3. ALGEBRA

1. $(x+a)(x+b) = x^2+(a+b)x+ab$
2. $(a+b)^2 = a^2+2ab+b^2$
3. $(a-b)^2 = a^2-2ab+b^2$
4. $(a+b)(a-b) = a^2-b^2$

Example 3.1

(i) $(x+3)(x+5)$

$(x+a)(x+b) = x^2+(a+b)x+ab$

$(x+3)(x+5) = x^2+(3+5)x+3\times5$

$= x^2+8x+15$

(ii) $(y+8)(y+6)$

$(x+a)(x+b) = x^2+(a+b)x+ab$

$(y+8)(y+6) = y^2+(8+6)y+8\times6$

$= y^2+14y+48$

(iii) 43×36

$(x+a)(x+b) = x^2+(a+b)x+ab$

$43\times36 = (40+3)(40-4)$

$= 40^2+(3-4)40+(3)(-4)$

$= 1600+(-1)40+(-12)$

$= 1600-40-12$

$= 1600-52$

$= 1548$

Example 3.2

(i) $(2x+5)^2$

$(a+b)^2 = a^2+2ab+b^2$

$(2x+5)^2 = (2x)^2+2(2x)(5)+(5)^2$

$= 4x^2+4x(5)+(5)^2$

$= 4x^2+20x+25$

(ii) $21^2 = (20+1)^2$

$(a+b)^2 = a^2+2ab+b^2$

$(20+1)^2 = 20^2+2(20)(1)+(1)^2$

$= (20)(20)+40(1)+(1)(1)$

$= 400+40+1$

$= 441$

Example 3.3

(i) $(3x-5y)^2$

$(a-b)^2 = a^2-2ab+b^2$

$(3x-5y)^2 = (3x)^2-2(3x)(5y)+(5y)^2$

$= (3x)(3x)-6x(5y)+(5y)(5y)$

$= 9x^2-30xy+25y^2$

(ii) 47^2

$(a-b)^2 = a^2-2ab+b^2$

$47^2 = (50-3)^2 = (50)^2-2(50)(3)+(3)^2$

$= (50)(50)-100(3)+(3)(3)$

$= 2500-300+9$

$= 2509-300$

$= 2209$

Example 3.4.

(i) $(3x+4)(3x-4)$

$(a+b)(a-b) = a^2-b^2$

$(3x+4)(3x-4) = (3x)^2-(4)^2$

$= (3x)(3x)-(4)(4)$

$= 9x^2-16$

(ii) 53×47.

$(a+b)(a-b) = a^2-b^2$

$53\times47 = (50+3)(50-3) = (50)^2-(3)^2$

$= (50)(50)-(3)(3)$

$= 2500-9$

$= 2491$

Example 3.5

Express the following algebraic expressions as the product of its factors.

(i) $ab^2 = a\times b\times b$

(ii) $-3pq^3 = -3\times p\times q\times q\times q$

(iii) $12m^2n^2p = 2\times2\times3\times m\times m\times n\times n\times p$

Example 3.6

Factorise by using the identity:

$a^2-b^2 = (a+b)(a-b)$

(i) a^2-1

$a^2-1 = a^2-1^2$

$= (a+1)(a-1)$

(ii) $9k^2-25$

$9k^2-25 = (3k)(3k)-(5)(5)$

$= (3k)^2-(5)^2$

$= (3k+5)(3k-5)$

(iii) $64x^2-81y^2$

$64x^2-81y^2 = (8x)(8x)-(9y)(9y)$

$= (8x)^2-(9y)^2$

$= (8x+9y)(8x-9y)$

Example 3.7

Factorise $9x^2+30xy+25y^2$

$9x^2+30xy+25y^2 = (3x)(3x)+2(3x)(5y)+(5y)(5y)$

$= (3x)^2+2(3x)(5y)+(5y)^2$

$= (3x+5y)^2$

$= (3x+5y)(3x+5y)$

Example 3.8

Factorise $4x^2-4xy+y^2$

$4x^2-4xy+y^2 = (2x)(2x)-2(2x)(y)+(y)(y)$

$= (2x)^2-2(2x)(y)+(y)^2$

$= (2x-y)^2$

$= (2x-y)(2x-y)$

Example 3.9

Factorise $x^2-2xy+y^2-z^2$

$x^2-2xy+y^2-z^2 = (x^2-2xy+y^2)-z^2$

$= (x-y)^2-z^2$

$= (x-y+z)(x-y-z)$

Exercise 3.1

1. Fill in the blanks.

(i) $(p-q)^2 = p^2-2pq+q^2$

(ii) The product of $(x+5)$ and $(x-5)$ is ____

$= (x+5)(x-5)$

$= x^2-5^2$

$= x^2-25$

(iii) The factors of x^2-4x+4 are ____

$= x^2-2(x)(2)+(2)(2)$

$= x^2-2(x)(2)+(2)^2$

$= (x-2)^2$

$= (x-2)(x-2)$

(iv) Express $24ab^2c^2$ as product of its factors is ____

$= 2\times2\times2\times3\times a\times b\times b\times c\times c$

2. Say whether the following statements are True or False.

(i) $(7x+3)(7x-4) = 49x^2-7x-12$

$(7x+3)(7x-4) = 49x^2-28x+21x-12$

$= 49x^2-7x-12$

True

(ii) $(a-1)^2 = a^2-1$

$(a-1)^2 = a^2-2(a)(1)+1^2$

$= a^2-2a+1$

$a^2-1 = a^2-1^2$

$= (a+1)(a-1)$

$(a-1)^2 \neq a^2-1$

False

(iii) $(x^2+y^2)(y^2+x^2) = (x^2+y^2)^2$

$(x^2+y^2)(y^2+x^2) = (x^2+y^2)(x^2+y^2)$

$= (x^2+y^2)^2$

True

(iv) $2p$ is the factor of $8pq$.

$8pq = 2\times2\times2\times p\times q$

$\therefore 2p$ is the factor of $8pq$.

True

3. Express the following as the product of its factors

(i) $24ab^2c^2 = 2\times2\times2\times3\times a\times b\times b\times c\times c$

(ii) $36x^3y^2z = 2\times2\times3\times3\times x\times x\times x\times y\times y\times z$

(iii) $56mn^2p^2 = 2\times2\times2\times7\times m\times n\times n\times p\times p$

