12

Reg No

Time 3.00 hrs

First Revision Examination - 2024

MATHEMATICS

Max Marks 90

PART - A

20 x 1 = 20

Choose the correct answer

1. If $A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$, B = adj A and C = 3A, then $\frac{|adj\ B|}{|C|}$ = a) $\frac{1}{3}$ b) $\frac{1}{9}$ c) d) 1
2. If P(A) = P([A|B]), then the system A x = B of linear equations is
   a) consistent and has a unique solution b) consistent c) consistent and has infinity many solution d) Inconsistent
3. If z is a non-zero complex number, such that $2iz^2 = \bar{z}$ then |z| is a) $\frac{1}{2}$ b) 1 c) 2 d) 3
4. The product of all four values of $\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^{\frac{3}{4}}$ is a) –2 b) –1 c) 1 d) 2
5. A polynomial equation in x of degree n always has
   a) n distinct roots b) n real roots c) n complex roots d) at most one root
6. The domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$ is a) [1, 2] b) [–1, 1] c) [0, 1] d) [–1, 0]
7. If $\sin^{-1} x + \cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$, then x is equal to a) $\frac{1}{2}$ b) $\frac{1}{\sqrt{5}}$ c) $\frac{2}{\sqrt{5}}$ d) $\frac{\sqrt{3}}{2}$
8. The radius of the circle $3x^2 + by^2 + 4bx - 6by + b^2 = 0$ is a) 1 b) 3 c) $\sqrt{10}$ d) $\sqrt{11}$
9. The eccentricity of the ellipse $(x-3)^2 + (y-4)^2 = \frac{y^2}{9}$ is a) $\frac{\sqrt{3}}{2}$ b) $\frac{1}{3}$ c) $\frac{1}{3\sqrt{2}}$ d) $\frac{1}{\sqrt{3}}$
10. The volume of the parallelapiped with its edges represented by the vectors $\hat{i}+\hat{j}, \hat{i}+2\hat{j}, \hat{i}+\hat{j}+\pi\hat{k}$
    a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$ c) π d) $\frac{\pi}{4}$
11. If the planes $\vec{r}\cdot(2\hat{i}-\lambda\hat{j}+\hat{k}) = 3$ and $\vec{r}\cdot(4\hat{i}+\hat{j}-\mu\hat{k}) = 5$ are parallel, then the value of λ and μ are
    a) $\frac{1}{2}, -2$ b) $-\frac{1}{2}, 2$ c) $-\frac{1}{2}, 2$ d) $\frac{1}{2}, 2$
12. The minimum value of the function |3 – x| + 9 is a) 0 b) 3 c) 6 d) 9
13. If f (x, y, z) = xy + yz + zx, then fx – fz is equal to a) z – x b) y – z c) x – z d) y – x
14. The volume of solid of revolution of the region bounded by $y^2 = x(a-x)$ about x axis is
    a) πa^3 b) $\frac{\pi a^3}{4}$ c) $\frac{\pi a^3}{5}$ d) $\frac{\pi a^3}{6}$
15. The order and degree of the differential equation $\sqrt{\sin x}(dx + dy) = \sqrt{\cos x}(dx - dy)$ is
    a) 1, 2 b) 2, 2 c) 1, 1 d) 2, 1
16. Integrating factor of the differential equation $\frac{dy}{dx} = \frac{x+y+1}{x+1}$ is
    a) $\frac{1}{x+1}$ b) x + 1 c) $\frac{1}{\sqrt{x+1}}$ d) $\sqrt{x+1}$
17. A random variable x has binomial distribution with n = 25 and p = 0.8 then standard deviation of x is
    a) 6 b) 4 c) 3 d) 2
18. If P(P x = 0) = 1 – P (x = 1), If E(x) = 3 var (x), then P(x = 0) is a) $\frac{2}{3}$ b) $\frac{2}{5}$ c) $\frac{1}{5}$ d) $\frac{1}{3}$
19. The operation * defined by $a * b = \frac{ab}{7}$ is not a binary operation on a) Q^* b) Z c) R d) C
20. Determine the truth value of each of the following statments :
    i) 4 + 2 = 5 and 6 + 3 = 9 ii) 3 + 2 = 5 and 6 + 1 = 7 iii) 4 + 5 = 9 and 1 + 2 = 4 iv) 3 + 2 = 5 and 4 + 7 = 11

	(a)	(b)	(c)	(d)		(a)	(b)	(c)	(d)		(a)	(b)	(c)	(d)		(a)	(b)	(c)	(d)
a)	F	T	F	T	b)	T	F	T	F	c)	T	T	F	F	d)	F	F	T	T

PART - B

Answer any 7questions. Q.No.30 is compulsory. **7 x 2 = 14**

21. If $adj(A) = \begin{bmatrix} 2 & -4 & 2 \\ -3 & 12 & -7 \\ -2 & 0 & 2 \end{bmatrix}$, Find A
22. Find the value of the real numbers x and y, if the complex number (2 + i) x + (1 – i)y + 2i – 3 and x + (–1 + 2i) y + 1 + i are equal.

