

# COMMON SECOND REVISION EXAMINATION - 2024

## Std - XII

### MATHEMATICS

Time : 3.00 Hours

Marks: 90

#### Part - I

20 x 1 = 20

Choose the correct answer:

- If  $|\text{adj}(\text{adj}A)| = |A|^9$ , then the order of the square matrix A is  
 a) 3       b) 4      c) 2      d) 5
- The area of the triangle formed by the complex numbers  $z$ ,  $iz$  and  $z+iz$  in the Argand's diagram is  
 a)   $\frac{1}{2}|z|^2$       b)  $|z|^2$       c)  $\frac{3}{2}|z|^2$       d)  $2|z|^2$
- The principal argument of  $\frac{3}{-1+i}$  is  
 a)  $-\frac{5\pi}{6}$       b)  $-\frac{2\pi}{3}$        c)  $-\frac{3\pi}{4}$       d)  $-\frac{\pi}{2}$
- The number of positive zeros of the polynomial  $\sum_{j=0}^n nC_j (-1)^j x^j$  is  
 a) 0       b)  $n$       c)  $<n$       d)  $r$
- If  $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$ , the value of  $x^{2017} + y^{2018} + z^{2019} - \frac{9}{x^{101} + y^{101} + z^{101}}$  is  
 a)  0      b) 1      c) 2      d) 3
- If the function  $f(x) = \sin^{-1}(x^2-3)$ , then  $x$  belongs to  
 a)  $[-1, 1]$       b)  $[\sqrt{2}, 2]$        c)  $[-2, \sqrt{2}] \cup [\sqrt{2}, 2]$       d)  $[-2, -\sqrt{2}]$
- The equation of the normal to the circle  $x^2 + y^2 - 2x - 2y + 1 = 0$  which is parallel to the line  $2x + 4y = 3$  is  
 a)  $x + 2y = 3$       b)  $x + 2y + 3 = 0$       c)  $2x + 4y + 3 = 0$       d)  $x - 2y + 3 = 0$
- If the coordinates at one end of a diameter of the circle  $x^2 + y^2 - 8x - 4y + c = 0$  are (11, 2), the coordinates of the other end are  
 a) (-5, 2)       b) (-3, 2)      c) (5, -2)      d) (-2, 5)
- Consider the vectors,  $\vec{a}, \vec{b}, \vec{c}, \vec{d}$  such that  $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = \vec{0}$ . Let  $P_1$  and  $P_2$  be the planes determined by the pairs of vectors  $\vec{a}, \vec{b}$  and  $\vec{c}, \vec{d}$  respectively. Then the angle between  $P_1$  and  $P_2$  is  
 a)  $0^\circ$       b)  $45^\circ$       c)  $60^\circ$       d)  $90^\circ$
- If the length of the perpendicular from the origin to the plane  $2x + 3y + \lambda z = 1, \lambda > 0$  is  $\frac{1}{5}$ , then the value of  $\lambda$  is  
 a)   $2\sqrt{3}$       b)  $3\sqrt{2}$       c) 0      d) 1

