11th STD

PUBLIC EXAMINATION - MARCH 2024

Reg. No.

PART - III

Business Mathematics And Statistics (with answers)

TIME ALLOWED : 3.00 Hours] [MAXIMUM MARKS : 90

Instructions :

(1) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.

(2) Use **Blue** or **Black** ink to write and underline and pencil to draw diagrams.

PART - I

Note : (i) Answer **all** the questions. **20 × 1 = 20**

(ii) Choose the most appropriate answer from the given **four** alternatives and write the option code and the corresponding answer.

1. The inventor of input-output analysis is :

(a) Prof. Wassily W. Leontief
(b) Sir Francis Galton
(c) Arthur Caylay
(d) Fisher

2. The number of permutation of n different things taken r at a time, when the repetition is allowed is :

(a) $\frac{n!}{(n-r)!}$ (b) r^n
(c) $\frac{n!}{(n+r)!}$ (d) n^r

3. If $y = e^{2x}$, then $\frac{d^2y}{dx^2}$ at $x = 0$ is :

(a) 2 (b) 4 (c) 0 (d) 9

4. The correlation coefficient :

(a) $r = b_{xy} \times b_{yx}$ (b) $r = \pm\sqrt{b_{xy} \times b_{yx}}$
(c) $r = \pm\sqrt{\frac{1}{b_{xy} \times b_{yx}}}$ (d) $r = \frac{1}{b_{xy} \times b_{yx}}$

5. Let P(A) = $\frac{3}{5}$ and P(B) = $\frac{1}{5}$. Find P(A∩B) if A and B are independent events.

(a) $\frac{3}{16}$ (b) $\frac{3}{25}$ (c) $\frac{4}{25}$ (d) $\frac{4}{10}$

6. If tanA = $\frac{1}{2}$ and tanB = $\frac{1}{3}$ then tan (2A +B) is equal to :

(a) 3 (b) 1 (c) 4 (d) 2

7. For the function $y = x^3 + 19$, find the values of x when its marginal value is equal to 27.

(a) ± 3 (b) ±1 (c) ±4 (d) ±2

8. If 'a' is the annual payment 'n' is the number of periods and 'i' is compound interest for ₹1 then future amount of the ordinary annuity is :

(a) $P = \frac{a}{i}$

(b) $A = \frac{a}{i}(1+i)[(1+i)^n - 1]$

(c) $P = \frac{a}{i}(1+i)[1-(1+i)^{-n}]$

(d) $A = \frac{a}{i}[(1+i)^n - 1]$

9. If $\begin{vmatrix} x & 2 \\ 8 & 5 \end{vmatrix} = 0$ then the value of x is :

(a) $\frac{-16}{5}$ (b) $\frac{-5}{6}$ (c) $\frac{16}{5}$ (d) $\frac{5}{6}$

10. If $y = 4ae^{4x}$, then find $\frac{dy}{dx}$:

(a) $16ae^x$ (b) ae^{4x}
(c) $16ae^{4x}$ (d) $4ae^{4x}$

11. The value of sin (–420°) is :

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{-1}{2}$ (d) $\frac{-\sqrt{3}}{2}$

12. If m_1 and m_2 are the slopes of the pair of lines given by $ax^2 + 2hxy + by^2 = 0$, then the value of $m_1 + m_2$ is :

(a) $\frac{2h}{a}$ (b) $\frac{2h}{b}$ (c) $-\frac{2h}{a}$ (d) $-\frac{2h}{b}$

13. One of the conditions for the activity (i, j) to lie on the critical path is :
(a) $E_j - E_i = L_i - L_j = t_{ij}$ (b) $E_j - E_i = L_j - L_i = t_{ij}$
(c) $E_j - E_i = L_j - L_i \neq t_{ij}$. (d) $E_i - E_j = L_j - L_i = t_{ij}$

