***A Valuable material from SS PRITHVI's***

**Class 12**

**2023-24**

PRIT EDUCATION
Practice ! Perform ! Perfect !

PRITEDUCATION
PRACTISE! PERFORM! PERFECT!

# PTA 2023 ANSWER KEYS

**SUBJECT:**

# M A T H

***MR. SS PRITHVI***

*CLICK TO JOIN US TO GET FREE MATERIALS*

www.Padasalai.Net

SS PRITHVI, PRIT~EDUCATION

Part-I

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

MODEL QUESTION PAPER-I

Std: XII

MATHEMATICS

One mark :-

1, If A and B are orthogonal, then $(AB)^T(AB)$ is

(c) I

2, If A is a non-singular matrix, such that $A^{-1}=\begin{bmatrix}5 & 3\\ -2 & -1\end{bmatrix}$, then $(A^T)^{-1}$ is

(d) $\begin{bmatrix}5 & -2\\ 3 & -1\end{bmatrix}$

3, $i^n + i^{n+1} + i^{n+2} + i^{n+3}$ is

(a) 0

4, The product of all four values of $\left[\cos\pi/3 + i\sin\pi/3\right]^{3/4}$ is

c) 1

5, The polynomial equation in x of degree n always has

c) n imaginary roots

6, If $\sin^{-1}x + \sin^{-1}y = 2\pi/3$, then $\cos^{-1}x + \cos^{-1}y$ is equal to

b) $\pi/3$

7, The equation of directrix of the parabola $y^2 = x+4$ is

c) $-17/4$

8, The circle $x^2+y^2 = 4x+8y+5$ intersects the line $3x-4y=m$ at two distinct points if

d) $-35 < m < 15$

9, If the line $\frac{x-1}{3} = \frac{y-2}{4} = \frac{z-3}{\lambda}$ is $\perp$r to the plane, $\vec{r}\cdot(2\hat{i}+3\hat{j}+4\hat{k}) = 0$, then the value of $\lambda$ is

$\lambda = -9/2$

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

10 Distance from the origin to the plane $3x-6y+2z+7=0$ is

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

b) 1

11) The curve $y = ax^4 + bx^2$ with $ab > 0$

d) has no point of inflection

12 The function $f(x) = \sqrt[3]{4-x^2}$ has a vertical tangent at

b) $x = 2$ and $x = -2$

13) If we measure the side of a cube to be 4 cm with an error of 0.1 cm, then error in our calculation of the volume is

d) 4.8 cu. cm

14) $\int_0^{\infty} e^{-3x} x^2 dx$

d) $2/27$

15) The value of $\int_0^{\pi} (\sin x + \cos x)\, dx$

b) 2

16) P is the amount of certain substance left in after time t. If the rate of evaporation of substance is proportional to the amount remaining, then [ where $k > 0$ ]

b) $P = ce^{-kt}$

17) The solution of differential equation $\frac{dy}{dx} = y/x + \frac{\phi(y/x)}{\phi'(y/x)}$ is

b) $\phi(y/x) = kx$

18) A rod of length $2l$ is broken into 2 pieces at random. The probability density function of shorter of 2 pieces is

$$f(x) = \begin{cases} 1/l & 0 < x < l \\ 0 & l \le x < 2l \end{cases}$$

The mean and variance of shorter of 2 pieces are respectively

d) $l/2, \frac{l^2}{12}$

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

19) If x is a binomial random variable with expected values 6 and variance 2.4. Then $P(X=5)$ is

d) $\binom{10}{5}\left[\frac{3}{5}\right]^5\left[\frac{2}{5}\right]^5$

20) the operation * is defined by $a*b=\frac{ab}{7}$. It is not a binary operation on

b) Z.

---

## part - II

21) prove that $\begin{bmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{bmatrix}$ is orthogonal.

Let $A = \begin{bmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{bmatrix}$

$AA^T = A^TA = I \Rightarrow$ orthogonal

$A^T = \begin{bmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{bmatrix}$

$$AA^T = \begin{bmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{bmatrix}$$

$$= \begin{bmatrix}\cos^2\theta+\sin^2\theta & \sin\theta\cos\theta-\sin\theta\cos\theta \\ \sin\theta\cos\theta-\sin\theta\cos\theta & \sin^2\theta+\cos^2\theta\end{bmatrix}$$

$$= \begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix}$$

$AA^T = I \quad \rightarrow (1)$

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

www.Padasalai.Net www.Trb Tnpsc.com of 2.

$$A^TA = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta + \sin^2\theta & -\sin\theta\cos\theta + \sin\theta\cos\theta \\ -\sin\theta\cos\theta + \cos\theta\sin\theta & \sin^2\theta + \cos^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^TA = I \quad \rightarrow (2)$$

From (1) and (2) we get,

$$AA^T = A^TA = I$$

$\therefore$ A is orthogonal.

22) Find the modulus of $\frac{1-i}{3+i} + \frac{4i}{5}$.

$$\left|\frac{1-i}{3+i} + \frac{4i}{5}\right| = \left|\frac{(1-i)5 + (4i)(3+i)}{(3+i)5}\right|$$

$$= \left|\frac{5-5i+12i-4}{15+5i}\right| \qquad i^2 = -1$$

$$= \left|\frac{1+7i}{15+5i}\right|$$

$$= \frac{\sqrt{1^2+7^2}}{\sqrt{15^2+5^2}}$$

$$= \frac{\sqrt{50}}{\sqrt{225+25}} = \frac{\sqrt{50}}{\sqrt{250}} = \frac{\sqrt{50}}{\sqrt{50\times 5}}$$

$$= \frac{\sqrt{50}}{\sqrt{50}\times\sqrt{5}}$$

$$= \frac{1}{\sqrt{5}}$$

PRIT EDUCATION Practice ! Perform ! Perfect ! www.Padasalai.Net

23) Find the value of $\tan^{-1}[\tan(-\pi/6)]$

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

$= \tan^{-1}[\tan(-\pi/6)] = -\pi/6$

$-\pi/6 \in [-\pi/2, \pi/2]$

24) Find the centre and radius of the circle.

$3x^2+3y^2-12x+6y-9=0 \Rightarrow x^2+y^2-4x+2y-3=0$

compare the equation with general form

$x^2+y^2+2gx+2fy+c=0$

$2g = -4$, $2f = 2$, $c = -3$

$g = -2$ $f = 1$

centre $(-g, -f) = (2, -1)$

radius $\Rightarrow \sqrt{g^2+f^2-c}$

$= \sqrt{(2)^2+(1)^2-(-3)} = \sqrt{4+1+3}$

$= \sqrt{36+9} = \sqrt{8}$

$= \sqrt{54}$ $= 2\sqrt{2}$

25) Verify whether the line $\frac{x-3}{-4} = \frac{y-4}{-7} = \frac{z+3}{12}$ lies in the plane $5x-y+z=8$.

$\frac{x-3}{-4} = \frac{y-4}{-7} = \frac{z+3}{12}$, $5x-y+z=8$

$(x_1, y_1, z_1) = (3, 4, -3)$

$\Rightarrow 5x-y+z$

$5(3)-4-3$

$15-7$

$= 8$

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

$(x_1, y_1, z_1) = (3, 4, -3)$

Satisfies the given plane $5x - y + z = 8$

www.Padasalai.Net. www.Trb-Tnpsc.com of 2.

$(a, b, c) = (-4, -7, 12)$

$(A, B, C) = (5, -1, 1)$

$aA + bB + cC = -20 + 7 + 12$

$= -1 \neq 0$

so, the normal to the plane is not $\perp r$ to the line. Hence, the given line does not lie in plane.

---

26. Evaluate $\int_{-\log 2}^{\log 2} e^{-|x|}\, dx$

Let $f(x) = e^{-|x|}$

Then, $f(-x) = e^{-|-x|}$

$= e^{-|x|}$

$= f(x)$

so, $f(x)$ is an even function

$$\int_{-\log 2}^{\log 2} e^{-|x|}\, dx = 2\int_0^{\log 2} e^{-|x|}\, dx$$

$$= 2\int_0^{\log 2} e^{-x}\, dx$$

$$= 2\left(-e^{-x}\right)_0^{\log 2}$$

$$= 2\left(-e^{-\log 2} + e^0\right)$$

$$= 2\left(-e^{\log 1/2} + 1\right)$$

$$= 2\left(-1/2 + 1\right)$$

$$= 1$$

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

27. Determine the order and degree [if exists] of differential equation

Padasalai.Net. www.Trb Tnpsc.com of 2.

$$\left[\frac{d^2y}{dx^2}\right]^2 + \left[\frac{dy}{dx}\right]^2 = x \sin\left[\frac{d^2y}{dx^2}\right]$$

The given differential equation, the highest order derivative appears is $\frac{d^2y}{dx^2}$

order => 2

degree => not defined

Because, the equation can't written in polynomial form.

28. Suppose a discrete random variable can only take values 0, 1 and 2. The probability mass function is defined by

$$f(x) = \begin{cases} \frac{x^2+1}{k} & \text{for } x = 0, 1, 2 \\ 0 & \text{otherwise} \end{cases}$$

Find value of $k$

SOL:

$$\sum_{x=-\infty}^{x=\infty} f(x) = 1$$

$$\sum_{x=-\infty}^{x=\infty} f(x) = f(0) + f(1) + f(2)$$

$$= \frac{1}{k} + \frac{2}{k} + \frac{5}{k}$$

$$\sum_{x=-\infty}^{x=\infty} f(x) = \frac{8}{k}$$

$$\frac{8}{k} = 1$$

$$\boxed{k = 8}$$

$f(0) = \frac{0^2+1}{k} = \frac{1}{k}$

$f(1) = \frac{1^2+1}{k} = \frac{2}{k}$

$f(2) = \frac{2^2+1}{k} = \frac{5}{k}$

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

29, verify the associative property under the binary operation * defined by $a*b = a^b, \forall a, b \in N$

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

Associative $\Rightarrow (a*b)*c = a*(b*c)$

Take $a=2, b=3, c=4$

L.H.S : $(a*b)*c$

$= (a^b)*c$

$= a^{bc}$

$= 2^{34}$

$= 2^{81}$

R.H.S : $a*(b*c)$

$= a*(b^c)$

$= a^{bc}$

$= a^{bc}$

$= 2^{(3)(4)}$

$= 2^{12}$

L.H.S $\neq$ R.H.S

so, it is not associative property on N

30, Evaluate $\lim_{x \to 0} \frac{xe^x - \sin x}{x}$

$\lim_{x \to 0} \frac{xe^x - \sin x}{x}$ ($\frac{0}{0}$ Form)

Apply 'l' Hopital rule.

$\lim_{x \to 0} \frac{xe^x - \sin x}{x} = \lim_{x \to 0} \frac{(xe^x + e^x) - \cos x}{1}$

$= \frac{(0e^0 + e^0) - \cos 0}{1}$

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

$= \frac{1-1}{1}$

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

$= 0$

3 mark :-

31, Find the rank of the matrix by row reduction method :

Let $A = \begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & 3 & 4 \\ 5 & -1 & 7 & 11 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & -3 & 1 & -2 \\ 0 & -6 & 2 & -4 \end{bmatrix}$ $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 5R_1$

$\sim \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & -3 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $R_3 \to R_3 - 2R_2$

$P(A) = 2$

32, Show that the equation $z^3 + 2\bar{z} = 0$ has five solution

$|z|^3 = |-2||\bar{z}| \to (1)$ $\boxed{z^3 = -2\bar{z}}$

$|z|^3 - 2|z| = 0$

$|z|[|z|^2 - 2] = 0$

$|z| = 0 \mid |z|^2 - 2 = 0$

$|z|^2 = 2$ $|z|^2 = z\bar{z}$

$|z|^2 = 2$

$z\bar{z} = 2$

$\bar{z} = \frac{2}{z} \Rightarrow$ sub in (1)

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

$z^3 = -2 - \frac{2}{z}$

$z^4 = -4$

$z^4 + 4 = 0$

It has 4 solution

$\therefore$ total number of solution is 5

Hence showed

33. Solve the equation $x^3 - 5x^2 - 4x + 20 = 0$.

$x^3 - 5x^2 - 4x + 20 = 0$

$p(x) = x^3 - 5x^2 - 4x + 20$

$p(2) = 2^3 - 5(2)^2 - 4(2) + 20$

$= 8 - 20 - 8 + 20$

$= 0$

$(x-2)$ is a factor of $p(x)$

| | | | | |
|---|---|---|---|---|
| 2 | 1 | -5 | -4 | 20 |
| | 0 | 2 | -6 | -20 |
| | 1 | -3 | -10 | 0 |

$x^2 - 3x - 10 = 0$

$(x-5)(x+2) = 0$

$x = 5, \; x = -2$

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

34. If the equation $3x^2+(3-p)xy+qy^2-2px=8pq$ represent a circle, find p and q. Also determine the centre and radius of the circle.

www.Padasalai.Net www.Trb Tnpsc.com of 2.

co-efficient of $xy = 0$

$3-p=0$

$\boxed{p=3}$

co-efficient of $x^2$ = co-efficient of $y^2$

$\boxed{q=3}$

The equation is $3x^2+3y^2-6x-72=0$

$\div 3 \Rightarrow x^2+y^2-2x-24=0$

$(x-1)^2+(y+0)^2=5^2$

centre $\Rightarrow (1,0)$

radius $\Rightarrow 5$

35. Find the points on curve $y=x^3-6x^2+x+3$ where the normal is parallel to the line $x+y=17$.

PRIT EDUCATION Practice ! Perform ! Perfect !

The slope of normal $\frac{-1}{m}=-1$

$\therefore$ slope of tangent, $m=1=\frac{dy}{dx}$

$y=x^3-6x^2+x+3 \Rightarrow \frac{dy}{dx}=3x^2-12x+1=1$

$3x^2-12x=0$

$x=0, \quad x=4$

$x=0 \Rightarrow y=3$

$x=4 \Rightarrow y=4^3-6(4)^2+4+3$

$=-25$

$\therefore$ points are $(0,3)$ and $(4,-25)$

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

36. If $u(x,y) = \frac{x^2+y^2}{\sqrt{x+y}}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{3}{2}u$

www.Padasalai.Net www.Trb Tnpsc.com of 2.

$$u(x,y) = \frac{x^2+y^2}{\sqrt{x+y}}$$

$$u(\lambda x, \lambda y) = \frac{\lambda^2(x^2+y^2)}{\sqrt{\lambda}(\sqrt{x+y})}$$

$$= \lambda^{3/2}\, u(x,y)$$

$u$ is homogenous function of degree $3/2$.

$\therefore$ By Eulers thm,

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3/u\, u$$

37. Evaluate $\int_0^1 \frac{2x}{1+x^2}\,dx$

Let $I = \int_0^1 \frac{2x}{1+x^2}\,dx$

$= [\log(1+x^2)]_0^1$

$= \log 2 - \log 1$

$$\boxed{I = \log 2}$$

PRIT EDUCATION
Practice Platform ! Perfect !
www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

38. Solve : $\cos x \cos y \, dy - \sin x \sin y \, dx = 0$

$\cos x \cos y \, dy - \sin x \sin y \, dx = 0$

$\cos x \cos y \, dy = \sin x \sin y \, dx$

$$\frac{\cos y \, dy}{\sin y} = \frac{\sin x \, dx}{\cos x}$$

$$\int \cot y \, dy = \int \tan x \, dx$$

$$\log|\sin y| = \log|\sec x| + \log c$$

$$\log|\sin y| = \log|c \sec x|$$

$$\boxed{\sin y = c \sec x}$$

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

www.Padasalai.Net www.TRB Tnpsc.com of 2.

39. If $\mu$ and $\sigma^2$ are mean and variance of discrete random variable $X$, and $E(X+3)=10$ and $E(X+3)^2=116$, Find $\mu$ and $\sigma^2$.

$E(X+3)=10$

$E(X+3)^2=116$

$E(X)+3=10$

mean, $\mu \Rightarrow E(X)=7$

$E(X+3)^2=116$

$E(X^2+6X+9)=116$

$E(X^2)+E(6X)+E(9)=116$

$E(X^2)+6E(X)+9=116$

$E(X^2)+6(7)+9=116$

$E(X^2)=116-51$

$=65$

Variance, $\sigma^2 = E(X^2)-[E(X)]^2$

$=65-7^2$

$=65-49$

$=16$

40. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors, prove that

$[\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}] = 2[\vec{a}, \vec{b}, \vec{c}]$

$[\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}] = [\vec{a}\cdot\vec{b}+\vec{c}\ \ \vec{c}+\vec{a}] + [\vec{b}\ \ \vec{b}+\vec{c}\ \ \vec{c}+\vec{a}]$

$= [\vec{a}\ \ \vec{b}\ \ \vec{c}+\vec{a}] + [\vec{a}\ \ \vec{c}\ \ \vec{c}+\vec{a}] + [\vec{b}\ \ \vec{b}\ \ \vec{c}+\vec{a}]$

$+ [\vec{b}\ \ \vec{c}\ \ \vec{c}+\vec{a}]$

$= [\vec{a}\ \ \vec{b}\ \ \vec{c}] + [\vec{a}\ \ \vec{b}\ \ \vec{a}] + [\vec{a}\ \ \vec{c}\ \ \vec{c}] + [\vec{a}\ \ \vec{c}\ \ \vec{a}]$

$+ [\vec{b}\ \ \vec{b}\ \ \vec{c}] + [\vec{b}\ \ \vec{b}\ \ \vec{a}] + [\vec{b}\ \ \vec{c}\ \ \vec{c}] + [\vec{b}\ \ \vec{c}\ \ \vec{a}]$

$= [\vec{a}\ \ \vec{b}\ \ \vec{c}] + 0 + 0 + 0 + 0 + 0 + 0 + [\vec{b}\ \ \vec{c}\ \ \vec{a}]$

$= 2[\vec{a}\ \ \vec{b}\ \ \vec{c}]$

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

5 mark :- www.Padasalai.Net. www.Trb Tnpsc.com of 2.

41 a) Examine the consistency of system of equations $4x+3y+6z=25$, $x+5y+7z=13$, $2x+9y+z=1$. If it is consistent then solve.

$$4x+3y+6z=25$$

$$x+5y+7z=13$$

$$2x+9y+z=1$$

$$[A|B]=\begin{bmatrix}4&3&6&25\\1&5&7&13\\2&9&1&1\end{bmatrix}$$

$$\sim\begin{bmatrix}1&5&7&13\\4&3&6&25\\2&9&1&1\end{bmatrix} \quad R_1 \leftrightarrow R_2$$

$$\sim\begin{bmatrix}1&5&7&13\\0&-17&-22&-27\\0&-1&-13&-25\end{bmatrix} \quad R_2 \to R_2-4R_1,\ R_3 \to R_3-2R_1$$

$$\sim\begin{bmatrix}1&5&7&13\\0&-17&-22&-27\\0&0&-199&-398\end{bmatrix} \quad R_3 \to 17R_3-R_2$$

$\ell[A]=\ell[A|B]=3=$ No. of unknown

The system has a unique solution.

$$x+5y+7z=13$$

$$-17y-22z=-27$$

$$-199z=-398$$

$$z=\frac{-398}{-199}$$

$$z=2$$

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

$z = 2 \Rightarrow$

$$-17y - 22z = -27$$

$$-17y - 44 = -27$$

$$-17y = 44 - 27$$

$$-17y = 17$$

$$\boxed{y = -1}$$

$y = -1, z = 2 \Rightarrow$

$$x + 5y + 7z = 13$$

$$x + 5(-1) + 7(2) = 13$$

$$x - 5 + 14 = 13$$

$$x = 13 - 9$$

$$\boxed{x = 4}$$

$\therefore (x, y, z) = (4, -1, 2)$

PRIT EDUCATION
Practice | Perform | Perfect |
www.Padasalai.Net

SS PRITHVI, PRIT~EDUCATION

42)

b) If $z=x+iy$ and $\arg\left[\frac{z-i}{z+2}\right]=\frac{\pi}{4}$ then show that $x^2+y^2+3x-3y+2=0$

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

$$\frac{z-i}{z+2}=\frac{x+iy-i}{x+iy+2}$$

$$=\frac{x+i(y-1)}{(x+2)+iy}$$

$$=\frac{x+i(y-1)}{(x+2)+iy}\times\frac{(x+2)-iy}{(x+2)-iy}$$

$$=\frac{[x(x+2)+y(y-1)]+i[-xy+(x+2)(y-1)]}{(x+2)^2+y^2}$$

$$=\frac{(x^2+2x+y^2-y)+i(-xy+xy-x+2y-2)}{(x+2)^2+y^2}$$

$$=\frac{x^2+2x+y^2-y}{(x+2)^2+y^2}+i\frac{(-x+2y-2)}{(x+2)^2+y^2}$$

$$\arg\left[\frac{z-i}{z+2}\right]=\pi/4$$

$$\tan^{-1}\left(\frac{\frac{-x+2y-2}{(x+2)^2+y^2}}{\frac{x^2+2x+y^2-y}{(x+2)^2+y^2}}\right)=\pi/4$$

$$\tan^{-1}\left(\frac{-x+2y-2}{x^2+2x+y^2-y}\right)=\pi/4$$

$$\frac{-x+2y-2}{x^2+2x+y^2-y}=\tan\pi/4$$

$$\frac{-x+2y-2}{x^2+2x+y^2-y}=1$$

$$-x+2y-2=x^2+2x+y^2-y$$

$$x^2+y^2+3x-3y+2=0$$

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

42. a) solve the equation: $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$

www.Padasalai.Net www.Trb Tnpsc.com of 2.

$\div x^2$

$$\frac{6x^4}{x^2} - \frac{35x^3}{x^2} + \frac{62x^2}{x^2} - \frac{35x}{x^2} + \frac{6}{x^2} = 0$$

$$6x^2 - 35x + 62 - \frac{35}{x} + \frac{6}{x^2} = 0$$

$$6\left(x^2 + \frac{1}{x^2}\right) - 35\left(x + \frac{1}{x}\right) + 62 = 0$$

$$6(y^2 - 2) - 35(y) + 62 = 0$$

$$6y^2 - 12 - 35y + 62 = 0$$

$$6y^2 - 35y + 50 = 0$$

$$(2y - 5)(3y - 10) = 0$$

$$y = 5/2, \quad y = 10/3$$

$x + 1/x = y$

$\frac{x^2+1}{x} = 5/2$

$2x^2 + 2 = 5x$

$2x^2 - 5x + 2 = 0$

$(2x - 1)(x - 2) = 0$

$x = 1/2, \quad x = 2$

$x + 1/x = y$

$\frac{x^2+1}{x} = 10/3$

$3x^2 + 3 = 10x$

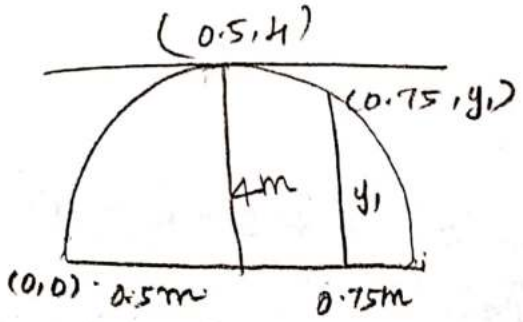
$3x^2 - 10x + 3 = 0$

$(3x - 1)(x - 3) = 0$

$x = 1/3, \quad x = 3$

---

42) b) At a water fountain, water attains a maximum height of 4m horizontal distance of 0.5 m from its origin. The flow is from the origin and the path of water is a parabola open upwards. The height of water at a horizontal distance of 0.75 m from the point of origin.




kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

Let the water fountain attains maximum height 4 m. when it is 0.5 m away from origin.

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

Let parabola open downward.

$(x-h)^2 = -4a(y-k)$

$(x-0.5)^2 = -4a(y-4) \rightarrow (1)$

(0, 0) lies on parabola.

$(-1/2)^2 = -4a(-4)$

$1/4 = 16a$

$a = 1/64$

sub a in eq (1)

$(x-0.5)^2 = -4(1/64)(y-4)$

$(0.75, y_1)$ lies on parabola,

$(0.75-0.5)^2 = \frac{-4}{64}(y_1-4)$

$(1/4)^2 = \frac{-1}{16}(y_1-4)$

$\frac{1}{16} = \frac{-1}{16}(y_1-4)$

$-1 = y_1 - 4$

$y_1 = 4-1$

$y_1 = 3$ m

---

43. a) If $\vec{a} = \hat{i}-\hat{j}$, $\vec{b} = \hat{i}-\hat{j}-4\hat{k}$, $\vec{c} = 3\hat{j}-\hat{k}$ and $\vec{d} = 2\hat{i}+5\hat{j}+\hat{k}$, verify that $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}) = [\vec{a}\ \vec{b}\ \vec{d}]\vec{c} - [\vec{a}\ \vec{b}\ \vec{c}]\vec{d}$.

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 0 \\ 1 & -1 & -4 \end{vmatrix}$$

$= \hat{i}(4-0) - \hat{j}(-4-0) + \hat{k}(-1+1)$

$= 4\hat{i} + 4\hat{j}$

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

$$\vec{c}\times\vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 3 & -1 \\ 2 & 5 & 1 \end{vmatrix}$$

$$= \hat{i}(3+5) - \hat{j}(0+2) + \hat{k}(0-6)$$

$$= 8\hat{i} - 2\hat{j} - 6\hat{k}$$

$$(\hat{a}\times\hat{b})\times(\vec{c}\times\vec{d}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 4 & 0 \\ 8 & -2 & -6 \end{vmatrix}$$

$$= \hat{i}(-24+0) - \hat{j}(-24+0) + \hat{k}(-8-32)$$

$$= -24\hat{i} + 24\hat{j} - 40\hat{k} \rightarrow (1)$$

$$[\vec{a}\ \vec{b}\ \vec{d}]\vec{c} - [\vec{a}\ \vec{b}\ \vec{c}]\vec{d} = 28(3\hat{j}-\hat{k}) - 12(2\hat{i}+5\hat{j}+\hat{k})$$

$$= -24\hat{i} + 24\hat{j} - 40\hat{k} \rightarrow (2)$$

From (1), (2), we get

$$(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}) = [\vec{a}\ \vec{b}\ \vec{d}]\vec{c} - [\vec{a}\ \vec{b}\ \vec{c}]\vec{d}$$

Hence verified.

---

43) b, Find vector and cartesian equations of the plane $\vec{r} = (6\hat{i}-\hat{j}+\hat{k}) + s(-\hat{i}+2\hat{j}+\hat{k}) + t(-5\hat{i}-4\hat{j}-5\hat{k})$.

$\vec{a} = 6\hat{i}-\hat{j}+\hat{k}$, $\vec{b} = -\hat{i}+2\hat{j}+\hat{k}$, $\vec{c} = -5\hat{i}-4\hat{j}-5\hat{k}$

Non-parametric form:

$$(\vec{r}-\vec{a})\cdot(\vec{b}\times\vec{c}) = 0$$

$$(\vec{r}\cdot(\vec{b}\times\vec{c}) - \vec{a}\cdot(\vec{b}\times\vec{c})) = 0$$

$$\vec{b}\times\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 1 \\ -5 & -4 & -5 \end{vmatrix}$$

$$= \hat{i}(-10+4) - \hat{j}(5+5) + \hat{k}(4+10)$$

$$\hat{b}\times\vec{c} = -6\hat{i} - 10\hat{j} + 14\hat{k}$$

$\div -2$

$$\hat{b}\times\vec{c} = 3\hat{i} + 5\hat{j} - 7\hat{k}$$

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

$\vec{r}\,(3\hat{i}+5\hat{j}-7\hat{k})\cdot(\hat{i}-\hat{j}+\hat{k})\cdot(3\hat{i}+5\hat{j}-7\hat{k})=0$

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

$\vec{r}\,(3\hat{i}+5\hat{j}-7\hat{k})-(18-5-7)=0$

$\vec{r}\,(3\hat{i}+5\hat{j}-7\hat{k})=6$

Cartesian form:

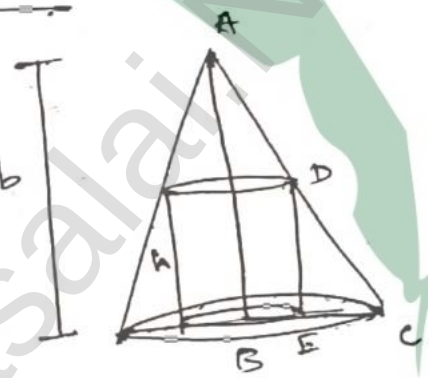
Let $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$

$(x\hat{i}+y\hat{j}+z\hat{k})\cdot(3\hat{i}+5\hat{j}-7\hat{k})=6$

$3x+5y-7z-6=0$

---

44 a) A hollow cone with base radius a cm and height b cm is placed on table. Show that the volume of largest cylinder that can be hidden underneath is $4/9$ times volume of cone.




Given: AB = b, BC = a, BE = r

In $\triangle ABC$ and DEC are similar.

$$\frac{AB}{BC}=\frac{DE}{EC}$$

$$\frac{b}{a}=\frac{h}{a-r}$$

$$h=(a-r)\frac{b}{a}$$

$$V=\pi r^2 h$$

$$=\pi r^2(a-r)\frac{b}{a}$$

$$V=\frac{\pi b}{a}(ar^2-r^3)$$

$$V'=\frac{\pi b}{a}(2ar-3r^2)$$

$$V''=\frac{\pi b}{a}(2a-6r)$$

$V'=0$

$2ar-3r^2=0$

$r(2a-3r)=0$

$r=0 \quad | \quad r=\frac{2a}{3}$

$r=\frac{2a}{3} \Rightarrow V''<0$

The volume is maximum

$$V=\frac{\pi b}{a}\left(\frac{4a^2}{3}\right)\left(a-\frac{2a}{3}\right)$$

$$=\frac{4}{9}\left(\frac{1}{3}\pi a^2 b\right)$$

$V=\frac{4}{9}$ (volume of cone)

PRIT EDUCATION
Practice ! Perform ! Perfect
www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

b) Expand $\log(1+x)$ as maclaurin series upto 4 non-zero terms for $-1 < x \leq 1$

www.Padasalai.Net www.Trb Tnpsc.com of 2.