11. The volume of a sphere is increasing in volume at the rate of  $3\pi \text{ cm}^3/\text{sec}$ .  
The rate of change of its radius when radius is  $1/2 \text{ cm}$   
a)  $3 \text{ cm/s}$                       b)  $2 \text{ cm/s}$                       c)  $1 \text{ cm/s}$                       d)  $1/2 \text{ cm/s}$
12. The number given by the mean value of theorem for the function  $1/x$ ,  $x \in [1,9]$  is  
a) 2                      b) 2.5                      c) 3                      d) 3.5
13. The percentage error of fifth root of 31 is approximately how many times the percentage error in 31?  
a)  $1/31$                       b)  $1/5$                       c) 5                      d) 31
14. The value of  $\int_{-1}^2 1/x^2 dx$  is  
a)  $-3/2$                       b)  $3/2$                       c)  $5/2$                       d) none of the above
15. If  $\int_0^x f(t) dt = x + \int_x^1 t f(t) dt$ , then the value of  $f(1)$  is  
a)  $1/2$                       b) 2                      c) 1                      d)  $3/4$
16. The integrating factor of the differential equation  $\frac{dy}{dx} + P(x)y = Q(x)$  is  $x$ , then  $P(x)$   
a)  $x$                       b)  $\frac{x^2}{2}$                       c)  $\frac{1}{x}$                       d)  $\frac{1}{x^2}$
17. If the solution of the differential equation  $\frac{dy}{dx} = \frac{ax+3}{2y+f}$  represents a circle, then the value of  $a$  is  
a) 2                      b) -2                      c) 1                      d) -1
18. If the function  $f(x) = \frac{1}{12}$  for  $a < x < b$ , represents a probability density function of a continuous random variable  $X$ , then which of the following cannot be the value of  $a$  and  $b$ ?  
a) 0 and 12                      b) 5 and 17                      c) 7 and 19                      d) 16 and 24
19. Suppose that  $X$  takes on one of the values 0, 1 and 2. If for some constant  $k$ ,  $P(X = i) = kP(X = i - 1)$  for  $i = 1, 2$  and  $P(X = 0) = 1/7$ , then the value of  $k$  is  
a) 1                      b) 2                      c) 3                      d) 4
20. Which one of the following statement has the truth value T?  
a)  $\sin x$  is an even function                      b) Every square matrix is non-singular  
c) the product of complex number and its conjugate is purely imaginary  
d)  $\sqrt{5}$  is an irrational number

Part - II

7 x 2 = 14

Answer any seven Questions. Question No. 30 is compulsory.

21. If  $\text{adj}(A) = \begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$ , find  $A^{-1}$ .

22. Find the square root of  $6-8i$ .

and  $\delta$  are the roots of the polynomial equation  $2x^4 + 5x^3 - 7x^2 + 8 = 0$ , find a cubic equation with integer coefficients whose roots are  $\alpha + \beta + \gamma + \delta$  and  $\alpha\beta\gamma\delta$ .

- 4. Prove that  $\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{3} = \frac{\pi}{4}$
- 5. Find the volume of the parallelepiped whose coterminal edges are represented by the vectors  $-6\hat{i} + 14\hat{j} + 10\hat{k}$ ,  $14\hat{i} - 10\hat{j} - 6\hat{k}$  and  $2\hat{i} + 4\hat{j} - 2\hat{k}$  720
- 6. Prove that the function  $f(x) = x^2 - 2x - 3$  is strictly increasing in  $(2, \infty)$
- 7. Use the linear approximation to find approximate values of  $(123)^{2/3}$  24.73
- 28. Find the differential equation corresponding to the family of curves represented by the equation  $y = Ae^{8x} + Be^{-8x}$ ; where A and B are arbitrary constants.
- 29. Let  $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$  be any two boolean matrices of the same type. Find  $A \vee B$  and  $A \wedge B$ .
- 30. Identify the type of the conic for the following equations.  
 $4x^2 + 4xy + y^2 + 4x + 32y + 16 = 0$  - ellipse  
ex 5.3  
7 x 3 = 21

Part - III

Answer any seven questions. Question No. 40 is compulsory.

- 31. Solve the following systems of linear equations by Cramer's rule.  $5x - 2y + 16 = 0$ ,  $x + 3y - 7 = 0$
- 32. Show that the equation  $z^3 + 2\bar{z} = 0$  has five solutions.
- 33. Determine k and solve the equation  $2x^3 - 6x^2 + 3x + k = 0$ . If one of its roots is twice the sum of the other two roots.
- 34. The equation of the ellipse is  $\frac{(x-11)^2}{484} + \frac{y^2}{64} = 1$ . (x and y are measured in centimeters) where to the nearest centimeter, should the patient's kidney stone be placed so that the reflected sound hits the kidney stone.
- 35. Find the angle between the straight line  $\vec{r} = (2\hat{i} + 3\hat{j} + \hat{k}) + t(\hat{i} - \hat{j} + \hat{k})$  and the plane  $2x - y + z = 5$ .
- 36. Evaluate :  $\lim_{x \rightarrow 1} x^{1-x}$
- 37. Let  $U(x,y,z) = xyz$ ,  $x = e^{-t}$ ,  $y = e^{-t} \cos t$ ,  $z = \sin t$ ,  $t \in \mathbb{R}$  find  $\frac{dU}{dt}$
- 38. Solve  $\frac{dy}{dx} = \frac{x-y+5}{2(x-y)+7}$
- 39. The probability density function of x is given by  $f(x) = \begin{cases} kxe^{-2x} & \text{for } x > 0 \\ 0 & \text{for } x \leq 0 \end{cases}$  find the value of k.