14. If $q = 1000 + 8p_1 - p_2$, then $\frac{\partial q}{\partial p_1}$ is :
(a) 1000 (b) –1
(c) $1000 - P_2$ (d) 8

15. The dividend received on 200 shares of Face Value ₹ 100 at 8% stock is :
(a) ₹1500 (b) ₹1600 (c) ₹800 (d) ₹1000

16. If clockwise and anticlockwise circular permutations are considered to be same, the number of circular permutation of n objects taken all at a time is :
(a) $\frac{n!}{2}$ (b) $\frac{(n+1)!}{2}$
(c) $\frac{(2n+1)!}{2}$ (d) $\frac{(n-1)!}{2}$

17. If two events A and B are dependent, then the conditional probability of P(B/ A) is :
(a) $\frac{P(A \cap B)}{P(A)}$ (b) P(A) P(B/ A)
(c) P(A) P(A/B) (d) $\frac{P(A \cap B)}{P(B)}$

18. Correlation co-efficient lies between :
(a) –1 to 0 (b) 0 to ∞.
(c) – 1 to ∞ (d) –1 to +1

19. Find the equation of the circle with centre at (4, 5) and radius 3 units.
(a) $x^2 + y^2 - 8x - 10y + 32 = 0$
(b) $x^2 + y^2 - 6x + 2y - 6 = 0$
(c) $x^2 + y^2 - 8x - 6y = 0$
(d) $x^2 + y^2 + 8x - 10y = 0$

20. Find the value of $^{100}C_{99}$:
(a) 1 (b) 100 (c) 0 (d) 99

PART - II

Note : Answer **any seven** questions. Question No. **30** is **Compulsory.** **7 × 2 = 14**

21. Evaluate $\begin{vmatrix} 1 & 3 & 4 \\ 102 & 18 & 36 \\ 17 & 3 & 6 \end{vmatrix}$

22. If $^nC_4 = {^nC_6}$, find $^{12}C_n$.

23. If $f(x) = x^3 - \frac{1}{x^3}, x \neq 0$, then show that $f(x) + f\left(\frac{1}{x}\right) = 0$

24. Find D_2 and D_6 for the following series 22, 4, 2, 12, 16, 6, 10, 18, 14, 20, 8.

25. Which is better investment? 20% stock at ₹140 (or) 10% stock at ₹70.

26. A manufacturing company has a contract to supply 4000 units of an item per year at uniform rate. The storage cost per unit per year amounts to ₹50 and the set-up cost per production run is ₹160. If the production run can be started instantaneously and shortages are not permitted, determine the number of units which should be produced per run to minimize the total inventory cost.

27. If the centre of the circle $x^2 + y^2 + 2x - 6y + 1 = 0$ lies on a straight line $ax + 2y + 2 = 0$, then find the value of 'a'.

28. Find the 5th term in the expansion of $(x - 2y)^{13}$.

29. If $f(x) = 2x$, then show that $f(x).f(y) = f(x + y)$

30. Find the rank of the word "TABLE" in English dictionary.

PART - III

Note : Answer **any seven** questions. Question No. **40** is **Compulsory**. **7 × 3 = 21**

31. Find the values of a and b if the equation $(a-1)x^2 + by^2 + (b-8)xy + 4x + 4y - 1 = 0$ represents a circle .

32. Evaluate : $\lim_{x \to 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x}$.

33. Draw the event oriented network for the following data.

Events	1	2	3	4	5	6	7
Immediate Predecessors	-	1	1	2,3	3	4,5	5,6

34. Solve the following linear programming problems by graphical method.
Maximize $z = 40x_1 + 50x_2$ subject to constraints $3x_1 + x_2 \leq 9; x_1 + 2x_2 \leq 8$ and $x_1, x_2 \geq 0$.

35. Find the interval in which the function $f(x) = x^2 - 4x + 6$ is strictly increasing and strictly decreasing.

36. Find the value of tan 75°.

37. A person purchases tomatoes from each of the 4 places at the rate of 1 kg., 2 kg., 3 kg. and 4 kg. per rupee respectively. On the average, how many kilograms has he purchased per rupee?