$f(x) = \log(1+x)$ $\quad f(0) = \log(1+0) = 0$

$f'(x) = \frac{1}{1+x}$ $\quad f'(0) = \frac{1}{1+0} = 1$

$f''(x) = \frac{-1}{(1+x)^2}$ $\quad f''(0) = \frac{-1}{(1+0)^2} = -1$

$f'''(x) = \frac{2}{(1+x)^3}$ $\quad f'''(0) = \frac{2}{(1+0)^3} = 2$

$f^{IV}(x) = \frac{-6}{(1+x)^4}$ $\quad f^{IV}(0) = \frac{-6}{(1+0)^4} = -6$

maclaurin series Expansion,

$$f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots$$

$$= 0 + \frac{1}{1!}x + \frac{-1}{2!}x^2 + \frac{2}{3!}x^3$$

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$$

---

a) A multiple choice examination has 10 questions, each question has four distractors with exactly 1 correct answer. Suppose a student answers by guessing and if X denotes the number of correct answer, find i) binomial distribution ii) probability that the student will get 7 correct answer iii) atleast one correct answer.

PRIT EDUCATION Practice Perform Perfect

i) X denotes the number of sucess. X be 0, 1, 2, ... 10.

the probability,

$p = 1/4$ and $q = 1 - p = 1 - 1/4 = 3/4$, $n = 10$

$p$ ⇓ Sucess $\quad$ $q$ ⇓ Failure

X follows the binomial distribution $X \sim B(10, 1/4)$

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

$$f(x) = \begin{bmatrix}10\\x\end{bmatrix}\left[\frac{1}{4}\right]^x\left[\frac{3}{4}\right]^{10-x} \quad x = 0, 1, 2, \ldots 10$$

ii) probability for 7 correct answers is

$$P(X=7) = f(7) = \begin{bmatrix}10\\7\end{bmatrix}\left[\frac{1}{4}\right]^7\left[\frac{3}{4}\right]^{10-7}$$

$$= 120\left[\frac{3^3}{4^{10}}\right]$$

iii) probability for atleast 1 correct answer is

$$P(X \geq 1) = 1 - P(X < 1)$$

$$= 1 - P(X=0)$$

$$= 1 - \begin{bmatrix}10\\0\end{bmatrix}\left[\frac{1}{4}\right]^0\left[\frac{3}{4}\right]^{10}$$

$$= 1 - \left[\frac{3}{4}\right]^{10}$$

4b

b) Show that $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

By equivalence property,

$p \leftrightarrow q \equiv (\neg p \vee q) \wedge (\neg q \vee p)$

$\equiv (\neg p \vee q) \wedge (p \vee \neg q)$ by commutative law

$\equiv (\neg p \wedge (p \vee \neg q)) \vee (q \wedge (p \vee \neg q))$ (by distributive law)

$\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q) \vee (q \wedge p) \vee (q \wedge \neg q)$ (by distributive law)

$\equiv F \vee (\neg p \wedge \neg q) \vee (q \wedge p) \vee F$ : (by complement law)

$\equiv (\neg p \wedge \neg q) \vee (q \wedge p)$ : by identity law

$\equiv (p \wedge q) \vee (\neg p \wedge \neg q)$ ; by commutative law.

Finally

$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

PRIT EDUCATION Practice ! Perform ! Perfect !
www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

45 a) If $u = xyz$, $x = e^{-t}$, $y = e^{-t}\sin t$, $z = \sin t$, find $\frac{du}{dt}$.

$u = xyz$

$\frac{\partial u}{\partial x} = yz$ ; $\frac{\partial u}{\partial y} = xz$ ; $\frac{\partial u}{\partial z} = xy$.

$\frac{dx}{dt} = -e^{-t}$ ; $\frac{dy}{dt} = e^{-t}[\cos t] + \sin t(-e^{-t})$

$\frac{dz}{dt} = \cos t$

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} + \frac{\partial u}{\partial z}\frac{dz}{dt}$$

$= yz(-e^{-t}) + xz[e^{-t}(-\sin t + \cos t)]$

$+ xy(\cos t)$

$= +e^{-t}\sin t(\sin t)(-e^{-t}) + e^{-t}\sin t[e^{-t}(\sin t + \cos t)]$

$+ e^{-t}(e^{-t}\sin t)(\cos t)$

$= e^{-2t}[-\sin^2 t + \sin^2 t + \sin t\cos t + \sin t\cos t]$

$= e^{-2t}[2\sin t\cos t - 2\sin^2 t]$

$$\boxed{\frac{du}{dt} = e^{-2t}[\sin 2t - 2\sin^2 t]}$$

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

Ans)

b, Solve: $(y-e^{\sin^{-1}x})\frac{dx}{dy}+\sqrt{1-x^2}=0$

www.Padasalai.Net www.Trb Tnpsc.com of 2.

$$(y-e^{\sin^{-1}x})\frac{dx}{dy}=-\sqrt{1-x^2}$$

$$\frac{y-e^{\sin^{-1}x}}{\sqrt{1-x^2}}=-\frac{dy}{dx}$$

$$\frac{dy}{dx}=\frac{-y}{\sqrt{1-x^2}}+\frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}}$$

$$\frac{dy}{dx}+\frac{y}{\sqrt{1-x^2}}=\frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}}$$

This is linear differential equation in $y$ ds of the form.

$$\frac{dy}{dx}+Py=Q$$

Here $P=\frac{1}{\sqrt{1-x^2}}$ $Q=\frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}}$

$$I.F \Rightarrow e^{\int P dx}=e^{\int \frac{1}{\sqrt{1-x^2}}dx}$$

$$=e^{\sin^{-1}x}$$

The solution is,

$$y\,e^{\int P dx}=\int Q\,e^{\int P dx}dx+C$$

$$=\int \frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}}\cdot e^{\sin^{-1}x}dx+C$$

Let $\sin^{-1}x=t$

Diff $\frac{1}{\sqrt{1-x^2}}dx=dt$

$$y\,e^{\sin^{-1}x}=\int e^t e^t dt+C$$

$$=\int e^{2t}dt+C$$

$$=\frac{e^{2t}}{2}+C$$

$$y\,e^{\sin^{-1}x}=\frac{e^{2(\sin^{-1}x)}}{2}+C$$

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

47, a, Find area enclosed by the curve $y = -x^2$ and st. line $x + y + 2 = 0$.

$y = -x^2$

| $x$ | 0 | 1 | -1 | 2 | -2 |
|---|---|---|---|---|---|
| $y$ | 0 | -1 | -1 | -4 | -4 |

$x + y + 2 = 0$

$y = -x - 2$

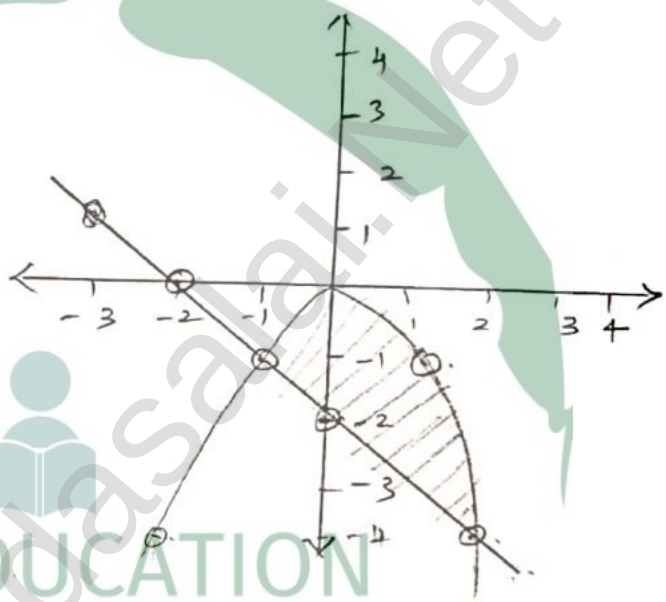
| $x$ | 0 | -1 | -2 | -3 | 2 |
|---|---|---|---|---|---|
| $y$ | -2 | -1 | 0 | 1 | -4 |

$-x^2 = -x - 2$

$x^2 - x - 2 = 0$

$(x+1)(x-2) = 0$

$x = -1, x = 2$




$$\text{Area} = \int_a^b (y_1 - y_2)\,dx$$

$$= \int_{-1}^{2} (-x^2) - (-x-2)\,dx$$

$$= \int_{-1}^{2} -x^2 + x + 2\,dx$$

$$= \left[\frac{-x^3}{3} + \frac{x^2}{2} + 2x\right]_{-1}^{2}$$

$$= \left[\left(-\frac{8}{3} + \frac{4}{2} + 4\right) - \left(\frac{1}{3} + \frac{1}{2} - 2\right)\right]$$

$$= \left[\left(\frac{-16+12+24}{6}\right) - \left(\frac{2+3-12}{6}\right)\right]$$

$$= \frac{20}{6} + \frac{7}{6}$$

$$= 27/6$$

Area $= 9/2$

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

SS PRITHVI, PRIT~EDUCATION

47)

b) $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$ and $0 < x, y, z < 1$, then show that $x^2+y^2+z^2+2xyz=1$

www.Padasalai.Net www.Trb Tnpsc.com of 2.

$\cos^{-1}x = \alpha, \quad \cos^{-1}y = \beta$

$x = \cos\alpha \qquad y = \cos\beta$

$\sin\alpha = \sqrt{1-\cos^2\alpha} = \sqrt{1-x^2}$

$\sin\beta = \sqrt{1-\cos^2\beta} = \sqrt{1-y^2}$

$\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$

$\alpha + \beta + \cos^{-1}z = \pi$

$\alpha + \beta = \pi - \cos^{-1}z \rightarrow (1)$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= xy - \sqrt{1-x^2}\sqrt{1-y^2}$

(1) $\Rightarrow$

$\cos(\alpha+\beta) = \cos(\pi - \cos^{-1}(z))$

$xy - \sqrt{1-x^2}\sqrt{1-y^2} = -\cos(\cos^{-1}(z))$

$xy - \sqrt{1-x^2}\sqrt{1-y^2} = -z$

$xy + z = \sqrt{1-x^2}\sqrt{1-y^2}$

squaring,

$(xy+z)^2 = (1-x^2)(1-y^2)$

$x^2+y^2+z^2+2xyz = 1-y^2-x^2+x^2y^2$

$x^2+y^2+z^2+2xyz = 1$

PRIT EDUCATION Practice ! Perform ! Perfect !
www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION


www.Padasalai.Net. www.Trb Tnpsc.com of 2.

(1) If $A=\begin{bmatrix}7 & 3\\ 4 & 2\end{bmatrix}$, then $9I-A=$ $2A^{-1}$ (d)

(2) If $A=\begin{bmatrix}3 & -3 & 4\\ 2 & -3 & 4\\ 0 & -1 & 1\end{bmatrix}$, then $\text{adj}(\text{adj}\,A)$ is $\begin{bmatrix}3 & -3 & 4\\ 2 & -3 & 4\\ 0 & -1 & 1\end{bmatrix}$ (a)

(3) The area of the triangle formed by the complex number $z$, $iz$ and $z+iz$ in the Argand's diagram is $\frac{1}{2}|z|^2$ (a)

(4) All the complex numbers $z$ which satisfy the equation $\left|\frac{z-6i}{z+6i}\right|=1$ lies on the real axis (a)

(5) If $\cot^{-1}(2)$ and $\cot^{-1}(3)$ are two angle of a triangle, then the third angle is $\frac{3\pi}{4}$ (b)

The ranges of $\sec^{-1}x$ is $[0,\pi]\setminus\pi/2$ (a)

(6) $P(x,y)$ be any point on $16x^2+25y^2=400$ with foci $F_1(3,0)$ & $F_2(-3,0)$ then $PF_1+PF_2$ is ~~6~~ (b) (c) 10

(7) If the two tangent drawn from a point P to the parabola $y^2=4x$ are at right angles then locus of P is: $x=-1$ (b)

(8) The locus of the point whose distance from $(-2,0)$ is $\frac{2}{3}$ times its distance from the line $x=-\frac{9}{2}$ is an ellipse (c)

(9) The angles between the lines $\vec{r}=(\hat{i}+2\hat{j}-3\hat{k})+t(2\hat{i}+\hat{j}-2\hat{k})$ and the plane $\vec{r}\cdot(\hat{i}+\hat{j})+4=0$ 45° (c)

(10) If the rate of increase of the radius of a circle is 5cm/sec. then the rate of Increase of its area when the radius is 20cm, will be $200\pi$ (c)

(11) Angle between $y^2=x$ & $x^2=y$ at origin is, $\pi/2$ (c)

(12) If $u(x,y)=x^y$, then $\frac{\partial u}{\partial x}$ is equal to $yx^{y-1}$ (c)

PRIT EDUCATION Practice ! Perform ! Perfect !
www.Padasalai.Net
Kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

(14) $\int_0^a \frac{1}{4+x^2}\,dx = \frac{\pi}{8}$ then $a$ is 1 (b).

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

(15) The area bounded by the curve $y^2 = 4x$ & the lines $x = 1, x = 4$ & $x$-axis in the first quadrant is $28/3$ (c)

(16) If solution of the differential equation $\frac{dy}{dx} = \frac{ax+3}{2y+f}$ represent a circle, then the value of $a$ is $-2$ (b)

(17) A random variable $X$ has binomial distribution with $n = 25$ & $P = 0.8$ then standard deviation 2 (d)

(18) Suppose that $X$ takes on one of the value 0, 1 & 2 If for some constant $C$. 2 (b)

(19) which one is the inverse of the statement.
(d) $(\neg P \wedge \neg q) \rightarrow (\neg P \vee \neg q)$

(20) which one of the following statement has truth value T?
(a) $\sqrt{5}$ is a irrational number.

PART – II

21. find $z^{-1}$, If $z = (2+3i)(1-i)$

Soln :

$$z = (2+3i)(1-i)$$
$$= (2+3)+(3-2)i$$
$$\boxed{z = 5+i}$$
$$z^{-1} = \frac{1}{z} = \frac{1}{5+i}$$
$$= \frac{1}{5+i} \times \frac{5-i}{5-i}$$
$$= \frac{5-i}{5^2+1^2} = \frac{5}{26} - i\frac{1}{26}$$
$$\boxed{z^{-1} = \frac{5}{26} - i\frac{1}{26}}$$

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

22. Find the square root of $6-8i$

www.Padasalai.Net www.Trb Tnpsc.com of 2.

$|z| = |6-8i| = \sqrt{6^2+(-8)^2} = 10.$

$|z| = 10$

$$\sqrt{a-ib} = \sqrt{\frac{|z|+a}{2}} + i\frac{b}{|b|}\sqrt{\frac{|z|-a}{2}}$$

Now:-

$$\sqrt{6-8i} = \pm\left[\sqrt{\frac{10+6}{2}} + i\sqrt{\frac{10-6}{2}}\right] \quad [\because b \text{ is negative } \frac{b}{|b|} = -1$$

$$= \pm\left[\sqrt{\frac{16}{2}} - i\sqrt{\frac{4}{2}}\right]$$

$$= \pm\sqrt{8} - i\sqrt{2}$$

$$= \pm(2\sqrt{2} - i\sqrt{2})$$

23) Find the principle value of $\text{cosec}^{-1}(-\sqrt{2})$

$\alpha = \text{cosec}^{-1}(-\sqrt{2})$

$\text{cosec}\,\alpha = -\sqrt{2}$

$\sin\alpha = -\frac{1}{\sqrt{2}}$

$\sin\alpha = \sin\left(-\frac{1}{\sqrt{2}}\right)$

$\sin\alpha = \sin\left(-\frac{\pi}{4}\right)$

$\alpha = -\pi/4 \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

PRIT EDUCATION Passion ! Perfect !
www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

24) Find the equation of the parabola in each of the given latus rectum are (4, -8) & (4, 8) Centre is (0, 0) & Open rightward.

www.Padasalai.Net. www.Trb Tnpsc.com of 2

Soln :-

Vertex at (0, 0).

The distance between the end points is the length of the latus rectum.

$$4a = 16$$

Since the vertex is (0, 0) and the type is ~~open~~ rightward., the equation

$$y^2 = 4ax.$$

$$\boxed{y^2 = 16x}$$

25) A particulate is fired straight from the ground to reach a height of s feet in t second, where $s(t) = 128(t) - 16t^2$. compute the maximum height of the particle reached.

PRIT EDUCATION Practice ! Perform ! Perfect !

Soln :-

At maximum height, the velocity $v(t)$ of the particle is zero. ~~of~~=

$$v(t) = \frac{ds}{dt} = 128 - 32t$$

$$v(t) = 0$$

$$128 - 32t = 0$$

$$-32t = -128$$

$$t = \frac{128}{34} = 4$$

$$\boxed{t = 4.}$$

After 4 second, the particle reaches the max height

The height at $t = 4$ is $s(4) = 128(4) - 16(4)^2 = 256$ ft

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

26). If $f(x,y) = x^3 - 3x^2 + y^2 + 5x + 6$, then $f_x$ at $(1,-2)$

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

Soln

$$f(x) = 3x^2 - 6x + 5$$

at $(1,-2)$.

$$f(1) = 3(1)^2 - 6(1) + 5$$

$$= 3 - 6 + 5$$

$$= -3 + 5$$

$$\boxed{f(1) = 2}$$

---

27). Find the differential equation of the curve represented by $xy = ae^x + be^{-x} + x^2$.

Soln :-

$$xy = ae^x + be^{-x} + x^2$$

↓ V method

Diff on both sides.

$$xy' + y = ae^x - be^{-x} + 2x.$$

Again diff on both sides.

$$xy'' + y' + y' = ae^x + be^{-x} + 2.$$

$$xy'' + 2y' = (xy - x^2) + 2$$

$$\boxed{xy'' + 2y' + x^2 - xy - 2 = 0}$$

---

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

28) Establish the equivalence property: $P \to q \equiv \neg P \vee q$

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

| | | | LHS | RHS |
|---|---|---|---|---|
| P | q | $\neg P$ | $P \to q$ | $\neg P \vee q$ |
| T | T | F | T | T |
| T | F | F | F | F |
| F | T | T | T | T |
| F | F | T | T | T |

↳ ① ↳ ②

LHS = RHS

$P \to q \equiv \neg P \vee q$

Hence proved

29) Let * be defined on R by $(a * b) = a + b + ab - 7$, Is * binary on R? If so, find $3 * \left(\frac{-7}{15}\right)$

Soln

$a + b + ab - 7$ are real number & their sum is also a real number.

$\therefore$ * is binary.

$$3 * \left(\frac{-7}{15}\right) = 3 + \left(\frac{-7}{15}\right) - 3\left(\frac{-7}{15}\right) - 7$$

$$= 3 - \frac{7}{15} - \frac{21}{5} - 7$$

$$= \frac{17}{15} - 7 = \frac{17 - 105}{15}$$

$$\boxed{3 * \left(\frac{-7}{15}\right) = -\frac{88}{15}}$$

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

30) Evaluate $\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x}\, dx$.

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

Put $\cos x = t$

| $x$ | 0 | $\pi/2$ |
|---|---|---|
| $t$ | 1 | 0 |

$-\sin x = dt$

$$\sin x\, dx = -dt$$

$$I = \int_0^1 \frac{-dt}{1+t}$$

$$= \int_0^1 \frac{1}{1+t^2}\, dt$$

$$= [\tan^{-1}(t)]_0^1$$

$$= \tan^{-1}(1) - \tan(0)$$

$$= \frac{\pi}{4} - 0$$

$$I = \frac{\pi}{4}$$

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

PART – III

31) Find the value of $\sum_{k=1}^{8} \left[\cos\left(\frac{2k\pi}{9}\right) + i \sin\left(\frac{2k\pi}{9}\right)\right]$

$w = \text{cis}\, \frac{2\pi}{9}$

then $1, w, w^2, \ldots w^8$ are 9th root of unity.

$$1 + w + w^2 + \ldots + w^8 = 0.$$

$$w + w^2 + \ldots + w^8 = -1$$

$$\sum_{k=1}^{8} \left[\cos\left(\frac{2k\pi}{9}\right) + i \sin\left(\frac{2k\pi}{9}\right)\right] = -1$$

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

32. Show that the equation $x^9 - 5x^5 + 4x^4 + 2x^2 + 1 = 0$ has atleast 6 imaginary solution.

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

Soln:-

$$x^9 - 5x^5 + 4x^4 + 2x^2 + 1 = 0$$

$$P(x) = x^9 - \underset{2}{5x^5} + 4x^4 + 2x^2 + 1 = 0$$

$$+ \quad - \quad + \quad + \quad +$$

$P(x)$ has two sign changes

$\therefore$ $P(x)$ has atmost 2 +ve roots.

$$P(-x) = (-x)^9 - 5(-x)^5 + 4(-x)^4 + 2(-x)^2 + 1$$

$$= -x^9 + 5x^5 + 4x^4 + 2x^2 + 1$$

$$= - \quad + \quad + \quad + \quad +$$

$$\underset{1}{}$$

$P(-x)$ has one sign change.

$\therefore$ $P(x)$ has atmost one -ve roots

Minimum No. of imaginary root

$$= 9 - (2+1)$$

$$= 9 - 3$$

$$= 6$$

$\therefore$ $P(x)$ has atleast six imaginary roots

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

33) Prove that the length of the latus rectum of the hyperbola $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\frac{2b^2}{a}$.

Soln :-

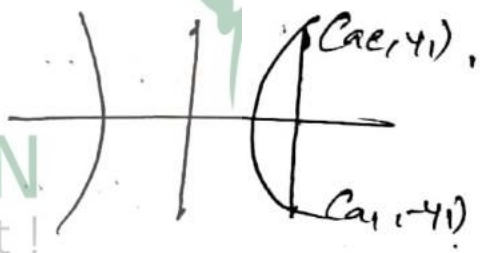
Let the equation of hyperbola is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$(ae, y_1)$ lies in the hyperbola.

$$\frac{(ae)^2}{a^2} - \frac{y_1^2}{b^2} = 0$$

$$\frac{a^2e^2}{a^2} - \frac{y_1^2}{b^2} = 1$$




$$e^2 - 1 = \frac{y_1^2}{b^2}$$

$$b^2(e^2 - 1) = y_1^2$$

$$\frac{b^2 - b^2}{a^2} = y_1^2 \Rightarrow y_1 = \pm \frac{b^2}{a}.$$

Since hyperbola.

length of the Intersection axis

$= 2y_1$

$= \frac{2b^2}{a}$.

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

www.Padasalai.Net www.Trb Tnpsc.com of 2.

34) Find the altitude of a parallelopiped determined by the vectors $\vec{a} = -2\hat{i} + 5\hat{j} + 3\hat{k}$, $\vec{b} = \hat{i} + 3\hat{j} - 2\hat{k}$ & $\vec{c} = -3\hat{i} + \hat{j} + 4\hat{k}$ if the base is taken as the parallelogram determined by $\vec{b}$ & $\vec{c}$.

Soln:-

$$[a, b, c] = \begin{vmatrix} -2 & 5 & 3 \\ 1 & 3 & -2 \\ -3 & 1 & 4 \end{vmatrix}$$

$$= -2(12+2) - 5(4-6) + 3(1+9)$$

$$= -2(14) - 5(-2) + 3(10)$$

$$= -28 + 10 + 30$$

$$\boxed{[a, b, c] = 12}$$

Volume of the parallelopiped = 12

Area of the parallelogram determined by $\vec{b}$ & $\vec{c}$ is $|\vec{b} \times \vec{c}|$.

$$\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & -2 \\ -3 & 1 & 4 \end{vmatrix} = 14\hat{i} + 2\hat{j} + 10\hat{k}.$$

$$|\vec{b} \times \vec{c}| = \sqrt{196 + 4 + 100} = \sqrt{300} = \sqrt{3 \times 100} = 10\sqrt{3}$$

$$\text{Altitude of Parallelopiped} = \frac{\text{Volume of parallelopiped}}{\text{Area of the base}} = \frac{12}{10\sqrt{3}} = \frac{2\sqrt{3}}{5}$$

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

37) Evaluate $\int_0^{\pi/2} \sin^2 x \cos^4 x \, dx$.

Soln:-

$$\int_0^{\pi/2} \sin^2 x \cos^4 x \, dx = \int_0^{\pi/2} \cos^4 x (1 - \cos^2 x) dx$$

$$= I_4 - I_6$$

$$= \frac{3}{4} \times \frac{1}{2} \cdot \frac{\pi}{2} - \frac{5}{6} \times \frac{3}{4} \times \frac{1}{2} \times \frac{\pi}{2}$$

$$\int_0^{\pi/2} \sin^2 x \cos^4 x \, dx = \frac{\pi}{32}.$$

38) Show that the value in the conclusion of the Mean value theorem for $f(x) = Ax^2 + Bx + C$ on any interval $[a, b]$ is $\frac{a+b}{2}$.

Soln :-

$$f'(x) = \frac{f(b) - f(a)}{b - a}$$

$$2Ax + B = \frac{(Ab^2 + Bb + C) - (Aa^2 + Ba + C)}{b - a}$$

$$2Ax + B = \frac{A(a+b) + B}{b - a}$$

$$x = \frac{a+b}{2} \in (a, b).$$

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

38) Assume that a spherical rain drop evaporates at a rate proportional to its surface area. Form a differential equation involving the rate of change of the radius of the rain drop.

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

Sol:- Let $r \rightarrow$ radius of rain drop at $t$

$V \rightarrow$ volume of the raindrop.

$S \rightarrow$ surface area of the rain drop.

We know that

Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of a sphere $= 4\pi r^3$

It is Given that, $\frac{dr}{dt} \propto S$

$$\frac{dr}{dt} = -kS$$

$$\frac{4}{3}\pi r^3 \frac{dr}{dt} = -k 4\pi r^2$$

$$\boxed{\frac{dr}{dt} = -k}$$ $k$ is proportionality constant

39) Probability distribution of random variable is given that

| $X = x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $P(X=x)$ | 0 | $k$ | $2k$ | $2k$ | $3k$ | $k^2$ | $2k^2$ | $7k^2+k$ |

From the given data

$$0+k+2k+2k+3k+k^2+2k^2+7k^2+k = 1$$

$$10k^2 + 9k = 1$$

$$10k^2 + 9k - 1 = 0$$

$$(10k-1)(k+1) = 0$$

$k = \frac{1}{10}, -1$ (positive)

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

$k = \frac{1}{10}$

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

$$P(0 < x < 4) = P(x=1) + P(x=2) + P(x=3)$$

$$= \frac{1}{10} + \frac{2}{10} + \frac{2}{10}$$

$$= \frac{1+2+2}{10}$$

$$= \frac{5}{10}$$

$$\boxed{P(0 < x < 4) = \frac{1}{2}}$$

---

40) Given find $A^2 = \lambda A - 2I$

$$A = \begin{bmatrix} 3 & -2 \\ \lambda & -2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 3 & -2 \\ \lambda & -2 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ \lambda & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 9-2\lambda & -6+4 \\ 3\lambda - 2\lambda & -2\lambda + 4 \end{bmatrix} = \begin{bmatrix} 9-2\lambda & -2 \\ 3\lambda - 2\lambda & -2\lambda + 4 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 9-2\lambda & -2 \\ \lambda & -2\lambda + 4 \end{bmatrix}$$

$$\lambda A = \lambda \begin{bmatrix} 3 & -2 \\ \lambda & -2 \end{bmatrix} = \begin{bmatrix} 3\lambda & -2\lambda \\ \lambda^2 & -2\lambda \end{bmatrix}$$

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

SS PRITHVI, PRIT~EDUCATION

$$2I = 2\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} = \begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}$$

www.Padasalai.Net www.Trb Tnpsc.com of 2.

$$\lambda A - 2I = \begin{bmatrix}3\lambda & -2\lambda\\ \lambda^2 & -2\lambda\end{bmatrix} - \begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}$$

$$= \begin{bmatrix}3\lambda-2 & -2\lambda-0\\ \lambda^2-0 & -2\lambda-2\end{bmatrix}$$

$$\lambda A - 2I = \begin{bmatrix}3\lambda-2 & -2\lambda\\ \lambda^2 & -2\lambda-2\end{bmatrix}$$

$$A^2 = \lambda A - 2I$$

$$\begin{bmatrix}9-2\lambda & -2\\ \lambda & -2\lambda+4\end{bmatrix} = \begin{bmatrix}3\lambda-2 & -2\lambda\\ \lambda^2 & -4\end{bmatrix}$$

$$-2\lambda = -2$$

$$\boxed{\lambda = 1}$$

35)

$f(x) = x^4 - 4x^3$

$f'(x) = 4x^3 - 12x^2$

$f''(x) = 12x^2 - 24x$

$f''(x) = 0$

$12x^2 - 24x = 0$

$12x(x-2) = 0$

$x = 0 \quad | \quad x - 2 = 0$

$x = 2$




| | $f''(x)$ | concavity |
|---|---|---|
| $(-\infty, 0)$ | + | concave upward |
| $(0, 2)$ | − | concave downward |
| $(2, \infty)$ | + | concave upward. |

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

42) a)

Given:-

$$x+2y+z=7$$
$$x+y+\lambda z=\mu$$
$$x+3y-5z=5$$

$$AX=B$$

$$\begin{bmatrix}1&2&1\\1&1&\lambda\\1&3&-5\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}7\\\mu\\5\end{bmatrix}$$

By Augment:-

$$[A|B]=\left[\begin{array}{ccc|c}1&2&1&7\\1&1&\lambda&\mu\\1&3&-5&5\end{array}\right]$$

$$\xrightarrow{R_2\leftrightarrow R_3}\left[\begin{array}{ccc|c}1&2&1&7\\1&3&-5&5\\1&1&\lambda&\mu\end{array}\right]$$

$$\xrightarrow[R_3\to R_3-R_1]{R_2\to R_2-R_1}\left[\begin{array}{ccc|c}1&2&1&7\\0&1&-6&-2\\0&-1&\lambda-1&\mu-7\end{array}\right]$$

$$\xrightarrow{R_3\to R_3+R_2}\left[\begin{array}{ccc|c}1&2&1&7\\0&1&-6&-2\\0&0&\lambda-7&\mu-9\end{array}\right]$$

(i) No solution.

$$\lambda=7 \text{ & } \lambda\neq 9.$$

$$\rho(A)=2,\ \rho([A|B])=3$$

$$\rho(A)\neq\rho([A|B])$$

Hence it is inconsistent & has no solution.

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

(ii) unique solution :-

$\lambda \neq 7$, $\mu$ is real number.

$P(A) = 3$, $P[A|B] = 3$

$P(A) = P(A|B) = 3$

Hence the given is consistent & has a unique solution.

(or)

A1)

b). Let $\alpha, \beta, \gamma$ be the root of the equation.

$2x^4 - 8x^3 + 6x^2 - 3 = 0$.

