

FIRST REVISION TEST - 2024

B

Standard XII

Reg.No.

MATHEMATICS

Time : 3.00 hrs

Part - I

Marks : 90

$20 \times 1 = 20$

I. Choose the correct answer:

1. If $A^T A^{-1}$ is symmetric, then $A^2 =$
 - a) A^{-1}
 - b) A^T
 - c) $(A^{-1})^2$
 - d) $(A^T)^2$
2. In the case of Cramer's rule which of the following are correct?
 - i) $\Delta = 0$
 - ii) $\Delta \neq 0$
 - iii) the system has unique solution
 - iv) the system has Infinitely many solutions
 a) (i) and (iv) b) (ii) and (iii) c) all d) none
3. If $\frac{z-1}{z+1}$ is purely imaginary, then $|Z|$ is
 - a) $\frac{1}{2}$
 - b) 2
 - c) 1
 - d) 3
4. If the direction cosines of a line are $\frac{1}{c}, \frac{1}{c}, \frac{1}{c}$ then
 - a) $c = \pm\sqrt{3}$
 - b) $c = \pm 3$
 - c) $c > 0$
 - d) $0 < c < 1$
5. Integrating factor of the differential equation $\frac{dy}{dx} = \frac{x+y+1}{x+1}$ is
 - a) $\frac{1}{x+1}$
 - b) $x+1$
 - c) $\frac{1}{\sqrt{x+1}}$
 - d) $\sqrt{x+1}$
6. The tangent to the curve $y^2 - xy + 9 = 0$ is vertical when
 - a) $y = 0$
 - b) $y = \pm\sqrt{3}$
 - c) $y = \frac{1}{2}$
 - d) $y = \pm 3$
7. The number of positive zeros of the polynomial $\sum_{r=0}^n nC_r (-1)^r x^r$ is
 - a) n
 - b) 0
 - c) $< n$
 - d) r
8. The volume of solid of revolution of the region bounded by $y^2 = x(a-x)$ about x axis
 - a) πa^3
 - b) $\frac{\pi a^3}{4}$
 - c) $\frac{\pi a^3}{5}$
 - d) $\frac{\pi a^3}{6}$
9. Rolle's theorem is applicable only when
 - a) $f'(x) \neq 0$
 - b) $f'(x) < 0$
 - c) $f'(x) > 0$
 - d) $f'(x) = 0$
10. Linear approximation for $g(x) = \cos x$ at $x = \frac{\pi}{2}$ is
 - a) $x - \frac{\pi}{2}$
 - b) $x + \frac{\pi}{2}$
 - c) $-x + \frac{\pi}{2}$
 - d) $-x - \frac{\pi}{2}$
11. A binary operation on a set S is a function from
 - a) $S \rightarrow S$
 - b) $(S \times S) \rightarrow S$
 - c) $S \rightarrow (S \times S)$
 - d) $(S \times S) \rightarrow (S \times S)$

12. If a compound statement involves 3 simple statements then the number of rows in the truth table is
 a) 9 b) 8 c) 6 d) 3
13. A random variable X has binomial distribution with $n = 25$ and $p = 0.8$ then standard deviation of X is
 a) 4 b) 3 c) 6 d) 2
14. The product of the roots of n^{th} roots of unity is
 a) $(-1)^n$ b) 1 c) $(-1)^{n-1}$ d) 0
15. $\int_0^a \frac{f(x)}{f(x) + f(a-x)} dx =$
 a) 0 b) a c) $\frac{a}{2}$ d) $2a$
16. For the parabola $(x - h)^2 = -4a(y - k)$, the equation of the directrix is
 a) $y = k$ b) $x = k + a$ c) $y = a$ d) $y = k + a$
17. If $X \sim B(n, p)$ then
 a) $\mu = np$, $\sigma^2 = np(1-p)$ b) $\mu = nq$, $\sigma = np(1-p)$
 c) $\mu = np$, $\sigma = np(1-p)$ d) $\mu = npq$, $\sigma = npq$
18. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + \hat{j}$, $\vec{c} = \hat{i}$ and $(\vec{a} \times \vec{b}) \times \vec{c} = \lambda \vec{a} + \mu \vec{b}$, then the value $\lambda + \mu$ is
 a) 0 b) 6 c) 1 d) 3
19. $\cos(\cos^{-1}x) = x$ if
 a) $|x| < 1$ b) $|x| \leq 1$ c) $|x| \geq 1$ d) $|x| = 0$
20. Identify the incorrect statement.
 a) $\operatorname{Re}(Z) \leq |Z|$ b) $|Z|^2 = 1 \Rightarrow \frac{1}{Z} = \bar{Z}$
 c) $||Z_1| - |Z_2|| \geq |Z_1 + Z_2|$ d) $|Z^n| = |Z|^n$

Part - II

II. Answer any 7 questions. (Q.No.30 is compulsory)

 $7 \times 2 = 14$

21. If A is a symmetric matrix, prove that $\operatorname{adj} A$ is also symmetric.

22. If $\left(\frac{1-i}{1+i}\right)^{100} = a+ib$, find the values of a and b .

23. If $x^2 + 2(k+2)x + 9k = 0$ has equal roots, Find k .

24. Find the principal solution of $\sqrt{3} \sec x = -2$

25. Evaluate : $\int_0^1 x^3 (1 - x^4) dx$

26. Show that $y = mx + \frac{7}{m}$, $m \neq 0$ is a solution of the differential equation $xy' + 7 \frac{1}{y'} - y = 0$

27. The probability function of X is given by $f(x) = \begin{cases} kxe^{-2x} & \text{for } x > 0 \\ 0 & \text{for } x \leq 0 \end{cases}$

Find the value of k .