38. The technology matrix of an economic system of two industries is $\begin{bmatrix} 0.8 & 0.2 \\ 0.9 & 0.7 \end{bmatrix}$. Test whether the system is viable as per Hawkins - Simon conditions.

39. If the payment of ₹ 2,000 is made at the end of every quarter for 10 years at the rate of 8% per year, then find the amount of annuity. $[(1.02)^{40} = 2.2080]$.

40. Prove that $\tan^{-1}\left(\frac{2}{11}\right) + \tan^{-1}\left(\frac{7}{24}\right) = \tan^{-1}\left(\frac{1}{2}\right)$

PART - IV

Note : Answer **all** the questions. **7 × 5 = 35**

41. (a) An economy produces only coal and steel. These two commodities serve as intermediate inputs in each other's production. 0.4 tonne of steel and 0.7 tonne of coal are needed to produce a tonne of steel, Similarly 0.1 tonne of steel and 0.6 tonne of coal are required to produce a tonne of coal. No capital inputs are needed. Do you think that the system is viable? 2 and 5 labour days are required to produce a tonnes of coal and steel respectively. If economy needs 100 tonnes of coal and 50 tonnes of steel, calculate the gross output of the two commodities and the total labour days required.

(OR)

(b) Verify Euler's theorem for the function $u = x^3 + y^3 + 3xy^2$.

42. (a) Show that the equation $2x^2 + 7xy + 3y^2 + 5x + 5y + 2 = 0$ represent two straight lines and find their separate equations.

(OR)

(b) Solve : $\tan^{-1}(x + 1) + \tan^{-1}(x - 1) = \tan^{-1}\left(\frac{4}{7}\right)$.

43. (a) A project schedule has the following characteristics.

Activity	1 – 2	1 – 3	2–4	3–4	3–5	4–9	5–6	5–7	6–8	7–8	8–10	9–10
Time	4	1	1	1	6	5	4	8	1	2	5	7

Construct the network and calculate the earliest start time, earliest finish time, latest start time and latest finish time of each activity and determine the Critical path of the project and duration to complete the project.

(OR)

(b) Compute Quartile deviation from the following data.

CI	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 -70	70 - 80
f	12	19	5	10	9	6	6

44. (a) By Mathematical Induction, prove that $1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$, for all $n \in \mathbb{N}$.

(OR)

(b) If $\sin y = x\sin(a + y)$, then prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$.

45. (a) The following are the ranks obtained by 10 students in Commerce and Accountancy are given below :

Commerce	6	4	3	1	2	7	9	8	10	5
Accountancy	4	1	6	7	5	8	10	9	3	2

To what extent is the knowledge of students in two subjects related?

(OR)

(b) A person sells a 20% stocks of Face Value ₹ 10,000 at a premium of 42%. With the money obtained he buys a 15% stock at a discount of 22%. What is the change in his income if the brokerage paid is 2%.

46. (a) The sum of three numbers is 20. If we multiply the first by 2 and add the second number and subtract the third, we get 23. If we multiply the first by 3 and add second and third to it, we get 46. By using matrix inversion method find the numbers.

(OR)

(b) A factory has 3 machines A_1, A_2, A_3 producing 1000, 2000, 3000 screws per day respectively. A_1 produces 1% defectives, A_2 produces 1.5% and A_3 produces 2% defectives. A screw is chosen at random at the end of a day and found defective. What is the probability that it comes from machine A_1 ?

47. (a) Find the equation of the circle passing through the points (1, 0), (–1, 0) and (0, 1).

