Example 3.68

13. If $A=\begin{pmatrix}1 & -1 & 2\end{pmatrix}$, $B=\begin{pmatrix}1 & -1\\ 2 & 1\\ 1 & 3\end{pmatrix}$ and $C=\begin{pmatrix}1 & 2\\ 2 & -1\end{pmatrix}$ show that(AB)C = A(BC).

Solution :

LHS (AB) C

$$A=\begin{pmatrix}1 & -1 & 2\end{pmatrix}_{1\times3}\times\begin{pmatrix}1 & -1\\ 2 & 1\\ 1 & 3\end{pmatrix}_{3\times2}$$

$$=(1-2+2-1-1+6)=(1\ \ 4)$$

$$(AB)C=(1\ \ 4)_{1\times2}\times\begin{pmatrix}1 & 2\\ 2 & -1\end{pmatrix}$$

$$=(1+8\ \ 2-4)=(9\ \ -2) \quad \text{.........(1)}$$

RHS A(BC)

$$BC=\begin{pmatrix}1 & -1\\ 2 & 1\\ 1 & 3\end{pmatrix}_{3\times2}\times\begin{pmatrix}1 & 2\\ 2 & -1\end{pmatrix}_{2\times2}$$

$$=\begin{pmatrix}1-2 & 2+1\\ 2+2 & 4-1\\ 1+6 & 2-3\end{pmatrix}=\begin{pmatrix}-1 & 3\\ 4 & 3\\ 7 & -1\end{pmatrix}$$

$$A(BC)=(1\ \ -1\ \ 2)_{1\times3}\times\begin{pmatrix}-1 & 3\\ 4 & 3\\ 7 & -1\end{pmatrix}_{3\times2}$$

$$A(BC)=(-1-4+14\ \ \ 3-3-2)$$

$$=(9\ \ -2) \quad \text{...........(2)}$$

From (1) and (2), (AB)C = A(BC).

12. If $A=\begin{pmatrix}3 & 1\\ -1 & 2\end{pmatrix}$ show that $A^2-5A+7I_2=0$

Solution :

Given $A=\begin{pmatrix}3 & 1\\ -1 & 2\end{pmatrix}$

To verify : $A^2-5A+7I_2=0$

$$A^2=\begin{pmatrix}3 & 1\\ -1 & 2\end{pmatrix}\begin{pmatrix}3 & 1\\ -1 & 2\end{pmatrix}$$

$$=\begin{pmatrix}9-1 & 3+2\\ -3-2 & -1+4\end{pmatrix}$$

$$=\begin{pmatrix}8 & 5\\ -5 & 3\end{pmatrix}$$

$$\therefore A^2-5A+7I_2$$

$$=\begin{pmatrix}8 & 5\\ -5 & 3\end{pmatrix}-\begin{pmatrix}15 & 5\\ -5 & 10\end{pmatrix}+7\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

$$=\begin{pmatrix}8 & 5\\ -5 & 3\end{pmatrix}-\begin{pmatrix}15 & 5\\ -5 & 10\end{pmatrix}+\begin{pmatrix}7 & 0\\ 0 & 7\end{pmatrix}$$

$$=\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}$$

$$=0$$

11. If $A=\begin{pmatrix}\cos\theta & 0\\ 0 & \cos\theta\end{pmatrix}$, $B=\begin{pmatrix}\sin\theta & 0\\ 0 & \sin\theta\end{pmatrix}$ then show that $A^2+B^2=I$.

Solution :

Given $A=\begin{pmatrix}\cos\theta & 0\\ 0 & \cos\theta\end{pmatrix}$, $B=\begin{pmatrix}\sin\theta & 0\\ 0 & \sin\theta\end{pmatrix}$