4. Using the identity $(x+a)(x+b)=x^2+x(a+b)+ab$ find the following product

(i) $(x+3)(x+7)$

$(x+a)(x+b) = x^2+(a+b)x+ab$

$(x+3)(x+7) = x^2+(3+7)x+(3)(7)$

$= x^2+10x+21$

(ii) $(6a+9)(6a-5)$

$(x+a)(x+b) = x^2+(a+b)x+ab$

$(6a+9)(6a-5) = (6a)^2+(9-5)(6a)+(9)(-5)$

$= 36a^2+4(6a)+(-45)$

$= 36a^2+24a-45$

(iii) $(4x+3y)(4x+5y)$

$(x+a)(x+b) = x^2+(a+b)x+ab$

$(4x+3y)(4x+5y) = (4x)^2+(3y+5y)(4x)+(3y)(5y)$

$= 16x^2+(8y)(4x)+15y^2$

$= 16x^2+32xy+15y^2$

(iv) $(8+pq)(pq+7)$

$(x+a)(x+b) = x^2+(a+b)x+ab$

$(pq+8)(pq+7) = (pq)^2+(8+7)(pq)+(8)(7)$

$= p^2q^2+(15)pq+56$

$= p^2q^2+15pq+56$

5. Expand the following squares, using suitable identities

(i) $(2x+5)^2$

$(a+b)^2 = a^2+2ab+b^2$

$(2x+5)^2 = (2x)^2+2(2x)(5)+(5)^2$

$= 4x^2+20x+25$

(ii) $(b-7)^2$

$(a-b)^2 = a^2-2ab+b^2$

$(b-7)^2 = b^2-2(b)(7)+(7)^2$

$= b^2-14b+49$

(iii) $(mn+3p)^2$

$(a+b)^2 = a^2+2ab+b^2$

$(mn+3p)^2 = (mn)^2+2(mn)(3p)+(3p)^2$

$= m^2n^2+2mn(3p)+9p^2$

$= m^2n^2+6mnp+9p^2$

(iv) $(xyz-1)^2$

$(a-b)^2 = a^2-2ab+b^2$

$(xyz-1)^2 = (xyz)^2-2(xyz)(1)+(1)^2$

$= x^2y^2z^2-2xyz+1$

6. Using the identity $(a+b)(a-b)=a^2-b^2$, find the following product.

(i) $(p+2)(p-2)$

$(a+b)(a-b) = a^2-b^2$

$(p+2)(p-2) = p^2-2^2$

$= p^2-4$

(ii) $(1+3b)(3b-1)$

$(a+b)(a-b) = a^2-b^2$

$(3b+1)(3b-1) = (3b)^2-(1)^2$

$= (3b)(3b)-(1)(1)$

$= 9b^2-1$

(iii) $(4-mn)(mn+4)$

$(a+b)(a-b) = a^2-b^2$

$(4-mn)(4+mn) = (4)^2-(mn)^2$

$= 16-m^2n^2$

(iv) $(6x+7y)(6x-7y)$

$(a+b)(a-b) = a^2-b^2$

$(6x+7y)(6x-7y) = (6x)^2-(7y)^2$

$= (6x)(6x)-(7y)(7y)$

$= 36x^2-49y^2$

7. Evaluate the following, using suitable identity.

(i) 51^2

$(a+b)^2 = a^2+2ab+b^2$

$51^2 = (50+1)^2 = (50)^2+2(50)(1)+(1)^2$

$= (50)(50)+100(1)+(1)(1)$

$= 2500+100+1$

$= 2601$

(ii) 103^2

$(a+b)^2 = a^2+2ab+b^2$

$103^2 = (100+3)^2 = (100)^2+2(100)(3)+(3)^2$

$= 10000+600+9$

$= 10609$

(iii) 998^2

$(a-b)^2 = a^2-2ab+b^2$

$998^2 = (1000-2)^2 = (1000)^2-2(1000)(2)+(2)^2$

$= 1000000-4000+4$

$= 1000004-4000$

$= 996004$

(iv) 47^2

$(a-b)^2 = a^2-2ab+b^2$

$47^2 = (50-3)^2 = (50)^2-2(50)(3)+(3)^2$

$= 2500-100(3)+9$

$= 2500-300+9$

$= 2509-300$

$= 2209$

(v) 297×303

$(a+b)(a-b) = a^2-b^2$

$297\times303 = (300-3)(300+3)$

$= 300^2-3^2$

$= 90000-9$

$= 89991$

(vi) 990×1010

$(a+b)(a-b) = a^2 - b^2$

$990 \times 1010 = 1010 \times 990$

$= (1000+10)(1000-10)$

$= (1000)^2 - (10)^2$

$= 1000000 - 100$

$= 999900$

(vii) 51×52

$(x+a)(x+b) = x^2 + (a+b)x + ab$

$51 \times 52 = (50+1)(50+2) = (50)^2 + (1+2)(50) + (1)(2)$

$= (50)(50) + 3(50) + 2$

$= 2500 + 150 + 2$

$= 2652$

8. Simplify: $(a+b)^2 - 4ab$