$\sum_1 = \alpha+\beta+\gamma+\delta = \frac{8}{2} = 4$

$\sum_2 = \alpha\beta+\alpha\gamma+\alpha\delta+\beta\gamma+\beta\delta+\gamma\delta = \frac{6}{2} = 3$

$\sum_3 = \alpha\beta\gamma+\alpha\beta\delta+\alpha\gamma\delta+\beta\gamma\delta = \frac{0}{2} = 0$.

$\sum_4 = \alpha\beta\gamma\delta = \frac{3}{2}$.

Sum of the roots is $\alpha^2, \beta^2, \gamma^2, \delta^2$

$\sum_1 = \alpha^2+\beta^2+\gamma^2+\delta^2 = (\alpha+\beta+\gamma+\delta)^2 - 2(\alpha\beta+\alpha\gamma+\alpha\delta+\beta\gamma+\beta\delta+$

$= 4^2 - 2(3)$

$= 16 - 6$

$= 10$

PRIT EDUCATION Practice | Perform | Perfect

www.Padasalai.Net

42).

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

(a)

$$\sin\left[\sin^{-1}\left(\frac{3}{5}\right)+\sec^{-1}\left(\frac{5}{4}\right)\right]$$

$\sec^{-1}\frac{5}{4}=\theta$, $\sec\theta=\frac{5}{4}$, $\cos\theta=\frac{4}{5}$.

$$\sin\theta=\sqrt{1-\cos^2}=\sqrt{1-\left(\frac{4}{5}\right)^2}=\sqrt{1-\frac{16}{25}}=\sqrt{\frac{25-16}{25}}=\sqrt{\frac{9}{25}}=\frac{3}{5}$$

$$\theta=\sin^{-1}(3/5)$$

$\sec^{-1}(5/4)=\sin^{-1}(3/5)$ & $\sin^{-1}3/5+\sec^{-1}\left(\frac{5}{4}\right)=2\sin^{-1}\left(\frac{3}{5}\right)$

$\sin^{-1}(2x\sqrt{1-x^2})=2\sin^{-1}x$.

$2\sin^{-1}(3/5)=\left[2\times\frac{3}{5}\sqrt{1-(3/5)^2}\right]=\sin^{-1}\left[\frac{24}{25}\right]$

$\sin\left[\sin^{-1}(3/5)+\sec^{-1}(5/4)\right]=\sin\left[\sin^{-1}\left(\frac{24}{25}\right)\right]$

$\sin\left[\sin^{-1}(3/5)+\sec^{-1}(5/4)\right]=\frac{24}{25}\in[-1,1]$

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

42)
B)

www.Padasalai.Net www.Trb Tnpsc.com of 2.

$4x^2-24x-25y^2+250y=-489+20$

$4(x^2-6x)-25(y^2-10y)=-489+20$

$4(x^2-6x)-25(y^2-10y)=489$

$4(x^2-6x+3^2-3^2)-25(y^2-10y+25-25)=489$

$4(x^2-6x+9-9)-25(y^2-10y+25-25)=489$

$4[(x-3)^2-9]-25[(y-5)^2-25]=489$

$4(x-3)^2-36-25(y-5)^2+625=489$

$4(x-3)^2-25(y-5)^2=100$

$\div 100$

$$\frac{4(x-3)^2}{-100}-\frac{25(y-5)^2}{-100}=\frac{100}{100}$$

$$\frac{(x-3)^2}{-25}+\frac{(y-5)^2}{4}=1$$

$$\frac{(y-5)^2}{4}-\frac{(x-3)^2}{25}=1$$

Centre $(h,k)=(3,5)$

$a^2=4$ | $b^2=25$

$a=2$ | $b=5$

Vertices $A(h,k+a)=A(3,7)$

$A'(h,k-a)=A(3,3)$

$c^2=a^2+b^2=2^2+5^2$

$c^2=4+25=29$

$c=\sqrt{29}$

Foci $S(h,k+c)=S(3,5+\sqrt{29})$

$S'(h,k-c)=S(3,5-\sqrt{29})$

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

(a) $\vec{a} = \hat{i} - 2\hat{j} + 4\hat{k}$

$\vec{u} = \hat{i} + 2\hat{j} - 3\hat{k}$

$\vec{v} = 3\hat{i} - \hat{j} + \hat{k}$

Vector equation of the plane

$$\vec{r} = \vec{a} + s\vec{u} + t\vec{v}$$

$$= (\hat{i} - 2\hat{j} + 4\hat{k}) + s(\hat{i} + 2\hat{j} - 3\hat{k}) + t(3\hat{i} - \hat{j} + \hat{k})$$

Cartesian equation.

$(x_1, y_1, z_1) = (1, -2, 4)$

$(l_1, m_1, n_1) = (1, 2, -3)$

$(l_2, m_2, n_2) = (3, -1, 1)$

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-1 & y+2 & z-4 \\ 1 & 2 & -3 \\ 3 & -1 & 1 \end{vmatrix} = 0$$

$$-(x-1) - 10(y+2) - 7(z-4) = 0$$

$$x + 10y + 7z - 9 = 0$$

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

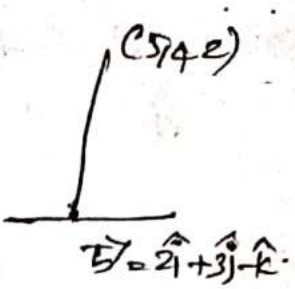
43)

(b)

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

The given point (5, 4, 2) by D & the point on the st. line

$\vec{b} = 2\hat{i} + 3\hat{j} - \hat{k}$.




$F(2t-1, 3t+3, -t+1)$.

$\overrightarrow{DF} = \overrightarrow{OF} - \overrightarrow{OD}$

$= (2t-6)\hat{i} + (3t-1)\hat{j} + (-t-1)\hat{k}$.

$\vec{b} \perp DF$

$(2t-6)(2) + (3t-1)(3) + (-t-1)(-1) = 0$

$4t - 12 + 9t - 3 + t + 1 = 0$

$14t - 14 = 0$

$14t = 14$

$\boxed{t = 1}$

$F = (1, 6, 0)$

Cartesian equation

$\frac{x-5}{-4} = \frac{y-4}{2} = \frac{z-2}{-2}$.

PRIT EDUCATION
Practice ! Perform ! Perfect
www.Padasalai.Net

SS PRITHVI, PRIT~EDUCATION

(42) (a) Soln:- $f(x) = \frac{1}{2}(e^x - e^{-x})$ www.Padasalai.Net. www.Trb Tnpsc.com of 2.

$$f(x) = \frac{1}{2}(e^x + e^{-x})$$

$$f''(x) = \frac{1}{2}(e^x - e^{-x})$$

$f'' = 0$

$e^x - e^x = 0$.

$\boxed{x = 0}$




Consider $(-\infty, 0)$. Let $\boxed{x = -1}$

$$f''(x) = \frac{1}{2}(e^{-1} - e) < 0$$

f is concave upward in $(-\infty, 0)$.

Consider $(0, \infty)$.

Let $\boxed{x = 1}$

$$f''(x) = \frac{1}{2}\left(e - \frac{1}{e}\right) > 0$$

f is concave upward in $(0, \infty)$

The point $x = 0$ changes the sign of $f''(x)$ from Negative to positive. $\therefore x = 0$ given point of Inflection. The point $(0, f(0))$ i.e., $(0, 0)$

PRIT EDUCATION Practice ! Platform ! Perfect !
www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

44)

(B)

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

$$\int_0^{2a} x^2 \sqrt{2ax - x^2}\, dx.$$

$x = 2a\cos^2\theta.$

$dx = -4a\sin\theta\cos\theta\, d\theta.$

$x = 0.$

$2a\cos^2\theta = 0$

$\theta = \frac{\pi}{2}$

$x = 2a,\ 2a\cos^2\theta = 2a,\ \theta = 0.$

We get

$$I = \int_0^{2a} x^2 \sqrt{2ax - x^2}\, dx.$$

$$= \int_{\pi/2}^{0} 4a^2\cos^2\theta\, 2a\cos\theta\sin\theta\,(4a\cos\theta\sin\theta)\, d\theta.$$

$$= 32a^4 \int_0^{\pi/2} \cos^4\theta\sin^2\theta\, d\theta.$$

$$= 32a^4 \times \frac{1}{2} \times \frac{3}{4} \times \frac{1}{2} \times \frac{\pi}{2}$$

$$= 32a^4 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{4} \times \frac{\pi}{2}$$

$$= \frac{32 \times 4 \times 6 \times 1 \times \pi}{24 \times 2}$$

$$= 32a^4 \times \frac{1}{4} \times \frac{1}{4} \times \frac{\pi}{2}$$

$$= 2\pi a^4 \times \frac{1}{16} \times \frac{\pi}{2}$$

$$= \pi a^4$$

PRIT EDUCATION Practice ! Perform ! Perfect !
www.Padasalai.Net

(45)

a) Sol:-

Both the curves are symmetrical about y-axis

The curve $y=|x|$ is $y=\begin{cases} x & \text{if } x\geq 0 \\ -x & \text{if } x\leq 0 \end{cases}$

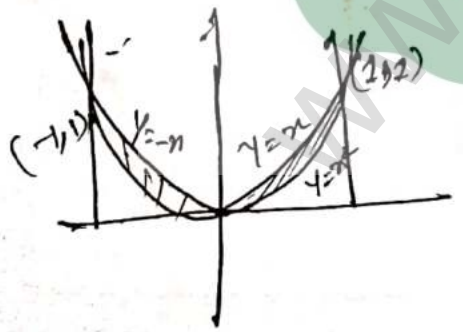
$y=x^2,\ 0\leq x\leq 1$

$$A=2\int_0^1 [y_u-y_L]\,dx.$$

$$=2\int_0^1 [x-x^2]\,dx.$$

$$=2\left[\frac{x^2}{2}-\frac{x^3}{3}\right]_0^1$$

$$=2\left[\frac{1}{2}-\frac{1}{3}\right]=\frac{1}{3}.$$




(45) b).

We are given that :

$$E=Ri+L\frac{di}{dt}$$

$$\frac{di}{dt}+\frac{R}{L}i=\frac{E}{L}.$$

This is a linear differential equation in $i$, where $P=\frac{R}{L}$ & $Q=\frac{E}{L}$.

$$I.F=e^{\int P\,dt}$$

$$=e^{\int \frac{R}{L}dt}$$

$$=e^{\frac{R}{L}t}$$

This solution is $ie^{\frac{R}{L}t}=\int \frac{E}{L}e^{\frac{R}{L}t}dt+c.$

$$ie^{R/L\,t}=\frac{E}{L}\left[\frac{e^{\frac{R}{L}t}}{R/L}\right]+c.$$

$$\boxed{i=\frac{E}{R}+Ce^{-\frac{R}{L}t}}$$

When $E=0$

$$\boxed{\text{We get } i=Ce^{-R/L\,t}}$$

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

46)

$$f(x) = \begin{cases} ke^{-\frac{x}{3}} & ; \text{for } x > 0 \\ 0 & ; \text{for } x \le 0 \end{cases}$$

PDF

(i)

$$\int_{-\infty}^{0} e^{x}\,dx + \int_{0}^{\infty} ke^{-2x}\,dx = 1$$

$$0 + k\left[\frac{e^{-2x}}{-2}\right]_0^{\infty} = 1$$

$$\Rightarrow k\left[\frac{e^{-\infty} - e^{0}}{-2}\right] = 1$$

$$k = 2$$

(ii) $P(x < 3)$

46)

(a) $$f(x) = \begin{cases} ke^{-\frac{x}{3}} & \text{for } x > 0 \\ 0 & \text{for } x \le 0 \end{cases}$$

(i) $$\int_0^{\infty} ke^{-\frac{x}{3}}\,dx = 1$$

$$k\left[-3e^{-\frac{x}{3}}\right]_0^{\infty} = 1$$

$$-3k[0-1]$$

$$\boxed{k = \frac{1}{3}}$$

(ii) $P(x < 3)$

$= F(3) = 1 - e^{-3/3}$

$$\boxed{P(x < 3) = 1 - e^{-1}}$$

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

46)
b) $(p \rightarrow q) \leftrightarrow (\neg p \rightarrow q)$

| $p$ | $q$ | $\neg p$ | $p \rightarrow q$ | $\neg p \rightarrow q$ | $(p \rightarrow q) \leftrightarrow (\neg p \rightarrow q)$ |
|---|---|---|---|---|---|
| T | T | F | T | T | T |
| T | F | F | F | T | F |
| F | T | T | T | T | T |
| F | F | T | T | F | F |

Hence it is contingency.

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

47) (a) $z^4 = 1 - \sqrt{3}i$

$z = (1-\sqrt{3}i)^{1/4}$

$z = [r(\cos\theta + i\sin\theta)]^{1/4}$

$r = \sqrt{1+3} = 2$

$\theta$ lies on the IV$^{th}$ quadrant

$$\alpha = \frac{\tan^{-1}|y|}{|x|}$$

$$= \tan^{-1}\frac{(\sqrt{3})}{1}$$

$$\alpha = \frac{\pi}{3}$$

$$\boxed{\theta = -\frac{\pi}{3}}$$

$$z = \left[2\left(\cos\left(-\tfrac{\pi}{3}\right) + i\sin\left(-\tfrac{\pi}{3}\right)\right)\right]^{1/4}$$

$$= 2^{1/4}\left[\cos\left(2k\pi - \frac{\pi}{3}\right) + i\sin\left(2k\pi - \frac{\pi}{3}\right)\right]^{1/4}$$

$$= 2^{1/4}\left[\cos\left(\frac{6k\pi - \pi}{12}\right) + i\sin\left(\frac{6k\pi - \pi}{12}\right)\right]$$

put $k = 0, 1, 2, 3$

$$z = 2^{1/4}\,\mathrm{cis}\left(-\frac{\pi}{12}\right);\ 2^{1/4}\cos\left(\frac{5\pi}{12}\right);\ 2^{1/4}\mathrm{cis}\left(\frac{11\pi}{12}\right)$$

$$;\ 2^{1/4}\,\mathrm{cis}\left(\frac{17\pi}{12}\right)$$

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

SS PRITHVI, PRIT~EDUCATION

47) b)

$f(x,y) = \log(\sqrt{x^2+y^2})$, show that $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$

www.Padasalai.Net www.Trb Tnpsc.com of 2.

$$\frac{\partial f}{\partial x} = \frac{1}{\sqrt{x^2+y^2}} \times \frac{1}{2\sqrt{x^2+y^2}} \times 2x$$

$$\frac{\partial f}{\partial x} = \frac{x}{(x^2+y^2)}$$ → U/V method.

$(\log x)' = \frac{1}{x}$

$(\log\sqrt{x})' = \frac{1}{2\sqrt{x}}$

$$\frac{\partial^2 f}{\partial x^2} = \frac{(x^2+y^2)(1) - x(2x)}{(x^2+y^2)^2}$$

$$= \frac{x^2+y^2-2x^2}{(x^2+y^2)}$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{y^2-x^2}{(x^2+y^2)^2} \longrightarrow (1)$$

$$\frac{\partial f}{\partial y} = \frac{1}{\sqrt{x^2+y^2}} \times \frac{1}{2\sqrt{x^2+y^2}} \times 2y$$

$$= \frac{y}{(x^2+y^2)^2}$$

$$= \frac{(x^2+y^2)(1) - y(2y)}{(x^2+y^2)^2}$$

$$= \frac{x^2-y^2}{(x^2+y^2)^2} \longrightarrow (2)$$

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = \frac{y^2-x^2+x^2-y^2}{(x^2+y^2)^2} = 0$$

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

# PTA MODEL QUESTION PAPER

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

## No. - 3

I. Choose the correct answer:

1) If $A = \begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}$ and $AB = I$, then $B =$

(b) $(\cos^2\theta/2)\, A^T$

2) If $x^a y^b = e^m$, $x^c y^d = e^n$, $\Delta_1 = \begin{vmatrix} m & b \\ n & d \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} a & m \\ c & n \end{vmatrix}$, $\Delta_3 = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$, then the values of $x$ and $y$ are respectively

(d) $e^{(\Delta_1/\Delta_3)}, e^{(\Delta_2/\Delta_3)}$

3) If $z = x + iy$ is a complex number such at $|z+2| = |z-2|$ then the locus of $z$ is

(b) Imaginary axis

4) The Principal argument of the complex number $\frac{(1+i\sqrt{3})^2}{4i(1-i\sqrt{3})}$ is

(d) $\pi/2$

5) The Polynomial equation $x^3 + 2x + 3 = 0$ has

one negative and two imaginary roots

6) $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right)$ is equal to

(d) $\tan^{-1}(1/2)$

7) The vertex of the Parabola $x^2 = 8y - 1$ ~~has~~ is

$\left(0, \frac{1}{8}\right)$

8) Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

(a) $2ab$

9) If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + \hat{j}$, $\vec{c} = \hat{i}$ and $(\vec{a} \times \vec{b}) \times \vec{c} = \lambda\vec{a} + \mu\vec{b}$, then the value of $\lambda + \mu$ is

(a) 0

10) If the distance of the point (1,1,1) from the origin is half of its distance from the plane $x + y + z = k$, then the values of k are

(d) 3, -9

11) If $x + y = k$ is a normal to the parabola $y^2 = 16x$, then the value of k is

(c) 12

PRIT EDUCATION Practice ! Perform ! Perfect !

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

12) The number given by the Mean value theorem for the function $\frac{1}{x}$, $x \in [1,9]$ is

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

(c) 3

13) If $f(x) = \frac{x-1}{x+1}$, then its differential is given by

(a) $\frac{2}{(x+1)^2}\,dx$

14) If $\int_0^x f(t)\,dt = x + \int_x^1 t f(t)\,dt$, then the value of $f(1)$ is

(a) $\frac{1}{2}$

15) The value of $\int_0^1 \log\left(\frac{x}{1-x}\right)dx$

(a) 0

16) The degree of the differential equation $y(x) = 1 + \frac{dy}{dx} + \frac{1}{1.2}\left(\frac{dy}{dx}\right)^2 + \frac{1}{1.2.3}\left(\frac{dy}{dx}\right)^3 + \cdots$ is (c) 1

17) The Population P in any year t is such that the rate of Increase in the population is proportional to the population. Then ($K > 0$).

(a) $P = ce^{kt}$

18) If the mean of a binomial distribution is 5 and its variance is 4, then the value of n and p are

(b) $(25, \frac{1}{5})$

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

19) The Probability function of a random variable is defined as:

| x | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| f(x) | K | 2K | 3K | 4K | 5K |

(d) 2/3

Then E(x) is equal to:

20) Which one is the Inverse of the statement $(p \vee q) \to (p \wedge q)$ is?

d) $(\sim p \wedge \sim q) \to (\sim p \vee \sim q)$

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

II. Answer the following:

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

21) Show that $(2+i\sqrt{3})^{10} + (2-i\sqrt{3})^{10}$ is real.

Sol:

Let, $z = (2+i\sqrt{3})^{10} + (2-i\sqrt{3})^{10}$

$$\bar{z} = \overline{(2+i\sqrt{3})^{10} + (2-i\sqrt{3})^{10}}$$

$$\bar{z} = \overline{(2+i\sqrt{3})^{10}} + \overline{(2-i\sqrt{3})^{10}}$$

$$\bar{z} = \overline{(2+i\sqrt{3})}^{10} + \overline{(2-i\sqrt{3})}^{10}$$

$$\bar{z} = (2-i\sqrt{3})^{10} + (2+i\sqrt{3})^{10}$$

$$\bar{z} = (2+i\sqrt{3})^{10} + (2-i\sqrt{3})^{10}$$

$$\bar{z} = z.$$

Hence, Proved that $z$ is real.

22) Find the value of $\sin^{-1}\left(\sin\left(\frac{5\pi}{4}\right)\right)$.

Sol:

$$\sin^{-1}\left(\sin\left(\frac{5\pi}{4}\right)\right) = \sin^{-1}\left(\sin\left(\pi + \frac{\pi}{4}\right)\right) \quad \text{since, } \frac{5\pi}{4} \notin \left[-\frac{\pi}{2}, \frac{\pi}{2}\right].$$

$$= \sin^{-1}\left(-\sin\left(+\frac{\pi}{4}\right)\right).$$

$$= -\frac{\pi}{4} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right].$$

PRIT EDUCATION
Practice ! Perform ! Perfect !

23) Obtain the equation for the circle for which (3,4) and (2,-7) are the ends of a diameter.

Sol:

Equation of the circle is,

$$(y-y_1)(y-y_2) + (x-x_1)(x-x_2) = 0.$$

$$(y-4)(y+7) + (x-3)(x-2) = 0.$$

$$y^2+7y-4y-28+x^2-2x-3x+6 = 0.$$

$$x^2+y^2+3y-5x-22 = 0.$$

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

24) Find the intercepts cut off by the plane $\vec{r}\cdot(6\hat{i}+4\hat{j}-3\hat{k})=12$ on the coordinate axes.

www.Padasalai.Net www.TrbTnpsc.com

Sol:

Let, $\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$.

$(x\hat{i}+y\hat{j}+z\hat{k})\cdot(6\hat{i}+4\hat{j}-3\hat{k})=12$.

$6x+4y-3z=12$.

$\div 12$

$$\frac{6x}{12}+\frac{4y}{12}-\frac{3z}{12}=\frac{12}{12}$$

$$\frac{x}{2}+\frac{y}{3}-\frac{z}{4}=1.$$

Comparing with,

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1.$$

x-intercept is 2.
y-intercept is 3
z-intercept is -4.

25) For the function $f(x)=x^4-2x^2$, find all the values of c in $(-2,2)$ such that $f'(c)=0$.

Sol:

$f(x)=x^4-2x^2$.

$f'(x)=4x^3-4x$.

Given, $f'(c)=0$.

$4c^3-4c=0$.

$4c(c^2-1)=0$.

$c=0$ | $c^2-1=0$. $c^2=1$. $c=\pm 1$.

$\therefore c=0,-1,+1.$

PRIT EDUCATION Practice ! Perform ! Perfect !
www.Padasalai.Net

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

26) Evaluate : $\int_0^{\frac{\pi}{2}} \sin^{10} x \, dx$.

Sol:

let, $I = \int_0^{\frac{\pi}{2}} \sin^{10} x \, dx$.

$$I_{10} = \frac{9}{10} \times \frac{7}{8} \times \frac{5}{6} \times \frac{3}{4} \times \frac{1}{2} \times \frac{\pi}{2}.$$

$$= \frac{63\pi}{512}.$$

27) Show that the differential equation for the function $y = e^{-x} + mx + n$, where m and n are arbitrary constants is $e^x\left(\frac{d^2y}{dx^2}\right) - 1 = 0$.

Sol:

$$y = e^{-x} + mx + n.$$

diff. respect to x,

$$\frac{dy}{dx} = -e^{-x} + m.$$

again diff.

$$\frac{d^2y}{dx^2} = -(e^{-x}) + 0.$$

$$\frac{d^2y}{dx^2} = e^{-x}$$

$$\frac{d^2y}{dx^2} = \frac{1}{e^x}$$

$$e^x\left(\frac{d^2y}{dx^2}\right) = 1.$$

$$e^x\left(\frac{d^2y}{dx^2}\right) - 1 = 0.$$

Hence, proved.

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

SS PRITHVI, PRIT~EDUCATION

28) Find the mean of the distribution $f(x) = \begin{cases} 3e^{-3x}, & 0 < x < \infty \\ 0, & \text{elsewhere.} \end{cases}$

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

Sol:

$$E(x) = \int_{-\infty}^{\infty} x f(x)\,dx.$$

$$= \int_{-\infty}^{0} x f(x)\,dx + \int_{0}^{\infty} x f(x)\,dx.$$

$$= 0 + \int_{0}^{\infty} x\, 3e^{-3x}\,dx.$$

$$= 3\int_{0}^{\infty} x e^{-3x}\,dx.$$

$\because \int_0^\infty x^n e^{-ax}\,dx = \frac{n!}{a^{n+1}}.$

$$= 3\left(\frac{1!}{3^{1+1}}\right)$$

$$= 3\left(\frac{1}{3^2}\right).$$

$$= \frac{1}{3}.$$

29) On Z, define $\otimes$ by $(m \otimes n) = m^n + n^m \;\forall m, n \in Z$. Is $\otimes$ binary on Z?

Sol:

$$m \otimes n = m^n + n^m \quad \forall m, n \in Z.$$

Addition is binary on Z.

let $m, n \in Z$.

$m = 2,\ n = 3.$

$m \otimes n = 2^3 + 2^3$

$= 8 + 9 = 17 \in Z.$

Also let $m = 2,\ n = -3.$

$m \otimes n = 2^{-3} + (-3)^2.$

$= \frac{1}{2^3} + 9.$

$= \frac{1}{8} + 9$

$= \frac{73}{8} \notin Z.$

$\otimes$ is not Binary on Z.

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

www.Padasalai.Net www.Trb Tnpsc.com of 2.

30) If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 0 \\ 2 & 5 \end{bmatrix}$, find adj(AB).

Sol:

$$AB = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 4 & 0 \\ 2 & 5 \end{bmatrix}.$$

$$= \begin{bmatrix} 8+6 & 0+15 \\ 4+4 & 0+10 \end{bmatrix}.$$

$$AB = \begin{bmatrix} 14 & 15 \\ 8 & 10 \end{bmatrix}.$$

$$adj(AB) = \begin{bmatrix} 10 & -15 \\ -8 & 14 \end{bmatrix}.$$

III. Answer the following:

31) Solve the following system of linear equations by Matrix Inversion method. $2x - y = 8$ ; $3x + 2y = -2$.

Sol:

$$A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 8 \\ -2 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \end{bmatrix}.$$

PRIT EDUCATION
Practice ! Perform ! Perfect !

$$AX = B.$$

$$\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -2 \end{bmatrix}.$$

$$|A| = \begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix}.$$

$$= 4+3$$

$$= 7.$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -2 \end{bmatrix} \frac{1}{|A|} adj(A).$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ -2 \end{bmatrix} \frac{1}{7} \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}.$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{7} \begin{bmatrix} 16 & -24 \\ -2 & -4 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 8 \\ -2 \end{bmatrix}.$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{7} \begin{bmatrix} 16-2 \\ -24-4 \end{bmatrix}.$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 14 \\ -26 \end{bmatrix}.$$

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \end{bmatrix}.$$

$x = 2, \quad y = -4.$

32) Find the value of $\dfrac{\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}}{\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}} \cdot = \dfrac{\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}}{\cos\left(\frac{-\pi}{3}\right) + i\sin\left(\frac{-\pi}{3}\right)}$

Sol:

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\left[\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2)\right].$$

$$= \frac{1}{1}\left[\cos\left(\frac{\pi}{6} - \left(\frac{-\pi}{3}\right)\right) + i\sin\left(\frac{\pi}{6} - \left(\frac{-\pi}{3}\right)\right)\right].$$

$$= \left[\cos\left(\frac{\pi}{6} + \frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{6} + \frac{\pi}{3}\right)\right].$$

$$= \cos\left(\frac{\pi+2\pi}{6}\right) + i\sin\left(\frac{\pi+2\pi}{6}\right).$$

$$= \cos\left(\frac{3\pi}{6}\right) + i\sin\left(\frac{3\pi}{6}\right).$$

$$= \cos\frac{\pi}{2} + i\sin\frac{\pi}{2}.$$

$$= 0 + i(1).$$

$$= i.$$

33) Find the equation of the hyperbola with foci $(\pm 3, 5)$ and eccentricity $e = 2$.

Sol:

Clearly major axis is along y-axis.

centre of the midpoint of the two foci,

$$C(h,k) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right). = \left(\frac{-3+3}{2}, \frac{5+5}{2}\right) = (0, 5).$$

PRIT EDUCATION Practice ! Perform ! Perfect !
www.Padasalai.Net

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

Foci $(\pm 3, 5)$ and $e = 2$.

$2ae = 6$.

$ae = 3$.

$a(2) = 3$.

$\boxed{a = 3/2}$

$a^2 = \frac{9}{4}$.

$b^2 = a^2(e^2 - 1)$. $\therefore$ Centre : $(0, 5)$.

$= \frac{9}{4}(4 - 1)$.

$b^2 = \frac{27}{4}$.

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1.$$

$$\frac{x^2}{(9/4)} - \frac{(y-5)^2}{(27/4)} = 1.$$

$$\frac{4x^2}{9} - \frac{4(y-5)^2}{27} = 1.$$

34) Find the cartesian equation equation of the plane passing through the line of intersection of the planes $\vec{r} \cdot (2\hat{i} - 7\hat{j} + 4\hat{k}) = 3$ and $3x - 5y + 4z + 11 = 0$ and the point $(-2, 1, 3)$.

Sol:

Given,

$\vec{r} \cdot (2\hat{i} - 7\hat{j} + 4\hat{k}) = 3$.

$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (2\hat{i} - 7\hat{j} + 4\hat{k}) = 3$.

$\boxed{2x - 7y + 4z - 3 = 0}$

$\boxed{3x - 5y + 4z + 11 = 0}$

$(\vec{r} \cdot \vec{n_1} - d_1) + \lambda(\vec{r} \cdot \vec{n_2} - d_2) = 0$.

$(2x - 7y + 4z - 3) + \lambda(3x - 5y + 4z + 11) = 0$.

——— (1).

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

www.Padasalai.Net www.Trb Tnpsc.com of 2.

To find $\lambda$, the lines are passing through (-2, 1, 3).

$$[2(-2)-7(1)+4(3)-3]+\lambda[3(-2)-5(1)+4(3)+11]=0.$$

$$[-4-7+12-3]+\lambda[-6-5+12+11]=0.$$

$$-2+12\lambda=0.$$

$$12\lambda=2.$$

$$\boxed{\lambda=1/6}.$$

Sub $\lambda$ in (1),

$$(2x-7y+4z-3)+\frac{1}{6}(3x-5y+4z+11)=0.$$

$$\frac{6(2x-7y+4z-3)+(3x-5y+4z+11)}{6}=0.$$

$$12x-42y+24z-18+3x-5y+4z+11=0.$$

$$15x-47y+28z-7=0.$$

35) Prove that the function $f(x)=x-\sin x$ is increasing but not strictly on the real line. Also discuss for the existence of local extrema.

Sol:

$$f(x)=x-\sin x.$$

$$f'(x)=1-\cos x \geq 0.$$

And zero at the points $x=2n\pi$, $n\in Z$ and hence the function is increasing on the real line.

Since, there is no sign change in $f(x)$ when passing through $x=2n\pi$, $n\in Z$ by the first derivative test there is no local extrema.

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

www.Padasalai.Net. www.Trb.Tnpsc.com of 2.

36) If $v(x,y,z) = \log(x^3+y^3+z^3)$, find $\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial v}{\partial z}$.

