SECOND REVISION TEST - 2024	12 - STD	
MATHEMATICS	Marks 90	Time 3.00 Hrs.

PART - I

All questions are compulsory. **20 x 1 = 20**

1. A Zero of $x^3 + 64$ is
   a) 0 b) 4 c) 4i d) -4
2. arg(o) is
   a) π b) 0 c) ∞ d) undefined
3. The value of $\int_0^{\pi} \sin^4 x\, dx$ is
   a) $\frac{3\pi}{2}$ b) $\frac{3\pi}{10}$ c) $\frac{3\pi}{8}$ d) $\frac{3\pi}{4}$
4. The domain a of the function which is defined by $f(x)=\sin^{-1}\sqrt{x-1}$ is
   a) [1, 2] b) [−1, 1] c) [0, 1] d) [−1, 0]
5. If p(x, y) be any point on$16x^2 + 25y^2 = 400$ with foci $F_1(3, 0)$ and $F_2(-3, 0)$ then $PF_1 + PF_2$ -is
   a) 8 b) 6 c) 10 d) 12
6. Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is
   a) 2ab b) ab c) $\sqrt{ab}$ d) a/b
7. Distance from the origin to the plane 3x − 6y + 2z + 7 = 0 is
   a) 0 b) 1 c) 2 d) 3
8. The non - parametric form of vector euqation of a straight line passing through a point whose position vector is $\vec{a}$ and parallel to $\vec{U}$ is
   a) $\vec{r}=\vec{a}+t\vec{u}$ b) $\vec{r}=\vec{u}+t\vec{a}$ c) $(\vec{r}-\vec{u})X\,\vec{a}=\vec{o}$ d) $(\vec{r}-\vec{a})X\,\vec{u}=\vec{o}$
9. The number given by the mean value theorem for the function $\frac{1}{x}$, $x\in[1,9]$ is
   a) 2 b) 2.5 c) 3 d) 3.5
10. The slope of the tangent to the curve f(x) = 2 cos 4x at $x=\frac{\pi}{12}$ is
    a) $-4\sqrt{3}$ b) −4 c) $\frac{\sqrt{3}}{12}$ d) $4\sqrt{3}$
11. If $U(x,y)=e^{x^2+y^2}$ then $\frac{\partial u}{\partial x}$ is equal to
    a) $e^{x2+y2}e^{x2+y2}$ b) 2xu c) x^2u d) y^2u
12. If f and g are differentiable functions then d(fg) is
    a) fdg + gdf b) f.df − gdg c) fdf + gdg d) fdg − gdf

13. The value of $\int_0^{\infty} e^{-3x}\, x^2\, dx$ is

a) $\frac{7}{27}$ b) $\frac{5}{27}$ c) $\frac{4}{27}$ d) $\frac{2}{27}$

14. The value of $\int_0^{2/3} \frac{dx}{\sqrt{4-9x^2}}$ is a) $\frac{\pi}{6}$ b) $\frac{\pi}{2}$ c) $\frac{\pi}{2}$ d) π

15. The solution of the differential equation $\frac{dy}{dx} + \frac{1}{\sqrt{1-x^2}} = 0$ is

a) $y \sin^{-1}x = c$ b) $x + \sin^{-1}y = 0$ c) $y^2 + 2\sin^{-1}x = c$ d) $x^2 + 2\sin^{-1}y = 0$

16. If the function $f(x) = \frac{1}{12}$, for $a < x < b$ represents a probability density function of a continuous random variable X then which of the following can not be the value of a and b?

a) 0 and 12 b) 5 and 17 c) 7 and 19 d) 16 and 24

17. If $f(x) = \begin{cases} 2x & 0 \le x \le a \\ 0 & otherwise \end{cases}$ is a probability density function of a random variable then the value f a is

a) 1 b) 2 c) 3 d) 4

18. If a compound statement involves 3 simple statements, then the number of rows in the truth table is

a) 9 b) 8 c) 6 d) 3

19. In the case of Cramer's rule which of the following are correct?

i) $\Delta = 0$ ii) $\Delta \neq 0$ iii) The system has unique solution

iv) The system has infinitely many solution

a) (i) and (iv) b) (ii) and (iii) c) all d) none

20. Identify the in correct statement

a) $|z|^2 = 1 \Rightarrow \frac{1}{z} = \bar{z}$ b) $\mathrm{Re}(z) \le |z|$

c) $\big||z_1| - |z_2|\big| \ge |z_1 + z_2|$ d) $|z^n| = |z|^n$

PART - II

Answer any seven of the following.Q.No. 30 is compulsory. **7 x 2 = 14**

21. Find the rank of the matrix $\begin{bmatrix} -2 & 2 & -1 \\ 0 & 5 & 1 \\ 2 & 0 & 0 \end{bmatrix}$

22. If $|Z| = 1$ show that $2 \le |Z^2 - 3| \le 4$.

23. If $x^2 + 2(k+2)x + 9k = 0$ has equal roots find K.

24. If $\cos^{-1}(-x) = \pi - \cos^{-1}(x)$ true? Justify your answer.

25. Prove that the function $f(x) = x^2 - 2x - 3$ is strictly increasing in $(2, \infty)$.

26. Find df if $f(x) = x^2 + 3x$, $x = 3$ and $dx = 0.02$.

27. Find the area of the region bounded by the line $6x + 5y = 30$, x-axis and the lines $x = -1$ and $x = 3$.

28. Solve : $\frac{dy}{dx} + 2y = e^{-x}$

29. Write the statements in words corresponding to Γp and $q \vee \Gamma p$ where P is 'It is cold' and q is 'It is raining'

