12

FIRST REVISION EXAMINATION, JANUARY - 2024

MATHEMATICS

Time Allowed : 3.00 Hours] [Max. Marks : 90

PART – A (Answer All the questions) 20 X 1=20

1. If $A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & -1 \end{bmatrix}$ and $A^{-1} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ then the value of a_{23} is
   [1]0 [2]-2 [3]-3 [4]-1
2. According to the rational root theorem, which number is not possible rational zero of $4x^7 + 2x^4 - 10x^3 - 5?$
   [1]-1 [2]$\frac{5}{4}$ [3]$\frac{4}{5}$ [4]5
3. Distance from the origin to the plane $3x - 6y + 2z + 7 = 0$ is
   (1)0 [2]1 [3]2 [4]3
4. The value of $\sum_{i=1}^{100}(i^n - i^{n+1})$
   [1]$1 + i$ [2]i [3]$1 - i$ [4]$-i$
5. If $\cot^{-1} x = \frac{2\pi}{5}$ for some $x \in R$, the value of $\tan^{-1} x$ is
   [1]$-\frac{\pi}{10}$ [2]$\frac{\pi}{5}$ [3]$\frac{\pi}{10}$ [4]$-\frac{\pi}{5}$
6. Consider the vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ such that $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = \vec{0}$. Let P_1 and P_2 be the planes determined by the pairs of vectors a, b and c, d respectively. Then the angle between P_1 and P_2 is
   [1]0° [2]45° [3]60° [4]90°
7. The solution of the differential equation $\frac{dy}{dx} - \frac{1}{\sqrt{1-x^2}} = 0$ is
   1]$y + \sin^{-1} x = c$ [2]$y - \sin^{-1} x = c$ [3]$x - \sin^{-1} y = c$ [4]$x + \sin^{-1} y = c$
8. If $(x, y) = \log(e^x + e^y)$, then the value of $\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y}$ is
   [1]$e^x + e^y$ [2]$\frac{1}{e^x+e^y}$ [3]$\frac{e^x e^y}{e^x+e^y}$ [4] 1
9. Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is
   [1]$2ab$ [2]ab [3]$\sqrt{ab}$ [4]$\frac{a}{b}$
10. The order and degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{\frac{1}{3}} + x^{\frac{1}{4}} = 0$
    [1]2, 3 [2]3,3 [3]2, 6 [4]2, 4
11. If $v(x, y) = (e^x + e^y)$, then the valve of $\frac{\partial v}{\partial x} - \frac{\partial v}{\partial y}$ is
    [1]$e^x + e^y$ [2]$\frac{1}{e^x+e^y}$ [3]$e^x - e^y$ [4] $\frac{1}{e^x-e^y}$
12. The number given by the Rolle's theorem for the function $x^3 - 3x^2, x \in [0, 3]$ is
    [1]1 [2]$\sqrt{2}$ [3]$\frac{3}{2}$ [4]2
13. z_1, z_2 and z_3 are complex numbers such that $z_1 + z_2 + z_3 = 0$ and $|z_1| = |z_2| = |z_3| = 1$ then $z_1^2 + z_2^2 + z_3^2$ is
    [1]3 [2]2 [3]1 [4]0

14. If $(AB)^{-1} = \begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$, then $B^{-1} =$

[1] $\begin{bmatrix} 2 & -5 \\ -3 & 8 \end{bmatrix}$ [2] $\begin{bmatrix} 8 & 5 \\ 3 & 2 \end{bmatrix}$ [3] $\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ [4] $\begin{bmatrix} 8 & -5 \\ -3 & 2 \end{bmatrix}$

15. *The value of* $\lim\limits_{x \to \infty} \frac{e^x}{x^m}$, $m \in N$

[1] 0 [2] 1 [3] 2 [4] ∞

16. If the normal of the parabola $y^2 = 4x$ drawn at the end points of its latus rectum are tangents to the circle $(x-3)^2 + (y+2)^2 = r^2$, then the value of r^2 is

[1] 2 [2] 3 [3] 1 [4] 4

17. If $f(x) = \int_0^x t \cot t \, dt$, then $\frac{df}{dx} =$

[1] $\cos x - x \sin x$ [2] $\sin x + x \cos x$ [3] $x \cos x$ [4] $x \sin x$

18. Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed n times. Then the possible values of X are

[1] $i + 2n, i = 0, 1, 2, \dots n$ [2] $2i - n, i = 0, 1, 2, \dots n$
[3] $n - i, i = 0, 2, \dots n$ [4] $2i + 2n, i = 0, 1, 2, \dots n$

19. The dual of $\neg(p \vee q) \vee [p \vee (p \wedge \neg r)]$ is

[1] $\neg(p \wedge q) \wedge [p \vee (p \wedge \neg r)]$ [2] $(p \wedge q) \wedge [p \wedge (p \vee \neg r)]$
[3] $\neg(p \wedge q) \wedge [p \wedge (p \wedge r)]$ [4] $\neg(p \wedge q) \wedge [p \wedge (p \vee \neg r)]$

20. *The value of* $\int_0^\infty e^{-3x} x^5 dx$ *is*

[1] $\frac{3!}{5^3}$ [2] $\frac{5!}{3^6}$ [3] $\frac{4!}{3^6}$ [4] $\frac{3!}{5^6}$

PART – B Answer Any Seven Question. Question No. 30 is compulsory **7 X 2 = 14**

21. If $adj(A) = \begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$, find A^{-1}. $\frac{1}{6}$

22. The orbit of Halley's comet is an ellipse 36.18 astronomical units long and by 9.12 astronomical units wide. Find its eccentricity. 0.97