28. Let * be defined on \mathbb{R} by $(a * b) = a + b + ab - 7$. Is * binary on \mathbb{R} ? If so, find $3 * \left(\frac{-7}{15}\right)$

29. Show that the percentage error in the n^{th} root of a number is approximately $\frac{1}{n}$ times the percentage error in the number.

30. Evaluate : $[2\hat{i} \hat{j} \hat{k}] + [\hat{i} \hat{k} \hat{j}] + [\hat{k} \hat{j} 2\hat{i}]$

Part - III

III. Answer any 7 questions. (Q.No.40 is compulsory)

31. Solve the following system of linear equations by matrix inversion method :

$$5x + 2y = 3, \quad 3x + 2y = 5$$

$$7 \times 3 = 21$$

32. Find the quotient $\frac{(-\frac{3\pi}{2}, -\frac{3\pi}{2})}{(\frac{4}{2}, \frac{4}{2})}$ in rectangular form.

33. Find all real numbers satisfying $4^x - 3(2^{x+2}) + 2^5 = 0$

34. Prove that $\frac{\pi}{2} \leq \sin^{-1} x + 2 \cos^{-1} x \leq \frac{3\pi}{4}$

35. Find the equation of the hyperbola whose foci are $(\pm 2, 0)$ and eccentricity = $\frac{3}{2}$

36. Find the angle between the line $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$ and the plane $10x + 2y - 11z = 3$

37. Evaluate: $\lim_{x \rightarrow 1^-} \left(\frac{\log(1-x)}{\cot(\pi x)} \right)$

38. If $X \sim B(n, p)$ such that $4P(X=4) = P(X=2)$ and $n=6$. Find the distribution.

39. Using truth table prove that $q \rightarrow p \equiv \neg p \rightarrow \neg q$

40. Find the area of the region bounded by the line $5x + 3y = 15$, x -axis and the lines $x = -1$ and $x = 2$.

Part - IV

IV. Answer all the questions.

$$7 \times 5 = 35$$

b) Prove that : $\tan\left[\frac{1}{2}\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \frac{1}{2}\cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)\right] = \frac{2a}{1-a^2}$

42. a) Solve the equation $(2x - 3)(6x - 1)(3x - 2)(x - 2) - 5 = 0$
(OR)

b) Find the area of the region in the first quadrant enclosed by x-axis. Line $x = \sqrt{3} y$ and the circle $x^2 + y^2 = 4$

43. a) Find the equation of the circle passes through the points $(0, 0)$, $(-2, 1)$ and $(-3, 2)$
(OR)

b) Find the probability mass function $f(x)$ of the discrete random variable X whose cumulative distribution function $F(X)$ is given by

$$F(x) = \begin{cases} 0 & , -\infty < x < -2 \\ 0.25 & , -2 \leq x < -1 \\ 0.60 & , -1 \leq x < 0 \\ 0.90 & , 0 \leq x < 1 \\ 1 & , 1 \leq x < \infty \end{cases}$$

Also find (i) $P(X < 0)$ and (ii) $P(X \geq -1)$

44. a) Find the foot of the perpendicular drawn from the point $(5,4,2)$ to the line

$\frac{x+1}{2} = \frac{y-3}{3} = \frac{z-1}{-1}$. Also find the equation of the perpendicular.

(OR)

- b) If $z = x + iy$ and $\arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$, show that $x^2 + y^2 + 3x - 3y + 2 = 0$

45. a) Using vector method, prove that $\cos(\alpha - \beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta$
(OR)

- b) If $f(x, y) = \cos(x^2 - 3xy)$, find f_x , f_y and show that $f_{xy} = f_{yx}$

46. a) A tank contains 1000 litres of water in which 100 grams of salt is dissolved. Brine (Brine is a high-concentration solution of salt [usually sodium chloride] in water) runs in at a rate of 10 litres per minute and each litre contains 5 grams of dissolved salt. The mixture of the tank is kept uniform by stirring. Brine runs out at 10 litres per minute. Find the amount of salt at any time t .

(OR)

- b) Verify

47. a) A metal box with a square base and vertical sides is to contain 1024 cm^3 of water. The material for the top and bottom costs ₹5 per cm^2 and the material for the sides costs ₹2.50 per cm^2 . Find the least cost of the box.

(OR)

- b) Solve : $(1 + y^2) + (x - e^{\tan^{-1} y}) \frac{dy}{dx} = 0$
