CLASSICAL MECHANICS -TEST 5

1. **Whenever the Lagrangian for a system does not contain q_k a coordinate explicity…**
   a) q_k is cyclic coordinate b) p_k is cyclic coordinate
   c) p_k the generalized momentum , is a constant of motion d) q_k is always zero

2. **The dimension of generalized momentum**
   a) are always those of linear momentum b) are always those of angular momentum
   c) may be those of linear momentum d) may be those of angular momentum

3. **The generalized momentum p_k of a particle of mass m with velocity v_k in an electromagnetic field is given by…**
   a) $p_k = mv_k$ b) $p_k = mv_k + qA_k$ c) $p_k = mv_k - qA_k$ d) $p_k = qv_kA_k$

4. **Choose the incorrect statements:**
   a) The angular momentum is conserved for systems possess rotational symmetry.
   b) If the Lagrangian of the system is invariant under translation along a direction, the corresponding linear momentum is conserved.
   c) If the Lagrangian of the system is invariant under translation along a direction we cannot say anything about the corresponding linear momentum.
   d) For a conservative system, the Hamiltonian is equal to the sum of K.E and P.E.

5. **If the Lagrangian does not contain time explicity…**
   a) The Hamiltonian is constant b) The Hamiltonian cannot be constant
   c) The K.E is constant d) The P.E is constant.

6. **The product of generalized coordinate and its conjugate momentum has the dimensions of**
   a) Force b) energy c) Linear momentum d) angular momentum

7. **The Lagrangian of a particle of mass m moving in a plane is given by L=$\frac{1}{2}m(v_x^2 + v_y^2) + a(xv_y - yv_x)$ where v_x and v_y are velocity components and a is a constant . The canonical momenta are given by…**
   a) $P_x = mv_x \; and \; P_y = mv_y$ b) $P_x = mv_x + ay \; and \; P_y = mv_y + ax$
   c) $P_x = mv_x - ay \; and \; P_y = mv_y + ax$ d) $P_x = mv_x - ay \; and \; P_y = mv_y - ax$

8. **Hamilton canonical equation of motion for a conservative system are**
   a) $-\frac{dq_i}{dt} = \frac{\partial H}{\partial P_i} \; and \; -\frac{dP_i}{dt} = \frac{\partial H}{\partial q_i}$ b) $\frac{dP_i}{dt} = \frac{\partial H}{\partial P_i} \; and \; \frac{dP_i}{dt} = \frac{\partial H}{\partial q_i}$
   c) $-\frac{dq_i}{dt} = \frac{\partial H}{\partial P_i} \; and \; \frac{dP_i}{dt} = \frac{\partial H}{\partial q_i}$ d) $\frac{dq_i}{dt} = \frac{\partial H}{\partial P_i} \; and \; -\frac{dP_i}{dt} = \frac{\partial H}{\partial q_i}$

9. **The Lagrangian of a particle moving in a plane under the influence of a central potential is given by $L = \frac{1}{2} m\left(\dot{r}^2 + r^2\dot{\theta}^2\right) - V(r)$. the generalized momenta corresponding to r and θ are given by**
a) $m\dot{r}$ and $mr^2\dot{\theta}$ b) $m\dot{r}$ and $mr\dot{\theta}$ c) $m\dot{r}^2$ and $mr^2\dot{\theta}$ d) $m\dot{r}^2$ and $mr^2\dot{\theta}^2$

10. **The Hamiltonian corresponding to the Lagrangian $L = a\dot{x}^2 + b\dot{y}^2 - kxy$ is**
a) $\frac{P_x^2}{2a} + \frac{P_y^2}{2b} + kxy$ b) $\frac{P_x^2}{2a} + \frac{P_y^2}{2b} - kxy$ c) $\frac{P_x^2}{4a} + \frac{P_y^2}{4b} + kxy$ d) $\frac{P_x^2 + P_y^2}{4ab} + kxy$

11. **For a conservative system , Hamiltonian is**
a) $H = T + V$ b) $H = T - V$ c) $H = 2T - V$ d) $H = 2T + V$

12. **If a space is homogeneous in space, it follows**
a) conservation of mass. b) Conservation of linear momentum
c) Conservation of angular momentum d) Conservation of energy

13. **If a space is isotropic in space, it follows**
a) conservation of mass b) Conservation of linear momentum
c) Conservation of angular momentum d) Conservation of energy