23. If $x^2 + 2(k+2)x + 9k = 0$ has equal roots, find k. $k=1$, $k=4$

24. Evaluate $\int_0^{\frac{\pi}{2}} \cos^7 x \, dx$

25. Is $\cos^{-1}(-x) = \pi - \cos^{-1}(x)$ true? Justify your answer.

26. Write into rectangular form. $\frac{10-5i}{6+2i}$ $\frac{5}{4}(1-i)$

27. Three fair coins are tossed simultaneously. Find the probability mass function for number of heads occurred.

28. If $w(x, y) = e^{2xy}$ then find the value of $\frac{\partial^2 w}{\partial x \partial y}$.

29. A particle acted upon by constant forces $2\hat{i} + 5\hat{j} + 6\hat{k}$ and $-\hat{i} - 2\hat{j} - \hat{k}$ is displaced from the point $(4, -3, -2)$ to the point $(6, 1, -3)$. Find the total work done by the forces. 12 u

30. Solve $\frac{dy}{dx} = \frac{-(1+y^2)}{\sqrt{1+x^2}}$

PART – C Answer Any Seven Question. Question No. 40 is compulsory 7 X 3 = 21

31. Verify whether the following compound proposition are tautology or contradiction or contingency $((p \vee q) \wedge \neg p) \rightarrow q$

32. Evaluate : $\int_0^1 |5x - 3| dx$

33. If $A = \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$, Verify that $A(adj A) = (adj\ A)A = |A| I_2$

34. Evaluate : $\tan\left(\cos^{-1}\left(\frac{1}{2}\right) - \sin^{-1}\left(-\frac{1}{2}\right)\right)$ = 0

35. The parabolic communication antenna has a focus at $2m$ distance from the vertex of the antenna. Find the width of the antenna $3m$ from the vertex. $4\sqrt{6}$ m

36. Find the slope of the tangent to the following curve at the respective given points.
$x = a\cos^3 t, y = b\sin^3 t$ at $t = \frac{\pi}{2}$

37. Show that the equation $z^3 + 2\bar{z} = 0$ has five solutions.

38. Find the mean and variance of a random variable X, whose probability density function is $f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{for } x \geq 0 \\ 0 & \text{Other wise} \end{cases}$

39. Show that the lines $\frac{x-1}{4} = \frac{2-y}{6} = \frac{z-4}{12}$ and $\frac{x-3}{-2} = \frac{y-3}{3} = \frac{5-z}{6}$ are parallel. $\vec{b} = -2\vec{d}$

40. In a newly developed city, it is estimated that the voting population (in thousands) will increase according to $V(t) = 30 + 12t^2 - t^3, 0 \leq t \leq 8$ where t is the time in years. Find the approximate change in voters for the time change from 4 to $4\frac{1}{6}$ year 4.8 $4\frac{4}{3}$

PART – D (Answer All the Questions) 7 X 5 = 35

41. a) Identify the type of conic and find centre, foci, vertices and directories of each of the following. $18x^2 + 12y^2 - 144x + 48y + 120 = 0$ $\vec{r}.(9\vec{i}+6\vec{j}-13\vec{k}) = -4$

(OR) $9x+6y-13z = -4$

b) Find the Non parametric form of vector equation and Cartesian equations of a straight line passing through (4,2,4) and is perpendicular to the straight lines
$2x + 5y + 4z = 6$ and $4x + 7y + 6z = 2$

42. a) Test for consistency and if possible, solve the following systems of equations
$x - y + 2z = 2,$ $2x + y + 4z = 7,$ $4x - y + z = 4$ $x, y, z = 1$

(OR)

b) Sketch the curve $y = f(x) = x^2 - x - 6$

43. a) Let X be a random variable denoting the life time of an electrical equipment having probability density function $f(x) = \begin{cases} ke^{-2x} & for\ x > 0 \\ 0 & for\ x \leq 0 \end{cases}$

Find (i) the value of k (ii)Distribution function (iii)$P(X < 2)$
(iv) calculate the probability that X is at least for four unit of time (v) $(P(X = 3)$

OR)

b) Water at temperature 100°C cools in 10 minutes to 80°C in a room temperature of 25°C. Find (i) The temperature of water after 20 minutes.
(ii) The time when the temperature is 40°C. $\left[\log_e \frac{11}{15} = -0.3101;\ \log_e 5 = 1.6094\right]$

44. a) A rod of length 1.2m moves with its ends always touching the coordinate axes. the locus of a point P on the rod, which is 0.3m from the end in contact with x-axis is an ellipse. find the eccentricity.

(OR)

b) Solve the equation $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$ if it is known that $\frac{1}{3}$ is a solution.

45. a) By vector method, prove that $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$.

(OR)

b) It the curves $ax^2 + by^2 = 1$ and $cx^2 + dy^2 = 1$ intersect each other orthogonally then, show that $\frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$

46. a) Find the area of the region bounded between the parabolas $y^2 = 4x$ and $x^2 = 4y$.

(OR)

b) If $z = x + iy$ and $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$, show that $x^2 + y^2 = 1$.

47. a) If $u = \sec^{-1}\left(\frac{x^3 - y^3}{x + y}\right)$, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\cot u$

(OR)

b) Verify (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity, and (v) existence of inverse for the operation $\times_{11}$ on a subset $A = \{1,3,4,5,9\}$ of the set of remainders $\{0,1,2,3,4,5,6,7,8,9,10\}$.