Sol:

$$\frac{\partial v}{\partial x} = \frac{1}{x^3+y^3+z^3}(3x^2).$$

$$= \frac{3x^2}{x^3+y^3+z^3}.$$

$$\frac{\partial v}{\partial y} = \frac{1}{x^3+y^3+z^2}(3y^2).$$

$$= \frac{3y^2}{x^3+y^3+z^3}.$$

$$\frac{\partial v}{\partial z} = \frac{1}{x^3+y^3+z^3}(3z^2).$$

$$= \frac{3z^2}{x^3+y^3+z^3}.$$

$$\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial v}{\partial z} = \frac{3x^2}{x^3+y^3+z^3} + \frac{3y^2}{x^3+y^3+z^3} + \frac{3z^2}{x^3+y^3+z^3}.$$

$$= \frac{3(x^2+y^2+z^2)}{x^3+y^3+z^3}.$$

37) Evaluate: $\int_0^{\pi} x^2 \cos nx\,dx$, where $n$ is a positive integer.

Sol:

let, $u = x^2$, $V = \cos nx$.

Bernoulli's formula,

$$I = \int_0^{\pi} x^2 \cos nx\,dx$$

$$= \left[(x^2)\left(\frac{\sin nx}{n}\right) - (2x)\left(-\frac{\cos nx}{n^2}\right) + (2)\left(-\frac{\sin nx}{n^3}\right)\right]_0^{\pi}$$

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

$= \frac{2\pi(-1)^n}{n^2}$

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

Since $\cos n\pi = (-1)^n$ and $\sin n\pi = 0$.

38) Solve : $\frac{dy}{dx} = e^{x+y} + x^2 e^{x^3+y}$.

Sol:

$$\frac{dy}{dx} = e^{x+y} + x^2 e^{x^3+y}$$

$$= e^x \cdot e^y + x^2 e^{x^3} \cdot e^y$$

$$\frac{dy}{dx} = e^y (e^x + x^2 e^{x^3})$$

$$e^{-y} \frac{dy}{dx} = e^x + x^2 e^{x^3}$$

$$\int e^{-y}\, dy = \int (e^x + x^2 e^{x^3})\, dx$$

$$-e^{-y} = e^x + \int x^2 e^{x^3} dx$$

$t = x^3$

$dt = 3x^2\, dx$

$\frac{dt}{3} = x^2\, dx$,

$$-e^{-y} = e^x + \int e^t \frac{dt}{3}$$

$$-e^{-y} = e^x + \frac{e^t}{3} + c$$

$$e^x + \frac{e^{x^3}}{3} + e^{-y} = c$$

$$\boxed{3e^x + e^{x^3} + 3e^{-y} = c}$$

"I math" youtube

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

39) The Mean and variance of a binomial variate X are respectively 2 and 1.5. Find P(X=0).

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

Sol:

Mean $= np = 2$ Variance $= npq = 1.5$.

$$\frac{npq}{np} = \frac{1.5}{2} = \frac{3}{4}.$$

$$q = 3/4 \Rightarrow p = 1 - \frac{3}{4} = \frac{1}{4}.$$

$$np = 2.$$

$$n = \frac{2}{p} = \frac{2}{1/4} = 8.$$

$$P(X=x) = f(x) = \binom{8}{x}\left(\frac{1}{4}\right)^x\left(\frac{3}{4}\right)^{8-x}$$

$$P(X=0) = f(0) = \binom{8}{0}\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^{8-0}$$

$$= \left(\frac{3}{4}\right)^8.$$

$P(x=r) = \binom{n}{r} p^r q^{n-r}$

40) Find the magnitude and direction cosines of the moment about the point $(0,-2,3)$ of a force $\hat{i}+\hat{j}+\hat{k}$ whose line of action passes through the origin.

Sol:

let A be the point $(0,-2,3)$.

$$\vec{r} = \vec{OA} = (-2\hat{j}+3\hat{k}).$$

$$\vec{F} = \hat{i}+\hat{j}+\hat{k}.$$

$$\vec{t} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -2 & 3 \\ 1 & 1 & 1 \end{vmatrix}.$$

$$= \hat{i}(-2-3) - \hat{j}(0-3) + \hat{k}(0+2).$$

$$\vec{t} = -5\hat{i}+3\hat{j}+2\hat{k}.$$

PRIT EDUCATION Practice ! Perform ! Perfect !
www.Padasalai.Net

Magnitude of moment $= \sqrt{(25)+(9)+(4)} = \sqrt{38}$.

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

Direction cosines $= \left(\frac{-5}{\sqrt{38}}, \frac{3}{\sqrt{38}}, \frac{2}{\sqrt{38}}\right)$.

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

IV. Answer the following:

41) a) If $ax^2+bx+c$ is divided by $x+3$, $x-5$ and $x-1$, the remainders are 21, 61 and 9 respectively. Find a, b and c.

Sol:

$f(x) = ax^2+bx+c$.

$f(-3) = a(-3)^2 + b(-3)+c = 21$.

$9a-3b+c = 21$. ——— (1).

$f(5) = 25a+5b+c = 61$ ——— (2)

$f(1) = a+b+c = 9$ ——— (3).

$x+3=0$. $x=-3$.

$x-5=0$. $x=5$.

$x-1=0$. $x=1$.

$$\Delta = \begin{vmatrix} 9 & -3 & 1 \\ 25 & 5 & 1 \\ 1 & 1 & 1 \end{vmatrix}.$$

$= 9(5-1) + 3(25-1) + 1(25-5)$.

$= 9(4) + 3(24) + 1(20)$.

$= 36 + 72 + 20$.

$\Delta = 128$.

$$\Delta_x = \begin{vmatrix} 21 & -3 & 1 \\ 61 & 5 & 1 \\ 9 & 1 & 1 \end{vmatrix}.$$

$= 21(5-1) + 3(61-9) + 1(61-45)$.

$= 21(4) + 3(52) + 1(16)$.

$= 84 + 156 + 16$.

$= 256$.

$$\Delta y = \begin{vmatrix} 9 & 21 & 1 \\ 25 & 61 & 1 \\ 1 & 9 & 1 \end{vmatrix}$$

$= 9(61-9) - 21(25-1) + 1(225-61)$.

$= 9(52) - 21(24) + 1(164)$.

$= 468 - 504 + 164$

$= 128$.

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

$\Delta_2 = \begin{vmatrix} 9 & -3 & 21 \\ 25 & 5 & 61 \\ 1 & 1 & 9 \end{vmatrix}$.

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

$= 9(45-61) + 3(225-61) + 21(25-5)$.

$= 9(-16) + 3(164) + 21(20)$.

$= -144 + 492 + 420$.

$= 768$

$x = \frac{\Delta x}{\Delta}$

$= \frac{256}{128}$

$x = 2$

$y = \frac{\Delta y}{\Delta}$

$= \frac{128}{128}$

$y = 1$

$z = \frac{768}{128}$

$z = 6$.

b) Simplify : $(-\sqrt{3}+3i)^{31}$.

Sol:

$-\sqrt{3}+3i = r(\cos\theta + i\sin\theta)$.

$r = \sqrt{(-\sqrt{3})^2+3^2}$

$= \sqrt{3+9} = \sqrt{12} = 2\sqrt{3}$.

$\alpha = \tan^{-1}\left|\frac{3}{-\sqrt{3}}\right|$

$= \tan^{-1}(\sqrt{3})$.

$\alpha = \frac{\pi}{3}$.

$\theta = \pi - \alpha$ (II Quadrant).

$= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$-\sqrt{3}+3i = 2\sqrt{3}\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$.

$(-\sqrt{3}+3i)^{31} = (2\sqrt{3})^{31}\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)^{31} = (2\sqrt{3})^{31}\left(\cos\frac{62\pi}{3} + i\sin\frac{62\pi}{3}\right)$.

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

$$= (2\sqrt{3})^{31}\left(\cos\left(20\pi+\frac{\pi}{3}\right)+i\sin\left(20\pi+\frac{\pi}{3}\right)\right).$$

$$= (2\sqrt{3})^{31}\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right).$$

$$= (2\sqrt{3})^{31}\left(\cos\left(\pi-\frac{\pi}{3}\right)+i\sin\left(\pi-\frac{\pi}{3}\right)\right).$$

$$= (2\sqrt{3})^{31}\left(-\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right).$$

$$= (2\sqrt{3})^{31}\left(-\frac{1}{2}+i\frac{\sqrt{3}}{2}\right).$$

42) a) Solve the equation $x^4-10x^3+26x^2-10x+1=0$.

Sol:

$$x^4-10x^3+26x^2-10x+1=0.$$

$\div x^2$

$$x^2-10x+26-\frac{10}{x}+\frac{1}{x^2}=0.$$

$$\left(x^2+\frac{1}{x^2}\right)-10\left(x+\frac{1}{x}\right)+26=0.$$

let $x+\frac{1}{x}=y$.

$$y^2-2-10y+26=0.$$

$$y^2-10y+24=0.$$

$24 < \begin{matrix}-6\\-4\end{matrix}$ $\quad -10$.

$$(y-6)(y-4)=0.$$

$y-6=0$. $\quad y-4=0$.

$\boxed{y=6}$ $\quad \boxed{y=4}$.

$x+\frac{1}{x}=6$.

$\frac{x^2+1}{x}=6$.

$x^2+1=6x$.

$x^2-6x+1=0$.

$x+\frac{1}{x}=4$.

$\frac{x^2+1}{x}=4$.

$x^2+1=4x$.

$x^2-4x+1=0$.

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{6 \pm \sqrt{36-4(1)(1)}}{2(1)}$$

$$= \frac{6 \pm \sqrt{36-4}}{2}$$

$$= \frac{6 \pm \sqrt{32}}{2}$$

$$= \frac{6 \pm 4\sqrt{2}}{2}$$

$$= \frac{2(3 \pm 2\sqrt{2})}{2}$$

$$\boxed{x = 3 \pm 2\sqrt{2}}.$$

$$x = \frac{4 \pm \sqrt{16-4(1)(1)}}{2(1)}.$$

$$= \frac{4 \pm \sqrt{16-4}}{2}$$

$$= \frac{4 \pm \sqrt{12}}{2}.$$

$$= \frac{4 \pm 2\sqrt{3}}{2}$$

$$= \frac{2(2 \pm \sqrt{3})}{2}$$

$$\boxed{x = 2 \pm \sqrt{3}}$$

b) Solve for $x$ : $\tan^{-1}x + 2\cot^{-1}x = \frac{2\pi}{3}$.

Sol:

$$\tan^{-1}x + 2\cot^{-1}x = \frac{2\pi}{3}.$$

$$\tan^{-1}x + \cot^{-1}x + \cot^{-1}x = \frac{2\pi}{3}. \quad \because \tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}.$$

$$\frac{\pi}{2} + \cot^{-1}x = \frac{2\pi}{3}.$$

$$\cot^{-1}x = \frac{2\pi}{3} - \frac{\pi}{2}.$$

$$\cot^{-1}x = \frac{4\pi - 3\pi}{6}$$

$$\cot^{-1}x = \frac{\pi}{6}.$$

$$\cot^{-1}x = \frac{\pi}{6}.$$

$$x = \cot\frac{\pi}{6}.$$

$$\boxed{x = \sqrt{3}} \in \mathbb{R}.$$

PRIT EDUCATION Practice ! Perform ! Perfect !
www.Padasalai.Net

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

43) a) For the ellipse $4x^2+y^2+24x-2y+21=0$, find the centre, vertices, foci and the length of latus rectum.

Sol:

Rearranging the terms, the equation of the ellipse is,

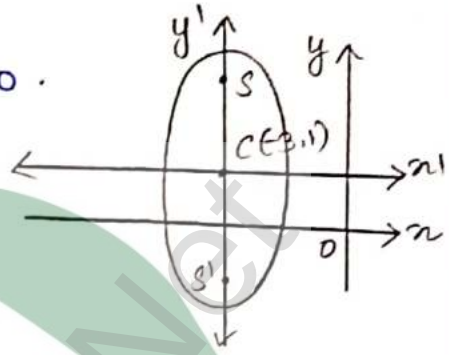
$$4x^2+24x+y^2-2y+21=0.$$

i.e., $4(x^2+6x+9-9)+(y^2-2y+1-1)+21=0.$

$$4(x+3)^2-36+(y-1)^2-1+21=0.$$

$$4(x+3)^2+(y-1)^2=16.$$

$$\frac{(x+3)^2}{4}+\frac{(y-1)^2}{16}=1.$$




Centre is $(-3,1)$ $a=4$, $b=2$ and the major axis is parallel to y-axis.

$$c^2=16-4=12.$$

$$c=\pm 2\sqrt{3}.$$

Therefore, the foci are $(-3, 2\sqrt{3}+1)$ and $(-3, -2\sqrt{3}+1)$.

Vertices are $(1, \pm 4+1)$. i.e., $(1,5)$ and $(1,-3)$.

length of latus rectum $= \frac{2b^2}{a}$

$= 2$ units.

b) By vector method, Prove that $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$.

Sol:

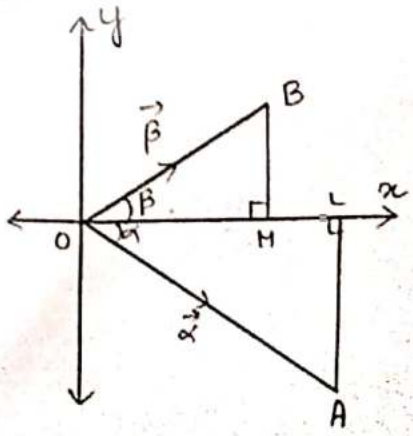
In $\Delta OCA$,

$$\cos\alpha=\frac{\vec{OL}}{\vec{OA}}$$

$$|\vec{OL}|=|\vec{OA}|\cos\alpha. \qquad \because |\vec{OA}|=1.$$

$$|\vec{OL}|=\cos\alpha.$$

$$\sin\alpha=\frac{\vec{LA}}{\vec{OA}}$$




PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI PRIT~EDUCATION

$|\vec{LA}| = |OA| \sin\alpha$.

$|\vec{LA}| = \sin\alpha$

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

$\vec{OA} = \vec{OL} + \vec{LA}$

$\vec{a} = \cos\alpha\hat{i} - \sin\alpha\hat{j}$ ——— (1).

$\vec{b} = \cos\beta\hat{i} + \sin\beta\hat{j}$ ——— (2).

By definition,

$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos(\alpha+\beta)$.

$\vec{a}\cdot\vec{b} = \cos(\alpha+\beta)$ ——— (3).

Dot product.

$\vec{a}\cdot\vec{b} = (\cos\alpha\hat{i} - \sin\alpha\hat{j})\cdot(\cos\beta\hat{i} + \sin\beta\hat{j})$.

$= \cos\alpha\cos\beta - \sin\alpha\sin\beta$ ——— (4).

(3) and (4),

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$.

Hence, proved.

44) a) Find the absolute extrema of the function $f(x) = 3x^4 - 4x^3$ on the interval $[-1, 2]$.

Sol:

$f(x) = 3x^4 - 4x^3$

$f'(x) = 12x^3 - 12x^2$

$f'(x) = 0$.

$12x^3 - 12x^2 = 0$.

$12x^2(x-1) = 0$.

$x^2 = 0$ $\quad x - 1 = 0$.

$\boxed{x = 0}$ $\quad \boxed{x = 1}$.

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

$f(-1) = 3(-1)^4 - 4(-1)^3 = 3(1) - 4(-1)$.

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

$= 3+4$

$= 7$.

$f(2) = 2(2)^4 - 4(2)^3 = 3(16) - 4(8)$.

$= 48 - 32$

$= 16$.

$f(0) = 0$.

$f(1) = 3(1)^4 - 4(1)^3 = 3 - 4$.

$= -1$.

Absolute Max. value $f(2) = 16$.

Absolute Min. value $f(1) = -1$.

b) For the function $f(x,y) = \frac{3x}{y+\sin x}$, find $f_x$, $f_y$ and show that $f_{xy} = f_{yx}$.

Sol:

$$f(x,y) = \frac{3x}{y+\sin x}.$$

$$f_x = \frac{(y+\sin x)(3) - (3x)(\cos x)}{(y+\sin x)^2}$$

$$f_x = \frac{3y + 3\sin x - 3x\cos x}{(y+\sin x)^2}$$

$$f_{yx} = \frac{(y+\sin x)^2(3) - (3y + 3\sin x - 3x\cos x)(2)(y+\sin x)(1+0)}{(y+\sin x)^4}$$

$$= \frac{(y+\sin x)\left[(y+\sin x)(3) - [6y + 6\sin x - 6x\cos x]\right]}{(y+\sin x)^{4-3}}$$

$$= \frac{3y + 3\sin x - 6y - 6\sin x + 6x\cos x}{(y+\sin x)^3}$$

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

$$= \frac{-3y - 3x\sin x + 6x\cos x}{(y+x\sin x)^3}$$

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

$$f_y = \frac{(y+x\sin x)(0) - 3x(1+0)}{(y+x\sin x)^2}$$

$$= \frac{-3x}{(y+x\sin x)^2}.$$

$$f_{xy} = \frac{(y+x\sin x)^2(-3) - (-3x)(y+x\sin x)(2)(0+\cos x)}{(y+x\sin x)^4}$$

$$= \frac{(y+x\sin x)\left[-3y - 3x\sin x - [-6x\cos x]\right]}{(y+x\sin x)^{4-3}}$$

$$= \frac{-3y - 3x\sin x + 6x\cos x}{(y+x\sin x)^3}$$

$$\therefore f_{xy} = f_{yx}.$$

Hence, proved.

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

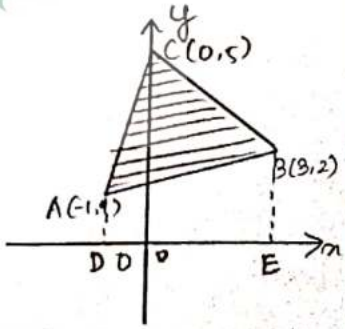
45) a) Using integration find the area of the region bounded by $\Delta ABC$, whose vertics A, B and C are $(-1,1)$, $(3,2)$ and $(0,5)$ respectively.

Sol:

Equation of AB is $\frac{y-1}{2-1} = \frac{x+1}{3+1}$ or $y = \frac{1}{4}(x+5)$.

Equation of BC is $\frac{y-5}{2-5} = \frac{x-0}{3-0}$ or $y = -x+5$.

Equation of AC is $\frac{y-1}{5-1} = \frac{x+1}{0+1}$ or $y = 4x+5$.



SS PRITHVI, PRIT~EDUCATION

$\therefore$ Area of $\triangle ABC$ = Area DACO + Area of OCBE − Area of DABE.

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

$$= \int_{-1}^{0}(4x+5)dx + \int_{0}^{3}(-x+5)dx - \frac{1}{4}\int_{-1}^{3}(x+5)dx.$$

$$= \left[\frac{4x^2}{2}+5x\right]_{-1}^{0} + \left[-\frac{x^2}{2}+5x\right]_{0}^{3} - \frac{1}{4}\left[\frac{x^2}{2}+5x\right]_{-1}^{3}$$

$$= 0-(+2-5) + \left(-\frac{9}{2}+15\right)-0 - \frac{1}{4}\left[\frac{9}{2}+15\right] + \frac{1}{4}\left[\frac{1}{2}-5\right].$$

$$= \frac{15}{2} \text{ sq. units}.$$

b) Solve : $y^2 - x^2\frac{dy}{dx} = xy\frac{dy}{dx}$.

Sol:

$$y^2 = x^2\frac{dy}{dx} + xy\frac{dy}{dx}$$

$$y^2 = \frac{dy}{dx}(x^2+xy).$$

$$\frac{dy}{dx} = \frac{y^2}{(x^2+xy)} \quad \text{——— (1)}.$$

Put $y = vx \Rightarrow v = \frac{y}{x}$.

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \quad \text{——— (2)}.$$

(1) = (2),

$$\frac{y^2}{(x^2+xy)} = v + x\frac{dv}{dx}.$$

$$\frac{(vx)^2}{x^2+x(vx)} = v + x\frac{dv}{dx}.$$

$$\frac{v^2x^2}{x^2+x^2v} = v + x\frac{dv}{dx}.$$

$$\frac{x^2v^2}{x^2(1+v)} = v + x\frac{dv}{dx}.$$

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

$$v + x\frac{dv}{dx} = \frac{v^2}{1+v}.$$

$$x\frac{dv}{dx} = \frac{v^2}{1+v} - v.$$

$$x\frac{dv}{dx} = \frac{v^2 - v - v^2}{1+v}$$

$$\int (1+v)\frac{dv}{v} = \int -\frac{dx}{x}$$

$$\int \frac{1}{v} + 1\, dv = -\int \frac{dx}{x}.$$

$$\log|v| + v = -\log|x| + \log|c|.$$

$$v = \log\left|\frac{c}{x}\right| - \log v.$$

$$v = \log\left|\frac{c/x}{v}\right|.$$

$$v = \log\left|\frac{c}{x\left(\frac{y}{x}\right)}\right|.$$

$$\frac{y}{x} = \log\left|\frac{c}{y}\right|.$$

$$e^{y/x} = \frac{c}{y}.$$

$$\boxed{y = ce^{-y/x}}$$

46) a) The cumulative distribution function of a discrete random variable is given by.

$$F(x) = \begin{cases} 0 & -\infty < x < -1 \\ 0.15 & -1 \le x < 0 \\ 0.35 & 0 \le x < 1 \\ 0.60 & 1 \le x < 2 \\ 0.85 & 2 \le x < 3 \\ 1 & 3 \le x < \infty \end{cases}$$

Find i) the Probability mass function ii) $P(X<1)$ and iii) $P(X \ge 2)$.

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

Sol:

Probability mass function.

$f(x) = P(X = x)$.

$f(-1) = P(x = -1) = 0.15$.

$f(0) = F(0) - F(-1)$.

$= 0.35 - 0.15$

$= 0.20$.

$f(1) = F(1) - F(0)$.

$= 0.60 - 0.35$

$= 0.25$.

$f(2) = F(2) - F(1)$.

$= 0.85 - 0.60$.

$= 0.25$.

$f(3) = F(3) - F(2)$.

$= 1 - 0.85$.

$= 0.15$.

i) Probability mass function:

| $x$ | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|
| $f(x)$ | 0.15 | 0.20 | 0.25 | 0.25 | 0.15 |

ii) $P(X < 1)$.

$= P(X = -1) + P(X = 0)$.

$= 0.15 + 0.20$

$= 0.35$.

iii) $P(X \geq 2)$

$= P(X = 2) + P(X = 3)$.

$= 0.25 + 0.15$

$= 0.40$.

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

SS PRITHVI, PRIT~EDUCATION

b) Using truth table check whether the statements $\neg(p \vee q) \vee (\neg p \wedge q)$ and $\neg p$ are logically equivalent.

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

Sol:

| p | q | $p \vee q$ | $\neg(p \vee q)$ | $\neg p$ | $\neg p \wedge q$ | $\neg(p \vee q) \vee (\neg p \wedge q)$ | $\neg p$ |
|---|---|---|---|---|---|---|---|
| T | T | T | F | F | F | F | F |
| T | F | T | F | F | F | F | F |
| F | T | T | F | T | T | T | T |
| F | F | F | T | T | F | T | T |

↓ LHS ↓ RHS

LHS = RHS.

Hence, Proved.

47) a) Find the shortest distance between the straight lines $\frac{x-6}{1} = \frac{2-y}{2} = \frac{z-2}{2}$ and $\frac{x+4}{3} = \frac{y}{-2} = \frac{1-z}{2}$.

PRIT EDUCATION
Practice ! Perform ! Perfect !

Sol:

$\frac{x-6}{1} = \frac{y-2}{-2} = \frac{z-2}{2}$ and $\frac{x+4}{3} = \frac{y}{-2} = \frac{z-1}{-2}$

$\vec{r} = (6\hat{i} + 2\hat{j} + 2\hat{k}) + t(\hat{i} - 2\hat{j} + 2\hat{k})$

$\vec{r} = (-4\hat{i} + \hat{k}) + s(3\hat{i} - 2\hat{j} - 2\hat{k})$

$\vec{a} = 6\hat{i} + 2\hat{j} + 2\hat{k}$ | $\vec{c} = -4\hat{i} + \hat{k}$

$\vec{b} = \hat{i} - 2\hat{j} + 2\hat{k}$ | $\vec{d} = 3\hat{i} - 2\hat{j} - 2\hat{k}$

$$S = \frac{|(\vec{c} - \vec{a}) \cdot (\vec{b} \times \vec{d})|}{|\vec{b} \times \vec{d}|}$$

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

$$\vec{b}\times\vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 3 & -2 & -2 \end{vmatrix}$$

$$= \hat{i}(4+4) - \hat{j}(-2-6) + \hat{k}(-2+6)$$

$$= 8\hat{i} + 8\hat{j} + 4\hat{k}.$$

$$|\vec{b}\times\vec{d}| = \sqrt{64+64+16} = \sqrt{144} = 12$$

$$|(\vec{c}-\vec{a})\cdot(\vec{b}\times\vec{d})| = |(-10\hat{i} - 2\hat{j} - \hat{k})\cdot(8\hat{i}+8\hat{j}+4\hat{k})|$$

$$= |-80-16-4|$$

$$= |-100|$$

$$= 100$$

$$\delta = \frac{100}{12} = \frac{25}{3}$$

$$\boxed{\delta = 25/3}$$

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

SS PRITHVI, PRIT~EDUCATION

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

47) b) Prove that among all the rectangles of the given area square has the least perimeter.

Sol:

let $x, y$ be the sides of the rectangle.

Area of the rectangle, $xy = K$. $\Rightarrow y = \frac{K}{x}$.

Perimeter of the rectangle, $P = 2(x+y)$.

$$P(x) = 2\left(x + \frac{K}{x}\right).$$

$$P'(x) = 2\left(1 - \frac{K}{x^2}\right).$$

$$P'(x) = 0.$$

$$2\left(1 - \frac{K}{x^2}\right) = 0.$$

$$1 - \frac{K}{x^2} = 0.$$

$$\frac{K}{x^2} = 1.$$

$$\boxed{x = \pm\sqrt{K}}$$

$x = \sqrt{K}$ is critical number.

$$P''(x) = \frac{4K}{x^3} > 0.$$

$$\boxed{x = \sqrt{K}}.$$

$$xy = K.$$

$$\boxed{y = \sqrt{K}}.$$

Therefore the minimum perimeter rectangle of given area is a square.

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

# PTA MODEL QUESTION PAPER

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

## No. - 4

Full Answer key

I. choose the correct answer:

1) The adjoint of 3x3 matrix P is $\begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, then the possible values(s) of the determinant P is (c) $\pm 3$

2) If $x = \frac{-1+i\sqrt{3}}{2}$ then the value of $x^2+x+1$ (c) 0

3) The value of $\left(\frac{1+i\sqrt{3}}{1-\sqrt{3}i}\right)^{10}$ is (a) $cis\frac{2\pi}{3}$

4) A Polynomial equation is x of degree n always has (c) Imaginary roots

5) $\sin^{-1}(\cos x) = \frac{\pi}{2} - x$ is valid for (b) $0 \le x \le \pi$

6) If $\cot^{-1}(\sqrt{\sin\alpha}) + \tan^{-1}(\sqrt{\sin\alpha}) = u$, then $\cos 2u$ is equal to (c) -1

7) The locus of a point whose distance from (-2,0) is $\frac{2}{3}$ times its distance from the line $x = -\frac{9}{2}$ is (c) an ellipse

8) If P(x,y) be any point on $4x^2+9y^2=36$, then the sum of the distances of P from the points $(\sqrt{5},0)$ and $(-\sqrt{5},0)$ is (c) 6

9) If the plane $x+2y+z-8=0$ has equal intercepts on the coordinate axes, the value of $\alpha$ is (a) 1

10) If the planes $\vec{r}.(2\hat{i}-\lambda\hat{j}+\hat{k})=3$ and $\vec{r}.(4\hat{i}+\hat{j}-\mu\hat{k})=5$ are parallel, then the value of $\lambda$ and $\mu$ are (c) $-\frac{1}{2}, -2$

11) The position of a particle moving along a horizontal line of any time t is given by $s(t) = 3t^2-2t-8$. For what values of t the particle is not moving? (b) $\frac{1}{3}$.

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

12) The minimum value of the function $|3-x|+9$ is (d) 9

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

13) The abscissa of the point on the curve $f(x)=\sqrt{8-2x}$ at which the slope of the tangent is $-0.25$ ? (b) $-4$

14) If $f(x)=\frac{x}{x+1}$, then its differential is given by (b) $\frac{1}{(x+1)^2}dx$

15) The solution of the differential equation $\frac{dy}{dx}+\frac{1}{\sqrt{1-x^2}}=0$ is

(a) $y+\sin^{-1}x=c$

16) The differential equation of the family of curves $y=Ae^x+Be^{-x}$, where A and B are arbitary constants is

(b) $\frac{d^2y}{dx^2}-y=0$

17) The solution of the differential equation $\frac{dy}{dx}=e^x+2$ is

(b) $y=2x+e^x+c$

18) The random variable X has the probability density function

$$f(x)=\begin{cases} ax+b & 0<x<1 \\ 0 & \text{otherwise} \end{cases}$$ and $E(X)=\frac{7}{12}$, then a and b are respectively

(a) 1 and $\frac{1}{2}$

PRIT EDUCATION
Practice | Perform | Perfect

19) In the set $\mathbb{R}$ of real numbers '$*$' is defined as follows. Which one of the following is not a binary operation on $\mathbb{R}$?

(d) $a*b=a^b$

20) If $a*b=\sqrt{a^2+b^2}$ on the real numbers then $*$ is

(c) both commutative and associative

www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION



www.Padasalai.Net. www.Trb Tnpsc.com of 2.

21) Let A be the non-singular matrix of odd order $2m+1$, where $m = 0, 1, 2, \ldots$. Then, we get $|A| \neq 0$

$$|\text{adj } A| = |A|^{(2m+1)-1} = |A|^{2m}$$

$|A|^{2m}$ is always positive, we get that $|\text{adj } A|$ is positive.