14. **If the Lagrangian does not depend on time explicity, then the conserved quantity is**
a) Linear momentum b) Generalized momentum
c) Angular momentum d) Mechanical energy

15. **Hamiltonian is the function of**
A) Generalized coordinates and generalized velocities
b) generalized coordinates and generalized momenta
c) Generalized velocities and generalized momenta d)constants

16. **For a conservative system the potential energy does not depend upon**
a)Force b) generalized coordinate c)generalized velocities d) all

17. **In δ *variation***
a) Time t fixed at end points only b) Time t is fixed at all positions of path
c) Time t varies at all positions of path including end points
d) Time t is varied at end points only

18. **A physical system is invariant under rotation about a fixed axis. Then which of the following quantity is conserved:**
a) Total linear momentum
b) Linear momentum along the axis of rotation c) Total angular momentum
d) Angular momentum along the axis of rotation.

19. **A system of particles contain n interacting particles and its equations of constraints are represented by l equations, then the no. of generalized coordinates is**

$a)\ (n-l)$ $b)\ 3n$ $c)\ (3n-l)$ $d)\ 3(n-l)$

20. **The Hamiltonian is defined as**

a) $H = \sum_k P_k\, \dot{q}_k + L$ b) $H = \sum_k P_k\, \dot{q}_k - L$ c) $H = \sum_k q_k\, \dot{P}_k + L$ d) $H = \sum_k q_k\, \dot{P}_k - L$

21. **The Lagrangian of a free particle in spherical polar coordinates is $\frac{1}{2}m(\dot{r}^2 + r\dot{\theta}^2 + r^2\dot{\phi}^2 sin^2\theta)$ the conserved quantity is**

a) $\frac{\partial L}{\partial \dot{r}}$ b) $\frac{\partial L}{\partial \dot{\theta}}$ c) $\frac{\partial L}{\partial \dot{\phi}}$ d) $\frac{\partial L}{\partial \dot{\phi}} + r\theta$

22. **For a particle moving in a central field**

a) KE is constant b)PE is velocity dependent
c)Motion is confined in a plane d)Total energy is not conserved

23. **A particle is moving under the action of a generalized potential $V(q,\dot{q})=\frac{(1+\dot{q})}{q^2}$, the magnitude of generalized force is**

a) $\frac{2(1+\dot{q})}{q^3}$ b) $\frac{2(1-\dot{q})}{q^3}$ c) $\frac{2}{q^3}$ d) $\frac{\dot{q}}{q^3}$

24. **The Lagrangian of a system is given by $L = \frac{1}{2}\dot{q}^2 + q\dot{q} - \frac{1}{2}q^2$ it describes motion of a**

a) Harmonic oscillator b) Damped Harmonic
c)Anharmonic d)system with un bounded motion

25. **If a particles moves outwards in a plane along a curved trajectory described by $r = a\theta$, $\theta = \omega t$ where a and ω constants., then its**

a) KE is conserved b)Angular momentum conserved
c)Total momentum conserved d)Radial Momentum conserved

26. **A particle is placed in a region with the potential $V(x) = \frac{1}{2}kx^2 - \frac{\lambda}{3}x^3$, where k, $\lambda > 0$ then,**

(a) $x = 0$ and $x = \frac{k}{\lambda}$ are points of stable equilibrium
(b) $x = 0$ is a point of stable equilibrium and $x = \frac{k}{\lambda}$ is a point of unstable equilibrium
(c) $x = 0$ and $x = \frac{k}{\lambda}$ are points of stable unequilibrium
(d) There are no points of stable or unstable equilibrium

27. **The Lagrangian for a simple pendulum is given by $L = \frac{1}{2}ml^2\dot{\theta}^2 - mgk(1-\cos\theta)$ Hamilton's equations are then given by**

a) $\dot{p}_\theta = -mgl\sin\theta\ ;\ \dot{\theta} = \frac{p_\theta}{ml^2}$
b) $\dot{p}_\theta = mgl\sin\theta\ ;\ \dot{\theta} = \frac{p_\theta}{ml^2}$
c) $\dot{p}_\theta = -m\ddot{\theta}\ ;\ \dot{\theta} = \frac{p_\theta}{m}$
d) $\dot{p}_\theta = -\frac{g}{l}\theta\ ;\ \dot{\theta} = \frac{p_\theta}{ml}$