$(a+b)^2 - 4ab = a^2 + 2ab + b^2 - 4ab$

$= a^2 - 2ab + b^2$

$= (a-b)^2$

9. Show that $(m-n)^2 + (m+n)^2 = 2(m^2+n^2)$

LHS $= (m-n)^2 + (m+n)^2$

$= m^2 - 2mn + n^2 + m^2 + 2mn + n^2$

$= 2m^2 + 2n^2$

$= 2(m^2+n^2)$

$=$ RHS

$\therefore (m-n)^2 + (m+n)^2 = 2(m^2+n^2)$

10. If $a+b=10$ and $ab=18$, find the value of a^2+b^2.

$a+b=10$

$ab=18$

$(a+b)^2 = a^2 + 2ab + b^2$

$a^2 + 2ab + b^2 = (a+b)^2$

$a^2 + b^2 = (a+b)^2 - 2ab$

$a^2 + b^2 = (10)^2 - 2(18)$

$= 10 \times 10 - 2 \times 18$

$= 100 - 36$

$a^2 + b^2 = 64$

11. Factorise the following algebraic expressions by using the identity $a^2 - b^2 = (a+b)(a-b)$

(i) $z^2 - 16$

$a^2 - b^2 = (a+b)(a-b)$

$z^2 - 16 = z^2 - 4^2 = (z+4)(z-4)$

(ii) $9 - 4y^2$

$a^2 - b^2 = (a+b)(a-b)$

$9 - 4y^2 = 3^2 - (2y)^2 = (3+2y)(3-2y)$

(iii) $25a^2 - 49b^2$

$a^2 - b^2 = (a+b)(a-b)$

$25a^2 - 49b^2 = (5a)^2 - (7b)^2$

$= (5a+7b)(5a-7b)$

(iv) $x^4 - y^4$

$a^2 - b^2 = (a+b)(a-b)$

$x^4 - y^4 = (x^2)^2 - (y^2)^2$

$= (x^2+y^2)(x^2-y^2)$

$= (x^2+y^2)(x+y)(x-y)$

$\therefore x^4 - y^4 = (x^2+y^2)(x+y)(x-y)$.

12. Factorise the following using suitable identity.

(i) $x^2 - 8x + 16$

$x^2 - 8x + 16 = x^2 - 2(x)(4) + 4^2$

$= (x-4)^2$

(ii) $y^2 + 20y + 100$

$y^2 + 20y + 100 = y^2 + 2(y)(10) + (10)^2$

$= (y+10)^2$

(iii) $36m^2 + 60m + 25$

$36m^2 + 60m + 25 = (6m)^2 + 2(6m)(5) + (5)^2$

$= (6m+5)^2$

(iv) $64x^2 - 112xy + 49y^2$

$64x^2 - 112xy + 49y^2 = (8x)^2 - 2(8x)(7y) + (7y)^2$

$= (8x - 7y)^2$

(v) $a^2 + 6ab + 9b^2 - c^2$

$a^2 + 6ab + 9b^2 - c^2 = a^2 + 2(a)(3b) + (3b)^2 - c^2$

$= (a+3b)^2 - c^2$

$= (a+3b+c)(a+3b-c)$

$\therefore a^2 + 6ab + 9b^2 - c^2 = (a+3b+c)(a+3b-c)$

Objective Type Questions.

13. If $a+b=5$ and $a^2+b^2=13$, then $ab = ?$

$(a+b)^2 = a^2 + 2ab + b^2$

$a^2 + 2ab + b^2 = (a+b)^2$

$a^2 + b^2 + 2ab = (a+b)^2$

$13 + 2ab = (5)^2$

$13 + 2ab = 25$

$2ab = 25 - 13$

$2ab = 12$

$ab = \frac{12}{2}$

$\boxed{ab = 6}$

Ans: (ii) 6

14. $(5+20)(-20-5) = ?$

$= (-)(5+20)(20+5)$

$= (-)(5+20)(5+20)$

$= -(5+20)^2$

$= -[(5)^2 + 2(5)(20) + (20)^2]$

$= -[25 + 200 + 400]$

$= -(625)$

$= -625$

Ans: (iii) -625

15. The factors of $x^2 - 6x + 9$ are

$x^2 - 6x + 9 = x^2 - 2(x)(3) + 3^2$

$= (x-3)^2$

$= (x-3)(x-3)$

$\therefore x^2 - 6x + 9 = (x-3)(x-3)$

Ans: (i) $(x-3)(x-3)$

16. The common factors of the algebraic expressions ax^2y, bxy^2 and $cxyz$ is

$ax^2y = a \times x \times x \times y$

$bxy^2 = b \times x \times y \times y$

$cxyz = c \times x \times y \times z$

Ans: (iv) xy

Inequations:

Example 3.10:

Solve: $2x+4<18$, where x is a natural number.