---

22) $\tan^{-1}[\sin(-\pi/2)] = \tan^{-1}(-\sin\pi/2)$

$= \tan^{-1}(-1)$

$= -\frac{\pi}{4} \in (-\frac{\pi}{2}, \frac{\pi}{2})$

The principal value is $-\pi/4$

---

23) Type of conic :

$y^2 + 4x + 3y + 4 = 0$

* parabola

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

---

24) Given,

$9x^2 - 16y^2 = 144$

$\frac{x^2}{16} - \frac{y^2}{9} = 1$

$a^2 = 16, b^2 = 9$

$c^2 = a^2 + b^2$

$= 16 + 9 = 25$

$c = 5$

Hence the foci are $(-5, 0)$ and $(5, 0)$

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

www.Padasalai.Net www.Trb Tnpsc.com of 2.

25) $3x = 2y = -z$ and $6x + y + 4z = 0$

$$\frac{x}{2} = \frac{y}{3} = \frac{z}{-6} \quad \text{and} \quad \frac{x}{2} = \frac{y}{-12} = \frac{z}{-3}$$

direction ratios $(a_1, b_1, c_1) = (2, 3, -6)$

direction ratios $(a_2, b_2, c_2) = (2, -12, -3)$

$$\cos\theta = \left| \frac{0}{\sqrt{49}\sqrt{157}} \right| = 0$$

$$\theta = \frac{\pi}{2}$$

---

26) Intervals of monotonicity

$$f(x) = x^2 - 4x + 4$$

$$\Rightarrow f(x) = (x-2)^2$$

$$f'(x) = 2(x-2)$$

$$f'(x) = 0$$

$$x = 2$$

| Interval | $x$ | Sign of $f'(x)$ | Monotonicity |
|---|---|---|---|
| $(-\infty, 2)$ | 1 | $-$ | strictly decrease |
| $(2, \infty)$ | 3 | $+$ | strictly increase |

---

27) Let $I = \int_0^1 \frac{(\sin^{-1}x)^2}{\sqrt{1-x^2}}\,dx$

Put $\sin^{-1}x = t$

| $x$ | 0 | 1 |
|---|---|---|
| $t$ | 0 | $\pi/2$ |

$$\frac{1}{\sqrt{1-x^2}}\,dx = dt$$

$$I = \int_0^{\frac{\pi}{2}} (t)^2\,dt$$

$$I = \left[\frac{t^3}{3}\right]_0^{\pi/2}$$

$$= \frac{1}{3}\left[\left(\frac{\pi}{2}\right)^3 - 0\right]$$

$$\boxed{I = \frac{\pi^3}{24}}$$

PRIT EDUCATION
Practice | Perform | Perfect |
www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

28) $y\left(\frac{dy}{dx}\right) = \frac{x}{\left(\frac{dy}{dx}\right) + \left(\frac{dy}{dx}\right)}$

$y\frac{dy}{dx}\left[\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^3\right] = x$

$y\left(\frac{dy}{dx}\right)^{2} + y\left(\frac{dy}{dx}\right)^4 - x = 0$

order = 1, degree = 4

---

29)

$$F(x) = \begin{cases} 0 & -\infty < x < 0 \\ \frac{1}{2}(x^2+x) & 0 \le x < 1 \\ 1 & 1 \le x < \infty \end{cases}$$

i) probability density function

$$f(x) = \begin{cases} \frac{1}{2}(2x+1) & 0 \le x < 1 \\ 0 & \text{otherwise} \end{cases}$$

ii) $P(0.3 \le x \le 0.6)$

$= F(0.6) - F(0.3)$

$= \frac{1}{2}\left[(0.6)^2 + 0.6\right] - \frac{1}{2}\left[(0.3)^2 + 0.3\right]$

$= \frac{1}{2}\left[0.36 + 0.6\right] - \frac{1}{2}\left[0.09 + 0.3\right]$

$= \frac{1}{2}\left[0.96\right] - \frac{1}{2}\left[0.39\right]$

$= 0.48 - 0.195$

$= 0.285$

---

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

30) In cube root of unity

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

$\omega^3 = 1$

$1+\omega+\omega^2 = 0$

$(1+\omega)^7 = A+B\omega$

$(-\omega^2)^7 = A+B\omega$

$-\omega^{14} = A+B\omega$

$-\omega^2 = A+B\omega$

$1+\omega = A+B\omega$

$\boxed{A=1}$, $\boxed{B=1}$

---

PART - III

31) Given,

$$A = \begin{vmatrix} 4 & -2 & 6 & 8 \\ 1 & 1 & -3 & -1 \\ 15 & -3 & 9 & 21 \end{vmatrix}$$

The order is $3 \times 4$

$\rho(A) \leq \min(3,4)$

$$A = \begin{vmatrix} 4 & -2 & 6 \\ 1 & 1 & -3 \\ 15 & -3 & 9 \end{vmatrix}$$

$= 4(9-9)+2(9+45)+6(-3-15)$

$= 4(0)+2(54)+6(-18)$

$= 108-108$

$= 0$

$$A = \begin{vmatrix} 4 & -2 & 8 \\ 1 & 1 & -1 \\ 15 & -3 & 21 \end{vmatrix} = 4(21-3)+2(21+15)+8(-3-15)$$

$= 144-144 = 0$

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

we reduce the matrix into 2x2

$$A = \begin{vmatrix} 4 & -2 \\ 1 & 1 \end{vmatrix}$$

$= 4 + 2 - 6 \neq 0$

∴ the rank of the matrix is 2.

---

38) $2x^2 - 7x + 13 = 0$

the roots are $\alpha^2$ and $\beta^2$

since $\alpha$ and $\beta$ are the roots of the equation,

$\alpha + \beta = \frac{7}{2}$ and $\alpha\beta = \frac{13}{2}$

Sum of the roots $= \alpha^2 + \beta^2$

$= (\alpha+\beta)^2 - 2\alpha\beta$

$= -\frac{3}{4}$

product of the roots $= \alpha^2\beta^2 = (\alpha\beta)^2 = \frac{169}{4}$.

the required equation is $x^2 + \frac{3}{4}x + \frac{169}{4} = 0$

---

39) $3x + 4y = 12$

$\frac{x}{4} + \frac{y}{3} = 1$. Therefore the point A and B are $(4,0)$ and $(0,3)$

equation of circle

$(x-x_1)(x-x_2) + (y-y_1)(y-y_2) = 0$

$(x-4)(x-0) + (y-0)(y-3) = 0$

$x^2 + y^2 - 4x - 3y = 0$

---

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

34) Given,

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

The given force is $\vec{F} = 3\hat{i} + 4\hat{j} - 5\hat{k}$

The vector $\vec{OA} = 2\hat{i} - 3\hat{j} + 4\hat{k}$

$\vec{OB} = 4\hat{i} + 2\hat{j} - 3\hat{k}$

$\vec{r} = \vec{AB} = \vec{OB} - \vec{OA}$

$= 2\hat{i} + 5\hat{j} - 7\hat{k}$

$\vec{\tau} = \vec{r} \times \vec{F}$ (or) $M = \vec{r} \times \vec{F}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 5 & -7 \\ 3 & 4 & -5 \end{vmatrix}$$

$= (-25+28)\hat{i} - (-10+21)\hat{j} + (8-15)\hat{k}$

$\vec{\tau} = 3\hat{i} - 11\hat{j} - 7\hat{k}$

The magnitude $= |3\hat{i} - 11\hat{j} - 7\hat{k}| = \sqrt{179}$

The direction cosines of the torque are

$$\left( \frac{3}{\sqrt{179}}, \frac{-11}{\sqrt{179}}, \frac{-7}{\sqrt{179}} \right)$$

PRIT EDUCATION Practice ! Perform ! Perfect !

---

35) Given,

$f(x) = x^2 e^{-2x}$

$f'(x) = x^2(-2e^{-2x}) + e^{-2x}(2x)$

$= e^{-2x}(-2x^2 + 2x)$

$f''(x) = e^{-2x}(-4x+2) + (-2x^2+2x)(-2e^{-2x})$

$= e^{-2x}[4x^2 - 8x + 2]$

$f'(x) = 0 \Rightarrow x = 0, 1$

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

$x=0, f''(x)>0$

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

$x=0$, $f$ attains local minimum

the local minimum is 0

$x=1, f''(x)<0$

$x=1$, $f$ attains local maximum

the local maximum is $e^{-2}$.

---

36) Given,

$r=10.5$

$r+\Delta r=10.75$

$\Delta=0.25$

Area of circular plate $A=\pi r^2$

$dA \simeq 2\pi r\,dr$

$dA = 2\pi(10.5)(0.25)$

$= 5.25\pi$

Percentage change $= \frac{dA}{A}\times 100$

$= \frac{5.25\pi}{\pi(10.5)^2}\times 100$

$= \frac{105}{0.1\times 10.5} = \frac{10}{0.1} = 4.76\%$

---

37)

$u = e^{2(x-y)}$

$u$ is not homogeneous

$f = \log u = 2(x-y)$

$f(tx,ty) = 2(tx-ty)$

$= t^1 2(x-y)$

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

$f$ is homogeneous of degree 1.

www.Padasalai.Net www.Trb Tnpsc.com of 2.

By Euler's theorem,

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf$$

$$x\frac{\partial(\log u)}{\partial x} + y\frac{\partial(\log u)}{\partial y} = \log u.$$

$$\frac{1}{u}\left[x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right] = \log u$$

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = u\log u$$

---

32) Let $f(x) = \log\left(\frac{3+\cos x}{3-\cos x}\right)$

$$f(2\pi - x) = \log\left(\frac{3+\cos(2\pi - x)}{3-\cos(2\pi - x)}\right)$$

$$= \log\left(\frac{3+\cos x}{3-\cos x}\right) = f(x)$$

$$\therefore \int_0^{2\pi} x \log\left(\frac{3+\cos x}{3-\cos x}\right)dx = \frac{2\pi}{2}\int_0^{2\pi} \log\left(\frac{3+\cos x}{3-\cos x}\right)dx$$

$$= \pi\int_0^{2\pi} \log\left(\frac{3+\cos x}{3-\cos x}\right)dx$$

$$= \pi \times 2\int_0^{\pi} \log\left(\frac{3+\cos x}{3-\cos x}\right)dx$$

$$= 2\pi\int_0^{\pi} \log\left(\frac{3+\cos x}{3-\cos x}\right)dx$$

$$= 2\pi \times 0$$

$$= 0.$$

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

---

33) i) $p \vee (\sim p)$ is a tautology

| p | ~p | p V (~p) |
|---|---|---|
| T | F | T |
| F | T | T |

ii) $p \wedge (\sim p)$ is contradiction

| p | ~p | p ∧ (~p) |
|---|---|---|
| T | F | F |
| F | T | F |

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

IV. Answer the following :-

41) a) Determine the values of $\lambda$ for which the following system of equations $x+y+3z=0$ ; $4x+3y+\lambda z=0$ ; $2x+y+2z=0$ has i) a unique solution.

ii) a non-trivial solution.

Sol:

$$A=\begin{bmatrix}1&1&3\\4&3&\lambda\\2&1&2\end{bmatrix} \qquad B=\begin{bmatrix}0\\0\\0\end{bmatrix}.$$

$$[A|B]=\left[\begin{array}{ccc|c}1&1&3&0\\4&3&\lambda&0\\2&1&2&0\end{array}\right]$$

$$\sim\left[\begin{array}{ccc|c}1&1&3&0\\2&1&2&0\\4&3&\lambda&0\end{array}\right] \quad R_2\leftrightarrow R_3.$$

$$\sim\left[\begin{array}{ccc|c}1&1&3&0\\0&-1&-4&0\\0&-1&\lambda-12&0\end{array}\right] \quad \begin{matrix}R_2\to R_2-2R_1.\\R_3\to R_3-4R_1.\end{matrix}$$

$$\sim\left[\begin{array}{ccc|c}1&1&3&0\\0&-1&-4&0\\0&0&\lambda-8&0\end{array}\right] \quad R_3\to R_3-R_2.$$

i) For having trivial (unique) solution,

$$\lambda\neq 8.$$

ii) For having a non-trivial solution,

$$\lambda=8.$$

PRIT EDUCATION Practice ! Perform ! Perfect ! www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

b) If $\cos\alpha + \cos\beta + \cos\gamma = \sin\alpha + \sin\beta + \sin\gamma = 0$ then show that,

i) $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha+\beta+\gamma)$ and

ii) $\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha+\beta+\gamma)$.

Sol:

Let $a = \cos\alpha + i\sin\alpha$, $b = \cos\beta + i\sin\beta$, $c = \cos\gamma + i\sin\gamma$.

$a+b+c = 0$.

We know that If $a+b+c=0$, then $a^3+b^3+c^3 = 3abc$.

$\therefore (\cos\alpha + i\sin\alpha)^3 + (\cos\beta + i\sin\beta)^3 + (\cos\gamma + i\sin\gamma)^3$

$= 3[\cos(\alpha+\beta+\gamma) + i\sin(\alpha+\beta+\gamma)]$.

$\cos 3\alpha + \cos 3\beta + \cos 3\gamma + i[\sin 3\alpha + \sin 3\beta + \sin 3\gamma]$.

$= 3[\cos(\alpha+\beta+\gamma) + i\sin(\alpha+\beta+\gamma)]$.

Equating real and imaginary parts.

$\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha+\beta+\gamma)$.

$\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha+\beta+\gamma)$.

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

42) a) Solve the equation $z^3 + 8i = 0$, where $z \in C$

Sol:

$z^3 + 8i = 0$.

$z^3 = -8i$.

$= 8(-i)$.

$= 8\left(\cos\left(-\frac{\pi}{2} + 2k\pi\right) + i\sin\left(-\frac{\pi}{2} + 2k\pi\right)\right)$, $k \in Z$.

Therefore,

$z = \sqrt[3]{8}\left(\cos\left(\frac{-\pi+4k\pi}{6}\right) + i\sin\left(\frac{-\pi+4k\pi}{6}\right)\right)$, $k = 0, 1, 2$.

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

Taking $k=0, 1, 2$ we get,

$k=0$ $\quad z = 2\left(\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right)$

$= 2\left(\frac{-1}{2} - i\frac{\sqrt{3}}{2}\right)$.

$= 2\left(\frac{\sqrt{3}}{2} - i\frac{1}{2}\right) = \sqrt{3} - i$.

$k=1$, $\quad z = 2\left(\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right)$

$= 2(0 + i)$.

$= 2i$.

$k=2$, $\quad z = 2\left(\cos\left(\frac{4\pi}{6}\right) + i\sin\left(\frac{4\pi}{6}\right)\right)$.

$= 2\left(\cos\left(\pi + \frac{\pi}{6}\right) + i\sin\left(\pi + \frac{\pi}{6}\right)\right)$.

$= 2\left(-\cos\left(\frac{\pi}{6}\right) - i\sin\left(\frac{\pi}{6}\right)\right)$.

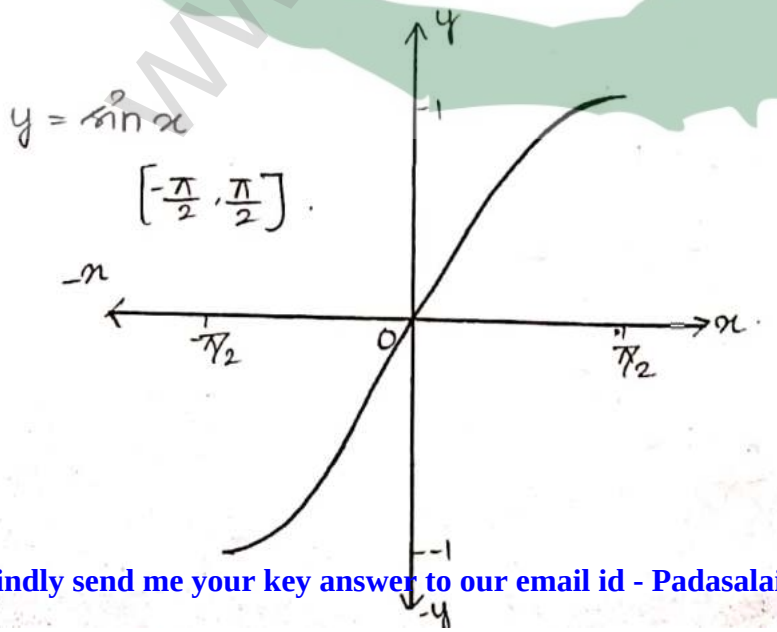
$= 2\left(-\frac{\sqrt{3}}{2} - i\frac{1}{2}\right)$

$= -\sqrt{3} - i$.

42)

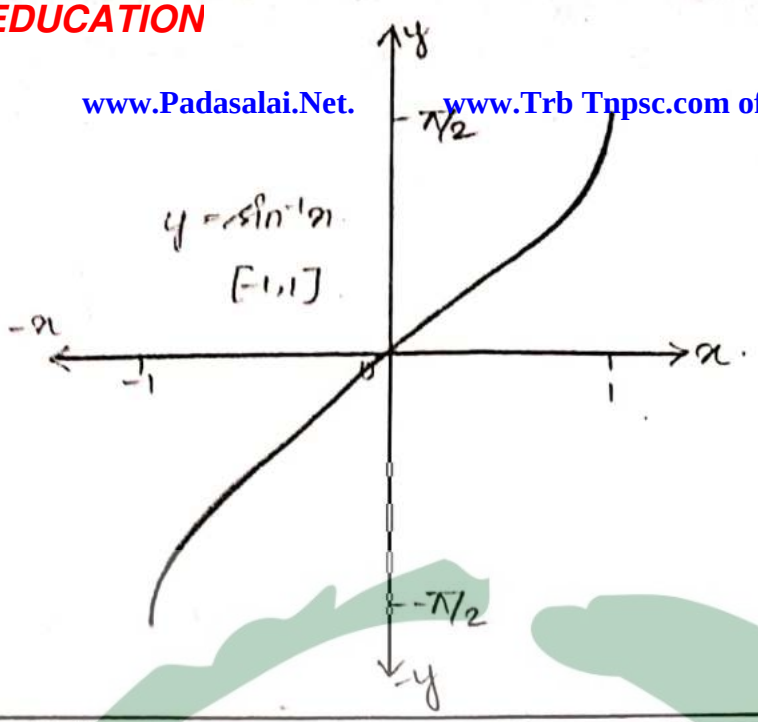
b) Draw the curve $\sin x$ in the domain $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $\sin^{-1} x$ in $[-1, 1]$.

Sol:




kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

www.Padasalai.Net. www.Trb Tnpsc.com of 2.




43) a) The eccentricity of an ellipse with its centre at the origin is $\frac{1}{2}$. If one of the directrix is $x=4$, then find the equation of the ellipse.

Sol:

Given: centre: (0,0)

$e = \frac{1}{2}$, $x = 4$

Directrix, $x = \pm a/e$

$$\frac{a}{e} = 4$$

$$a = 4 \times \frac{1}{2}$$

$$\boxed{a = 2}$$

$$b^2 = a^2(1-e^2)$$

$$= 4\left(1 - \frac{1}{4}\right)$$

$$= 4\left(\frac{3}{4}\right)$$

$$\boxed{b^2 = 3}$$

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION



The equation of ellipse.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\boxed{\frac{x^2}{4} + \frac{y^2}{3} = 1}$$

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

43) b) If the st. lines $\frac{x-1}{2} = \frac{y+1}{\lambda} = \frac{z}{2}$ and $\frac{x+1}{5} = \frac{y+1}{2} = \frac{z}{\lambda}$ are coplanar, find $\lambda$ and cartesian equations of the planes containing these two lines.

Sol:

The lines are coplanar.

$$\begin{vmatrix} 2 & 0 & 0 \\ 2 & \lambda & 2 \\ 5 & 2 & \lambda \end{vmatrix} = 0.$$

$$2(\lambda^2 - 4) = 0.$$

$$\lambda = -2, 2.$$

Equation of the plane containing the two given lines is.

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

$$\begin{vmatrix} x-1 & y+1 & z \\ 2 & \lambda & 2 \\ 5 & 2 & \lambda \end{vmatrix} = 0.$$

Case (I) when $\lambda = -2$,

$$\begin{vmatrix} x-1 & y+1 & z \\ 2 & -2 & 2 \\ 5 & 2 & -2 \end{vmatrix} = 0.$$

$$y+z+1=0.$$

Case (II) when $\lambda = 2$,

$$\begin{vmatrix} x-1 & y+1 & z \\ 2 & 2 & 2 \\ 5 & 2 & 2 \end{vmatrix} = 0.$$

$$y-z+1=0.$$

---

44) a) Find the angle between $y=x^2$ and $(x-3)^2=y$.

PRIT EDUCATION
Practice ! Perform ! Perfect !

Sol:

Equating,

$$x^2 = (x-3)^2.$$

$$x = 3/2$$

$$y = 9/4.$$

The point of intersection $\left(\frac{3}{2}, \frac{9}{4}\right)$.

$$y = x^2.$$

$$\frac{dy}{dx} = 2x.$$

$$m_1 a = \frac{dy}{dx} \text{ at } \left(\frac{3}{2}, \frac{9}{4}\right) = 3.$$

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

$y = (x-3)^2$.

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

$\frac{dy}{dx} = 2(x-3)$.

let $m_2 = \frac{dy}{dx}$ at $\left(\frac{3}{2}, \frac{9}{4}\right) = -3$.

we get,

$$\theta = \tan^{-1}\left(\frac{3-(-3)}{1-9}\right).$$

$$\theta = \tan^{-1}\left[\frac{3}{4}\right).$$

A4)

b) let $w(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$, $(x,y,z) \neq (0,0,0)$. Show that

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} = 0.$$

Sol:

$$\frac{\partial w}{\partial x} = \frac{-1}{2}(x^2+y^2+z^2)^{-3/2}(2x)$$

$$= -x(x^2+y^2+z^2)^{-3/2}.$$

$$\frac{\partial^2 w}{\partial x^2} = -x\left(\frac{-3}{2}\right)(x^2+y^2+z^2)^{-5/2}(2x) + (x^2+y^2+z^2)^{-3/2}(-1).$$

$$= \left[3x^2(x^2+y^2+z^2)^{-1} - 1\right](x^2+y^2+z^2)^{-3/2}.$$

$$\frac{\partial w}{\partial y} = \frac{-1}{2}(x^2+y^2+z^2)^{-3/2}(2y).$$

$$= -y(x^2+y^2+z^2)^{-3/2}.$$

$$\frac{\partial^2 w}{\partial y^2} = \left[3y^2(x^2+y^2+z^2)^{-1} - 1\right](x^2+y^2+z^2)^{-3/2}.$$

Similarly,

$$\frac{\partial^2 w}{\partial z^2} = \left[3z^2(x^2+y^2+z^2)^{-1} - 1\right](x^2+y^2+z^2)^{-3/2}.$$

PRIT EDUCATION Practice ! Perform ! Perfect !
www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

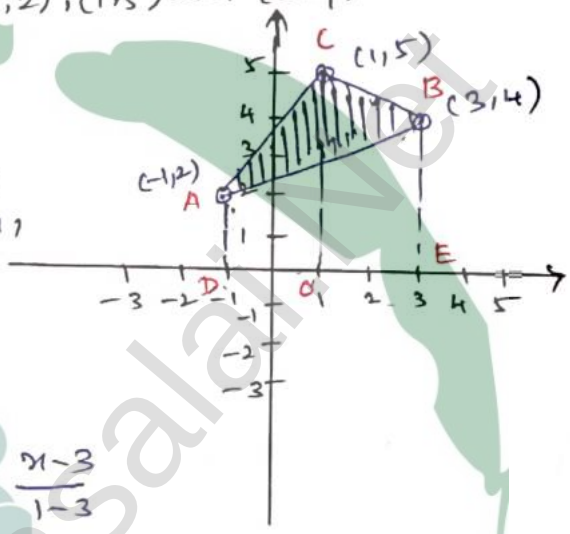


$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} = (x^2+y^2+z^2)^{-3/2}\left[\frac{3x^2}{x^2+y^2+z^2} + \frac{3y^2}{x^2+y^2+z^2} + \frac{3z^2}{x^2+y^2+z^2} - 3\right]$$

$$= (x^2+y^2+z^2)^{-3/2}\left[\frac{3x^2+3y^2+3z^2-3(x^2+y^2+z^2)}{x^2+y^2+z^2}\right].$$

45) a) Using integration, find the area of the region bounded by the triangle whose vertices are (-1,2), (1,5) and (3,4).

Sol. A(-1,2), C(1,5), B(3,4)




⇒ Equation of AB $\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$

Equation of AB, $y = \frac{1}{2}(x+5)$

$$\boxed{y = \frac{1}{2}(x+5)}$$

⇒ Equation of BC ⇒ $\frac{y-4}{5-4} = \frac{x-3}{1-3}$

$$\boxed{y = \frac{1}{2}(-x+11)}$$

⇒ Equation of AC ⇒ $\frac{y-2}{5-2} = \frac{x+1}{1+1}$

$$\boxed{y = \frac{1}{2}(3x+7)}$$

Area of $\Delta ABC$ = Area of DACO + Area of OCBE − Area of DABE.

$$= \int_{-1}^{1} \frac{1}{2}(3x+7)\,dx + \int_{1}^{3} \frac{1}{2}(-x+11)\,dx - \int_{-1}^{3} \frac{1}{2}(x+5)\,dx$$

$$= \frac{1}{2}\left\{\left[\left(\frac{3x^2}{2}+7x\right)\right]_{-1}^{1} + \left(\frac{-x^2}{2}+11x\right)_{1}^{3} - \left(\frac{x^2}{2}+5x\right)_{-1}^{3}\right]\}$$

$$= \frac{1}{2}\left[\frac{28}{2} + \frac{36}{2} - \frac{48}{2}\right]$$

$$= \frac{1}{2}\left[14+18-24\right]$$

$$= \frac{1}{2}(8) = 4.$$

PRIT EDUCATION Practice ! Perform ! Perfect !
www.Padasalai.Net

SS PRITHVI, PRIT~EDUCATION

www.Padasalai.Net www.Trb Tnpsc.com of 2.

b) Solve : $(1+2e^{x/y})dx + 2e^{x/y}\left(1-\frac{x}{y}\right)dy = 0$.

Sol^n

The given equation can be written as,

$$\frac{dx}{dy} = \frac{\left(\frac{x}{y}-1\right)2e^{x/y}}{1+2e^{x/y}} \quad \text{——— (1)}.$$

$$= g\left(\frac{x}{y}\right).$$

So, we substitute $x = vy$.

sub in (1), $y\frac{dv}{dy} = \frac{-2e^v+v}{1+2e^v}$

$$\frac{1+2e^v}{v+2e^v}dv = -\frac{dy}{y}$$

On Integration,

$$\log|2e^v+v| = -\log|y| + \log|c|.$$

$$\log|2ye^v+vy| = \log|c|.$$

$$2ye^v + vy = \pm c.$$

put $v = x/y$.

$$\boxed{2ye^{x/y} + x = K}$$

$K = \pm c$.

---

46) a) The Probability density function of $x$ is given by

$$f(x) = \begin{cases} 16xe^{-4x} & \text{for } x>0 \\ 0 & \text{for } x\le 0 \end{cases}$$ Find the mean and variance of $x$.

Sol^n

$$E(x) = \int_{-\infty}^{\infty} x f(x)dx$$

$$= \int_{-\infty}^{0} x(0)dx + \int_{0}^{\infty} 16x^2e^{-4x}dx.$$

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

$= 16 \times \frac{\lfloor 3}{4^3}$

$= \frac{1}{2}$.

$E(X^2) = \int_0^\infty 16x^3 e^{-4x}\, dx$.

$= 16 \cdot \frac{\lfloor 3}{4^4}$

$= \frac{3}{8}$.

$Var(X) = E(X^2) - [E(X)]^2$

$= \frac{3}{8} - \frac{1}{4}$

$= \frac{1}{8}$.

---

46) b) Verify i.) closure Property ii.) associative property iii.) existence of Identity iv.) existence of inverse and v.) Commutative property for the operation $+_5$ on $Z_5$ using table corresponding to addition modulo 5.

Sol:- It is known that $Z_5 = \{[0], [1], [2], [3], [4]\}$. The table corresponding to addition modulo 5 is as follows: We take remainders $\{0, 1, 2, 3, 4\}$ to represent the classes $\{[0], [1], [2], [3], [4]\}$

| $+_5$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 | 4 |
| 1 | 1 | 2 | 3 | 4 | 0 |
| 2 | 2 | 3 | 4 | 0 | 1 |
| 3 | 3 | 4 | 0 | 1 | 2 |
| 4 | 4 | 0 | 1 | 2 | 3 |
| | | | | | |

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

b) Solve : $(1+2e^{x/y})dx + 2e^{x/y}\left(1-\frac{x}{y}\right)dy = 0$

www.Padasalai.Net www.Trb-Tnpsc.com of 2.

Sol:

The given equation can be written as,

$$\frac{dx}{dy} = \frac{\left(\frac{x}{y}-1\right)2e^{x/y}}{1+2e^{x/y}} \quad \text{——— (1).}$$

$$= g\left(\frac{x}{y}\right).$$

So, we substitute $x = vy$.

Sub in (1), $y\frac{dv}{dy} = \frac{-2e^v + v}{1+2e^v}$

$$\frac{1+2e^v}{v+2e^v}dv = \frac{-dy}{y}$$

On Integration,

$$\log|2e^v + v| = -\log|y| + \log|c|$$

$$\log|2ye^v + vy| = \log|c|.$$

$$2ye^v + vy = \pm c.$$

put $v = x/y$.

$$\boxed{2ye^{x/y} + x = k}$$

$k = \pm c$.

PRIT EDUCAT. Practice ! Perform ! Perfect

46) a) The Probability density function of $x$ is given by

$$f(x) = \begin{cases} 16xe^{-4x} & \text{for } x > 0 \\ 0 & \text{for } x \le 0 \end{cases}$$ Find the mean and variance of $x$.

Sol:

$$E(x) = \int_{-\infty}^{\infty} x f(x)dx$$

$$= \int_{-\infty}^{0} x(0)dx + \int_{0}^{\infty} 16x^2 e^{-4x}dx.$$

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

i.) Since each box in the table is filled with exactly one element of $Z_5$, the output $a +_5 b$ is unique and hence $+_5$ is a binary operation.

www.Padasalai.Net www.Trb Tnpsc.com of 2

ii.) The entries are symmetrically placed with respect to the main diagonal. So $+_5$ has commutative property.

iii.) The table cannot be used directly for the verification of the associative property. So, it is to be verified as usual.

For instance, $(2 +_5 3) +_5 4 = 4 = 0 +_5 4 = 4 \pmod 5$.

and $2 +_5 (3 +_5 4) = 2 +_5 2 = 4 \pmod 5$.

Hence, $(2 +_5 3) +_5 4 = 2 +_5 (3 +_5 4)$.