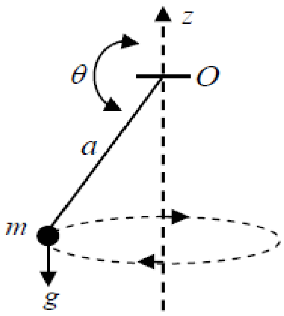
28. **A particle is moving under the action of a generalized potential $V(q,\dot{q}) = \frac{(1+\dot{q})}{q^2}$, the magnitude of generalized force is**

a) $\frac{2(1+\dot{q})}{q^3}$ b) $\frac{2(1-\dot{q})}{q^3}$ c) $\frac{2}{q^3}$ d) $\frac{\dot{q}}{q^3}$

29. **A particle of unit mass moves along the x-axis under the influence of a potential, $V(x) = x(x-2)^2$. . The particle is found to be in stable equilibrium at the point $x = 2$. The time period of oscillation of the particle is**

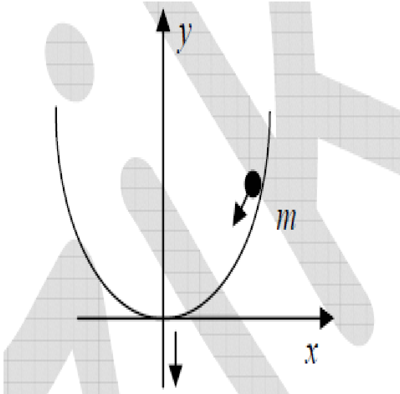
a) $\frac{\pi}{2}$ b) π c) $\frac{3\pi}{2}$ d) 2π

30. **A particle of mass m is attached to a fixed point O by a weightless inextensible string of length a . It is rotating under the gravity as shown in the figure. The Lagrangian of the particle is $L(\theta,\phi) = \frac{1}{2}ma^2\left(\dot{\theta}^2 + \sin^2\theta\,\dot{\phi}^2\right) - mga\cos\theta$ where θ and ϕ are the polar angles. The Hamiltonian of the particles is..**



a) $H = \frac{1}{2ma^2}\left(P_\theta^2 + \frac{P_\phi^2}{\sin^2\theta}\right) - mgacos\theta$
b)) $H = \frac{1}{2ma^2}\left(P_\theta^2 + \frac{P_\phi^2}{\sin^2\theta}\right) + mgacos\theta$
c)) $H = \frac{1}{2ma^2}\left(P_\theta^2 + P_\phi^2\right) - mgacos\theta$
d) $H = \frac{1}{2ma^2}\left(P_\theta^2 + P_\phi^2\right) + mgacos\theta$

31. **A particle of mass m slides under the gravity without friction along the parabolic path $y = ax^2$, as shown in the figure. Here a is a constant. The Lagrangian for this particle is given by**



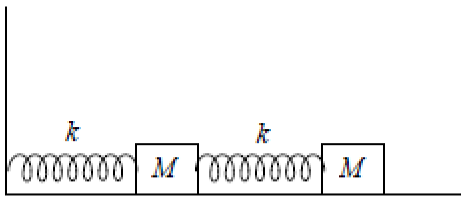
a) $L = \frac{1}{2}m\dot{x}^2 - mgax^2$
b) $L = \frac{1}{2}m\dot{x}^2 + mgax^2$
c) $L = \frac{1}{2}m(1+4a^2x^2)\dot{x}^2 - mgax^2$
d) $L = \frac{1}{2}m(1+4a^2x^2)\dot{x}^2 + mgax^2$

32. **The Lagrangian of a system with one degree of freedom q is given by $L = \alpha\dot{q}^2 + \beta q^2$, where α and β are non-zero constants. If P_q denotes the canonical momentum conjugate to q then which one of the following statements is CORRECT?**

(a) $P_q = 2\,\beta q$ and it is a conserved quantity.
(b) $P_q = 2\,\beta q$ and it is not a conserved quantity.
(c) $P_q = 2\,\alpha\dot{q}$ and it is a conserved quantity.
(d) $P_q = 2\,\alpha\dot{q}$ and it is not a conserved quantity.

