PMM

12 - Std

SECOND REVISION TEST - 2024

MATHEMATICS

Time : 3.00 Hrs Marks : 90

PART - I

Note : 1) Answer all the questions. 2) Choose the most appropriate answer from the given four alternatives and write the option code with the corresponding answer. 20 × 1 = 20

1. If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ then adj (adj A) is

   a) $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 6 & -6 & 8 \\ 4 & -6 & 8 \\ 0 & -2 & 2 \end{bmatrix}$ c) $\begin{bmatrix} -3 & 3 & -4 \\ -2 & 3 & -4 \\ 0 & 1 & -1 \end{bmatrix}$ d) $\begin{bmatrix} 3 & -3 & 4 \\ 0 & -1 & 1 \\ 2 & -3 & 4 \end{bmatrix}$

2. If $A^T A^{-1}$ is symmetric, then $A^2 =$ a) A^{-1} b) $(A^T)^2$ c) A^T d) $(A^{-1})^2$

3. If $\frac{z-1}{z+1}$ is purely imaginary, then $|Z|$ is a) $\frac{1}{2}$ b) 1 c) 2 d) 3

4. Identify the incorrect statement

   a) $|z|^2 = 1 \Rightarrow \frac{1}{z} = \bar{z}$ b) $Re(z) \le |z|$ c) $||z_1| - |z_2|| \ge |z_1 + z_2|$ d) $|z^n| = |z|^n$

5. According to the rational theorem which of the following is not possible rational root of $4x^7 + 2x^4 - 10x^3 - 5$.

   a) -1 b) $\frac{5}{4}$ c) $\frac{4}{5}$ d) 5

6. $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right)$ is equal to

   a) $\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$ b) $\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)$ c) $\frac{1}{2}\tan^{-1}\left(\frac{3}{5}\right)$ d) $\tan^{-1}\left(\frac{1}{2}\right)$

7. If the normals of the parabola $y^2 = 4x$ drawn at the end points of its latus rectum are tangents to the circle $(x-3)^2 + (y+2)^2 = r^2$, then the value of r^2 is

   a) 2 b) 3 c) 1 d) 4

8. The equation of directrix of the parabola $(x-h)^2 = -4a(y-k)$ is

   a) $y = k$ b) $y = a$ c) $x = k + a$ d) $y = k + a$

9. If a vector $\vec{\alpha}$ lies in the plane containing $\vec{\beta}$ and $\vec{\gamma}$ then

   a) $[\vec{\alpha}, \vec{\beta}, \vec{\gamma}] = 1$ b) $[\vec{\alpha}, \vec{\beta}, \vec{\gamma}] = -1$ c) $[\vec{\alpha}, \vec{\beta}, \vec{\gamma}] = 0$ d) $[\vec{\alpha}, \vec{\beta}, \vec{\gamma}] = 2$

10. The distance between the planes $x + 2y + 3z + 7 = 0$ and $2x + 4y + 6z + 7 = 0$ is

    a) $\frac{\sqrt{7}}{2\sqrt{2}}$ b) $\frac{7}{2}$ c) $\frac{\sqrt{7}}{2}$ d) $\frac{7}{2\sqrt{2}}$

11. The slope of the line normal to the curve $f(x) = 2\cos 4x$ at $x = \frac{\pi}{12}$ is

a) $-4\sqrt{3}$ b) -4 c) $\frac{\sqrt{3}}{12}$ d) $4\sqrt{3}$

12. The maximum product of two positive numbers, when their sum of the squares is 200 is a) 100 b) $25\sqrt{7}$ c) 28 d) $24\sqrt{14}$

13. Linear approximation for $g(x) = \cos x$ at $x = \frac{\pi}{2}$ is

a) $x + \frac{\pi}{2}$ b) $-x + \frac{\pi}{2}$ c) $x - \frac{\pi}{2}$ d) $-x - \frac{\pi}{2}$

14. The value of $\int_{-1}^{2} |x|\, dx$ is a) $\frac{1}{2}$ b) $\frac{3}{2}$ c) $\frac{5}{2}$ d) $\frac{7}{2}$

15. $\int_{a}^{b} f(a+b-x)\,dx =$ a) f(a) - f(b) b) $\int_{b}^{a} f(x)\,dx$ c) 0 d) $\int_{a}^{b} f(x)\,dx$

16. The general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$ is

a) $xy = k$ b) $y = k \log x$ c) $y = kx$ d) $\log y = kx$

17. The integrating factor of the differential equation $\frac{dy}{dx} + p(x)y = Q(x)$ is $\sin x$, then P(x) a) log sinx b) cos x c) tan x d) cot x

18. A random variable X has binormal distribution with $n = 25$ and $P = 0.8$, then standard deviation of x is

a) 6 b) 4 c) 3 d) 2

19. Determine the truth value of each of the following statements:

a) 4 + 2 = 5 and 6 + 3 = 9
b) 3 + 2 = 5 and 6 + 1 = 7
c) 4 + 5 = 9 and 1 + 2 = 4
d) 3 + 2 = 5 and 4 + 7 = 11

	(a)	(b)	(c)	(d)
a)	F	T	F	T
b)	T	F	T	F
c)	T	T	F	F
d)	F	F	T	T

20. Which of the following is not a binary operation on R a) + b) - c) ÷ d) X

PART - II

Note : 1) Answer any seven questions. 7 X 2 = 14
2) Question number 30 is compulsory.

21. Find the rank of the following matrix by minor method $\begin{bmatrix} 1 & -2 & -1 & 0 \\ 3 & -6 & -3 & 1 \end{bmatrix}$

22. Show that $\left(2+i\sqrt{3}\right)^{10} + \left(2-i\sqrt{3}\right)^{10}$ is purely real.

23. Find the principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$.

24. Determine whether the following three vectors $2\hat{i}+3\hat{j}+\hat{k}$, $\hat{i}-2\hat{j}+2\hat{k}$ and $3\hat{i}+\hat{j}+3\hat{k}$ are coplanar?

