4) Find the conjugate of $z_1 - z_2$ if $z_1 = 2+3i$, $z_2 = 5+2i$.
a) $-3+i$ b) $3+i$ c) $-3-i$ d) $3-i$
- 5) The polynomial x^3+2x+3 has
a) one negative and two imaginary zeros
b) one positive and two imaginary zeros
c) three real zeros
d) n zeros
- 6) If $\sin^{-1}x + \cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$ then x is equal to
a) $\frac{1}{2}$ b) $\frac{1}{\sqrt{5}}$ c) $\frac{2}{\sqrt{5}}$ d) $\frac{\sqrt{3}}{2}$
- 7) If the two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles then the locus of P is
a) $2x+1 = 0$ b) $x = -1$ c) $2x-1 = 0$ d) $x = 1$
- 8) If the conic is called a ellipse then the constant
a) $e = 1$ b) $e > 1$ c) $e < 1$ d) $e = 0$
- 9) Distance from the origin to the plane $3x-6y+2z+7 = 0$ is
a) 1 b) 2 c) 0 d) 3
- 10) If $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0$ then the value of $[\vec{a} \ \vec{b} \ \vec{c}]$ is
a) 1 b) $\frac{1}{3} |\vec{a}| |\vec{b}| |\vec{c}|$ c) -1 d) $|\vec{a}| |\vec{b}| |\vec{c}|$
- 11) The value of the limit $\lim_{x \rightarrow 0} \left(\cot x - \frac{1}{x} \right)$ is
a) 0 b) 1 c) 2 d) ∞
- 12) The maximum value of the product of two positive numbers, when their sum of the squares is 200 is
a) 100 b) $25\sqrt{7}$ c) 28 d) $24\sqrt{14}$
- 13) $f(x, y) = \frac{x^2 + 5xy - 10y^2}{3x + 7y}$ is a homogeneous function of degree _____
a) 1 b) 2 c) 0 d) 3
- 14) If $g(x) = x^2 + \sin x$ then $g''(x)$ is
a) $2x + \cos x$ b) $2 - \sin x$ c) $2 + \sin x$ d) $2x - \cos x$

15) If $\int_0^a \frac{1}{4+x^2} dx = \frac{\pi}{8}$ then a is

- a) 1 b) 3 c) 2 d) 4

16) The value of $\int_{-1}^2 |x| dx$ is

- a) $\frac{1}{2}$ b) $\frac{3}{2}$ c) $\frac{5}{2}$ d) $\frac{7}{2}$

17) Find the integrating factor of $\frac{dy}{dx} + 2y = e^{-x}$ is

- a) $2x$ b) e^{-2x} c) x d) e^{2x}

18) Let x have a Bernoulli distribution with mean 0.4 then the variance of $(2X-3)$ is

- a) 0.24 b) 0.48 c) 0.6 d) 0.96

19) Which one is the contrapositive of the statement $(p \vee q) \rightarrow r$?

- a) $\neg r \rightarrow (\neg p \wedge \neg q)$ b) $\neg r \rightarrow (p \vee q)$ c) $r \rightarrow (p \wedge q)$ d) $p \rightarrow (q \vee r)$

20) Which one of the following is a binary operation on \mathbb{N} ?

- a) Multiplication b) Division c) Subtraction d) All the above

PART - B

Answer any seven questions. Question No. 30 is compulsory.

7×2=14

21) Prove that $\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$ is orthogonal.

22) Find the equation of the hyperbola if foci $(\pm 2, 0)$ eccentricity = $\frac{3}{2}$

23) Find the value of $\tan^{-1}\left(\tan \frac{5\pi}{4}\right)$

24) Show that the equation $z^3 + 2\bar{z} = 0$ has five solutions.

25) Find a polynomial equation of minimum degree with rational coefficients having $2 - \sqrt{3}$ as a root.