To show : $A^2+B^2=I$

$$A^2=A.A$$

$$=\begin{pmatrix}\cos\theta & 0\\ 0 & \cos\theta\end{pmatrix}\begin{pmatrix}\cos\theta & 0\\ 0 & \cos\theta\end{pmatrix}$$

$$=\begin{pmatrix}\cos^2\theta+0 & 0+0\\ 0+0 & 0+\cos^2\theta\end{pmatrix}$$

$$=\begin{pmatrix}\cos^2\theta & 0\\ 0 & \cos^2\theta\end{pmatrix}$$

$$B^2=BB=\begin{pmatrix}\sin\theta & 0\\ 0 & \sin\theta\end{pmatrix}\begin{pmatrix}\sin\theta & 0\\ 0 & \sin\theta\end{pmatrix}$$

$$=\begin{pmatrix}\sin^2\theta+0 & 0+0\\ 0+0 & 0+\sin^2\theta\end{pmatrix}$$

$$=\begin{pmatrix}\sin^2\theta & 0\\ 0 & \sin^2\theta\end{pmatrix}$$

$$\therefore A^2+B^2=\begin{pmatrix}\cos^2\theta+\sin^2\theta & 0\\ 0 & \cos^2\theta+\sin^2\theta\end{pmatrix}$$

$$=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

$$=I$$

Hence proved.

10. Find X and Y if $X+Y=\begin{pmatrix}7 & 0\\ 3 & 5\end{pmatrix}$ and $X-Y=\begin{pmatrix}3 & 0\\ 0 & 4\end{pmatrix}$

Solution :

Given $X+Y=\begin{pmatrix}7 & 0\\ 3 & 5\end{pmatrix}$ .... (1)

$X-Y=\begin{pmatrix}3 & 0\\ 0 & 4\end{pmatrix}$ .... (2)

$(1)+(2) \Rightarrow 2X=\begin{pmatrix}10 & 0\\ 3 & 9\end{pmatrix}$

$\Rightarrow X=\begin{pmatrix}5 & 0\\ 3/2 & 9/2\end{pmatrix}$

$(1)-(2) \Rightarrow 2Y=\begin{pmatrix}4 & 0\\ 3 & 1\end{pmatrix}$

$\Rightarrow Y=\begin{pmatrix}2 & 0\\ 3/2 & 1/2\end{pmatrix}$

1. If $A=\begin{pmatrix}0 & 4 & 9\\ 8 & 3 & 7\end{pmatrix}$, $B=\begin{pmatrix}7 & 3 & 8\\ 1 & 4 & 9\end{pmatrix}$ find the value of i) B − 5A ii) 3A − 9B

Solution :

Given

$$A=\begin{pmatrix}0 & 4 & 9\\ 8 & 3 & 7\end{pmatrix}, B=\begin{pmatrix}7 & 3 & 8\\ 1 & 4 & 9\end{pmatrix}$$

i) $B-5A=\begin{pmatrix}7 & 3 & 8\\ 1 & 4 & 9\end{pmatrix}-\begin{pmatrix}0 & 20 & 45\\ 40 & 15 & 35\end{pmatrix}$

$$=\begin{pmatrix}7 & -17 & -37\\ -39 & -11 & -26\end{pmatrix}$$

ii) $3A-9B=\begin{pmatrix}0 & 12 & 27\\ 24 & 9 & 21\end{pmatrix}-\begin{pmatrix}63 & 27 & 72\\ 9 & 36 & 81\end{pmatrix}$

$$=\begin{pmatrix}-63 & -65 & -45\\ 15 & -27 & -60\end{pmatrix}$$

2. Find the values of x, y, z if

i) $\begin{pmatrix}x-3 & 3x-z\\ x+y+7 & x+y+z\end{pmatrix}=\begin{pmatrix}1 & 0\\ 1 & 6\end{pmatrix}$

ii) (x y − z z + 3) + (y 4 3) = (4 8 16)

Solution :

i) Given $\begin{pmatrix}x-3 & 3x-z\\ x+y+7 & x+y+z\end{pmatrix}=\begin{pmatrix}1 & 0\\ 1 & 6\end{pmatrix}$

$\Rightarrow x-3=1$, $\therefore x=4$ | $3x-z=0$, $12-z=0$, $z=12$ | $x+y+7=1$ $\Rightarrow x+y=-6$ $\Rightarrow 4+y=-6$ $\Rightarrow y=-10$

ii) (x y − z z + 3) + (y 4 3) = (4 8 16)