25. Evaluate. $\lim_{x \to 0} \left(\frac{\sin mx}{x} \right)$

26. Let $f(x, y) = \frac{y^2 - xy}{\sqrt{x} - \sqrt{y}}$ for (x, y) ≠ (0, 0) show that $\lim_{(x,y)\to(0,0)} f(x, y) = 0$.

27. Show that y = a cos bx is a solution of the differential equation $\frac{d^2y}{dx^2} + b^2y = 0$

28. If X is a random variable with distribution function F(x) given by

$$F(x) = \begin{cases} 0, & x < 0 \\ \frac{1}{2}(x^2 + x), & 0 \le x < 1 \\ 1, & x \ge 1 \end{cases}$$ then find the probability density function.

29. Determine whether * is a binary operation on R given by a * b = $a\sqrt{b}$?

30. If α and β are the roots of the quadratic equation $2x^2 - 7x + 13 = 0$, then find the value of $\alpha^2 + \beta^2 + 3\alpha\beta$.

PART - III

Note : 1) Answer any seven questions.
2) Question number 40 is compulsory.

7 X 3 = 21

31. Verify $(AB)^{-1} = B^{-1} A^{-1}$ with $A = \begin{bmatrix} 0 & -3 \\ 1 & 4 \end{bmatrix}, B = \begin{bmatrix} -2 & -3 \\ 0 & -1 \end{bmatrix}$

32. $w \neq 1$ is a cube root of unity show that $(1 - w + w^2)^6 + (1 + w - w^2)^6 = 128$.

33. Solve : $(1+x^2) \frac{dy}{dx} = 1 + y^2$

34. A circle of area 9π square units has two of its diameters along the lines x + y = 5 and x - y = 1. Find the equation of the circle.

35. Prove that the function f(x) = x – sinx is increasing on the real line. Also discuss for the existence of local extrema.

36. Prove that $q \to p \equiv \neg p \to \neg q$. q → p ≡ ¬p → ¬q

37. Find the constant C such that the function $f(x) = \begin{cases} cx^2, & 1 < x < 4 \\ 0, & otherwise \end{cases}$ is a probability density function, and compute

i) P (1.5 < X < 3.5) ii) $p(x \le 2)$ iii) P(3 < X)

38. If U = log $(x^3 + y^3 + z^3)$, find $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$

39. Evaluate : $\int_{2}^{3} \frac{\sqrt{x}}{\sqrt{5 - x} + \sqrt{x}} dx$

40. Find the torque of the force $3\hat{i} + 2\hat{j} - 4\hat{k}$ about the point (2, -1, 3) acting through the point (1, -1, 2)

PART- IV

Note : Answer all the questions $7 \times 5 = 35$

41. a) Solve the following system of linear equations by Cramer's rule.

$$\frac{3}{x}-\frac{4}{y}-\frac{2}{z}-1=0,\ \frac{1}{x}+\frac{2}{y}+\frac{1}{z}-2=0\ \ \frac{2}{x}-\frac{5}{y}-\frac{4}{z}+1=0$$ ***(OR)***

b) Find the area of the region bounded by $y = \cos x$, $y = \sin x$, the lines $x=\frac{\pi}{4}$ and $x=\frac{5\pi}{4}$.

42. a) The maximum and minimum distances of the earth from the sun respectively are 152×10^6 km and 94.5×10^6 km .The Sun is at one focus of the elliptical orbit. Find the distance from the sun to the other focus. ***(OR)***

b) If the probability that a fluorescent light has a useful life of at least 600 hours is 0.9, find the probabilities that among 12 such lights.
i) exactly 10 will have a useful life of atleast 600 hours. ii) atleast 11 will have a useful life of atleast 600. iii) atleast 2 will not have a useful life of atleast 600 hours.

43. a) Solve: $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$. ***(OR)***

b) Show that the altitudes of a triangle are concurrent by using vectors.

44. a) If $2\cos\alpha = x + \frac{1}{x}$ and $2\cos\beta = y + \frac{1}{y}$ Show that

i) $\frac{x}{y}+\frac{y}{x}=2\cos(\alpha-\beta)$ ii) $x^m y^n + \frac{1}{x^m y^n} = 2\cos(m\alpha+n\beta)$. ***(OR)***

b) Find the vertex, focus directrix, and length of the latus rectum of the parabola $x^2 - 4x - 5y - 1 = 0$.

45. a) Find the parametric form of vector equation and Cartesian equation of the plane passing through the points (2, 2, 1), (9, 3, 6) and perpendicular to the plane $2x + 6y + 6z = 9$. ***(OR)***

b) Find the population of a city at any time t, given that the rate of increase of population is proportional to the population at that instant and that in a period of 40 years the population increased from 3, 00, 000 to 4, 00, 000.

46. a) Find the value of $\tan^{-1}(-1)+\cos^{-1}\left(\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{2}\right)$ ***(OR)***

b) Prove that $f(x, y) = x^3 - 2x^2y + 3xy^2 + y^3$ is homogeneous; what is the degree? Verify Euler's theorem for f.

47. a) Salt is poured from a conveyer belt at a rate of 30 cubic meter per minute forming a conital pile with a circular base whose height and diameter of the base are always equal. How far is the height of the pile increasing when the pile is 10 metre high. ***(OR)***

b) Let A be Q \ {1}. Define * on A by x * y = x + y - xy. Is * binary on A? If so, examine the commutative, associative, the existance of identity the existance of inverse properties satisfied by * on A.