$2x+4<18$

$2x+4-4<18-4$

$2x<14$

$\frac{2x}{2}<\frac{14}{2}$

$\boxed{x<7}$

$x = 1, 2, 3, 4, 5, 6$

Example 3.11:

Solve: $5-7x \geq 33$, where x is an integer.

$5-7x \geq 33$

$5-5-7x \geq 33-5$

$-7x \geq 28$

$\frac{-7x}{-7} \geq \frac{28}{-7}$ → (-7) is negative number

$\boxed{x \leq -4}$

$x = -4, -5, -6, \ldots$

Example 3.12:

If one worker earns ₹200 per day, how many workers can be employed within a monthly budget of ₹3 lakh?

Let the no. of workers be x.

The wages that x workers will earn per day $=$ ₹$200x$

The wages that x workers will earn per month $=$ ₹$(200x \times 30)$

$=$ ₹$6000x$

$6000x \leq 3,00,000$

$\frac{6000x}{6000} \leq \frac{300000}{6000}$

$\boxed{x \leq 50}$

$\therefore$ 50 workers can be employed.

Example 3.13

Represent the solutions $-8<2x<10$ in a number line, where x is a natural number.

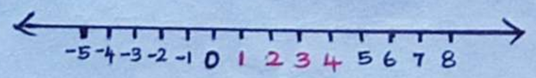
$-8<2x<10$

$\frac{-8}{2}<\frac{2x}{2}<\frac{10}{2}$

$-4<x<5$

$x =$ Natural number

$\therefore x = 1, 2, 3, 4$



Example 3.14

Represent the solutions of $3x+9 \leq 12$ in a number line, where x is an integer.

$3x+9 \leq 12$

$3x+9-9 \leq 12-9$

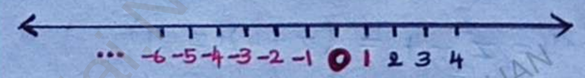
$3x \leq 3$

$\frac{3x}{3} \leq \frac{3}{3}$

$\boxed{x \leq 1}$

$x =$ Integer

$\therefore x = 1, 0, -1, -2, \ldots$



Example 3.15

Solve the inequation: $-2 \leq z+3 \leq 5$, where z is an integer. Also, represent the solution, graphically.

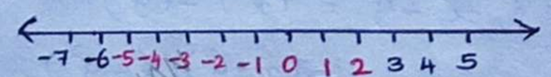
$-2 \leq z+3 \leq 5$

$-3-2 \leq z+3-3 \leq 5-3$

$-5 \leq z \leq 2$

$z =$ integer

$z = -5, -4, -3, -2, -1, 0, 1, 2$



Example 3.16

Solve graphically: $6y-5 \leq 2y+7$, where y is an integer.

$6y-5 \leq 2y+7$

$6y-2y-5 \leq 2y-2y+7$

$4y-5 \leq 7$

$4y-5+5 \leq 7+5$

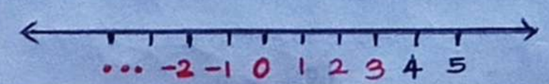
$4y \leq 12$

$\frac{4y}{4} \leq \frac{12}{4}$

$\boxed{y \leq 3}$

$y =$ integer

$y = 3, 2, 1, 0, -1, -2, \ldots$



Exercise 3.2

1. Given that $x \geq y$. Fill in the blanks with suitable inequality signs.

$x \geq y$

(i) $y \boxed{\leq} x$ If the expressions on both sides of an inequation are interchanged, the sign of inequality must be reversed

(ii) $x+6 \boxed{\geq} y+6$ $x \geq y$, $x+6 \geq y+6$

(iii) $x^2 \boxed{\geq} xy$ $x \geq y$, $x \times x \geq y \times x$, $x^2 \geq xy$

(iv) $-xy \boxed{\leq} -y^2$ → $x \geq y$, $x(-y) \leq y(-y)$, $-xy \leq -y^2$

(v) $x-y \boxed{\geq} 0$ ⇒ $x \geq y$, $x-y \geq y-y$, $x-y \geq 0$

Addition, Subtraction, Multiplication of the same number (+ve) → the inequality remains unchanged.