Proceeding like this one can verify this for all possible triples and ultimately it can that $+_5$ is associative.

iv.) The row headed by 0 and the column headed by 0 are identical. Hence the identity element is 0.

v.) The existence of inverse is guaranteed provided the identity 0 exists in each row and each column.

---

47) a) If $\vec{a} = \hat{i} + \hat{j} - \hat{k}$, $\vec{b} = 2\hat{i} + 3\hat{j}$ and $\vec{c} = \hat{j} - \hat{k}$, verify that $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$.

PRIT EDUCATION Practice ! Perform ! Perfect !

Sol:

LHS

$$\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 0 \\ 0 & 1 & -1 \end{vmatrix}.$$

$$= \hat{i}(-3) - \hat{j}(-2) + \hat{k}(2).$$

$$= -3\hat{i} + 2\hat{j} + 2\hat{k}.$$

$$\vec{a} \times (\vec{b} \times \vec{c}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ -3 & 2 & 2 \end{vmatrix} = \hat{i}(2+2) - \hat{j}(2-3) + \hat{k}(2+3).$$

$$= 4\hat{i} + \hat{j} + 5\hat{k}.$$

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

RHS

$\vec{a}\cdot\vec{c} = (\hat{i}+\hat{j}-\hat{k})\cdot(\hat{j}-\hat{k})$.

$= \hat{j}\cdot\hat{j} - \hat{k}\cdot\hat{k} = 1+1 = 2$.

$(\vec{a}\cdot\vec{c})\vec{b} = 2(2\hat{i}+3\hat{j})$.

$= 4\hat{i}+6\hat{j}$.

$\vec{a}\cdot\vec{b} = (\hat{i}+\hat{j}-\hat{k})\cdot(2\hat{i}+3\hat{j})$.

$= 2+3 = 5$.

$(\vec{a}\cdot\vec{b})\vec{c} = 5(\hat{j}-\hat{k})$.

$= 5\hat{j}-5\hat{k}$.

$(\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c} = 4\hat{i}+6\hat{j}-(5\hat{j}-5\hat{k})$.

$= 4\hat{i}+6\hat{j}-5\hat{j}+5\hat{k}$.

$= 4\hat{i}+\hat{j}+5\hat{k}$.

LHS = RHS.

Hence, proved.

---

b) Evaluate : $\int_0^{\pi/2} \frac{dx}{4\sin^2 x + 5\cos^2 x}$.

Sol:

Let $I = \int_0^{\pi/2} \frac{dx}{4\sin^2 x + 5\cos^2 x}$

$= \int_0^{\pi/2} \frac{\sec^2 x}{4\tan^2 x + 5}\,dx$ ∵ Dividing both denominator and numerator by $\cos^2 x$.

Let $u = \tan x$, $du = \sec^2 x\,dx$.

When $x = 0$, $u = \tan 0 = 0$.

When $x = \pi/2$, $u = \tan\frac{\pi}{2} = \infty$.

$\therefore I = \int_0^{\infty} \frac{du}{4u^2+5}$ (improper integral).

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

$$= \frac{1}{4} \int_0^{\infty} \frac{du}{\left[u^2 + \left(\frac{\sqrt{5}}{2}\right)\right]}$$

$$= \frac{1}{4} \times \frac{2}{\sqrt{5}} \left[\tan^{-1}\left(\frac{u}{\sqrt{5}/2}\right)\right].$$

$$= \frac{1}{2\sqrt{5}} (\tan^{-1}\infty - \tan^{-1}0).$$

$$= \frac{1}{2\sqrt{5}} \left(\frac{\pi}{2}\right).$$

$$= \frac{\pi}{4\sqrt{5}}.$$

---

40)

Let A be the population at any time t

$$\frac{dA}{dt} \propto A$$

$$\frac{dA}{dt} = kA \qquad k = 5\% = 0.05$$

$$\int \frac{dA}{A} = \int k\, dt$$

$$\log|A| = kt + c$$

$$A = e^{kt+c}$$

$$A = ce^{kt}$$

$$A = ce^{0.05t}$$

Put $t = 0$, $A = A_0$

$$A_0 = ce^0$$

$$\boxed{c = A_0}$$

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

$A = A_0 e^{0.05t}$



put $t = t_1$, $A = 2A_0$

$$2A_0 = A_0 e^{0.05t_1}$$

$$2 = e^{0.05t_1}$$

$$\log 2 = 0.05t_1$$

$$t_1 = \frac{0.6912}{0.05}$$

$$t_1 \approx 14 \text{ years}$$

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

SS PRITHVI, PRIT~EDUCATION

# MODEL QUESTION PAPER - 5

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

## PART-I.

CHOOSES :

1. If $A$ is a $3\times3$ matrix such that $|3\,adj\,A| = 3$ then $|A|$ is equal to (c) $\pm 1/3$

2. If $A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & -1 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ then the value of $a_{23}$ is (d) $-1$

3. $z_1, z_2$ and $z_3$ be complex numbers such that $z_1+z_2+z_3 = 0$ and $|z_1| = |z_2| = |z_3| = 1$ then $z_1^2 + z_2^2 + z_3^2$ is (d) 0

4. The value of $i^{201} + i^{202} + i^{203}$ is (d) $-1$

5. If $\frac{z-1}{z+1}$ is purely imaginary, then $|z|$ is (b) 1

6. If $f$ and $g$ are polynomials of degrees $m$ and $n$ respectively, and if $h(x) = (fog)(x)$, then the degree of $h$ is (a) $mn$

7. If $\sin^{-1}\frac{x}{5} + \operatorname{cosec}^{-1}\frac{5}{4} = \frac{\pi}{2}$, then the value of $x$ is (d) $\frac{x}{\sqrt{1+x^2}}$

8. The axis of the parabola $y^2 - 2y + 8x + 23 = 0$ is (d) $y = 1$

9. The eccentricity of the hyperbola whose latus rectum is 8 and conjugate axis is equal to half the distance between the foci is (c) $\frac{2}{\sqrt{3}}$

10. If the volume of the parallelopiped with $\vec{a}\times\vec{b}, \vec{b}\times\vec{c}, \vec{c}\times\vec{a}$ as coterminous edges is 8 cubic units, then the volume of the parallelopiped with $(\vec{a}\times\vec{b})\times(\vec{b}\times\vec{c}), (\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a})$

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

and $(\vec{c}\times\vec{a})\times(\vec{a}\times\vec{b})$ as coterminous edges is (c) 64 cu.units

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

11. The angle between the lines $\frac{x-2}{3}=\frac{y+1}{-2}, z=2$ and $\frac{x-1}{1}=\frac{2y+3}{3}=\frac{z+5}{2}$ is (b) $\pi/2$

12. The slope of the curve $y^3-xy^2=4$ at the point where $y=2$ is (a) $1/2$

13. The point of inflection of the curve $y=(x-1)^3$ is (c) $(1,0)$

14. If $f(x,y)=e^{xy}$, then $\frac{\partial^2 f}{\partial x\partial y}$ is equal to (b) $(1+xy)e^{xy}$

15. If $u(x,y)=e^{x^2+y^2}$, then $\frac{\partial u}{\partial x}$ is equal to (b) $2xu$

16. The general solution of the differential equation $\log\left(\frac{dy}{dx}\right)=x+y$ is (b) $e^x+e^{-y}=c$

17. The number of arbitrary constants in the general solutions of order $n$ and $n+1$ are respectively (b) $n, n+1$

18. If the function $f(x)=\frac{1}{12}$ for $a<x<b$, represents a probability density function of a continuous random variable $x$, then which of the following cannot be the value of a and b (d) 16 and 24

PRIT EDUCATION Practice ! Perform ! Perfect !

19. On a multiple-choice exam with 3 possible destructives for each of the 5 questions, the probability that a student will get 4 or more correct answers just by guessing is (a) $\frac{11}{243}$

20. In the last column of the truth table for $\neg(p\vee\neg q)$ the number of final outcomes of the truth value 'F' are (c) 3.

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

PART-II

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

2 marks:

21. Find the rank of the matrix: $\begin{bmatrix} 1 & -2 & -1 & 0 \\ 3 & -6 & -3 & 1 \end{bmatrix}$

Let $A = \begin{bmatrix} 1 & -2 & -1 & 0 \\ 3 & -6 & -3 & 1 \end{bmatrix}$

Order of matrix is $2 \times 4$

$P(A) \leq \min\{2, 4\}$

The highest order of matrix is 4.

Let us consider a $2 \times 2$ matrix

$$\begin{vmatrix} 1 & -2 \\ 3 & -6 \end{vmatrix} = -6+6 = 0$$

$$\begin{vmatrix} -1 & 0 \\ -3 & 1 \end{vmatrix} = -1+0 = -1$$

$\therefore \boxed{P(A) = 1}$

The rank of the word is 1.

22. If $z_1 = 2-i$ and $z_2 = -4+3i$, find the inverse of $\frac{z_1}{z_2}$.

$$\frac{z_1}{z_2} = \frac{2-i}{-4+3i} \times \frac{-4-3i}{-4-3i}$$

$$= \frac{-8-6i+4i+3i^2}{16+9}$$

$$= \frac{-8i-2i-3}{25}$$

$$\frac{z_1}{z_2} = \frac{-11-2i}{25}$$

Inverse of $\frac{z_1}{z_2}$

$$\Rightarrow \left(\frac{z_1}{z_2}\right)^{-1} = \frac{1}{z_1/z_2}$$

$$= \frac{1}{\frac{-11-2i}{25}}$$

$$= \frac{25}{-11-2i} \times \frac{-11+2i}{-11+2i}$$

$$= \frac{25(-11+2i)}{121+4}$$

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

$= \frac{25(-11+2i)}{125}$

$\left(\frac{z_1}{z_2}\right)^{-1} = \frac{1}{5}(-11+2i)$

23. State the reason for $\cos^{-1}[\cos(-\pi/6)] \neq \pi/6$

$\cos^{-1}(\cos x) = x, \quad x \in (0, \pi)$

$\cos^{-1}[\cos(-\pi/6)] = \cos^{-1}(\cos(\pi/6)) = \pi/6$

$\cos^{-1}[\cos(-\pi/6)] \neq -\pi/6$

25. Evaluate : $\lim_{x \to 0}\left[\frac{\sin x}{x^2}\right]$

$\lim_{x \to 0} \frac{\sin x}{x^2} = \frac{\sin 0}{0} = 0/0$

$\Rightarrow$ Indeterminate form.

Applying 'l' Hôpital rule,

$\lim_{x \to 0^+}\left(\frac{\sin x}{x^2}\right) = \lim_{x \to 0^+}\left(\frac{\cos x}{2x}\right)$

$= \frac{\cos(0)}{2(0)}$

$= 1/0$

$= \infty$

$\lim_{x \to 0^-}\left(\frac{\sin x}{x^2}\right) = \lim_{x \to 0^-}\left(\frac{\cos x}{2x}\right)$

$= \frac{\cos(-0)}{2(-0)}$

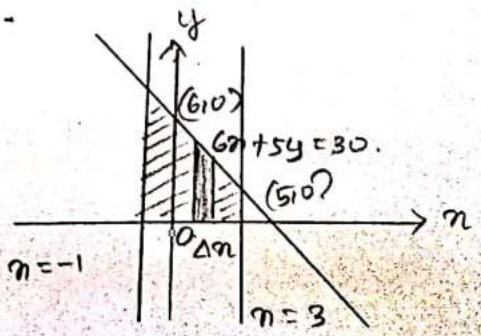
$= \frac{-1}{0}$

$= -\infty$

26. Find the area of the region bounded by the line $6x+5y=30$, x-axis and the lines $x=-1$ and $x=3$.

$A = \int_{-1}^{3} y\,dx = \int_{-1}^{3}\left(\frac{30-6x}{5}\right)dx$

$= \left(\frac{30x-3x^2}{5}\right)_{-1}^{3}$




kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

$= \left(\frac{90-27}{5}\right) - \left(\frac{-30-3}{5}\right)$

$= \frac{96}{5}$

27. solve: $\frac{dy}{dx} + y = e^{-x}$

Given: $\frac{dy}{dx} + y = e^{-x}$ ⇒ This is a linear differential equation.

$\frac{dy}{dx} + Py = Q$. Compare with this equation.

Here $P = 1$, $Q = e^{-x}$

$\int P dx = \int dx = x$

$I.F = e^{\int P dx}$

$= e^{x}$

⇒ $\boxed{ye^{\int P dx} = \int Q e^{\int P dx} dx + c}$.

$ye^{x} = \int e^{-x} e^{x} dx + c$. (or) $ye^{x} = e^{0} + c$ (or) $ye^{x} = x + c$

This is the required solution.

28. Find the mean and variance of $X$, for the probability mass function of $x$ given below:

$$f(x) = \begin{cases} 2(x-1) & 1 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

Mean, $E(x) = \int_{-\infty}^{\infty} x f(x) dx$

$= \int_{1}^{2} x 2(x-1) dx$

$= 2 \int_{1}^{2} (x^2 - x) dx$

$= 2 \left[\frac{x^3}{3} - \frac{x^2}{2}\right]_1^2$

$= 2 \left[(8/3 - 4/2) - (1/3 - 1/2)\right]$

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

$= 2\left[8/3 - 4/2 - 1/3 + 1/2\right]$

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

$= 2\left[7/3 - 3/2\right]$

$= 2\left[\frac{14-9}{6}\right]$

$= 2(5/6)$

$= 5/3$

$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx$

$= \int_1^2 x^2\, 2(x-1)\,dx$

$= 2\int_1^2 (x^3 - x^2)\,dx$

$= 2\left[\frac{x^4}{4} - \frac{x^3}{3}\right]_1^2$

$= 2\left[(16/4 - 8/3) - (1/4 - 1/3)\right]$

$= 2\left[16/4 - 8/3 - 1/4 + 1/3\right]$

$= 2(15/4 - 7/3)$

$= 2\left(\frac{45-28}{12}\right)$

$= 2(17/12)$

$= 17/6$

Variance $= E(X^2) - [E(X)]^2$

$= 17/6 - 25/9$

$= \frac{102-100}{36}$

$= \frac{2}{36}$

$= \frac{1}{18}$

29. Let $A = \{a + \sqrt{5}b : a, b \in Z\}$. Check whether the usual multiplication is a binary operation on $A$.

$A = \{a + \sqrt{5}b, a, b \in Z\}$

Let $a + \sqrt{5}b \in A$ | Also Let $c + \sqrt{5}d \in A$

$a, b \in Z$ | $c, d \in Z$

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

www.Padasalai.Net www.Trb Tnpsc.com of 2.

Addition is binary on Z.

Multiplication is binary on Z.

$(a+\sqrt{5}b)(c+\sqrt{5}d) = ac+ad\sqrt{5}+bc\sqrt{5}+5bd$

$= (ac+5bd)+\sqrt{5}(ad+bc) \in Z$

Yes, usual multiplication is binary on A.

PART-III

3 marks:

8. Solve by matrix inversion method: $5x+2y=4$ ; $7x+3y=5$

$5x+2y=4 \rightarrow (1)$

$7x+3y=5 \rightarrow (2)$

$AX=B$ $\quad A=\begin{bmatrix}5 & 2\\ 7 & 3\end{bmatrix}$, $X=\begin{bmatrix}x\\ y\end{bmatrix}$, $B=\begin{bmatrix}4\\ 5\end{bmatrix}$

$X = A^{-1}B.$

$A^{-1} = \frac{1}{|A|}\,adj\,A.$

$|A| = \begin{vmatrix}5 & 2\\ 7 & 3\end{vmatrix} = 15-14 = 1$

$adj\,A = \begin{bmatrix}3 & -2\\ -7 & 5\end{bmatrix}$

$\therefore A^{-1} = \begin{bmatrix}3 & -2\\ -7 & 5\end{bmatrix}$

$\begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}3 & -2\\ -7 & 5\end{bmatrix}\begin{bmatrix}4\\ 5\end{bmatrix}$

$= \begin{bmatrix}12-10\\ -28+25\end{bmatrix}$

$\begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}2\\ -3\end{bmatrix}$

$\therefore$ The solution is $\boxed{x=2}$ ; $\boxed{y=-3}$

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

32. Solve the equation $2x^3+11x^2-9x-18=0$

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

$2x^3+11x^2-9x-18=0$

Sum of odd power coefficients $= 2-9$
$= -7$

Sum of even power coefficients $= 11-18$
$= -7.$

Sum of odd power coeff. = sum of even power coeff.

$-1$ is a root of $p(x)$

$$\begin{array}{c|cccc} -1 & 2 & 11 & -9 & -18 \\ & 0 & -2 & -9 & 18 \\ \hline & 2 & 9 & -18 & 0 \end{array}$$

The other factor $2x^2+9x-18=0$

$(x+6)(2x-3)=0$

$x+6=0$ $\quad$ $2x-3=0$

$\boxed{x=-6}$ $\quad$ $2x=3$

$\boxed{x=3/2}$

∴ The 3 roots are $-1, -6, 3/2$

PRIT EDUCATION
Practice ! Perform ! Perfect !

33. The maximum and minimum distances of the Earth from the sun respectively are $152\times10^6$ km and $94.5\times10^6$ km. The sun is at one focus of the elliptical orbit. Find the distance from the sun to other focus.

$AS = 94.5\times10^6$ km, $SA' = 152\times10^6$ km

$a+c = 152\times10^6$

$a-c = 94.5\times10^6$

Subtracting $\Rightarrow 2c = 57.5\times10^6$

$= 5.75\times10^5$ km

**Distance $SS' = 5.75\times10^5$ km**

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

34. If $\vec{a} = \hat{i}+2\hat{j}+3\hat{k}$, $\vec{b} = 2\hat{i}-\hat{j}+\hat{k}$, $\vec{c} = 3\hat{i}+2\hat{j}+\hat{k}$ and $\vec{a}\times(\vec{b}\times\vec{c}) = l\vec{a}+m\vec{b}+n\vec{c}$, find the values of $l, m, n$.

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

$\vec{a} = \hat{i}+2\hat{j}+3\hat{k}$, $\vec{b} = 2\hat{i}-\hat{j}+\hat{k}$, $\vec{c} = 3\hat{i}+2\hat{j}+\hat{k}$

$\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c}$

$(\vec{a}\cdot\vec{c})\vec{b} = (3+4+3)(2\hat{i}-\hat{j}+\hat{k})$

$= 10(2\hat{i}-\hat{j}+\hat{k})$

$= 20\hat{i}-10\hat{j}+10\hat{k}$

$(\vec{a}\cdot\vec{b})\vec{c} = (2-2+3)(3\hat{i}+2\hat{j}+\hat{k})$

$= 9\hat{i}+6\hat{j}+3\hat{k}$

$(\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c} = 11\hat{i}-16\hat{j}+7\hat{k}$

$11\hat{i}-16\hat{j}+7\hat{k} = l(\hat{i}+2\hat{j}+3\hat{k}) + m(2\hat{i}-\hat{j}+\hat{k}) + n(3\hat{i}+2\hat{j}+\hat{k})$

Equating the coefficients of $\hat{i}, \hat{j}$ and $\hat{k}$

$l+2m+3n = 11 \rightarrow (1)$

$2l-m-2n = -16 \rightarrow (2)$

$3l+m+n = 7 \rightarrow (3)$

$(2)+(3) \Rightarrow 2l-m+2n = -16$

$3l+m+n = 7$

$5l+3n = -9 \rightarrow (4)$

$(2)\times 2 \Rightarrow 4l-2m+4n = -32 \rightarrow (5)$

$(1)+(5) \Rightarrow l+2m+3n = 11$

$4l-2m+4n = -32$

$5l+7n = -21 \rightarrow (6)$

$(4)-(6) \Rightarrow 5l+3n = -9$

$5l+7n = -21$

$-4n = 12$

$\boxed{n = -3}$

$n = -3 \Rightarrow 5l+3(-3) = -9 \Rightarrow 5l = 0 \Rightarrow \boxed{l = 0}$.

$(1) \Rightarrow 0+2m-9 = 11$

$2m = 20$

$\boxed{m = 10}$

∴ The values of $l, m, n$ are $0, -3$, and $10$

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

35. Find the two positive numbers whose product is 20 and their sum is minimum.

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

Let $x, y$ be the two positive numbers.

Given that product $xy = 20$

$$\boxed{y = \frac{20}{x}} \rightarrow (1)$$

Sum $= x + y$

$$S(x) = x + \frac{20}{x}$$

$$S'(x) = 1 - \frac{20}{x^2}$$

$$S'(x) = 0$$

$$1 - \frac{20}{x^2} = 0$$

$$\frac{20}{x^2} = 1$$

$$x^2 = 20$$

$$\boxed{x = 2\sqrt{5}}$$

$$S''(x) = 0 - 20\left(\frac{-2}{x^3}\right)$$

$$S''(x) = \frac{40}{x^3}$$

$$S''(2\sqrt{5}) = \frac{40}{(2\sqrt{5})^3}$$

$$= \frac{40}{8 \times 5\sqrt{5}}$$

$$= \frac{1}{\sqrt{5}} > 0$$

$\therefore$ Sum is minimum at $x = 2\sqrt{5}$

$x = 2\sqrt{5}$, (1) $\Rightarrow y = 20/2\sqrt{5} = \frac{4 \times 5}{2\sqrt{5}}$

$$\boxed{y = 2\sqrt{5}}$$

$\therefore$ The required two positive number are $2\sqrt{5}, 2\sqrt{5}$.

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

37. Find the area of the region for $2x-y+1=0$, $y=-1$, $y=3$.

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

$2x-y+1=0 \rightarrow (1)$

$y=-1$, $y=3$

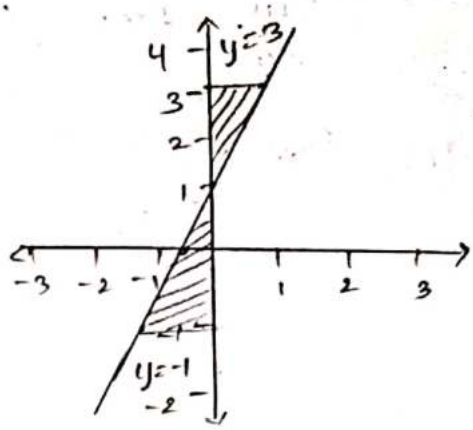
y axis put $x=0$, $(1) \Rightarrow -y+1=0$

$\boxed{y=1}$

$(1) \Rightarrow 2x-y+1=0$

$2x=y-1$

$x=\frac{y-1}{2}$




| $x$ | -1 | -0.5 | 0 | 1 |
|---|---|---|---|---|
| $y$ | -1 | 0 | 1 | 3 |

$A_1$ lies right side of y-axis between $y=1$ to $y=3$

$A_2$ lies left side of the y-axis between $y=-1$ to $y=1$

$$\text{Area} = \int_c^d x\,dy + \int_c^d -x\,dy$$

$$= \int_1^3 \frac{y-1}{2}dy - \int_{-1}^{1} \frac{y-1}{2}dy$$

$$= \frac{1}{2}\left[\int_1^3 (y-1)dy - \int_{-1}^{1}(y-1)dy\right]$$

$$= \frac{1}{2}\left[\left[\frac{y^2}{2}-y\right]_1^3 - \left[\frac{y^2}{2}-y\right]_{-1}^{1}\right]$$

$$= \frac{1}{2}\left[(9/2-3)-(1/2-1)\right] - \left[(1/2-1)-(1/2+1)\right]$$

$$= \frac{1}{2}\left[9/2-3-1/2+1-1/2+1+1/2+1\right]$$

$$= \frac{1}{2}\left[9/2-1/2\right]$$

$$= \frac{1}{2}(8/2)$$

$$= 2$$

38. Find the differential equation for the function $y=2(x^2-1)+ce^{-x^2}$ where c is an arbitrary constant.

Given, $y = 2(x^2-1)+ce^{-x^2}$

where c is an arbitrary constant.

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

Differentiate with respect to $x$,

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

$$\frac{dy}{dx} = 4x - 2x\,ce^{-x^2}$$

38 The differential equation is

$$\frac{dy}{dx} + 2xy - dx^3 = 0.$$

39. If $X \sim B(n,p)$ such that $4P(X=4) = P(X=2)$ and $n=6$. Find the distribution, mean and standard deviation.

$$n=6$$

$$4P(X=4) = P(X=2)$$

$$P(X=x) = \binom{n}{x} p^x (1-p)^{n-x}$$

$$4P(X=4) = P(X=2)$$

$$4\binom{6}{4} p^4 (1-p)^{6-4} = \binom{6}{2} p^2 (1-p)^{6-2}$$

$$4\binom{6}{2} p^4 (1-p)^2 = \binom{6}{2} p^2 (1-p)^4$$

$$4p^2 = (1-p)^2$$

$$4p^2 = 1 + p^2 - 2p$$

$$3p^2 + 2p - 1 = 0$$

$$(p+1)(3p-1) = 0$$

$p = -1$ — not possible

$3p = 1$

$p = 1/3$

$p = 1/3$ $\quad q = 2/3$

$p + q = 1$

$1/3 + q = 1$

$q = 1 - 1/3$

$q = 2/3$

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

Distribution:

www.Padasalai.Net www.Trb Tnpsc.com of 2.

$$P(X=n) = \binom{6}{n}\left(\frac{1}{3}\right)^{n}\left(1-\frac{1}{3}\right)^{6-n}$$

$$= \binom{6}{n}\left(\frac{1}{3}\right)^{n}\left(\frac{2}{3}\right)^{6-n}$$

Mean:

$$= np$$

$$= 6\left(\frac{1}{3}\right)$$

$$= 2$$

$$SD = \sqrt{npq}$$

$$= \sqrt{6\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)}$$

$$= \sqrt{\frac{4}{3}}$$

$$\boxed{SD. = \frac{2}{\sqrt{3}}}$$

PART-IV

5 marks.

41. (a) Test for consistency and if possible, solve the system of equations $2x-y+z=2$, $6x-3y+3z=6$, $4x-2y+2z=4$

Here the no. of unknowns is 3

The matrix system of equations is $AX=B$

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 6 & -3 & 3 \\ 4 & -2 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 2 \\ 6 \\ 4 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

The augmented matrix is

$$[A|B] = \left[\begin{array}{ccc|c} 2 & -1 & 1 & 2 \\ 6 & -3 & 3 & 6 \\ 4 & -2 & 2 & 4 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - 2R_1} \left[\begin{array}{ccc|c} 2 & -1 & 1 & 2 \\ 6 & -3 & 3 & 6 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

$R_2 \to R_2 - 3R_1$ www.Padasalai.Net www.Trb Tnpsc.com of 2.

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

The no. of non-zeros in rows in A is 1.

No. of non-zeros in [A|B] is 1

$\rho(A) = 1 \qquad \rho[A|B] = 1$

It is consistent with infinitely many solutions.

$$x - y + z = 2 \quad \to (1)$$

Take $z = t$ $\quad y = s$

(1) $\Rightarrow x - s + t = 2$

$x = 2 + s - t$

$x = \frac{2+s-t}{2}$

$x = \frac{1}{2}(s - t + 2)$

$s, t \in R.$

b) Find all the cubic roots of $\sqrt{3} + i$.

$z = \sqrt[3]{\sqrt{3}+i}$

$z = (\sqrt{3}+i)^{1/3}$

$\sqrt{3}+i = r(\cos\theta + i\sin\theta)$

$r = \sqrt{(\sqrt{3})^2 + 1^2}$

$r = \sqrt{3+1}$

$r = \sqrt{4}$

$r = 2$

$\theta = \tan^{-1}|y/x|$

$\theta = \tan^{-1}|1/\sqrt{3}|$

$\theta = \pi/6$

$\theta$ lies in the 1st quadrant

$\theta = A \Rightarrow \theta = \pi/6$

$\sqrt{3}+i = r(\cos\theta + i\sin\theta)$

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

$\sqrt{3}+i = 2\left(\cos\pi/6 + i\sin\pi/6\right)$

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

$z = (\sqrt{3}+i)^{1/3}$

$z = \left[2\left(\cos\pi/6 + i\sin\pi/6\right)\right]^{1/3}$

$z = 2^{1/3}\left[\cos\pi/6 + i\sin\pi/6\right]^{1/3}$

$z = 2^{1/3}\left[\cos(2k\pi+\pi/6) + i\sin(2k\pi+\pi/6)\right]^{1/3}$

$z = (2)^{1/3}\left[\cos\left(\frac{12k\pi+\pi}{6}\right) + i\sin\left(\frac{12k\pi+\pi}{6}\right)\right]^{1/3}$

$z = (2)^{1/3}\left[\cos\left(\frac{12k\pi+\pi}{18}\right) + i\sin\left(\frac{12k\pi+\pi}{18}\right)\right]$

When $k = 0, 1, 2$

Where $k=0$,

$(2^{1/3})\left[\cos\pi/18 + i\sin\pi/18\right]$

When $k=1$,

$(2^{1/3})\left[\cos 25\pi/18 + i\sin\frac{25\pi}{18}\right]$

$(2)^{1/3}\left[\cos\left(\pi+\frac{7\pi}{18}\right) + i\sin\left(\pi+\frac{7\pi}{18}\right)\right]$

$(2)^{1/3}\left[-\cos\frac{7\pi}{18} - i\sin\frac{7\pi}{18}\right]$

42. Find the polynomial equation of minimum degree with rational coefficients, having $\sqrt{5}-\sqrt{3}$ as a root.

Let one root be $\sqrt{5}-\sqrt{3}$ since the coefficient are rational.

$\sqrt{5}+\sqrt{3}$ is also other root

Sum $= \sqrt{5}-\sqrt{3}+\sqrt{5}+\sqrt{3}$

$= 2\sqrt{5}$

Product $= (\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})$

$= 5-3$

$= 2$

The corresponding factor is $x^2 - 2\sqrt{5}x + 2 = 0$.

Let the other roots are $-\sqrt{5}+\sqrt{3}$, $-\sqrt{5}-\sqrt{3}$.

Sum $\Rightarrow -\sqrt{5}+\sqrt{3}-\sqrt{5}-\sqrt{3}$

$= -2\sqrt{5}$

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

www.Padasalai.Net www.Trb Tnpsc.com of 2.

Product $\Rightarrow (-\sqrt{5}+\sqrt{3})(-\sqrt{5}-\sqrt{3})$

$= -3-\sqrt{15}+\sqrt{15}+5$

$= 2$

The other corresponding factor is $x^2+2\sqrt{5}x+2=0$

The polynomial of minimum degree with rational coefficient is

$\therefore (x^2+2\sqrt{5}x+2)(x^2-2\sqrt{5}x+2)=0$

$(x^2+2+2\sqrt{5}x)(x^2+2-2\sqrt{5}x)=0$

$x^4+4+4x^2-20x^2=0$

$\boxed{x^4-16x^2+4=0}$

(b) If D is the midpoint of the side BC of $\triangle ABC$, show by vector method that $|\vec{AB}|^2+|\vec{AC}|^2=2(|\vec{AD}|^2+|\vec{BD}|^2)$.