33. **Consider two small blocks, each of mass M, attached to two identical springs. One of the springs is attached to the wall, as shown in the figure. The spring constant of each spring is k . The masses slide along the surface and the friction is negligible. The frequency of one of the normal modes of the system is,**

a) $\sqrt{\left(\frac{3+\sqrt{2}}{2}\right)}\sqrt{\left(\frac{k}{M}\right)}$
b) $\sqrt{\left(\frac{3+\sqrt{3}}{2}\right)}\sqrt{\left(\frac{k}{M}\right)}$
c) $\sqrt{\left(\frac{3+\sqrt{5}}{2}\right)}\sqrt{\left(\frac{k}{M}\right)}$
d) $\sqrt{\left(\frac{3+\sqrt{6}}{2}\right)}\sqrt{\left(\frac{k}{M}\right)}$



34. **The Hamiltonian for a system of two particles of masses m_1 and m_2 at $\vec{r_1}$ and $\vec{r_2}$ having velocities $\vec{v_1}$ and $\vec{v_2}$ is given by $H = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + \frac{C}{\vec{r_1}-\vec{r_2}}\hat{z}\,.(\vec{r_1}\times\vec{r_2})$ where C is constant. Which one of the following statements is correct?**

(a) The total energy and total momentum are conserved
(b) Only the total energy is conserved
(c) The total energy and the z - component of the total angular momentum are conserved
(d) The total energy and total angular momentum are conserved

35. **The Lagrangian of a system is given by $L = \frac{1}{2}ml^2(\dot{\theta}^2 + \dot{\phi}^2 sin^2\theta) - mgl\cos\theta$ where m,l and g are constants. Which of the following is conserved?**

A) $\dot{\phi}\, sin^2\theta$ **B)** $\dot{\phi}\, sin\theta$ **C)** $\frac{\dot{\phi}}{sin\theta}$ **D)** $\frac{\dot{\phi}}{sin^2\theta}$

36. **If the Lagrangian $L_0 = \frac{1}{2}m\left(\frac{dq}{dt}\right)^2 - \frac{1}{2}m\omega^2 q^2$ is modified to $L = L_0 + \alpha q\left(\frac{dq}{dt}\right)$, which one of the following is TRUE? 2017**

(a) Both the canonical momentum and equation of motion do not change
(b) Canonical momentum changes, equation of motion does not change
(c) Canonical momentum does not change, equation of motion changes

(d) Both the canonical momentum and equation of motion chang

37. In the context of small oscillations, which one of the following does NOT apply to the normal coordinates? (2018)

(a) Each normal coordinate has an eigen-frequency associated with it
(b) The normal coordinates are orthogonal to one another
(c) The normal coordinates are all independent
(d) The potential energy of the system is a sum of squares of the normal coordinates with constant coefficients

38. Consider the Hamiltonian $H(q,p) = \frac{\alpha P^2 q^4}{2} + \frac{\beta}{q^2}$ where α and β are parameters with appropriate dimensions, and q and p are the generalized coordinate and momentum, respectively. The corresponding Lagrangian $L(q,\dot{q},t)$ is

A. $\frac{1}{2\alpha}\frac{\dot{q}^2}{q^4} - \frac{\beta}{q^2}$ B. $\frac{1}{2\alpha}\frac{\dot{q}^2}{q^4} + \frac{\beta}{q^2}$ C) $\frac{1}{\alpha}\frac{\dot{q}^2}{q^4} + \frac{\beta}{q^2}$ D) $-\frac{1}{2\alpha}\frac{\dot{q}^2}{q^4} + \frac{\beta}{q^2}$

39. If a particle is moving along a sinusoidal curve, the number of degree of freedom of the particle is (2020)

Ans. : 1

40. Consider the Lagrangian $L = a\left(\frac{dx}{dt}\right)^2 + b\left(\frac{dy}{dt}\right)^2 + cxy$ where a,b and c are constants. If P_x and P_y are the momenta conjugate to the coordinates x and y respectively, then the Hamiltonian is (2020)

a) $\frac{P_x^2}{4a} + \frac{P_y^2}{4b} - cxy$ b) $\frac{P_x^2}{2a} + \frac{P_y^2}{2b} - cxy$ c) $\frac{P_x^2}{2a} + \frac{P_y^2}{2b} + cxy$ d) $\frac{P_x^2}{a} + \frac{P_y^2}{b} + cxy$

41. Which one of the following matrices does NOT represent a proper rotation in a plane? (2020)

A $\begin{pmatrix} -sin\theta & cos\theta \\ -cos\theta & -sin\theta \end{pmatrix}$ *B.* $\begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix}$ *C.* $\begin{pmatrix} sin\theta & cos\theta \\ -cos\theta & sin\theta \end{pmatrix}$ *D.* $\begin{pmatrix} -sin\theta & cos\theta \\ -cos\theta & sin\theta \end{pmatrix}$