Multiplication (or) division of same (−ve) number → The sign of inequality is reversed.

2. Say True or False

(i) Linear inequation has atmost one solution. **False**

A linear inequation may have many solutions

(ii) When x is an integer, the solutions for $x \le 0$ are $-1, -2, \ldots$ **False**

$x \le 0$

$\Rightarrow x = 0, -1, -2, \ldots$

(iii) An inequation, $-3 < x < -1$, where x is an integer, cannot be represented in the number line **False**

$-3 < x < -1$

$x = -2$

(iv) $x < -y$ can be rewritten as $-y < x$. **False**

$x < -y$

$-y > x$

If the expressions on both sides of an inequation are interchanged, the sign of inequality must be reversed.

3. Solve the following inequations

(i) $x \le 7$, where x is a natural number.

$x = 1, 2, 3, 4, 5, 6, 7$

(ii) $x - 6 < 1$, where x is a natural number.

$x - 6 < 1$

$x - 6 + 6 < 1 + 6$

$x < 7$

$x = 1, 2, 3, 4, 5, 6$

(iii) $2a + 3 \le 13$, where a is a whole number

$2a + 3 \le 13$

$2a + 3 - 3 \le 13 - 3$

$2a \le 10$

$\frac{2a}{2} \le \frac{10}{2}$

$a \le 5$

$a = 0, 1, 2, 3, 4, 5$

(iv) $6x - 7 \ge 35$, where x is an integer.

$6x - 7 \ge 35$

$6x - 7 + 7 \ge 35 + 7$

$6x \ge 42$

$\frac{6x}{6} \ge \frac{42}{6}$

$x \ge 7$

$x = 7, 8, 9, \ldots$

(v) $4x - 9 > -33$, where x is a negative integer.

$4x - 9 > -33$

$4x - 9 + 9 > -33 + 9$

$4x > -24$

$\frac{4x}{4} > \frac{-24}{4}$

$x > -6$

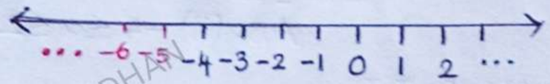
$x = -5, -4, -3, -2, -1$

4. Solve the following inequations and represent the ~~following~~ solution on the number line:

(i) $k > -5$, k is an integer.

k = integer

$k = -4, -3, -2, -1, 0, 1, 2, \ldots$



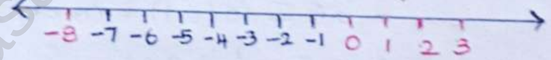
(ii) $-7 \le y$, y is a negative integer.

y = negative integer

$-7 \le y$

$y \ge -7$

$y = -7, -6, -5, -4, -3, -2, -1$



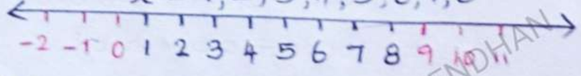
(iii) $-4 \le x \le 8$, x is a natural number

$-4 \le x \le 8$

$x = -4, -3, -2, \ldots, 6, 7, 8$

x = Natural number

$x = 1, 2, 3, 4, 5, 6, 7, 8$



(iv) $3m - 5 \le 2m + 1$, m is an integer.

$3m - 5 \le 2m + 1$

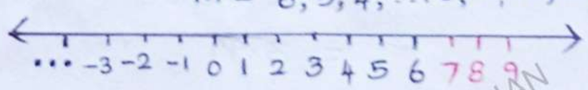
$3m - 2m - 5 \le 2m - 2m + 1$

$m - 5 \le 1$

$m - 5 + 5 \le 1 + 5$

$m \le 6$

$m = 6, 5, 4, \ldots 0, -1, -2, \ldots$



5. An artist can spend any amount between ₹80 to ₹200 on brushes. If cost of each brush is ₹5 and there are 6 brushes in each packet, then how many packets of brush can the artist buy?