Let A be the origin, $\vec{b}$ be the position vector of B and $\vec{c}$ be the position vector of C.




D is the midpoint of BC and so the position vector of D $\Rightarrow \frac{\vec{b}+\vec{c}}{2}$

$|\vec{AD}|^2 = \vec{AD}\cdot\vec{AD} = \left(\frac{\vec{b}+\vec{c}}{2}\right)\cdot\left(\frac{\vec{b}+\vec{c}}{2}\right)$

$= \frac{1}{4}(|\vec{b}|^2+|\vec{c}|^2+2\vec{b}\cdot\vec{c}) \rightarrow (1)$

$\vec{BD} = \vec{AD}-\vec{AB} = \frac{\vec{b}+\vec{c}}{2}-b$

$= \frac{\vec{b}+\vec{c}-2b}{2}$

$\vec{BD} = \frac{\vec{c}-\vec{b}}{2}$

$|\vec{BD}|^2 = \vec{BD}\cdot\vec{BD} = \left(\frac{\vec{c}-\vec{b}}{2}\right)\left(\frac{\vec{c}-\vec{b}}{2}\right)$

$= \frac{1}{4}(|\vec{b}|^2+|\vec{c}|^2-2\vec{b}\cdot\vec{c}) \rightarrow (2)$

$|\vec{AD}|^2+|\vec{BD}|^2 = \frac{1}{4}(|\vec{b}|^2+|\vec{c}|^2+2\vec{b}\cdot\vec{c})+\frac{1}{4}(|\vec{b}|^2+|\vec{c}|^2-2\vec{b}\cdot\vec{c})$.

$= \frac{1}{2}(|\vec{b}|^2+|\vec{c}|^2)$.

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

$|\vec{AD}|^2 + |\vec{BD}|^2 = \frac{1}{2}(|\vec{AB}|^2 + |\vec{AC}|^2)$

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

$$|\vec{AB}|^2 + |\vec{AC}|^2 = 2[|\vec{AD}|^2 + |\vec{BD}|^2]$$

Hence, proved.

43. a) Show that the straight lines $\vec{r} = (5\hat{i}+7\hat{j}-3\hat{k}) + s(4\hat{i}+4\hat{j}-5\hat{k})$ and $\vec{r} = (8\hat{i}+4\hat{j}+5\hat{k}) + t(7\hat{i}+\hat{j}+3\hat{k})$ are coplanar. Find the vector equation of the plane in which they lie.

Let $\vec{a} = 5\hat{i}+7\hat{j}-3\hat{k}$, $\vec{b} = 4\hat{i}+4\hat{j}-5\hat{k}$,

$\vec{c} = 8\hat{i}+4\hat{j}+5\hat{k}$, $\vec{d} = 7\hat{i}+\hat{j}+3\hat{k}$.

condition for coplanarity $\Rightarrow (\vec{c}-\vec{a}) \cdot (\vec{b}\times\vec{d}) = 0$.

$\vec{c}-\vec{a} = 3\hat{i}-3\hat{j}+8\hat{k}$

$$\vec{b}\times\vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 4 & -5 \\ 7 & 1 & 3 \end{vmatrix} = \hat{i}(12+5) - \hat{j}(12+35) + \hat{k}(4-28)$$

$$= 17\hat{i} - 47\hat{j} - 24\hat{k}$$

$(\vec{c}-\vec{a}) \cdot (\vec{b}\times\vec{d}) = (3\hat{i}-3\hat{j}+8\hat{k}) \cdot (17\hat{i}-47\hat{j}-24\hat{k})$

$= 51 + 141 - 192$

$= 192 - 192$

$= 0$.

$\therefore (\vec{c}-\vec{a}) \cdot (\vec{b}\times\vec{d}) = 0$

Hence, the given lines are coplanar.

To find the vector equation of the plane,

$\vec{r} = \vec{a} + t\vec{b} + s\vec{d}$. $(\vec{r}-\vec{a}) \cdot (\vec{b}\times\vec{d}) = 0$.

$\vec{r} - (5\hat{i}+7\hat{j}-3\hat{k}) \cdot (17\hat{i}-47\hat{j}-24\hat{k}) = 0$

$(\vec{r}-\vec{a}) \cdot (\vec{b}\times\vec{d}) = 0$

$\vec{r} \cdot (\vec{b}\times\vec{d}) - \vec{a}(\vec{b}\times\vec{d}) = 0$

$\vec{r}(17\hat{i}-47\hat{j}-24\hat{k}) - (5\hat{i}+7\hat{j}-3\hat{k})(17\hat{i}-47\hat{j}-24\hat{k}) = 0$

$\vec{r}(17\hat{i}-47\hat{j}-24\hat{k}) - (85 - 329 + 72) = 0$

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

www.Padasalai.Net www.Trb Tnpsc.com of 2.

$\vec{r}\cdot(17\hat{i}-4\hat{j}-24\hat{k})-17=0$

$$\boxed{\vec{r}\cdot(17\hat{i}-4\hat{j}-24\hat{k})=17}$$

44. (a) If the curves $ax^2+by^2=1$ and $cx^2+dy^2=1$ intersect each other orthogonally then $\frac{1}{a}-\frac{1}{b}=\frac{1}{c}-\frac{1}{d}$

$$ax^2+by^2=1 \rightarrow (1)$$
$$cx^2+dy^2=1 \rightarrow (2)$$

(1) $\times c \Rightarrow acx^2+bcy^2=c$

(2) $\times a \Rightarrow acx^2+ady^2=a$

$$(bc-ad)y^2=c-a$$

$$y^2=\frac{c-a}{bc-ad}$$

$$\boxed{y=\pm\sqrt{\frac{c-a}{bc-ad}}}$$

(1) $\times d \Rightarrow adx^2+bdy^2=d$

(2) $\times b \Rightarrow bcx^2+bdy^2=b$

$$(ad-bc)x^2=d-b$$

$$x^2=\frac{d-b}{ad-bc}$$

$$\boxed{x=\pm\sqrt{\frac{d-b}{ad-bc}}}$$

The point of intersection $\left(\pm\sqrt{\frac{d-b}{ad-bc}},\ \pm\sqrt{\frac{c-a}{bc-ad}}\right)$

$ax^2+by^2=1$

diff $a(2x)+b(2y)\frac{dy}{dx}=0$

$by\times\frac{dy}{dx}=-ax$

$$\boxed{\frac{dy}{dx}=\frac{-ax}{by}} \rightarrow m_1$$

$cx^2+dy^2=1$

diff $c(2x)+d(2y)\frac{dy}{dx}=0$

$dy\frac{dy}{dx}=-cx$

$$\boxed{\frac{dy}{dx}=\frac{-cx}{dy}} \rightarrow m_2$$

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

Given the curves intersect orthogonally

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

$m_1 \times m_2 = -1$

$$\frac{-ax}{by} \times \frac{-cx}{dy} = -1$$

$$acx^2 = -bdy^2$$

at $\left(\pm\sqrt{\frac{d-b}{ad-bc}}, \pm\sqrt{\frac{c-a}{bc-ad}}\right)$

$$ac\left(\frac{d-b}{ad-bc}\right) = -bd\left(\frac{c-a}{bc-ad}\right)$$

$$ac\left(\frac{d-b}{ad-bc}\right) = bd\left(\frac{c-a}{ad-bc}\right)$$

$$ac(d-b) = bd(c-a)$$

$$\frac{d-b}{bd} = \frac{c-a}{ac}$$

$$\frac{d}{bd} - \frac{b}{bd} = \frac{c}{ac} - \frac{a}{ac}$$

$$\frac{1}{b} - \frac{1}{d} = \frac{1}{a} - \frac{1}{c}$$

$$\boxed{\frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}}$$ Hence, proved.

45. a) Prove that $\int_0^{\pi/4} \log(1+\tan x)\,dx = \pi/8 \log 2$.

$$I = \int_0^{\pi/4} \log(1+\tan x)\,dx \quad \rightarrow (1)$$

Property $\Rightarrow$ $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$ in (1).

$$I = \int_0^{\pi/4} \log\left[1 + \tan(\pi/4 - x)\right] dx$$

$$= \int_0^{\pi/4} \log\left[1 + \frac{\tan\pi/4 - \tan x}{1 + \tan\pi/4 \tan x}\right] dx$$

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

$$= \int_0^{\pi/4} \log\left[1 + \frac{1-\tan x}{1+\tan x}\right] dx$$

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

$$= \int_0^{\pi/4} \log\left[\frac{1+\tan x + 1 - \tan x}{1+\tan x}\right] dx$$

$$= \int_0^{\pi/4} \log\left[\frac{2}{1+\tan x}\right] dx$$

$$= \int_0^{\pi/4} [\log 2 - \log(1+\tan x)] dx$$

$$= [\log 2 \int_0^{\pi/4} dx - \int_0^{\pi/4} \log(1+\tan x) dx$$

$$I = \pi/4 \log 2 - I$$

$$I + I = \pi/4 \log 2$$

$$2I = \pi/4 \log 2$$

$$\boxed{I = \pi/8 \log 2}$$

To solve : $\frac{dy}{dx} = \frac{x-y+5}{2(x-y)+7}$

Put $z = x - y$

$$\Rightarrow \frac{dy}{dx} = \frac{z+5}{2z+7} \quad (1)$$

$z = x - y$

$$\frac{dz}{dx} = 1 - \frac{dy}{dx}$$

$$\frac{dy}{dx} = 1 - \frac{dz}{dx}$$

Substitute in (1)

$$1 - \frac{dz}{dx} = \frac{z+5}{2z+7}$$

$$\frac{dz}{dx} = 1 - \frac{z+5}{2z+7}$$

$$\frac{dz}{dx} = \frac{z+2}{2z+7}$$

PRIT EDUCATION Practice ! Perform ! Perfect !
www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

$$\frac{2z+7}{z+2}\,dz = dx$$

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

$$\frac{2(z+2)+3}{(z+2)}\,dz = dx$$

Integrate on both sides

$$\int \frac{2(z+2)+3}{(z+2)}\,dz = \int dx$$

$$2z + 3\log|z+2| = x+c$$

$$\boxed{2(x-y)+3\log|x-y+2| = x+c}$$

46. b) Establish the equivalence property $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$.

| $p$ | $q$ | $p \to q$ | $q \to p$ | $p \leftrightarrow q$ | $(p \to q) \wedge (q \to p)$ |
|---|---|---|---|---|---|
| T | T | T | T | T | T |
| T | F | F | T | F | F |
| F | T | T | F | F | F |
| F | F | T | T | T | T |

$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

They are equivalent.

47. b) Find the equation of the circle through the points $(1,0)$, $(-1,0)$ and $(0,1)$.

General equation. $\Rightarrow x^2+y^2+2gx+2fy+c=0$.

substitute the points in this equation.

$(1,0) \Rightarrow 1+0+2g(1)+2f(0)+c=0$.

$$\boxed{2g+c=-1} \rightarrow (1)$$

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

$(-1,0) \Rightarrow (-1)^2 + (0)^2 + 2g(-1) + 2f(0) + c = 0$

$1 + c - 2g = 0$

$\boxed{-2g + c = -1} \rightarrow (2)$

$(0,1) \Rightarrow (0)^2 + (1)^2 + 2g(0) + 2f(1) + c = 0$

$1 + 2f + c = 0$

$\boxed{2f + c = -1} \rightarrow (3)$

Solve these equations,

(1) & (2) $\Rightarrow$ $2g + c = -1$

$-2g + c = -1$

$2c = -2$

$\boxed{c = -1}$

(2) $\Rightarrow$ $-2g - 1 = -1$

$-2g = 0$

$\boxed{g = 0}$

(3) $\Rightarrow$ $2f - 1 = -1$

$2f = 0$

$\boxed{f = 0}$

The equation of Circle, $\boxed{x^2 + y^2 = 1}$

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

Part - II

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

24) $16y^2 - 9x^2 = 144$

$144 \div \frac{y^2}{9} - \frac{x^2}{16} = 1$

$a^2 = 9 \Rightarrow a = 3.$

$b^2 = 16 \Rightarrow b = 4$

length of latus rectum $= \frac{2b^2}{a}$

$= \frac{2(16)}{3}$

$= \frac{32}{3}$

27) Solve: $\frac{dy}{dx} + y = e^{-x}$

Compare, $\frac{dy}{dx} + Py = Q$

$P = 1, \quad Q = e^{-x}.$

$I.F = e^{\int P dx}$

$= e^{\int 1 dx}$

$= e^x$

$\boxed{I.F = e^x}$

The general solution,

$y(I.F) = \int Q (I.F) dx + C$

$ye^x = \int e^{-x} \times e^x dx + C$

$ye^x = \int dx + C$

$ye^x = x + C$

$y = xe^{-x} + Ce^{-x}$ (or) $y = e^{-x}(x + C)$

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

30)

$\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

$\vec{n} = 3\hat{i} + 4\hat{j} + 3\hat{k}$

vector equation,

$\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$

$\vec{r} \cdot (3\hat{i} + 4\hat{j} + 3\hat{k}) = (2\hat{i} + 2\hat{j} + 3\hat{k}) \cdot (3\hat{i} + 4\hat{j} + 3\hat{k})$

$= 6 + 8 + 9$

$\boxed{\vec{r} \cdot (3\hat{i} + 4\hat{j} + 3\hat{k}) = 23}$

Part - III

36) Let $y = (x)^{1/5}$ $\sqrt[5]{31}$

$dy = \frac{1}{5} x^{\frac{1}{5}-1} dx$

$dy = \frac{1}{5} x^{-4/5} dx$

$x_0 = 32$, $\Delta x = -1$

$f'(x_0)\Delta x = dy = \frac{1}{5}(32)^{-4/5}(-1)$

$= \frac{-1}{5}(2^5)^{-4/5}$

$= -\frac{1}{5} \times \frac{1}{16}$

$f'(x_0)\Delta x = -0.0125$

$f(x + \Delta x) \approx f(x_0) + f'(x_0)\Delta x$

$\sqrt[5]{31} \approx 2 + (-0.0125)$

$\sqrt[5]{31} \approx 1.9875$

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

40) $x+iy = \sqrt{\frac{a+ib}{c+id}}$ www.Padasalai.Net. www.Trb Tnpsc.com of 2.

$$(x+iy)^2 = \frac{a+ib}{c+id}$$

$$x^2 - y^2 + i2xy = \frac{a+ib}{c+id} \times \frac{c-id}{c-id}$$

$$x^2 - y^2 + i2xy = \frac{ac - iad + ibc + bd}{c^2+d^2}$$

$$x^2 - y^2 + i2xy = \frac{(ac+bd)}{c^2+d^2} + \frac{i(bc-ad)}{c^2+d^2}$$

$$x^2 - y^2 = \frac{ac+bd}{c^2+d^2}, \quad 2xy = \frac{bc-ad}{c^2+d^2}$$

$$(x^2+y^2)^2 = (x^2-y^2)^2 + (2xy)^2$$

$$= \left(\frac{ac+bd}{c^2+d^2}\right)^2 + \left(\frac{bc-ad}{c^2+d^2}\right)^2$$

$$= \frac{a^2c^2 + 2abcd + b^2d^2 + b^2c^2 + a^2d^2 - 2abcd}{(c^2+d^2)^2}$$

$$= \frac{c^2(a^2+b^2) + d^2(a^2+b^2)}{(c^2+d^2)^2}$$

$$= \frac{(a^2+b^2)(c^2+d^2)}{(c^2+d^2)^2}$$

$$(x^2+y^2)^2 = \frac{a^2+b^2}{c^2+d^2}$$

$$\boxed{x^2+y^2 = \sqrt{\frac{a^2+b^2}{c^2+d^2}}}$$

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

Part - IV

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

44) (b) $z(x,y) = xe^{y} + ye^{-x}$, $x = e^{-t}$, $y = st^2$

$\frac{\partial z}{\partial x} = e^{y} - ye^{-x}$ ; $\frac{\partial x}{\partial t} = -e^{-t}$ ; $\frac{\partial y}{\partial t} = 2st$

$\frac{\partial z}{\partial y} = xe^{y} + e^{-x}$ ; $\frac{\partial x}{\partial s} = 0$ ; $\frac{\partial y}{\partial s} = t^2$

$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial s}$

$= (e^{y} - ye^{-x})(0) + (xe^{y} + e^{-x})(t^2)$

$= t^2(xe^{y} + e^{-x})$

$$\boxed{\frac{\partial z}{\partial s} = t^2\left(e^{-t}e^{st^2} + e^{-(e^{-t})}\right)}$$

$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial t}$

$= (e^{y} - ye^{-x})(-e^{-t}) + (xe^{y} + e^{-x})(2st)$

$= 2st\left(e^{-t}e^{st^2} + e^{-(e^{-t})}\right) - e^{-t}\left(e^{st^2} - (st^2)(e^{-e^{-t}})\right)$

$= 2ste^{-t}e^{st^2} + 2ste^{-(e^{-t})} - e^{-t}e^{st^2} + st^2e^{-t}e^{-e^{-t}}$

$$\boxed{\frac{\partial z}{\partial t} = e^{-t}e^{st^2}(2st - 1) + ste^{-(e^{-t})}(2 + te^{-t})}$$

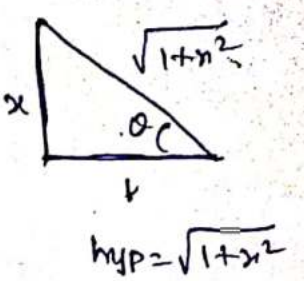
PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

47) www.Padasalai.Net. www.Trb Tnpsc.com of 2.

(a) Solve : $\cos(\tan^{-1}x) = \sin(\cot^{-1}(3/4))$

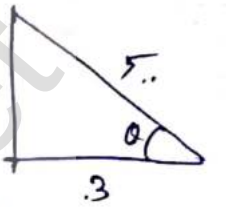
$\Rightarrow \tan^{-1}x = \cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)$




$\cos(\tan^{-1}x) = \cos\left(\cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)\right)$

$\cos(\tan^{-1}x) = \frac{1}{\sqrt{1+x^2}}$ —①

$\Rightarrow \cot^{-1}(3/4) = \sin^{-1}(4/5)$




$\sin(\cot^{-1}(3/4)) = \sin(\sin^{-1}(4/5))$

$\sin(\cot^{-1}(3/4)) = \frac{4}{5}$ —②

① = ②

$\frac{1}{\sqrt{1+x^2}} = \frac{4}{5}$

$\frac{1}{1+x^2} = \frac{16}{25}$

$1+x^2 = \frac{25}{16}$

$x^2 = \frac{25}{16} - 1$

$x^2 = \frac{25-16}{16}$

$x^2 = \frac{9}{16}$

$\boxed{x = \pm 3/4}$

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

46)

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

(a)

$np + npq = 1.8$ , $n = 5$

$np(1+q) = 1.8$

$5p(1+q) = 1.8$

$5p[1+(1-p)] = 1.8$

$5p + 5p - 5p^2 = 1.8$

$5p^2 - 10p + 1.8 = 0$

$5p^2 - p - 9p + 1.8 = 0$

$p(5p-1) - 1.8(5p-1) = 0$

$(5p-1)(p-1.8) = 0$

$5p = 1$ | $p = 1.8 > 1$ (not possible)

$p = \frac{1}{5}$ |

$q < 1$

$-q$

$-1 \; 0$

$\Rightarrow q = 1-p = 1-\frac{1}{5} = \frac{4}{5}$

$P(X=x) = {}^{n}C_{x}\, p^{x}\, q^{n-x}$ , $x = 0, 1, 2, \ldots n$,

$\boxed{P(X=x) = {}^{5}C_{x} \left(\frac{1}{5}\right)^{x} \left(\frac{4}{5}\right)^{5-x}}$ , $x = 0, 1, 2, \ldots 15$

(or)

$P(X=x) = \binom{5}{x} \left(\frac{1}{5}\right)^{x} \left(\frac{4}{5}\right)^{5-x}$ , $x = 0, 1, 2, \ldots 5$.

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

43) (b) The volume of a cylinder is equal to V cubic cm, where V is a constant. Find the condition that minimize the total surface area of the cylinder..

www.Padasalai.Net www.Pgtb.Trbsc.com of 2.

V be the volume of the cylinder.

$$V = \pi r^2 h$$

$$\boxed{h = \frac{V}{\pi r^2}}$$

Total surface area, $S = 2\pi rh + 2\pi r^2$

$$= 2\pi r\left(\frac{V}{\pi r^2}\right) + 2\pi r^2$$

$$S(r) = \frac{2V}{r} + 2\pi r^2$$

$$S'(r) = 2V\left(-\frac{1}{r^2}\right) + 4\pi r$$

$$S''(r) = -2V\left(\frac{-2}{r^3}\right) + 4\pi$$

$$S''(r) = \frac{4V}{r^3} + 4\pi$$

$$\Rightarrow S'(r) = 0$$

$$\frac{-2V}{r^2} + 4\pi r = 0$$

$$-2V + 4\pi r^3 = 0$$

$$4\pi r^3 = 2V$$

$$r^3 = \frac{V}{2\pi}$$

$$r = \left(\frac{V}{2\pi}\right)^{1/3}$$

Radius not in negative.

$$S''(r) = \frac{4V}{r^3} + 4\pi > 0$$

$S''(r) > 0$ Thus S is minimum when r is minimum.

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2

SS PRITHVI, PRIT~EDUCATION

# Model question paper - 6.

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

## PART - I

choose the best answer :-

1.) If $P(A) = P(A/B)$, then the system of linear equations $AX = B$ is b) consistent

2.) Let A be a non-singular matrix then which one is false :
c) $Adj(adj A) = |A|^n A$.

3.) If $I = \frac{(\sqrt{3}+i)^3 (3i+4)^2}{(8+6i)^2}$, then $|I|$ is c) 2

4) The continued product of the four values of $[\cos\pi/3 + i\sin\frac{\pi}{3}]^{3/4}$ is a) $\pi$

5) The value of $\sin^{-1}(2\cos^2 x - 1) + \cos^{-1}(1 - 2\sin^2 x)$ is a) $\pi/2$

6) The area of a quadrilateral formed with foci of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ & $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$ is b) $2(a^2+b^2)$

7.) An ellipse has OB as semi minor axes F and F', its foci and the angle FBF' is a right angle. Then the eccentricity of the ellipse. a) $1/\sqrt{2}$

8) If $\vec{a}, \vec{b}, \vec{c}$, are three non-coplanar vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{b}+\vec{c})/\sqrt{2}$, then the angle b/w $\vec{a}$ & $\vec{b}$ is b) $3\pi/4$

9) The equ. of plane passing through (3, 4, 5) & ||el to the plane $x + 2y - 2z - 9 = 0$ is. c) $x + 2y - 2z = 1$

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

www.Padasalai.Net. www.Trb.Tnpsc.com of 2.

10) If $[\vec{a},\vec{b},\vec{c}]=1$, then value of $\frac{\vec{a}.(\vec{b}\times\vec{c})}{(\vec{c}\times\vec{a}).\vec{b}} + \frac{\vec{b}.(\vec{c}\times\vec{a})}{(\vec{a}\times\vec{b})\vec{c}} + \frac{\vec{c}(\vec{a}\times\vec{b})}{(\vec{c}\times\vec{b}).\vec{a}}$ is **c) 2**

11) The slope of the line normal to the curve $f(x) = 2\cos 4x$ at $x = \pi/12$ is c) $\sqrt{3}/12$

12) The number given by the Rolle's theorem for the function $x^3 - 3x^2$, $x \in [0,3]$ is **d) 2**

13) If $f(x,y,z) = xy+yz+3x$, then $f_x - f_y$ is equal to **a) $z-x$.**

14) If $f(x) = \int_0^{\vec{x}} t\cot t\, dt$, then $\frac{df}{dx} \Rightarrow$ **c) $x\cos x$**

15) The value of $\int_{-\pi/2}^{\pi/2} \frac{\sin x}{2+\cos x}\, dx$ is **a) 0**

16) The order and degree of differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{1/3} + x^{1/4} = 0$ are respectively **a) 2, 3**

17) The solution of differential equations $2x\frac{dy}{dx} - y = 3$ represents **c) parabola**

18) A random variable X has binomial distribution with $n = 25$, $p = 0.8$, then standard deviation of X is **d) 2**

19) If $f(x) = \begin{cases} 2x & 0 \le x \le a \\ 0 & \text{otherwise} \end{cases}$ is a probability dense function of a random variable, the value then of a is **a) 1**

20) Subtraction is not a binary operation on **c) N**

PRIT EDUCATION Practice ! Perform ! Perfect
www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

## Part - II

**II) Answer the following (2 Marks).**

21) If $adj(A) = \begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$, find $A^{-1}$.

Sol

$$A^{-1} = \pm \frac{1}{\sqrt{|adj A|}} adj A$$

$$|adj A| = \begin{vmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{vmatrix} = 0(12+0) + 2(36-18) + 0(0+6)$$

$$= 2(18)$$

$$= 36 \neq 0.$$

$$\sqrt{|adj A|} = 6.$$

$$A^{-1} = \pm \frac{1}{\sqrt{|adj A|}} adj A$$

$$A^{-1} = \pm \frac{1}{6} \begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$$

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

23) Find the centre and radius of the circle $2x^2+2y^2-6x+4y+2=0$.

Sol $2g=-3$ ; $-f=1$.

$-g=3/2$

centre $(3/2, 1)$.

$R=\sqrt{\frac{9}{4}+1-1}$

$R=3/2$

24) Find the angle b/w st. lines $\vec{r}=(2\hat{i}+3\hat{j}+\hat{k})+t(\hat{i}-\hat{j}+\hat{k})$ & plane $2x-y+z=5$.

Sol

$$\theta=\sin^{-1}\left(\frac{|\vec{b}\cdot\vec{n}|}{|\vec{b}||\vec{n}|}\right).$$

$$=\sin^{-1}\left(\frac{\left|[\hat{i}-\hat{j}+\hat{k}]\cdot[2\hat{i}-\hat{j}+\hat{k}]\right|}{|\hat{i}-\hat{j}+\hat{k}|\,|2\hat{i}-\hat{j}+\hat{k}|}\right)$$

$$=\sin^{-1}\left(\frac{|2+1+1|}{\sqrt{1+1+1}\ \sqrt{4+1+1}}\right)$$

$$=\sin^{-1}\frac{4}{\sqrt{3}(\sqrt{6})}=\sin^{-1}\frac{4}{\sqrt{3}(\sqrt{3}\times\sqrt{2})}$$

$$=\sin^{-1}\frac{4}{3\sqrt{2}}$$

$$=\sin^{-1}\frac{2(2)}{\sqrt{2}(3)}$$

$$=\sin^{-1}\frac{(\sqrt{2})2}{3}$$

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

25) Explain why langrange mean value theorm is not applicable to function $f(x) = \left|\frac{1}{x}\right|$ ; $x \in [-1,1]$. [creative sum]

$f(x)$ is not continuous at $x = 0$.

Ans.

27) Find the differential equation of family of parabola $y^2 = 4ax$ where $a$ is an arbitary constant.

Sol

The equation of family of parabolas is given by $y^2 = 4ax$, $a$ is an arbitary constant $\rightarrow (1)$

$(1) \Rightarrow$ differentiating both sides,

$$2y\frac{dy}{dx} = 4a.$$

$$\boxed{a = \frac{y}{2}\frac{dy}{dx}.}$$

Sub "a" in eqn (1)

$$(1) \Rightarrow \boxed{\frac{dy}{dx} = \frac{y}{2x}.}$$

28) Compute $P(X = k)$ for binomial distribution $B(n, p)$, where $n = 10$; $p = 1/5$; $k = 5$.

Sol

$$P(X = x) = \binom{n}{x} p^k (1-p)^{n-k}.$$

$$P(X = 5) = \binom{10}{5}\left(\frac{1}{5}\right)^5 \left(1 - \frac{1}{5}\right)^{10-5}$$

$$= \frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2 \times 1}\left(\frac{1}{5}\right)^5\left(\frac{4}{5}\right)^5$$

$$= 252\left(\frac{1}{5}\right)^5\left(\frac{4}{5}\right)^5$$

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

www.Padasalai.Net www.Trb Tnpse.com of 2.

29) write the statement of $\neg p$, $q \vee \neg p$, where p is "It's cold", q is "It is raining".

Sol. * $\neg p \Rightarrow$ It is not cold

* $q \vee \neg p \Rightarrow$ It is raining (or) it is not cold.

30) Find the value of $\cos^{-1}\left(\frac{1}{2}\right) - \sin^{-1}\left(\frac{-1}{2}\right)$.

Sol Let $x = \cos^{-1}\left(\frac{1}{2}\right)$. $y = \sin^{-1}(-1/2)$

$\cos x = 1/2$ $\sin y = -1/2$.

$x = \pi/3$ $y = -\pi/6$

$\Rightarrow x - 2y$

$\Rightarrow \frac{\pi}{3} - 2\left(\frac{-\pi}{6}\right)$

$= \frac{\pi}{3} + \frac{\pi}{3}$

$= 2\pi/3$

---

Part - III

3-marks.

31) Find the adjoint of $A = \begin{bmatrix} 1 & 3 \\ 2 & -5 \end{bmatrix}$ & verify that $A(\text{adj}A) = (\text{adj}A)A = |A|I$

Sol $\text{adj}A = \begin{bmatrix} -5 & -3 \\ -2 & 1 \end{bmatrix}$

$A(\text{adj}A) = \begin{bmatrix} 1 & 3 \\ 2 & -5 \end{bmatrix}\begin{bmatrix} -5 & -3 \\ -2 & 1 \end{bmatrix}$

$= \begin{bmatrix} -5+(-6) & -3+3 \\ -10+10 & -6+(-5) \end{bmatrix}$

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

$$= \begin{bmatrix} -11 & 0 \\ 0 & -11 \end{bmatrix} \longrightarrow (1).$$

$$(\text{Adj } A)A = \begin{bmatrix} -5 & -3 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 2 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} 5-6 & -15+15 \\ -2+2 & -6-5 \end{bmatrix}$$

$$= \begin{bmatrix} -11 & 0 \\ 0 & -11 \end{bmatrix} \longrightarrow (2).$$

$$|A|I_2 \Rightarrow |A| = \begin{vmatrix} 1 & 3 \\ 2 & -5 \end{vmatrix} = -5-(6) = -11$$

$$\Rightarrow -11\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -11 & 0 \\ 0 & -11 \end{bmatrix} \longrightarrow (3)$$

From (1), (2) & (2).