30. If $E(X + 5) = 6$ then show that $E(x) = 1$.

PART- III

Answer any seven of the following.Q.No. 40 is compulsory. **7 x 3 = 21**

31. Solve the following system of linear equations using matrix inverse method $5x + 2y = 3$, $3x + 2y = 5$

32. Simplify. $\left(\sin\frac{\pi}{6} + i\cos\frac{\pi}{6}\right)^{18}$

33. Show that the equation of the parabola whose end points of latus rectum are (4, –8) and (4, 8) open right ward and the vertex is (0,0) is $y^2 = 16x$.

34. Prove by vector method that the area of quadrilateral ABCD having diagonals AC and BD is $\frac{1}{2}\left|\overrightarrow{AC} \times \overrightarrow{BD}\right|$.

35. The time T, taken for a complete oscillation of a single pendulum with length l, is given by the equation $T = 2\pi\sqrt{\frac{l}{g}}$, where g is constant. Find the approximate percentage error in the calculated value of T corresponding to an error of 2 percent in the value of t.

36. Evaluate. $\int_0^{\pi/2} \begin{vmatrix} \cos^4 x & 7 \\ \sin^5 x & 3 \end{vmatrix} dx$

37. Find the differential equation of the family of circles passing through the origin and having their centres on the x axis.

38. Suppose that f(x) given below represents a probability mass function.

x	1	2	3	4	5	6
f(x)	C^2	$2C^2$	$3C^2$	$4C^2$	C	2C

Find i) the value of C ii) Mean and variance.

39. Prove that $q \rightarrow p \equiv \neg p \rightarrow \neg q$.

40. Discuss the real and imaginary roots of $x^5 + x^3 + x^2 + 1 = 0$.

PART - IV

Answer all the questions. **7 x 5 = 35**

41. a) Find the value of k for which the equations kx – 2y + z = 1, x – 2ky + z =2, x – 2y + kz = 1 i) no solution ii) unique solution iii) infinitely many solution.

(OR)

b) The growth of population is proportional to the number present. If the population of a colony doubles in 50 years, in how many years will the population become triple?

42. a) Prove that $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \tan^{-1}\left(\frac{x+y+z-xyz}{1-xy-yz-zx}\right)$

(OR)

b) Find all the values of $\left(-\sqrt{3}-i\right)^{1/3}$

43. a)The mean and variance of a binomial variate x are respectively 2 and 1.5. Find i) p(x = 0) ii) p (x = 1) iii) p(x≥1)

(OR)

b) If $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$, $\vec{b}=3\hat{i}+5\hat{j}+2\hat{k}$, $\vec{c}=-\hat{i}-2\hat{j}+3\hat{k}$ verify that

i) $\left(\vec{a}\times\vec{b}\right)\times\vec{c} = (\vec{a}\cdot\vec{c})\vec{b}-(\vec{b}\cdot\vec{c})\vec{a}$.

44. a) Find the vertex, focus, equation of directrix and length of the latus rectum of the parabola $y^2 - 4y - 8x + 12 = 0$.

(OR)

b) A conical water tank with vertex down of 12 metres height has a radius of 5 metres at the top. If water flows into the tank at a rate 10 cubic m/min. How fast is the depth of the water increases when the water is 8 metres deep?

45. a) Find the dimensions of the rectangle with maximum area that can be inscribed in a circle of radius 10 cm.

(OR)

b) On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4 m when it is 6m away from the point of projection. Finally it reaches the ground 12m away from the starting point. Find the angle of projection.

46. a) Find the non - parametric form of vector and Cartesian equation of the plane. $\vec{r}=(6\hat{i}-\hat{j}+\hat{k})+s(-\hat{i}+2\hat{j}+\hat{k})+t(-5\hat{i}-4\hat{j}-5\hat{k})$

(OR)

b) Find the area of the region bounded between the curves y = sin x, y = cos x and x = 0 and the lines x = 0 and $x = \pi$

47. a) Verify whether the compound proposition $(p\rightarrow q)\leftrightarrow(\neg p\rightarrow q)$ is a tautology or contradiction or contingency.

(OR)

b) Solve the differential equation $xdy - ydx = \sqrt{x^2+y^2}\,dx$.