(OR)

(b) Resolve into partial fraction. $\dfrac{x-2}{(x+2)(x-1)^2}$

❋ ❋ ❋

ANSWERS

Part - I

1. (a) Prof. Wassily W. Leontief
2. (d) n^r
3. (b) 4
4. (b) $r = \pm\sqrt{b_{xy} \times b_{yx}}$
5. (b) $\dfrac{3}{25}$
6. (a) 3
7. (a) ± 3
8. (d) $A = \dfrac{a}{i}[(1+i)^n - 1]$
9. (c) $\dfrac{16}{5}$
10. (c) $16ae^{4x}$
11. (d) $\dfrac{-\sqrt{3}}{2}$
12. (d) $-\dfrac{2h}{b}$
13. (c) $E_j - E_i = L_j - L_i = t_{ij}$.
14. (d) 8
15. (b) ₹ 1600
16. (d) $\dfrac{(n-1)!}{2}$
17. (a) $\dfrac{P(A\cap B)}{P(A)}$
18. (d) –1 to + 1
19. (a) $x^2 + y^2 - 8x - 10y + 32 = 0$
20. (b) 100

Part - II

21. $$\begin{vmatrix} 1 & 3 & 4 \\ 102 & 18 & 36 \\ 17 & 3 & 6 \end{vmatrix} = 6\begin{vmatrix} 1 & 3 & 4 \\ 17 & 3 & 6 \\ 17 & 3 & 6 \end{vmatrix} = 0$$

(since $R_2 \equiv R_3$)

22. If $nC_x = nC_y$, then $x + y = n$.

Here $nC_4 = nC_6$

$\therefore n = 4 + 6 = 10$

$12C_n = 12C_{10} = 12C_2$

$= \dfrac{12\times 11}{1\times 2} = 66$

23. Given $f(x) = x^3 - \dfrac{1}{x^3}$ ... (1)

$$f\left(\frac{1}{x}\right) = \left(\frac{1}{x}\right)^3 - \frac{1}{\left(\frac{1}{x}\right)^3} = \frac{1}{x^3} - \frac{1}{\frac{1}{x^3}} = \frac{1}{x^3} - x^3 \quad ..(2)$$

Adding (1) and (2) we get,

$$f(x) + f\left(\frac{1}{x}\right) = x^3 - \frac{1}{x^3} + \frac{1}{x^3} - x^3 = 0$$

Hence proved.

24. Here $n = 11$ observations are arranged into ascending order

2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22

$D_2 = \text{size of } 2\left(\dfrac{n+1}{10}\right)^{\text{th}}$ value

$D_6 = \text{size of } 6\left(\dfrac{n+1}{10}\right)^{\text{th}}$ value

D_2 = size of 2.4th value ≈ size of 2nd value = 4

D_6 = size of 7.2th value ≈ size of 7th value =14

25. Let the investment in each case be ₹ (140×70)

Case (i) Income from 20% stock at ₹140

$= \dfrac{20}{140}\times(140\times 70) =$ ₹ 1400

Case (ii) Income from 10% stock at ₹70

$= \dfrac{10}{70}\times(140\times 70) =$ ₹ 1400

For the same investment both stocks fetch the same income. Therefore they are equivalent shares.

26. Annual demand: R = 4000

Storage cost: C_1 = ₹ 50

Setup cost per production: C_3= ₹160

$$\text{EOQ} = \sqrt{\frac{2Rc_3}{c_1}} = \sqrt{\frac{2\times4000\times160}{50}} = 160.$$

∴ To minimize the production cost, number of units produced per run is 160 units.

27. Centre C(–1, 3)

$$\text{It lies on } ax + 2y + 2 = 0$$
$$-a + 6 + 2 = 0$$
$$a = 8$$

28. Given $(x - 2y)^{13}$

Here $n = 13, x = x, a = -2y$

The general term is

$$\boxed{T_{r+1} = nCr\, x^{n-r}\, a^r}$$

$\therefore T_{r+1} = 13C_r\, x^{13-r}\, (-2y)^r = 13C_r\, x^{13-r}\, (-1)^r.\, 2^r\, y^r$

To find the 5^{th} term, put $r = 4$

$\therefore T_{4+1} = 13C_4 x^{13-4} (-1)^4 . 2^4 . y^4 = 13C_4\, x^9\, (16)\, y^4$

$$= \frac{13\times12\times11\times10}{4\times3\times2\times1} \times 16 \times x^9 y^4$$

$$T_5 = 11440\, x^9 y^4$$

29. $f(x) = 2^x$ (Given)