23 Discuss the maximum possible number of positive and negative roots of the polynomial equation $9x^8 - 4x^6 + 4x^7 - 3x^6 + 2x^5 + x^3 + 7x^2 + 7x + 2 = 0$

24 Find the equation of the parabola with vertex $(-1, -2)$ axis parallel to y-axis and passing through $(3, 6)$

25 Prove by vector method that the parallelograms on the same base and between the same parallels are equal in area.

26 Find the values in the interval $(1, 2)$ of the mean value theorem satisfied by the function $f(x) = x - x^2$ for $1 \le x \le 2$

27 Evaluate $\int_3^4 \frac{dx}{x^2 - 4}$

28 Find the differential equation of the family of parabolas $y^2 = 4ax$, where a is an arbitrary constant.

29 Three coins are tossed simultaneously. Find the probability mass function for number of heads occured.

30 Show that $p \rightarrow q$ and $q \rightarrow p$ are not equivalent.

PART - C

Answer any 7questions. Q.No.40 is compulsory. **7 x 3 = 21**

31 Solve : $5x - 2y + 16 = 0$, $x + 3y - 7 = 0$

32 If $|z| = 2$, show that $8 \le |z + 6 + 8i| < 12$.

33 Find the value of $\tan^{-1}(-1) + \cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$

34 Find the equations of tangents to the hyperbola $\frac{x^2}{16} - \frac{y^2}{64} = 1$ which are parallel to $10x - 3y + 9 = 0$.

35 Find the parametric form of vector equation and Cartesian equations of the straight line passing through the point $(-2, 3, 4)$ and parallel to the straight line $\frac{x-1}{-4} = \frac{y+3}{5} = \frac{8-z}{6}$

36 Expand $\log(1 + x)$ as a Maclaurin's series up to 4 non-zero terms for $-1 < x \le 1$.

37 Let $U(x, y, z) = x^2 - xy + 3 \sin z$, $x, y, z \in R$. Find the linear approximation for U at $(2, -1, 0)$

38 Evaluate $\int_0^{\pi/2} (\sin^2 x + \cos^4 x)\, dx$

39 Solve $(e^x + 1)\cos x\, dx + e^x \sin x\, dy = 0$

40 The probability that a certain kind of component will survive a electrical test is $\frac{3}{4}$. Find the probability that exactly 3 of the 5 components tested survive.

PART - D

Answer all the questions. **7 x 5 = 35**

41 a) Investigate the values of λ and μ the system of linear equations $2x + 3y + 5z = 9$, $7x + 3y - 5z = 8$, $2x + 3y + \lambda z = \mu$ here

i) no solution ii) a unique solution iii) an infinite number of solutions. **(OR)** b) Find all cube roots of $\sqrt{3} + i$

42 a) Solve the equation $(x - 2)(x - 7)(x - 3)(x + 2) + 19 = 0$ **(OR)**
b) Find the number of solutions of the equation $\tan^{-1}(x - 1) + \tan^{-1} x + \tan^{-1}(x + 1) = \tan^{-1}(3x)$

43 a) A bridge has a parabolic arch that is 10m high in the centre and 30m circle at the bottom. Find the height of the arch 6m from the centre on either sides. **(OR)** b) If $\vec{a} = -2\hat{i} + 3\hat{j} - 2\hat{k}$, $\vec{b} = 3\hat{i} - \hat{j} + 3\hat{k}$, $\vec{c} = 2\hat{i} - 5\hat{j} + \hat{k}$, Find $(\vec{a} \times \vec{b}) \times \vec{c}$ and $\vec{a} \times (\vec{b} \times \vec{c})$. State whether they are equal.

44 a) A police jeep, approaching an orthogonal intersection from the northern direction, is chasing a speeding car that has turned and moving straight east. When the Jeep is 0.6 km north of the intersection and the car is 0.8 km to the east. The police determine with a radar that the distance between them and the car is increasing at 20km/hr. If the jeep is moving at 60km/hr at the instant of measurement. What is the speed of the car? **(OR)**

b) If $U = \sin^{-1}\left(\frac{x + y}{\sqrt{x} + \sqrt{y}}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$

45 a) Find the area of the region bounded between the parabola $x^2 = y$ and the curve $y = |x|$ **(OR)**
b) In a murder investigation, a corpse was found by a detective at exactly 8 p.m. Being alert, the detective also measured the body temperature and found it to be 70°F. Two hours later, the detective measured the body temperature again and found it to be 60°F. If the room temperature is 50°F, and assuming that the body temperature of the person before death was 98.6°F, at what time did the murder occur? [log (2.43) = 0.88789 ; log(0.5) = –0.69315)

46 a) A six sided die is marked '1' on one face, '2' an two of its faces, and '3' on remaining three faces. The die is rolled twice. If x denotes the total scores in two throws.
i) Find the probability mass function. ii) Find the cumulative distribution function.iii) Find $P(3 \le x < 6)$ iv) Find $P(x \ge 4)$ **(OR)** b) Verify i) closure property ii) commutative property iii) associative property, iv) existence of identity and v) existence of inverse for the operation t_5 on z_5. Using table corresponding to addition modulo 5.

47 a) Find the non-parametric form of vector equation, and cartesian equation of the plane passing through the point $(2, 3, 6)$ and parallel to the straight lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-3}{1}$ and $\frac{x+3}{2} = \frac{y-3}{-5} = \frac{z+1}{-3}$ **(OR)**

b) A farmer plans to fence a rectangular pasture adjacent to a river. The paasture must contain 1,80,000 sq.mtrs in order to provide enough grass for herds. No fencing is needed along the river. What is the length of the minimum needed fencing material?