26) $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ are any two boolean matrices. Find $A \vee B$ and $A \wedge B$

27) If $\lim_{\theta \rightarrow 0} \left(\frac{1 - \cos m\theta}{1 - \cos n\theta} \right) = 1$ then prove that $m = \pm n$

28) If $\omega(x, y, z) = x^2y + y^2z + z^2x$, $x, y, z \in \mathbb{R}$ find the differential $d\omega$.

29) If x is the random with distribution function $F(x)$ given by $F(x) = \begin{cases} 0 & x < 0 \\ x & 0 \leq x < 1 \\ 1 & 1 \leq x \end{cases}$
find $P(0.2 \leq x \leq 0.7)$

30) Evaluate $\int_0^1 [2x] dx$ where $[\]$ is the greatest integer function

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PART - C

Answer any seven questions. Question No. 40 is compulsory.

7×3=21

31) If $\text{adj } A = \begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$ find A^{-1} .

32) Find the square root of $-5-12i$

33) Solve the equation $x^4-14x^2+45=0$

34) Find the angle between the straight line $\vec{r} = (2\vec{i} + 3\vec{j} + \vec{k}) + t(\vec{i} - \vec{j} + \vec{k})$ and the plane $2x-y+z=5$

35) Evaluate $\int_0^1 (5x+4) dx$ as the limit of a sum

36) Find the binomial distribution for five fair coins are tossed once and X denotes the number of heads.

37) Evaluate: $\lim_{x \rightarrow 0} x \log x$

38) Show that $x^2+y^2=r^2$ where r is a constant is a solution of the differential equation $\frac{dy}{dx} = \frac{-x}{y}$

39) $\vec{a} = -3\vec{i} - \vec{j} + 5\vec{k}$, $\vec{b} = \vec{i} - 2\vec{j} + \vec{k}$, $\vec{c} = 4\vec{j} - 5\vec{k}$ find $\vec{a} \cdot (\vec{b} \times \vec{c})$

40) $u(x, y, z) = \log(x^3+y^3+z^3)$ find $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$

PART - D

Answer all the questions:

7×5=35

41) a] If $z = x+iy$ and $\arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$ show that $x^2+y^2+3x-3y+2=0$

(OR)

b] Prove that the ellipse $x^2+4y^2=8$ and the hyperbola $x^2-2y^2=4$ intersect orthogonally

42) a] Solve using Gaussian Elimination method for the following system of linear equations. $2x-2y+3z=2$; $x+2y-z=3$; $3x-y+2z=1$

(OR)

b] Find the area of the region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

43) a] Using vector method prove that $\cos(\alpha-\beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$

(OR)

b] Prove $p \rightarrow (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$ without using truth table.

44) a] Solve $6x^6-35x^5+56x^4-56x^2+35x-6=0$

(OR)

b] Prove that $\sin^{-1} \frac{3}{5} - \cos^{-1} \frac{12}{13} = \sin^{-1} \frac{16}{65}$

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- 45) a) Find the foci, vertices and length of major and minor axis of the conic
 $4x^2 + 36y^2 + 40x - 288y + 532 = 0$

(OR)

b) Solve $(y + \sqrt{x^2 + y^2}) dx - x dy = 0, y(1) = 0$

- 46) a) If $\omega(x, y, z) = \log \left(\frac{5x^3y^4 + 7y^2xz^4 - 75y^3z^4}{x^2 + y^2} \right)$ find $x \frac{\partial \omega}{\partial x} + y \frac{\partial \omega}{\partial y} + z \frac{\partial \omega}{\partial z}$

(OR)

- b) The probability density function of X is given by $f(x) = \begin{cases} Ke^{-2x} & \text{for } x > 0 \\ 0 & \text{for } x \leq 0 \end{cases}$

- (i) Find the value of K.
 (ii) Distribution function
 (iii) $P(X < 2)$

- 47) a) Find the vector parametric, vector non-parametric and Cartesian form of the equation of the plane passing through the points $(-1, 2, 0)$ $(2, 2, -1)$

and parallel to the straight line $\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$

(OR)

b) Evaluate: $\int_0^{\frac{1}{2}} \frac{\sin^{-1} x}{(1-x^2)^{\frac{3}{2}}} dx$