$\therefore f(x + y) = 2^{x+y}$

$= 2^x . 2^y = f(x). f(y)$

30. The rank of the word TABLE

A ––––	=	4! = 24 ways
B ––––	=	4! = 24 ways
E ––––	=	4! = 24 ways
L ––––	=	4! = 24 ways
TABEL	=	1 way
TABLE	=	1 way

The rank of the word TABLE is 4 × 4! + 1 + 1 = 98:

Part - III

31. Equation of the circle is

$(a - 1)x^2 + by^2 + (b - 8)\, xy + 4x + 4y - 1 = 0$

Equation of the circle will not have (xy) term.

∴ Co-efficient of (xy) term is 0

$\Rightarrow \quad b - 8 = 0 \Rightarrow b = 8 \quad \ldots (1)$

Also, for any circle,

Co - efficient of x^2 = Co-efficient of y^2

$\Rightarrow \quad a - 1 = b$

$\Rightarrow \quad a - 1 = 8$

$\Rightarrow \quad a = 9$

$\therefore a = 9$ and $b = 8$.

32. $\lim\limits_{x\to0} \dfrac{\sqrt{1+x}-\sqrt{1-x}}{x}$.

Multiplying the numerator and denominator by the conjugate of the numerator, we get,

$$\lim_{x\to0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}\times\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}$$

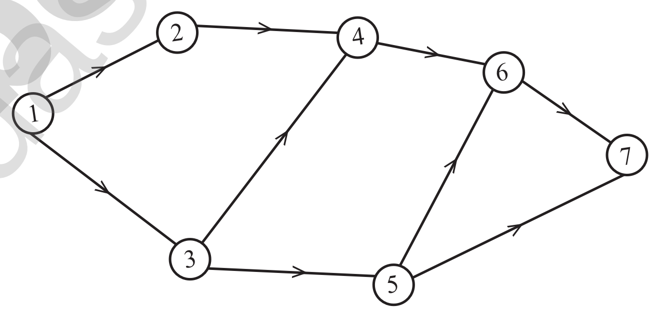
$$= \lim_{x\to0}\frac{(1+x)-(1-x)}{x\left(\sqrt{1+x}+\sqrt{1-x}\right)}$$

$$= \lim_{x\to0}\frac{2x}{x\left(\sqrt{1+x}+\sqrt{1-x}\right)}$$

$$= \lim_{x\to0}\frac{2}{\left(\sqrt{1+x}+\sqrt{1-x}\right)}$$

$$= 2\left(\frac{1}{\sqrt{1+0}+\sqrt{1-0}}\right) = 2\left(\frac{1}{2}\right) = 1$$

33.



34. Maximize $Z = 40x_1 + 50x_2$ subject to the constraints $3x_1 + x_2 = 9$, $x_1 + 2x_2 = 8$ and $x_1, x_2 = 0$

Since both the decision variables x_1 and x_2 are non-negative, the solution lies in the first quadrant of the plane.

Consider the equations

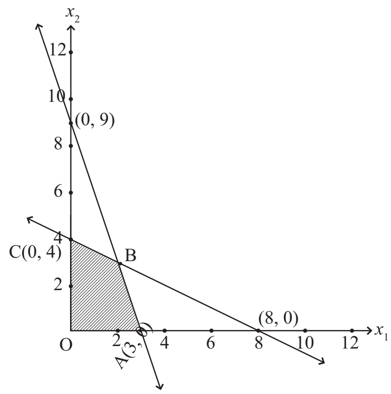
$3x_1 + x_2 = 9$

x_1	0	3
x_2	9	0

$x_1 + 2x_2 = 8$

x_1	0	8
x_2	4	0

Any point lying on or below the lines $3x_1 + x_2 = 9$ and $x_1 + 2x_2 = 8$ satisfies the constraints $3x_1 + x_2 = 9$ and $x_1 + 2x_2 = 8$.