$\Rightarrow x+y=4$ $\Rightarrow x+14=4$ $\Rightarrow x=-10$ | $y-z+4=8$, $y-z=4$, $y-10=4$, $\therefore y=14$ | $z+6=16$, $z=10$

$\therefore x=-10, y=14, z=10$

3. Solve for x, y $\begin{pmatrix}x^2\\ y^2\end{pmatrix}+2\begin{pmatrix}-2x\\ y\end{pmatrix}=\begin{pmatrix}5\\ 8\end{pmatrix}$

Solution :

Given $\begin{pmatrix}x^2\\ y^2\end{pmatrix}+2\begin{pmatrix}-2x\\ y\end{pmatrix}=\begin{pmatrix}5\\ 8\end{pmatrix}$

$\Rightarrow x^2-4x=5$ $\Rightarrow x^2-4x-5=0$ $\Rightarrow (x-5)(x+1)=0$ $\Rightarrow x=5,-1$ | $y^2-2y=8$ $\Rightarrow y^2-2y-8=0$ $\Rightarrow (y-4)(y+2)=0$ $\Rightarrow \therefore y=4, y=-2$

4. Find x and y if $x\begin{pmatrix}4\\ -3\end{pmatrix}+y\begin{pmatrix}-2\\ 3\end{pmatrix}=\begin{pmatrix}4\\ 6\end{pmatrix}$

Solution :

Given $x\begin{pmatrix}4\\ -3\end{pmatrix}+y\begin{pmatrix}-2\\ 3\end{pmatrix}=\begin{pmatrix}4\\ 6\end{pmatrix}$

$\Rightarrow 4x-2y=4 \quad -3x+3y=6$

$\Rightarrow 2x-y=2$ .... (1)

$-x+y=2$ .... (2)

(1) $\Rightarrow 2x-y=2$

(2) $\Rightarrow -x+y=2$

Adding, $x=4$

Sub x = 4 in (2)

$-4+y=2 \Rightarrow y=6$

$\therefore x=4, y=6$

5. Find the non-zero values of x satisfying the matrix equation

$$x\begin{pmatrix}2x & 2\\ 3 & x\end{pmatrix}+2\begin{pmatrix}8 & 5x\\ 4 & 4x\end{pmatrix}=2\begin{pmatrix}x^2+8 & 24\\ 10 & 6x\end{pmatrix}$$

Solution :

Given $x\begin{pmatrix}2x & 2\\ 3 & x\end{pmatrix}+2\begin{pmatrix}8 & 5x\\ 4 & 4x\end{pmatrix}=2\begin{pmatrix}x^2+8 & 24\\ 10 & 6x\end{pmatrix}$

$$\Rightarrow \begin{pmatrix}2x^2 & 2x\\ 3x & x^2\end{pmatrix}+\begin{pmatrix}16 & 10x\\ 8 & 8x\end{pmatrix}=\begin{pmatrix}2x^2+16 & 48\\ 20 & 12x\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}2x^2+16 & 12x\\ 3x+8 & x^2+8x\end{pmatrix}=\begin{pmatrix}2x^2+16 & 48\\ 20 & 12x\end{pmatrix}$$

$\therefore 12x=48 \Rightarrow x=4$; $3x+8=20 \Rightarrow 3x=12 \Rightarrow x=4$ | $x^2+8x=12x$ $\Rightarrow x^2-4x=0$ $\Rightarrow x(x-4)=0$ $\Rightarrow x=0, x=4$

$\therefore x=4$

6. If $A=\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}$ prove that $AA^T=I$

Solution :

Given $A=\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}$

To prove : $A.A^T=I$

LHS :

$$A.A^T=\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}$$

$$=\begin{pmatrix}\cos^2\theta+\sin^2\theta & -\cos\theta\sin\theta+\sin\theta\cos\theta\\ -\sin\theta\cos\theta+\cos\theta\sin\theta & \sin^2\theta+\cos^2\theta\end{pmatrix}$$

$$=I$$

Hence proved.

