FIRST REVISION TEST - 2024	12 - STD	
MATHEMATICS	Marks 90	Time 3.00 Hrs.

PART - I

I.Choose the correct Answer : **20 x 1 = 20**

1. The differential equation representing the family of curves y = A cos (x + B), where A and B are parameters, is

   a) $\frac{d^2y}{dx^2}-y=0$ b) $\frac{d^2y}{dx^2}+y=0$ c) $\frac{d^2y}{dx^2}=0$ d) $\frac{d^2x}{dy^2}=0$

2. Consider a game where the player tosses a six-sided fair die. If the face that comes up is 6. The player wins ₹ 36, otherwise he loses ₹ k^2, where k is the face that comes up k = { 1, 2, 3, 4, 5 }. The expected amount to win at this game in ₹ is

   a) $\frac{19}{6}$ b) $-\frac{19}{6}$ c) $\frac{3}{2}$ d) $-\frac{3}{2}$

3. $\begin{bmatrix}3 & 1\\5 & 2\end{bmatrix}$ the inverse of is

   a) $\begin{bmatrix}3 & -1\\-5 & -3\end{bmatrix}$ b) $\begin{bmatrix}2 & -1\\-5 & 3\end{bmatrix}$ c) $\begin{bmatrix}-3 & 5\\1 & -2\end{bmatrix}$ d) $\begin{bmatrix}-2 & 5\\1 & -3\end{bmatrix}$

4. The centre of the hyperbola $\frac{(x-1)^2}{16}-\frac{(y+1)^2}{25}=1$

   a) $\left(\frac{1}{2},\frac{-1}{2}\right)$ b) (-1, 1) c) (1, -1) d) (0, 0)

5. If $\left|z-\frac{3}{z}\right|=2$, then the least value of |Z| is a) 1 b) 2 c) 3 d) 5

6. For any value of $n\in\mathbb{Z}$, $\int_0^{\pi} e^{\cos^2 x}\cos^3[(2n+1)x]dx$ is a) $\frac{\pi}{2}$ b) π c) 0 d) 2

7. The function f(x) = x^2, in the interval [0, ∞) is

   a) cannot be determined b) increasing function
   c) increasing and decreasing function d) decreasing function

8. If $[\vec{a},\vec{b},\vec{c}]=1$ then the value of $\frac{\vec{a}.(\vec{b}\times\vec{c})}{(\vec{c}\times\vec{a}).\vec{b}}+\frac{\vec{b}.(\vec{c}\times\vec{a})}{(\vec{a}\times\vec{b}).\vec{c}}+\frac{\vec{c}.(\vec{a}\times\vec{b})}{(\vec{c}\times\vec{b}).\vec{a}}$ is

   a) 1 b) -1 c) 2 d) 3

9. Which one of the following is a binary operation on $\mathbb{N}$?

   a) Subtraction b) Multiplication c) Division d) All the above

10. A balloon rises straight up at 10 m/s. An observer is 40 m away from the spot where the balloon left the ground. The rate of change of the balloon's angle of elevation in radian per second when the balloon is 30 metres above the ground.

    a) $\frac{3}{25}$ radians / sec b) $\frac{4}{25}$ radians / sec c) $\frac{1}{5}$ radians / sec d) $\frac{1}{3}$ radians / sec

11. Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed n times. Then the possible value of x are
    a) i + 2n, i = 0, 1, 2 ...... n b) 2i - n, i = 0, 1, 2, ... n
    c) n - i, i = 0, 1, 2 .... n d) 2i + 2n ; i = 0, 1, 2, ... n
12. The equation of the normal to the circle $x^2 + y^2 - 2x - 2y + 1 = 0$ which is parallel to the line 2x + 4y = 3 is
    i) x + 2y = 3 b) x + 2y + 3 = 0 c) 2x + 4y + 3 = 0 d) x - 2y + 3 = 0
13. If the distance of the point (1, 1, 1) from the origin is half of its distance from the plane x + y + z + k = 0, then the values of k are
    a) ±3 b) ±6 c) -3, 9 d) 3, -9
14. If f and g are polynomials of degrees m and n respectively, and if h(x) = (f o g)(x), then the degree of h is
    a) mn b) m + n c) m^n d) n^m
15. The solution of the differential equation $2x\frac{dy}{dx} - y = 3$ represents
    a) straight line b) circles c) parabola d) ellipse
16. $\int_0^{\pi/2} \sin^7 x\, dx =$ a) $\frac{\pi}{2}$ b) $\int_0^{\pi/2} \cos^7 x\, dx$ c) 0 d) 1
17. The value of $\sin^{-1}\left(\frac{1}{2}\right) + \cos^{-1}\left(\frac{1}{2}\right)$ is a) 0 b) $\frac{\pi}{2}$ c) $\frac{\pi}{3}$ d) π
18. If $A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & -1 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ then the value of a_{23} is
    a) 0 b) -2 c) -3 d) -1
19. The value of the complex number $(i^{25})^3$ is equal to
    a) 1 2) i 3) -i 4) -1
20. If $f(x, y) = e^{xy}$, then $\frac{\partial^2 f}{\partial x \partial y}$ is equal to
    a) xye^{xy} b) $(1 + xy)e^{xy}$ c) $(1 + y) e^{xy}$ d) $(1 + x) e^{xy}$

PART - II

Answer any seven questions. Q.No. 30 is compulsory **7 x 2 = 14**

21. Simplify $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1+i}{1-i}\right)^3$ into rectangular form.
22. If α and β are the roots of $x^2 - 5x + 6 = 0$ then prove that $\alpha^2 - \beta^2 = 15$
23. Find the principal value of $\sin^{-1}(2)$, if it exists.
24. If $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} - 2\hat{k}$, $\vec{c} = 3\hat{i} + 2\hat{j} + \hat{k}$ find $\vec{a}.(\vec{b} \times \vec{c})$
25. Prove that $\lim_{x \to \infty}\left(\frac{e^x}{x^m}\right)$, where m is a positive integer is ∞
26. Let g(x) = x2 + sin x. Calculate the differential dg.