The feasible region is OABC and its co-ordinates are O(0, 0), A(3, 0), C(0, 4) and B is the point of intersection of the lines

$$3x_1 + x_2 = 9 \quad \ldots (1)$$

$$\text{and } x_1 + 2x_2 = 8 \quad \ldots (2)$$

Verification of B

$$(1) \Rightarrow \quad 3x_1 + x_2 = 9$$

$$(2) \times 3 \Rightarrow 3x_1 + 6x_2 = 24$$

$$\quad (-) \quad (-) \quad (-)$$

$$-5x_2 = -15 \quad \Rightarrow x_2 = 3$$

Substituting $x_2 = 3$ in (1) we get

$$3x_1 + 3 = 9$$

$$\Rightarrow \quad 3x_1 = 6$$

$$\Rightarrow \quad x_1 = 2$$

∴ B is (2, 3)

Corner Points	$Z = 40x_1 + 50x_2$
O(0, 0)	0
A(3, 0)	120
B(2, 3)	230
C(0, 4)	200

Maximum value occurs at B(2, 3).

∴ The solution is $x_1 = 2$, $x_2 = 3$ and $Z_{max} = 230$

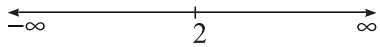
35. Given that $f(x) = x^2 - 4x + 6$

Differentiate with respect to x,

$f'(x) = 2x - 4$

When $f'(x) = 0 \Rightarrow 2x - 4 = 0 \Rightarrow x = 2$.

Then the real line is divided into two intervals namely $(-\infty, 2)$ and $(2, \infty)$



[To choose the sign of $f'(x)$ choose any values for x from the intervals ans substitute in $f'(x)$ and get the sign.]

Interval	Sign of $f'(x) = 2x - 4$	Nature of the function
$(-\infty, 2)$	< 0	$f(x)$ is strictly decreasing in $(-\infty, 2)$
$(2, \infty)$	> 0	$f(x)$ is strictly increasing in $(2, \infty)$

36. $$\tan 75° = \frac{1}{\tan 75°}$$

$$\tan 75° = \tan(45° + 30°) = \frac{\tan 45° + \tan 30°}{1 - \tan 45 . \tan 30}$$

$$\left[\because \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \quad A = 45°, B = 30°\right]$$

$$\left[\tan 45° = 1, \tan 30° = \frac{1}{\sqrt{3}}\right]$$

$$= \frac{1 + \frac{1}{\sqrt{3}}}{1 - 1\left(\frac{1}{\sqrt{3}}\right)} = \frac{\frac{\sqrt{3}+1}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

37. Since we are given rate per rupee, harmonic mean will give the correct answer.

$$HM = \frac{n}{\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}} = \frac{4}{\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}}$$

$$= \frac{4 \times 12}{25} = 1.92 \text{ kg per rupee.}$$

38. $$B = \begin{bmatrix} 0.8 & 0.2 \\ 0.9 & 0.7 \end{bmatrix}$$

$$1 - B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.8 & 0.2 \\ 0.9 & 0.7 \end{bmatrix}$$

$$= \begin{bmatrix} 0.2 & -0.2 \\ -0.9 & 0.3 \end{bmatrix}$$

$$= (0.2)(0.3) - (0.2)(-0.9)$$

$$= 0.06 - 0.18 = -0.12 < 0$$

Since |I – B| is negative, Hawkins – Simon conditions are not satisfied.

Therefore, the given system is not viable.