40. Show that  $\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

Part - IV

Answer all the questions.

41. a) Determine the values of  $\lambda$  for which the following system of equations  $x + y + \lambda z = 0$ ,  $4x + 3y + \lambda z = 0$ ,  $2x + y + 2z = 0$  has i) a unique solution ii) a non-trivial solution. (OR)

b) If  $z = x + iy$  is a complex number such that  $\text{Im} \left( \frac{2z+1}{iz+1} \right) = 0$  show that the locus of  $z$  is

$$2x^2 + 2y^2 + x - 2y = 0.$$

42. a) If  $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$ , show that  $x + y + z = xyz$ . (OR)

b) Assume that water issuing from the end of a horizontal pipe, 7.5m above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position 2.5m below the line of the pipe, the flow of water has curved outward 3m beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground.

43. a) Prove by vector method that  $\sin(\alpha+\beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$ . (OR)

b) Find the non-parametric form of vector equation and cartesian equation of the plane

passing through the point (2, 3, 6) and parallel to the straight lines  $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-3}{1}$

$$\text{and } \frac{x+3}{2} = \frac{y-3}{-5} = \frac{z+1}{-3}$$

44. a) A conical water tank with vertex down of 12 metres height has a radius of 5 metres at the top. If water flows into the tank at a rate 10 cubic m/min, how fast is the depth of the water increases when the water is 8 metres deep? (OR)

b) A random variable  $X$  has the following probability mass function.

$x$  1 2 3 4 5 6

$f(x)$   $k$   $2k$   $6k$   $5k$   $6k$   $10k$  Find i)  $P(2 < x < 6)$  ii)  $P(2 \leq x < 5)$  iii)  $P(x \leq 4)$  iv)  $P(3 < x)$

45. a) The time  $T$ , taken for a complete oscillation of a single pendulum with length  $l$ , is

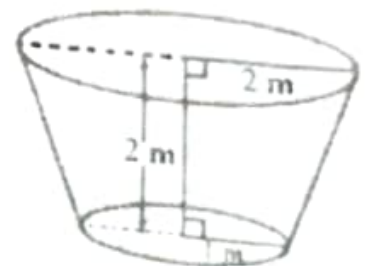
given by the equation  $T = 2\pi \sqrt{\frac{l}{g}}$ , where  $g$  is a constant. Find the approximate percentage error in the calculated value of  $T$  corresponding to an error of 2 percent in the value of  $l$ . (OR)

b) Find the area of the region common to the circle  $x^2 + y^2 = 16$  and the parabola  $y^2 = 6x$ .

46. a) Let  $A$  be  $\mathbb{Q} \setminus \{1\}$ . Define  $*$  on  $A$  by  $x*y = x + y - xy$ , is  $*$  binary on  $A$ ? If so, examine the existence of identity, existence of inverse properties for the operation  $*$  on  $A$ . (OR)

b) For the function  $f(x) = 4x^3 + 3x^2 - 6x + 1$  find the intervals of monotonicity, local extrem, intervals of concavity and points of inflection.

47. a) Find the integration, the volume of the container which is in the shape of a right circular conical frustum. (OR)



b) The engine of a motor boat moving at, 10m/s is shut off.

Given that the retardation at any subsequent time (after shutting off the engine) equal to the velocity at that time. Find the velocity after 2 seconds of

switching off the engine.