7. Verify that $A^2=I$ when $A=\begin{pmatrix}5 & -4\\ 6 & -5\end{pmatrix}$

Solution :

Given $A=\begin{pmatrix}5 & -4\\ 6 & -5\end{pmatrix}$

To prove : $A^2=I$

$$A^2=A.A=\begin{pmatrix}5 & -4\\ 6 & -5\end{pmatrix}\begin{pmatrix}5 & -4\\ 6 & -5\end{pmatrix}$$

$$=\begin{pmatrix}25-24 & -20+20\\ 30-30 & -24+25\end{pmatrix}$$

$$=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

$$=I$$

Hence proved.

Example 3.67

8. Solve $\begin{pmatrix}2 & 1\\ 1 & 2\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}4\\ 5\end{pmatrix}$

Solution :

$$\begin{pmatrix}2 & 1\\ 1 & 2\end{pmatrix}_{2\times2}\times\begin{pmatrix}x\\ y\end{pmatrix}_{2\times1}=\begin{pmatrix}4\\ 5\end{pmatrix}$$

By matrix multiplication $\begin{pmatrix}2x+y\\ x+2y\end{pmatrix}=\begin{pmatrix}4\\ 5\end{pmatrix}$

Rewriting $2x+y=4$ .... (1)

$x+2y=5$ .... (2)

(1) − 2 × (2) gives $2x+y=4$

$2x+4y=10$ (−)

$-3y=-6$ gives y = 2

Substituting y = 2 in (1), 2x + 2 = 4 gives x = 1

Therefore, x = 1, y = 2.

9. Show that the matrices $A=\begin{pmatrix}1 & 2\\ 3 & 1\end{pmatrix}$, $B=\begin{pmatrix}1 & -2\\ -3 & 1\end{pmatrix}$ satisfy commutative property AB = BA

Solution :

Given $A=\begin{pmatrix}1 & 2\\ 3 & 1\end{pmatrix}$, $B=\begin{pmatrix}1 & -2\\ -3 & 1\end{pmatrix}$

$$AB=\begin{pmatrix}1 & 2\\ 3 & 1\end{pmatrix}\begin{pmatrix}1 & -2\\ -3 & 1\end{pmatrix}$$

$$=\begin{pmatrix}1-6 & -2+2\\ 3-3 & -6+1\end{pmatrix}$$

$$=\begin{pmatrix}-5 & 0\\ 0 & -5\end{pmatrix}$$

$$BA=\begin{pmatrix}1 & -2\\ -3 & 1\end{pmatrix}\begin{pmatrix}1 & 2\\ 3 & 1\end{pmatrix}$$

$$=\begin{pmatrix}1-6 & -2+2\\ -3+3 & -6+1\end{pmatrix}$$

$$=\begin{pmatrix}-5 & 0\\ 0 & -5\end{pmatrix}$$

$\therefore AB=BA$

$\therefore$ Commutative property is true.

Example 3.69

14. If $A=\begin{pmatrix}1&1\\-1&3\end{pmatrix}, B=\begin{pmatrix}1&2\\-4&2\end{pmatrix}, C=\begin{pmatrix}-7&6\\3&2\end{pmatrix}$ verify that A (B + C) = AB + AC.

Solution :

LHS A (B + C)

$$B+C=\begin{pmatrix}1&2\\-4&2\end{pmatrix}+\begin{pmatrix}-7&6\\3&2\end{pmatrix}=\begin{pmatrix}-6&8\\-1&4\end{pmatrix}$$

$$A(B+C)=\begin{pmatrix}1&1\\-1&3\end{pmatrix}\times\begin{pmatrix}-6&8\\-1&4\end{pmatrix}$$

$$=\begin{pmatrix}-6-1&8+4\\6-3&-8+12\end{pmatrix}=\begin{pmatrix}-7&12\\3&4\end{pmatrix} \quad ...(1)$$

RHS AB + AC

$$AB=\begin{pmatrix}1&1\\-1&3\end{pmatrix}\times\begin{pmatrix}1&2\\-4&2\end{pmatrix}=\begin{pmatrix}1-4&2+2\\-1-12&-2+6\end{pmatrix}$$

$$=\begin{pmatrix}-3&4\\-13&4\end{pmatrix}$$

$$AC=\begin{pmatrix}1&1\\-1&3\end{pmatrix}\times\begin{pmatrix}-7&6\\3&2\end{pmatrix}=\begin{pmatrix}-7+3&6+2\\7+9&-6+6\end{pmatrix}$$

$$=\begin{pmatrix}-4&8\\16&0\end{pmatrix}$$

Therefore, $AB+AC=\begin{pmatrix}-3&4\\-13&4\end{pmatrix}+\begin{pmatrix}-4&8\\16&0\end{pmatrix}$

$$=\begin{pmatrix}-7&12\\3&4\end{pmatrix} \quad ......(2)$$

From (1) and (2), A (B + C) = AB + AC.