Hence it is proved that

$$\boxed{(\text{Adj } A)A = A(\text{adj } A) = |A|I_2}$$

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

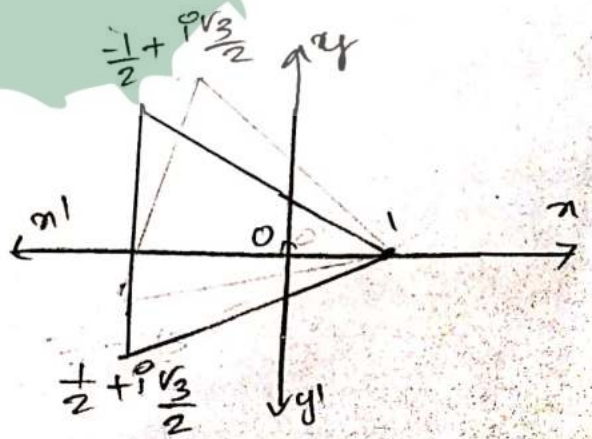
32) S.T the points $1, \frac{-1}{2} + \frac{i\sqrt{3}}{2}$ & $\frac{-1}{2} - \frac{i\sqrt{3}}{2}$ are vertices of the equilateral triangle.

Sol.

$z_1 = 1$

$z_2 = -1/2 + i\sqrt{3}/2$

$z_3 = -1/2 - i\sqrt{3}/2$




kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

$|z_1 - z_2| = \left| 1 - \left(\frac{-1}{2} + i\frac{\sqrt{3}}{2}\right) \right|$

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

$= \left| \frac{3}{2} + \left(-\frac{\sqrt{3}}{2} i\right) \right|$

$= \sqrt{\frac{9}{4} + \frac{3}{4}}$

$= \frac{2\sqrt{3}}{2} = \sqrt{3}. \longrightarrow (1)$

$|z_2 - z_3| = \left| -\frac{1}{2} + i\frac{\sqrt{3}}{2} + \frac{1}{2} + \frac{i\sqrt{3}}{2} \right|$

$= \left| \left[ 2\frac{i\sqrt{3}}{4} \right] \right|$

$= \sqrt{0 + (3)}$

$= \sqrt{3} \longrightarrow (2)$

$|z_3 - z_1| = \left| \left[ \left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) \right] - 1 \right|$

$= \left| \frac{-3}{2} - \frac{\sqrt{3}}{2} i \right|$

$= \sqrt{\frac{9}{4} + \frac{3}{4}} = \sqrt{3} \longrightarrow (3)$

eq(1) = eq(2) = eq(3).

Hence, proved.

33) Find the equ of hyperbola with vertices $(0, \pm 4)$ & foci $(0, \pm 6)$.

Sol.

$AA' = 2a$ ; $SS' \Rightarrow 2c = 12$

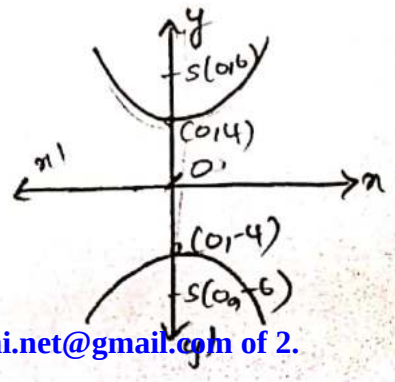
$= 8$

$c = 6$

$a = 4$

$b^2 = c^2 - a^2 = 36 - 16$

$b^2 = 20$




kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

The equation of hyperbola is www.Padasalai.Net. www.Trb Tnpsc.com of 2.

$$\frac{y^2}{16} - \frac{x^2}{20} = 1$$

34) If the two lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-1}{2} = z$ intersect at a point, find m.

Sol. $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4} = t$.

$x = 2t+1$ $\quad y = 3t-1$ $\quad z = 4t+1$

The points are $(2t+1, 3t-1, 4t+1)$,

$\frac{x-3}{1} = \frac{y+1}{s} = \frac{z+0}{1} = s$

$x = s+3$ $\quad y = 2s+m$ $\quad z = s$

$(2t+1, 3t-1, 4t+1) = (s+3, 2s+m, s)$.

$2t+1 = s+3$

$2t-2 = s \rightarrow (1)$

$4t+1 = s \rightarrow (2)$.

$(1)+(2) \Rightarrow$ $t = -3/2$.

sub "t" in (1)

$s = 2(-3/2) - 2$

$s = 7\left(\frac{-3}{7}\right) - 2$

$s = -5$

to find m $\Rightarrow$ $2s+m = 3t-1$

$2(-5)+m = 3\left(\frac{-3}{2}\right) - 1$

$= \frac{-11}{2} + 10$

$m = 9/2$

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

35) If $w(x,y) = xy + \sin(xy)$ P.T $\frac{\partial^2 w}{\partial y \partial x} = \frac{\partial^2 w}{\partial x \partial y}$

www.Padasalai.Net www.Trb Tnpsc.com of 2.

Sol $\frac{\partial w}{\partial x} = y + \cos(xy)y$.

$\frac{\partial^2 w}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial w}{\partial x}\right)$

$= \frac{\partial}{\partial y}(y + \cos(xy)y)$. $\Rightarrow u' = -\sin xy(x)y$ ; $v' = 1$

$= [1] + [(-\sin(xy)(y).(x)] + \cos(xy)(1)$.

$= 1 - xy\sin(xy) + \cos(xy) \longrightarrow (1)$.

$\frac{\partial w}{\partial y} = x + \cos(xy)x$

$= \frac{\partial}{\partial x}\left(\frac{\partial w}{\partial y}\right)$.

$= \frac{\partial}{\partial x}(x + \cos(xy)x)$.

$= 1 + -\sin(xy)y(x) + \cos xy(1)$. $\longrightarrow (2)$

eq(1) = eq(2)

Hence, proved.

36) $\int_0^{\pi/3} \frac{\sec x \tan x}{1+\sec^2 x}\, dx$

Sol. Let $u = \sec x$.

$\frac{du}{dx} = \tan x \sec x$.

$\int_0^{\pi/3} \frac{\sec x \tan x}{1+\sec^2 x} dx \Rightarrow \int_0^{\pi/3} \frac{du}{1+u^2}$

$= [1 \tan^{-1}(u)]_0^{\pi/3}$

$\frac{1}{1^2+x^2} = \frac{1}{a}\tan^{-1}(x)$.

| $x$ | 0 | $\pi/3$ |
|---|---|---|
| $u = \sec x$ | 1 | 2 |

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

$= [\tan^{-1}(x)]_1^2$

$= \tan^{-1}(2) - \tan^{-1}(1)$

$= \tan^{-1}(2) - \pi/4.$

37) Solve: $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

$\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

$\frac{1}{\sqrt{1-y^2}}dx = \frac{1}{\sqrt{1-x^2}}dx.$

$\sin^{-1}(y) = \sin^{-1}(x) + c$

39) Solve: $y^2 + x^2\frac{dy}{dx} = xy\frac{dy}{dx}$

Sol.

Rewriting the equation,

$\frac{dy}{dx} = \frac{y^2}{xy - x^2}$

This is homogeneous differential eqn:-

put $y = vx$; $x\frac{dv}{dx} = \frac{v}{v-1}$.

$\frac{v-1}{v}dv = \frac{dx}{x}.$

Integrating on both sides

$\int \frac{v-1}{v}dv = \int \frac{1}{x}dx.$

$v - \log|v| = \log|x| + \log|c|.$

$v = \log|vxc|$

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

www.Padasalai.Net. by www.Trb Tnpsc.com of 2.

by separating variables we get

$$\frac{y}{x} = \log|cy| \text{ (or)}.$$

$$|xy| = e^{y/x} \text{ (or)}.$$

$$y = ke^{y/x}$$

40) Find the equation of tangent to curve $x^2y - x = y^3 - 8$ at $x=0$. (creative sum).

Part-IV.

41) a) Find the inverse of a non-singular matrix, $A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ by elementary transformation.

Sol

$$[A|I_3] = \left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 3 & 2 & 1 & 0 & 1 & 0 \\ 2 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$$

$R_1 \to \frac{1}{2}R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 1/2 & 1/2 & 1/2 & 0 & 0 \\ 3 & 2 & 1 & 0 & 1 & 0 \\ 2 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$$

$R_2 \to R_2 - 3R_1$, $R_3 \to R_3 - 2R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 1/2 & 1/2 & 1/2 & 0 & 0 \\ 0 & 1/2 & -1/2 & -3/2 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

$R_2 \to 2R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 1/2 & 1/2 & 1/2 & 0 & 0 \\ 0 & 1 & -1 & -3 & 2 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

$R_1 \to R_1 - \frac{1}{2}R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 2 & -1 & 0 \\ 0 & 1 & -1 & -3 & 2 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

$R_1 \to R_1 - R_3$, $R_2 \to R_2 + R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -1 & -1 \\ 0 & 1 & 0 & -4 & 2 & 1 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 3 & -1 & -1 \\ -4 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

b) $z = x+iy$ and $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$; show that $x^2+y^2=1$.

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

Sol $\frac{z-1}{z+1} = \left[\frac{x+iy-1}{x+iy+1}\right]$.

$\arg\left(\frac{z-1}{z+1}\right) = \arg(z-1) - \arg(z+1)$.

$= \arg(x+iy-1) - \arg(x+iy+1)$

$= \arg(x-1+iy) - \arg(x+1+iy)$

$= \tan^{-1}\left(\frac{y}{x-1}\right) - \tan^{-1}\left(\frac{y}{x+1}\right)$.

$\frac{\pi}{2} = \tan^{-1}\left[\frac{\frac{y}{x-1} - \frac{y}{x+1}}{1+\left(\frac{y}{x-1}\right)\left(\frac{y}{x+1}\right)}\right]$.

$= \tan^{-1}\left[\frac{\frac{y(x+1)-y(x-1)}{x-1(x+1)}}{\frac{x^2-1^2+y^2}{(x^2-1^2)}}\right]$

$= \tan^{-1}\left(\frac{xy+y-xy+y}{x^2+y^2-1}\right)$

$\frac{\pi}{2} = \tan^{-1}\left(\frac{2y}{x^2+y^2-1}\right)$.

$\tan\frac{\pi}{2} = \frac{2y}{x^2+y^2-1}$

$\frac{1}{0} = \frac{2y}{x^2+y^2-1}$

$1(x^2+y^2-1) = 2y(0)$.

$\boxed{x^2+y^2=1}$

Hence, proved

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

42) a) solve: $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$, if $\frac{1}{3}$ is one known solution.

www.Padasalai.Net www.Trb-tnpsc.com of 2.

Sol $\div x^2 \Rightarrow 6x^2 - 5x - 38 - \frac{5}{x} + \frac{6}{x^2} = 0.$

$$6\left(x^2 + \frac{1}{x^2}\right) - 5\left(x + \frac{1}{x}\right) - 38 = 0.$$

$\therefore$ Let $y = x + 1/x$.

$$6(y^2 - 2) - 5(y) - 38 = 0$$

$$6y^2 - 12 - 5y - 38 = 0$$

$$6y^2 - 5y - 50 = 0.$$

$$(3y - 10)(2y + 5) = 0$$

$y = 10/3$ $\quad y = -5/2$.

$-300$, $-20$, $15$

$x + 1/x = 10/3$

$3x^2 - 10x + 3 = 0$

$x = 3 \quad x = 1/3$

$\frac{x^2+1}{x} = -\frac{5}{2}$

$2x^2 + 2 + 5x = 0$

$x = -1/2 \quad x = -2$

Hence, found the solution.

42 b) Solve $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$.

$$\tan^{-1}\left(\frac{2x + 3x}{1 - 2x(3x)}\right) = \frac{\pi}{4}.$$

$$\frac{5x}{1 - 6x^2} = \tan^{-1}\left(\frac{\pi}{4}\right).$$

$$5x / 1 - 6x^2 = 1$$

$$1 - 6x^2 = 5x.$$

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

$6x^2+5x-1=0 \Rightarrow (x+1)(6x-1)$

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

$x=1/6 ; x=-1.$

$\therefore$ Since $x=-1$ doesn't satisfy $6x^2<1$,

So, $\boxed{x=1/6}$ is the only solution.

44) a) Show that the st. lines $x+1=2y=-12z$ ; $x=y+2=6z-6$ are skew and hence find the shortest distance.

Sol $\frac{x+1}{1}=\frac{2y}{(1/2)}=\frac{z}{(-1/12)}$ ; $\frac{x}{1}=\frac{y+2}{1}=\frac{z-1}{(1/6)}$

$\vec{a}=-\hat{i}$ $\quad \vec{b}=\hat{i}+\frac{1}{2}\hat{j}-\frac{1}{12}\hat{k}$

$\vec{c}=-2\hat{j}+\hat{k}$ $\quad \vec{d}=\hat{i}+\hat{j}+\frac{1}{6}\hat{k}.$

$\vec{c}-\vec{a}=+\hat{i}-2\hat{j}+\hat{k}$

$$\vec{b}\times\vec{d}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1/2 & -1/12 \\ 1 & 1 & 1/6 \end{vmatrix}$$

$=\hat{i}\left(\frac{1}{12}+\frac{1}{12}\right)-\hat{j}\left(\frac{1}{6}+\frac{1}{12}\right)+\hat{k}\left(1-\frac{1}{2}\right).$

$=\hat{i}\left(\frac{2}{12}\right)-\hat{j}\left(\frac{1}{4}\right)+\hat{k}\left(\frac{1}{2}\right).$

$=\frac{1}{6}\hat{i}-\frac{\hat{j}}{4}+\frac{1}{2}\hat{k}.$

$(\vec{b}\times\vec{d})(\vec{c}-\vec{a})=\left(\frac{1}{6}\hat{i}-\frac{1}{4}\hat{j}+\frac{1}{2}\hat{k}\right)\left(\hat{i}-2\hat{j}+\hat{k}\right).$

$=\frac{1}{6}+\frac{1}{2}+\frac{1}{12}$

$=\frac{2+6+6}{12}=\frac{14}{6}\neq 0$

$\therefore$ The given lines are skew lines.

PRIT EDUCATION Practice Platform ! Perfect !
www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

b) If we blow air into a balloon of spherical shape at rate of $1000 cm^3/sec$. At what rate the radius of balloon changes when $r = 7cm$? Also find the rate at which the surface area changes

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

Sol. $V = \frac{4}{3}\pi r^3$.

$$\frac{dv}{dt} = \frac{4}{3}\pi 3r^2 \frac{dr}{dt}.$$

$$1000 = \frac{4}{3}\pi 3r^2 \left(\frac{dr}{dt}\right)$$

$$1000 = \frac{4}{3}\pi 3(7)^2 \left(\frac{dr}{dt}\right).$$

$$\frac{dr}{dt} = \frac{1000}{4 \times 49 \times \pi}.$$

$$\frac{dr}{dt} = \frac{250}{49\pi}$$

The surface area of balloon, $S = 4\pi r^2$

$$\frac{ds}{dt} = 8\pi \times r \times \frac{dr}{dt}$$

$$\frac{250}{49\pi} = 8 \times \pi \times 7 \left(\frac{250}{49\pi}\right).$$

$$= \frac{2000}{7} cm^2/sec.$$

45) a) If an initial amount $A_0$ of money is invested at interest rate r compounded n times a year. The value of investment after t years is $A = A_0\left(1 + \frac{r}{n}\right)^{nt}$. If the interest is compounded continuously, show that the amount after t years is $A = A_0 e^{rt}$.

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

Let $y = \left(1+\frac{r}{n}\right)^{nt}$

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

$$\log y = nt \cdot \log\left(1+\frac{r}{n}\right)$$

$$\log y = \frac{\log\left(1+\frac{r}{n}\right)}{1/nt}$$

$$\lim_{n\to\infty} \log y = \lim_{n\to\infty} \frac{\log\left(1+\frac{r}{n}\right)}{1/nt}$$

$$= \lim_{n\to\infty} \frac{\frac{1}{1+\frac{r}{n}}\left(-\frac{(r)}{n^2}\right)}{\frac{1}{t}\left(-\frac{1}{n^2}\right)}$$

$$= \lim_{n\to\infty} \frac{rt}{1+\frac{r}{n}}$$

$$= rt$$

By composite theorem :

$$\log \lim_{n\to\infty} y = rt$$

$$\Rightarrow \lim_{n\to\infty} y = e^{rt}$$

$$A = A_0\left(1+\frac{r}{n}\right)^{nt}$$

$$A \to A_0 e^{rt}$$

$$\boxed{A = A_0 e^{rt}}$$

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

SS PRITHVI, PRIT~EDUCATION

47) b) Four fair coins are tossed once. Find the Probability mass function, mean & variance of the no. of heads occured.

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

Sol: The sample space = 16.

Let X denote the no. of heads.

$P(X=0) = 1/16$

$P(X=1) = 1/4$

$P(X=2) = 3/8$

$P(X=3) = 1/4$

$P(X=4) = 1/16.$

Pmf:

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $P(X=x)$ | 1/16 | 4/16 | 6/16 | 4/16 | 1/16. |

$n = 4$

$p = 1/2$

$q = 1/2$

$np = (4)\left(\frac{1}{2}\right) = 2$

(i.e) mean = 2

$npq = 1$ (i.e) variance, $\Rightarrow npq$

$= 4\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$

$= 1.$

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

45) b) $u = \sec^{-1}\left(\frac{x^3+y^3}{x+y}\right)$ ; show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\cot u$.

Sol. Let $x = xt$ ; $y = yt$

$$u = \sec^{-1}\left(\frac{(xt)^3 - (yt)^3}{xt+yt}\right)$$

$$= \sec^{-1}\left(\frac{x^3t^3 - y^3t^3}{t(x+y)}\right)$$

$$= \sec^{-1}\left(\frac{t^3(x^3-y^3)}{t(x+y)}\right)$$

$$= t^2 f(x,y).$$

∴ Hence, 2 is the degree of homogenous function.

By Euler's theorem:

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf$$

∴ let $f = \sec u$.

$$x\frac{\partial \sec u}{\partial x} + y\frac{\partial(\sec u)}{\partial x} = 2f(x,y)$$

$$x\sec u\tan u\frac{\partial u}{\partial x} + y\sec u\tan u\frac{\partial u}{\partial x} = 2\sec u$$

$\div \sec u$

$$x\tan u\frac{\partial u}{\partial x} + y\tan u\frac{\partial u}{\partial x} = 2 \quad (1)$$

$\div \tan u$

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial x} = \frac{2}{\tan u}$$

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial x} = 2\cot u.$$

Hence, proved.

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

46) a) The curve $y=(x-2)^2+1$ has the minimum point at P. A point Q on the curve is such that slope of PQ = 2. Find the area bounded by curve and chord PQ.

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

Sol

$$y=(x^2+4-4x+1) \longrightarrow (1).$$

$$y=x^2-4x+5$$

$$(y')=2x-4$$

$$0=2x-4$$

$$\boxed{x=2}$$

$y''=2>0$ (It is minimum at $x=2$)

$y''(2)=2>0$.

$(1) \Rightarrow y=4-8+5$

$$\boxed{y=1}$$

(2,1) is the point of P.

slope of PQ = 2.

$$\boxed{m=2}$$

Equation of PQ,

$$y-y_1=m(x-x_1)$$

$$y-1=2(x-2).$$

$$y-1=2x-4$$

$$2x-y-3=0$$

$$y=2x-3 \longrightarrow (2).$$

sub (2) in (1)

$$2x-3=(x^2+5-4x).$$

$$x^2-6x+8=0$$

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

$(x-2)(x+2)=0$

$x=2 \Rightarrow (2) \Rightarrow y=1$

$x=4 \Rightarrow (2) \Rightarrow y=5$.

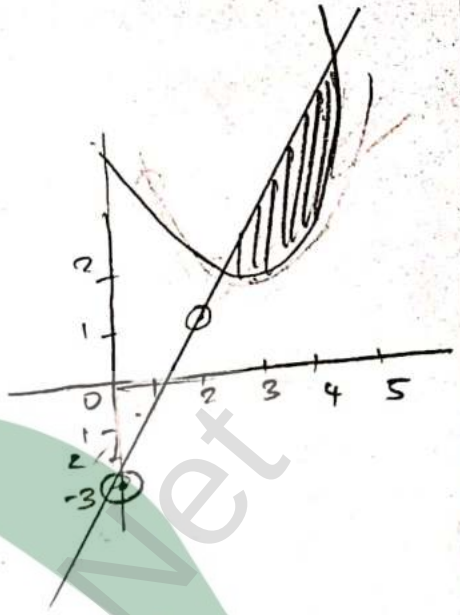
Point of Intersection (2,1) & (4,5).

$y=2x-3$

| $x$ | 0 | 2 | 4 |
|---|---|---|---|
| $y$ | -3 | 1 | 5 |

$y=x^2-4x-5$

| $x$ | 0 | 2 | 3 | 4 |
|---|---|---|---|---|
| $y$ | -5 | -1 | -2 | -5 |




$f(x)=2x-3$

$g(x)=x^2-4x+5$

$$\text{Area} = \int_a^b (f(x)-g(x))\,dx.$$

$$= \int_3^4 [2x-3-x^2+4x+(-5)]\,dx$$

$$= \int_3^4 -x^2+6x-8\,dx$$

$$= \left[-\frac{x^3}{3}+\frac{6x^2}{2}-8x\right]_3^4 = \left(-\frac{64}{3}+3(16)-32\right)-\left(-\frac{8}{3}+12-16\right)$$

$$= 4/3 \text{ Sq. units}.$$

46) b) P.T $P \to (\neg q \vee r) \equiv \neg P \vee (\neg q \vee r)$

| $P$ | $q$ | $r$ | $\neg q$ | $\neg q \vee r$ | $P \to (\neg q \vee r)$ | $\neg P$ | $\neg r$ | $\neg q \vee r$ | $\neg P \vee (\neg q \vee r)$ |
|---|---|---|---|---|---|---|---|---|---|
| T | T | T | F | T | T | F | F | T | T |
| T | T | F | F | F | F | F | T | F | F |
| T | F | T | T | T | T | F | F | T | T |
| T | F | F | T | T | T | F | T | T | T |
| F | T | T | F | T | T | T | F | T | T |
| F | T | F | F | F | T | T | T | F | F |
| F | F | T | T | T | T | T | F | T | T |
| F | F | F | T | T | T | T | T | T | T |

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

www.Padasalai.Net www.Trb Tnpsc.com of 2.

24) Creative sums... (2 Mark)

28) Express $(-1+i\sqrt{3})$ in polar form

$-1+i\sqrt{3} = r(\cos\theta + i\sin\theta)$

$r = \sqrt{(-1)^2+(\sqrt{3})^2} = \sqrt{1+3} = 2$

It is in 2nd quadrant.

$\theta = \pi - \alpha$

$\alpha = \tan^{-1}\left|\frac{y}{x}\right|$.

$= \tan^{-1}\frac{\sqrt{3}}{1}$

$\alpha = \pi/3$

$\theta = \pi - \pi/3$

$\theta = 2\pi/3$.

$-1+i\sqrt{3} = 2(\cos 2\pi/3 + i\sin 2\pi/3)$.

$= 2\left(\cos\left[2k\pi + \frac{2\pi}{3}\right) + i\sin\left(2k\pi + \frac{2\pi}{3}\right)\right]$, $k \in 2, 3$

PRIT EDUCATION
Practice ! Perform ! Perfect !

25) Explain why lagrange mean value theorem is not applicable to function $f(x) = |1/x|$; $x \in [-1, 1]$.

If $f(x) = |1/x|$, $x \in [-1, 1]$

$f(x)$ is not continous at $x = 0$

∴ Hence, it is not applicable.

26) Evaluate: $\int_0^1 \frac{|x|}{x}\,dx$.

$= \int_0^1 1\,dx$

$= [x]_0^1$

$I = 1$

$f(x) = \begin{cases} +x/x = +1, & x > 0 \\ -x/x = -1, & x \le 0 \end{cases}$

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION



Part-III (Creative Questions):

38) Form the differential equation of $y = e^{3x}(C\cos 2x + D\sin 2x)$, C and D are arbitrary constants.

Sol $y \times e^{-3x} = C\cos 2x + D\sin 2x$

$u \quad v$

diff $\Rightarrow y(e^{-3x})(-3) + e^{-3x}y' = -C2\sin 2x + 2D\cos 2x$

$e^{-3x}(-3y + y') = -2[C\sin 2x - D\cos 2x]$

diff $\Rightarrow e^{-3x}(-3y' + y'') + (-3y + y')(e^{-3x} \times -3) = -2[2C\cos 2x + D\sin 2x]$.

$e^{-3x}[-3y' + y'' + 9y - 3y'] = -4[\cos 2x + D\sin 2x]$

$y'' - 6y' + 9y = -4e^{-3x}[\cos 2x + D\sin 2x]$

$y'' - 6y' + 9y = -4y$

$\boxed{y'' - 6y' + 13y = 0}$

40) Find the eqn of tangent $x^2y - x = y^3 - 8$ at $x = 0$

Sol $x = 0 \Rightarrow 0(y) - 0 = y^3 - 8$

$y^3 = 8$

$\boxed{y = 2}$

Diff with r.to x.

$x^2y' + 2x(y) - 1 = 3y^2(y') - 0$

$x^2y' + 2xy - 1 - 3y^2y' = 0.$

$y' = \frac{1 - 2xy}{x^2 - 3y^2}$

$\frac{dy}{dx} = \frac{1 - 2xy}{x^2 - 3y^2}.$

$m = \frac{dy}{dx}_{(0,2)} = \frac{1 - 2(0)(2)}{0 - 3(4)} = \frac{-1}{12}$

PRIT EDUCATION Practice ! Perform ! Perfect !
www.Padasalai.Net

SS PRITHVI, PRIT~EDUCATION

www.Padasalai.Net www.Trb Tnpsc.com of 2.

The eqn of tangent (0, 2)

$y - y_1 = m(x - x_1)$

$y - 2 = -1/12(x - 0)$

$\boxed{x + 12y - 24 = 0}$

Part IV (Creative sums)...

43) a) An ellipse whispering room of height 5m & width 26m. Where should two persons stand if they would like whisper back & forth be heard.

If $2a = 26$

$a = 13m$

$b = 5m$

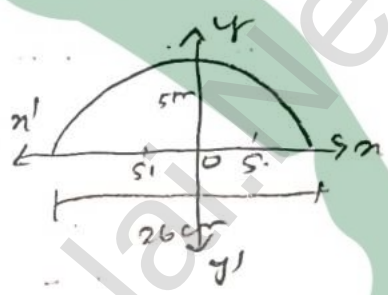
$c^2 = a^2 - b^2$

$= 13^2 - 5^2$

$= 169 - 25$

$= 144$

$\boxed{c = +12}$




* The two person are located on either side about 12m from the centre along major axis

47) a) An eqn. of relating to stability of aircraft given by $\frac{dv}{dt} = gravity - kv$, where g and k are constant, obtain expression in terms of v, v=0, t=0 (when).

$\frac{dv}{dt} = gravity - kv$

$\boxed{\frac{dv}{dt} + Pv = Q}$

$\frac{dv}{dt} + kv = gravity$

$P = k$ ; $Q = gravity$

$I.F = e^{\int P dt}$

$= e^{\int k dt}$

$I.F = e^{kt}$

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

www.Padasalai.Net. www.Trb Tnpsc.com of 2.

The general solution,

$$y(I.F) = \int Q\,(I.F)\,dx + c.$$

$$v(I.F) = \int Q(I.F)\,dt + c$$

$$ve^{kt} = \int g\cos\alpha\, e^{kt}\,dt + c$$

$$ve^{kt} = g\cos\alpha\left(\frac{e^{kt}}{k}\right) + c$$

put $v=0$, $t=0$

$$0 = g\cos\alpha\left(\frac{e^{0}}{k}\right) + c$$

$$\boxed{c = -\frac{g\cos\alpha}{k}}$$

$$ve^{kt} = g\cos\alpha\left(\frac{e^{kt}}{k}\right) - \frac{g\cos\alpha}{k}$$

$$ve^{kt} = \frac{g\cos\alpha}{k}\,(e^{kt}-1)$$

$$\boxed{v = \frac{g\cos\alpha}{k}\,(1-e^{-kt})}$$

PRIT EDUCATION
Practice ! Perform ! Perfect !
www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.

SS PRITHVI, PRIT~EDUCATION

43)(b) Show that the four points (6,-7,0) (16,-19,-4) (0,3,-6) (2,-5,10) lie on a same plane.

www.Padasalai.Net www.Trb Tnpsc.com of 2.

Let $\vec{OA} = 6\hat{i} - 7\hat{j}$, $\vec{OB} = 16\hat{i} - 19\hat{j} - 4\hat{k}$, $\vec{OC} = 3\hat{j} - 6\hat{k}$

$\vec{OD} = 2\hat{i} - 5\hat{j} + 10\hat{k}$.

TO Show that the four points A, B, C, D lie on a we have to prove that the three vectors $\vec{AB}$, $\vec{AC}$, $\vec{AD}$ are coplanar.

$$\vec{AB} = \vec{OB} - \vec{OA} = (16\hat{i} - 19\hat{j} - 4\hat{k}) - (6\hat{i} - 7\hat{j}) = 10\hat{i} - 12\hat{j} - 4\hat{k}$$

$$\vec{AC} = \vec{OC} - \vec{OA} = (3\hat{j} - 6\hat{k}) - (6\hat{i} - 7\hat{j}) = -6\hat{i} + 10\hat{j} - 6\hat{k}$$

$$\vec{AD} = \vec{OD} - \vec{OA} = (2\hat{i} - 5\hat{j} + 10\hat{k}) - (6\hat{i} - 7\hat{j}) = -4\hat{i} + 2\hat{j} + 10\hat{k}$$

$$[\vec{AB}, \vec{AC}, \vec{AD}] = \begin{vmatrix} 10 & -12 & -4 \\ -6 & 10 & -6 \\ -4 & 2 & 10 \end{vmatrix} = 0$$

PRIT EDUCATION
Practice | Perform | Perfect |

∴ The three vectors $\vec{AB}$, $\vec{AC}$, $\vec{AD}$ are coplanar and hence the four points A, B, C and D lies on a plane.

www.Padasalai.Net

kindly send me your key answer to our email id - Padasalai.net@gmail.com of 2.