Cost of one brush = ₹5

Cost of one packet (6 brushes) = ₹5 × 6 = ₹30

Cost of x packets = ₹30x

The inequation becomes

₹80 < ₹30x < ₹200

Cost of 3 packets = ₹30 × 3 = ₹90

Cost of 6 packets = ₹30 × 6 = ₹180

$x = 3, 4, 5, 6$

$3 \le x \le 6$

Objective Type Questions:

6. The solution of the inequation $3 \le p \le 6$ are (where p is a natural number).

$3 \le p \le 6$

$p = 3, 4, 5, 6$

Ans: (iv) 3, 4, 5 and 6.

7. The solution of the inequation $5x+5 \le 15$ are (where x is a natural number)

$5x+5 \le 15$

$5x+5-5 \le 15-5$

$5x \le 10$

$\frac{5x}{5} \le \frac{10}{5}$

$\boxed{x \le 2}$

$x = 1, 2$ (natural numbers)

Ans: (i) 1 and 2

8. The cost of one pen is ₹8 and it is available in a sealed pack of 10 pens. If Swetha has only ₹500, how many packs of pens can she buy at the maximum?

Cost of one pen = ₹8

Cost of a pack of 10 pens = ₹8×10
= ₹80

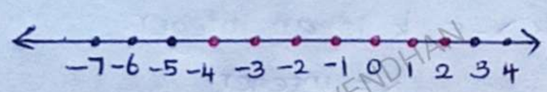
$x < 500$

Cost of 6 pack of pens = ₹80×6
= ₹480

She can buy 6 pack of pens.

Ans: (iii) 6

9. The inequation that is represented on the number line as shown below is _____



$x = -4, -3, -2, -1, 0, 1, 2$

$-4 \le x \le 2$

Ans: (ii) $-4 \le x \le 2$

Exercise 3.3

Miscellaneous Practice problems

1. Using identity, find the value of

(i) $(4.9)^2$

$(4.9)^2 = (5-0.1)^2$
$= (5)^2 - 2(5)(0.1) + (0.1)^2$
$= 25 - 10(0.1) + (0.01)$
$= 25 - 1 + 0.01$
$= 24 + 0.01$
$= 24.01$

(ii) $(100.1)^2 = (100+0.1)^2$
$= (100)^2 + 2(100)(0.1) + (0.1)^2$
$= 10000 + 200(0.1) + 0.01$
$= 10000 + 20 + 0.01$
$= 10020 + 0.01$
$= 10020.01$

(iii) $(1.9) \times (2.1)$

$(1.9) \times (2.1) = (2-0.1)(2+0.1)$
$= (2)^2 - (0.1)^2$
$= 4 - 0.01$
$= 3.99$

2. Factorise: $4x^2 - 9y^2$

$4x^2 - 9y^2 = (2x)^2 - (3y)^2$
$= (2x+3y)(2x-3y)$

3. Simplify using identities

(i) $(3p+q)(3p+r)$

$(3p+q)(3p+r) = (3p)^2 + (3p)(q+r) + qr$
$= 9p^2 + 3p(q+r) + qr$

(ii) $(3p+q)(3p-q)$

$(3p+q)(3p-q) = (3p)^2 - (q)^2$
$= 9p^2 - q^2$

4. Show that $(x+2y)^2 - (x-2y)^2 = 8xy$

LHS $= (x+2y)^2 - (x-2y)^2$
$= x^2 + 2(x)(2y) + (2y)^2 - [x^2 - 2(x)(2y) + (2y)^2]$
$= x^2 + 4xy + 4y^2 - [x^2 - 4xy + 4y^2]$
$= x^2 + 4xy + 4y^2 - x^2 + 4xy - 4y^2$
$= 4xy + 4xy$
$= 8xy$
$=$ RHS
$\therefore (x+2y)^2 - (x-2y)^2 = 8xy$

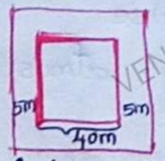
5. The pathway of a square paddy field has 5m width and length of its sides is 40m. Find the total area of its pathway.