Hence proved.

15. Given that $A=\begin{pmatrix}1&3\\5&-1\end{pmatrix}, B=\begin{pmatrix}1&-1&2\\3&5&2\end{pmatrix}, C=\begin{pmatrix}1&3&2\\-4&1&3\end{pmatrix}$ **verify that A(B + C) = AB + AC**.

Solution :

Given

$$A=\begin{pmatrix}1&3\\5&-1\end{pmatrix}, B=\begin{pmatrix}1&-1&2\\3&5&2\end{pmatrix}, C=\begin{pmatrix}1&3&2\\-4&1&3\end{pmatrix}$$

To verify : A (B + C) = AB + AC

LHS : A (B + C)

$$=\begin{pmatrix}1&3\\5&-1\end{pmatrix}\begin{pmatrix}2&2&4\\-1&6&5\end{pmatrix}$$

$$=\begin{pmatrix}1\times2+3\times-1&1\times2+3\times6&1\times4+3\times5\\5\times2+(-1)\times(-1)&5\times2-1\times6&5\times4-1\times5\end{pmatrix}$$

$$=\begin{pmatrix}2-3&2+18&4+15\\10+1&10-6&20-5\end{pmatrix}$$

$$=\begin{pmatrix}-1&20&19\\11&4&15\end{pmatrix} \quad ....(1)$$

RHS : AB + AC

$$=\begin{pmatrix}1&3\\5&-1\end{pmatrix}\begin{pmatrix}1&-1&2\\3&5&2\end{pmatrix}+\begin{pmatrix}1&3\\5&-1\end{pmatrix}\begin{pmatrix}1&3&2\\-4&1&3\end{pmatrix}$$

$$=\begin{pmatrix}1+9&-1+15&2+6\\5-3&-5-5&10-2\end{pmatrix}+\begin{pmatrix}1-12&3+3&2+9\\5+4&15-1&10-3\end{pmatrix}$$

$$=\begin{pmatrix}10&14&8\\2&-10&8\end{pmatrix}+\begin{pmatrix}-11&6&11\\9&14&7\end{pmatrix}$$

$$=\begin{pmatrix}-1&20&19\\11&4&15\end{pmatrix} \quad ....(2)$$

∴ From (1) & (2) LHS = RHS

Example 3.70

16. If $A=\begin{pmatrix}1&2&1\\2&-1&1\end{pmatrix}$ and $B=\begin{pmatrix}2&-1\\-1&4\\0&2\end{pmatrix}$ show that $(AB)^T=B^TA^T$.

Solution :

LHS $(AB)^T$

$$AB=\begin{pmatrix}1&2&1\\2&-1&1\end{pmatrix}_{2\times3}\times\begin{pmatrix}2&-1\\-1&4\\0&2\end{pmatrix}_{3\times2}$$

$$=\begin{pmatrix}2-2+0&-1+8+2\\4+1+0&-2-4+2\end{pmatrix}=\begin{pmatrix}0&9\\5&-4\end{pmatrix}$$

$$(AB)^T=\begin{pmatrix}0&9\\5&-4\end{pmatrix}^T=\begin{pmatrix}0&5\\9&-4\end{pmatrix} \quad ....(1)$$

RHS (B^TA^T)

$$B^T=\begin{pmatrix}2&-1&0\\-1&4&2\end{pmatrix}, A^T=\begin{pmatrix}1&2\\2&-1\\1&1\end{pmatrix}$$

$$B^TA^T=\begin{pmatrix}2&-1&0\\-1&4&2\end{pmatrix}_{2\times3}\begin{pmatrix}1&2\\2&-1\\1&1\end{pmatrix}_{3\times2}$$

$$=\begin{pmatrix}2-2+0&4+1+0\\-1+8+2&-2-4+2\end{pmatrix}$$

$$B^TA^T=\begin{pmatrix}0&5\\9&-4\end{pmatrix} \quad ....(2)$$

From (1) and (2), $(AB)^T=B^TA^T$.