27. Solve the following differential equation $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$

28. If x is the random variable with probability density function f(x) given by

$$f(x)=\begin{cases} x+1, & -1\le x<0 \\ -x+1, & 0\le x<1 \\ 0 & otherwise \end{cases}$$ then find

i) the distribution function F(x) ii) P ($-0.5\le x\le 0.5$)

29. Find the constant C such that the function $f(x)=\begin{cases} Cx^2 & 1<x<4 \\ 0 & otherwise \end{cases}$ is a density function.

Find the value C.

30. Show that the differential equation corresponding to y = A sin x, where A is an arbitrary constent is y = $y=y'\tan x$

PART - III

Answer any seven questions. Q.No. 40 is compulsory **7 x 3 = 21**

31. Find the rank of the following matrices by minor method $\begin{bmatrix} 1 & -2 & 3 \\ 2 & 4 & -6 \\ 5 & 1 & -1 \end{bmatrix}$

32. If $A=\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$, show that $A^2 - 3A - 7I_2 = O_2$. Hence find A^{-1}.

33. Show that the points 1, $\frac{-1}{2}+i\frac{\sqrt{3}}{2}$ and $\frac{-1}{2}-i\frac{\sqrt{3}}{2}$ are the vertices of an equilateral triangle.

34. If the sides of a cubic box are increased by 1, 2, 3 units respectively to form a cuboid, then the volume is increased by 52 cubic units. Show that the volume of the cuboid is 60 cubic units.

35. Find the equation of the parabola with focus $(-\sqrt{2},0)$ and directrix $x=\sqrt{2}$.

36. A particle acted upon by constant forces $2\vec{i}+5\vec{j}+6\vec{k}$ and $-\vec{i}-2\vec{j}-\vec{k}$ is displaced from the point (4, -3, -2) to the point (6, 1, -3). Find the total work done by the forces.

37. Find the points on the curve $y = x^3 - 6x^2 + x + 3$ where the normal is parallel to the line x + y = 1729.

38. A circular plate expands uniformly under the influence of heat. If it's radius increases from 10.5 cm to 10.75 cm, then find an approximate change in the area and the approximate percentage change in the area.

39. Let A = { a + $\sqrt{5}b$: a, b∈ $\mathbb{Z}$ }. Check whether the usual multiplication is a binary operation on A.

40. Show that $\int_0^1 \frac{\sqrt{x}}{\sqrt{1-x}+\sqrt{x}}dx=\frac{1}{2}$

PART - IV

Answer all the questions. **7 x 5 = 35**

41. a) Test for consistency and if possible solve the following systems of equations by rank method. $3x + y + z = 2$, $x - 3y + 2z = 1$, $7x - y + 4z = 5$

(OR)

b) If the volume of a cube of side length x is $v = x^3$. Find the rate of change of the volume with respect to x when x = 5 units.

42. a) If $z = x + iy$ is a complex number such that $\text{Im}\left[\frac{2z+1}{iz+1}\right] = 0$, show that the locus of z is $2x^2 + 2y^2 + x - 2y = 0$.

(OR)

b) Find the area of the region bounded between the curves $y = \sin x$ and $y = \cos x$ and the lines $x = 0$ and $x = \pi$

43. a) Prove that $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$

(OR)

b) On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4m when it is 6m away from the point of projection. Finally it reaches the ground 12m away from the starting point. Find the angle of projection.

44. a) The rate of increase in the number of bacteria in a certain bacteria culture is proportional to the number present. Given that number triples in 5 hours. Find how many bacteria will be present after 10 hours?

(OR)

b) Show that $\neg(p \to q) \equiv p \wedge (\neg q)$

45. a) Using vector method, prove that cos (A - B) = cos A Cos B + sin A Sin B

(OR)

b) Prove that among all the rectangles of the given perimeter, the square has the maximum area.

46. a) A semielliptical archway over a one-way road has a height of 3m and a width of 12m. the truck has a width of 3m and a height of 2.7m. Will the truck clear the opening of the archway?

(OR)

b) A random variable x has the following probability mass function.

x	1	2	3	4	5	6
f(x)	k	2k	6k	5k	6k	10k

Find (i) $P(2 < X < 6)$ (ii) $P(2 \leq X < 5)$ (iii) $P(X \leq 4)$ (iv) $P(3 < X)$

47. a) Find the area of the region bounded by $2x - y + 1 = 0$, $y = -1$, $y = 3$ and y-axis.

(OR)

b) Find the non-parametric form of vector equation and cartesian equations of the plane passing through the points (2, 2, 1), (9, 3, 6) and perpendicular to the plane $2x + 6y + 6z = 9$.