Side of the square = 40m

Width of the pathway = 5m

Side of the larger square = 40m + 5m + 5m
= 50m

Area of the pathway = Area of large square − Area of small square



$= 50^2 - 40^2 = (50+40)(50-40)$
$= 2500 - 1600 = (90)(10)$
$= 900 m^2$

$\therefore$ Area of the pathway $= 900 m^2$

Challenge problems:

6. If $X = a^2 - 1$ and $Y = 1 - b^2$, then find $X+Y$ and factorize the same.

$X = a^2 - 1$, $Y = 1 - b^2$

$X + Y = a^2 - 1 + 1 - b^2$
$= a^2 - b^2$
$X + Y = (a+b)(a-b)$

7. Find the value of $(x-y)(x+y)(x^2+y^2)$

$(x-y)(x+y)(x^2+y^2) = (x^2-y^2)(x^2+y^2)$
$= (x^2)^2 - (y^2)^2$
$= x^4 - y^4$

8. Simplify $(5x-3y)^2 - (5x+3y)^2$

$(5x-3y)^2 - (5x+3y)^2 = [5x - \cancel{3y} + 5x + \cancel{3y}]$
$[\cancel{5x} - 3y - \cancel{5x} - 3y]$

$= (5x+5x)(-3y-3y)$
$= (10x)(-6y)$
$= -60xy$

9. Simplify

(i) $(a+b)^2 - (a-b)^2$

$(a+b)^2 - (a-b)^2 = a^2 + 2ab + b^2 - (a^2 - 2ab + b^2)$
$= a^2 + 2ab + b^2 - a^2 + 2ab - b^2$
$= 2ab + 2ab$
$= 4ab$

(ii) $(a+b)^2 + (a-b)^2$

$(a+b)^2 + (a-b)^2 = a^2 + 2ab + b^2 + a^2 - 2ab + b^2$
$= a^2 + b^2 + a^2 + b^2$
$= 2a^2 + 2b^2$
$= 2(a^2 + b^2)$

10. A square lawn has a 2m wide path surrounding it. If the area of the path is $136m^2$, find the area of lawn.

Let the side of the lawn = am

Side of big square = (a+2+2)m

= (a+4)m

Area of the path = Area of large square − Area of small square

$136 = (a+4)^2 - a^2$

$(a+4)^2 - a^2 = 136$

$a^2 + 2(a)(4) + (4)^2 - a^2 = 136$

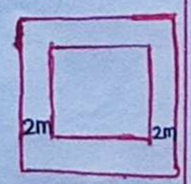
$8a + 16 = 136$

$8a = 136 - 16$

$8a = 120$

$a = \frac{120}{8}$

$\boxed{a = 15}$



Side of the lawn = 15m

Area of the lawn = a^2

= 15×15

= $225m^2$

∴ Area of the lawn = $225m^2$

11. Solve the following inequalities.

(i) $4n+7 \geq 3n+10$, n is an integer.

$4n+7 \geq 3n+10$

$4n-3n+7 \geq 3n-3n+10$

$n+7 \geq 10$

$n+7-7 \geq 10-7$

$\boxed{n \geq 3}$

$n = 3, 4, 5, 6, \ldots$

(ii) $6(x+6) \geq 5(x-3)$, x is a whole number.

$6(x+6) \geq 5(x-3)$

$6x + 6\times6 \geq 5x - 5\times3$

$6x + 36 \geq 5x - 15$

$6x - 5x + 36 \geq 5x - 5x - 15$

$x + 36 \geq -15$

$x + 36 - 36 \geq -15 - 36$

$\boxed{x \geq -51}$

x = whole number

$x = 0, 1, 2, 3, \ldots$

(iii) $-13 \leq 5x + 2 \leq 32$, x is an integer

$-13 \leq 5x + 2 \leq 32$

$-13 - 2 \leq 5x + 2 - 2 \leq 32 - 2$

$-15 \leq 5x \leq 30$

$\frac{-15}{5} \leq \frac{5x}{5} \leq \frac{30}{5}$

$-3 \leq x \leq 6$

$x = -3, -2, -1, 0, 1, 2, 3, 4, 5, 6$