Hence proved.

17. If $A=\begin{pmatrix}5&2&9\\1&2&8\end{pmatrix}, B=\begin{pmatrix}1&7\\1&2\\5&-1\end{pmatrix}$ **verify that $(AB)^T=B^TA^T$**

Solution :

Given $A=\begin{pmatrix}5&2&9\\1&2&8\end{pmatrix}, B=\begin{pmatrix}1&7\\1&2\\5&-1\end{pmatrix}$

To verify : $(AB)^T=B^TA^T$

$$AB=\begin{pmatrix}5&2&9\\1&2&8\end{pmatrix}\begin{pmatrix}1&7\\1&2\\5&-1\end{pmatrix}$$

$$=\begin{pmatrix}5\times1+2\times1+9\times5&5\times7+2\times2+9\times-1\\1\times1+2\times1+8\times5&1\times7+2\times2+8\times-1\end{pmatrix}$$

$$=\begin{pmatrix}5+2+45&35+4-9\\1+2+40&7+4-8\end{pmatrix}$$

$$=\begin{pmatrix}52&30\\43&3\end{pmatrix}$$

$$\therefore (AB)^T=\begin{pmatrix}52&43\\30&3\end{pmatrix} \quad ....(1)$$

$$B^TA^T=\begin{pmatrix}1&1&5\\7&2&-1\end{pmatrix}\begin{pmatrix}5&1\\2&2\\9&8\end{pmatrix}$$

$$=\begin{pmatrix}5+2+45&1+2+40\\35+4-9&7+4-8\end{pmatrix}$$

$$=\begin{pmatrix}52&43\\30&3\end{pmatrix} \quad ....(2)$$

∴ From (1) & (2), $(AB)^T=B^TA^T$

Hence proved.

18. If $A=\begin{pmatrix}a&b\\c&d\end{pmatrix}$ and $I=\begin{pmatrix}1&0\\0&1\end{pmatrix}$ **show that $A^2-(a+d)A=(bc-ad)I_2$.**

Solution :

Given $A=\begin{pmatrix}a&b\\c&d\end{pmatrix}, I=\begin{pmatrix}1&0\\0&1\end{pmatrix}$

To prove : $A^2-(a+d)A=(bc-ad)I$

$$A^2=A.A$$

$$=\begin{pmatrix}a&b\\c&d\end{pmatrix}\cdot\begin{pmatrix}1&0\\0&1\end{pmatrix}$$

$$=\begin{pmatrix}a^2+bc&ab+bd\\ac+cd&bc+d^2\end{pmatrix}$$

$$(a+d)A=(a+d)\begin{pmatrix}a&b\\c&d\end{pmatrix}$$

$$=\begin{pmatrix}a^2+ad&ab+bd\\ca+cd&ad+d^2\end{pmatrix}$$

$$A^2-(a+d)A$$

$$=\begin{pmatrix}a^2+bc&ab+bd\\ac+cd&bc+d^2\end{pmatrix}-\begin{pmatrix}a^2+ad&ab+bd\\ca+cd&ad+d^2\end{pmatrix}$$

$$=\begin{pmatrix}bc+ad&0\\0&bc-ad\end{pmatrix}$$

$$=(bc-ad)\begin{pmatrix}1&0\\0&1\end{pmatrix}$$

$$=(bc-ad)(I)$$

= RHS

Hence proved.

19. Let $A=\begin{pmatrix}1&2\\1&3\end{pmatrix}, B=\begin{pmatrix}4&0\\1&5\end{pmatrix}, C=\begin{pmatrix}2&0\\1&2\end{pmatrix}$

Show that **i) A(BC) = (AB)C**

ii) (A − B) C = AC − BC

iii) $(A-B)^T=A^T-B^T$

Solution :

Given $A=\begin{pmatrix}1&2\\1&3\end{pmatrix}, B=\begin{pmatrix}4&0\\1&5\end{pmatrix}, C=\begin{pmatrix}2&0\\1&2\end{pmatrix}$

i) To show : A(BC) = (AB)C

$$BC=\begin{pmatrix}4&0\\1&5\end{pmatrix}\begin{pmatrix}2&0\\1&2\end{pmatrix}=\begin{pmatrix}8+0&0+0\\2+5&0+10\end{pmatrix}$$

$$=\begin{pmatrix}8&0\\7&10\end{pmatrix}$$

$$A(BC)=\begin{pmatrix}1&2\\1&3\end{pmatrix}\begin{pmatrix}8&0\\7&10\end{pmatrix}=\begin{pmatrix}8+14&0+20\\8+21&0+30\end{pmatrix}$$

$$=\begin{pmatrix}22&20\\29&30\end{pmatrix} \quad ....(1)$$

$$AB=\begin{pmatrix}1&2\\1&3\end{pmatrix}\begin{pmatrix}4&0\\1&5\end{pmatrix}$$

$$=\begin{pmatrix}4+2&0+10\\4+3&0+15\end{pmatrix}$$

$$=\begin{pmatrix}6&10\\7&15\end{pmatrix}$$

$$(AB)C=\begin{pmatrix}6&10\\7&15\end{pmatrix}\begin{pmatrix}2&0\\1&2\end{pmatrix}$$

$$=\begin{pmatrix}12+10&0+20\\14+15&0+30\end{pmatrix}$$

$$=\begin{pmatrix}22&20\\29&30\end{pmatrix} \quad ....(2)$$

∴ From (1) & (2) LHS = RHS

ii) To show (A − B) C = AC − BC

(A − B) C

$$=\begin{pmatrix}-3&2\\0&-2\end{pmatrix}\begin{pmatrix}2&0\\1&2\end{pmatrix}$$

$$=\begin{pmatrix}-6+2&0+4\\0-2&0-4\end{pmatrix}$$

$$=\begin{pmatrix}-4&4\\-2&-4\end{pmatrix} \quad ....(1)$$

$$AC=\begin{pmatrix}1&2\\1&3\end{pmatrix}\begin{pmatrix}2&0\\1&2\end{pmatrix}$$

$$=\begin{pmatrix}2+2&0+4\\2+3&0+6\end{pmatrix}$$

$$=\begin{pmatrix}4&4\\5&6\end{pmatrix}$$

$$BC=\begin{pmatrix}4&0\\1&5\end{pmatrix}\begin{pmatrix}2&0\\1&2\end{pmatrix}$$

$$=\begin{pmatrix}8+0&0+0\\2+5&0+10\end{pmatrix}$$

$$=\begin{pmatrix}8&0\\7&10\end{pmatrix}$$

$$\therefore AC-BC=\begin{pmatrix}4&4\\5&6\end{pmatrix}-\begin{pmatrix}8&0\\7&10\end{pmatrix}$$

$$=\begin{pmatrix}-4&4\\-2&-4\end{pmatrix} \quad ....(2)$$

∴ From (1) & (2), LHS = RHS

iii) To show : $(A-B)^T=A^T-B^T$

$$(A-B)^T=\begin{pmatrix}-3&2\\0&-2\end{pmatrix}^T=\begin{pmatrix}-3&0\\2&-2\end{pmatrix} \quad ....(1)$$

$$A^T-B^T=\begin{pmatrix}1&1\\2&3\end{pmatrix}-\begin{pmatrix}4&1\\0&5\end{pmatrix}$$

$$=\begin{pmatrix}-3&0\\2&-2\end{pmatrix} \quad ....(2)$$

∴ From (1) & (2)

$(A-B)^T=A^T-B^T$

அணிகள்.

S. G. BALASUBRAMANI, M.Sc.,B.Ed.,
B. T. ASST. (MATHS)
GOVT. HIGH SCHOOL,
MACHAMBUT - 635 808,
VELLORE DISTRICT.

MATRICES.