39. Given

$$a = ₹\,2{,}000$$

$$i = 8\% = \frac{\frac{8}{100}}{4} = \frac{2}{100} = 0.02$$

$$n = 10 \times 4 = 40$$

$$A = \frac{a}{i}\left[(1+i)^n - 1\right] = \frac{2000}{0.02}\left[(1+0.02)^{40} - 1\right]$$

$$= \frac{2000}{0.02}\left[(1.02)^{40} - 1\right] = \frac{2000}{0.02}[2.2080 - 1] \quad [\because (1.02)^{40} = 2.2080]$$

$$= 100{,}000\,[1.2080] = ₹\,1{,}20{,}800$$

40. To prove : $\tan^{-1}\left(\frac{2}{11}\right) + \tan^{-1}\left(\frac{7}{24}\right) = \tan^{-1}\left(\frac{1}{2}\right)$

$$\text{LHS} = \tan^{-1}\left(\frac{2}{11}\right) + \tan^{-1}\left(\frac{7}{24}\right)$$

$$= \tan^{-1}\frac{\left(\frac{2}{11} + \frac{7}{24}\right)}{1 - \frac{2}{11} \times \frac{7}{24}} \qquad \left[\tan^{-1} x + \tan^{-1} y = \tan^{-1}\frac{x+y}{1-xy}\right]$$

$$= \tan^{-1}\frac{\left(\frac{48+77}{264}\right)}{\frac{11 \times 24 - 14}{11 \times 24}} = \tan^{-1}\frac{\left(\frac{48+77}{264}\right)}{\frac{250}{264}} = \tan^{-1}\frac{\left(\frac{125}{264}\right)}{\frac{250}{264}} = \tan^{-1}\frac{\overset{1}{\cancel{125}}}{\cancel{264}} \times \frac{\cancel{264}}{\underset{2}{\cancel{250}}}$$

$$= \tan^{-1}\frac{1}{2} = \text{R.H.S} \qquad \therefore \text{Hence proved.}$$

Part - IV

41. (a) Here the technology matrix is given under

	Steel	Coal	Final demand
Steel	0.4	0.1	50
Coal	0.7	0.6	100
Labour days	5	2	-

The technology matrix is $B = \begin{bmatrix} 0.4 & 0.1 \\ 0.7 & 0.6 \end{bmatrix}$

$$I - B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.4 & 0.1 \\ 0.7 & 0.6 \end{bmatrix} = \begin{bmatrix} 0.6 & -0.1 \\ -0.7 & 0.4 \end{bmatrix}$$

$$|I - B| = \begin{vmatrix} 0.6 & -0.1 \\ -0.7 & 0.4 \end{vmatrix} = (0.6)(0.4) - (-0.7)(-0.1) = 0.24 - 0.07 = 0.17$$

Since the diagonal elements of I – B are positive and value of (I – B) is positive, the system is viable.

$$\text{adj}\,(I-B) = \begin{bmatrix} 0.4 & 0.1 \\ 0.7 & 0.6 \end{bmatrix}$$

$$(I-B)^{-1} = \frac{1}{|I-B|}\,\text{adj}\,(I-B) = \frac{1}{0.17}\begin{bmatrix} 0.4 & 0.1 \\ 0.7 & 0.6 \end{bmatrix}$$

$$X = (I-B)^{-1} D, \text{ where } D = \begin{bmatrix} 50 \\ 100 \end{bmatrix} = \frac{1}{0.17}\begin{bmatrix} 0.4 & 0.1 \\ 0.7 & 0.6 \end{bmatrix}\begin{bmatrix} 50 \\ 100 \end{bmatrix} = \frac{1}{0.17}\begin{bmatrix} 30 \\ 95 \end{bmatrix} = \begin{bmatrix} 176.5 \\ 558.8 \end{bmatrix}$$

Steel output = 176.5 tonnes

Coal output = 558.8 tonnes

Total labour days required = 5(steel output) + 2(coal output) = 5(176.5) + 2(558.8)

= 882.5 + 1117.6 = 2000.1 ≃ 2000 labour days.

(OR)

(b) Given $u(x, y) = x^3 + y^3 + 3xy^2$

$$u(tx, ty) = (tx)^3 + (ty)^3 + 3(tx)(ty)^2 = t^3(x^3 + y^3 + 3xy^2) = t^3u(x, y)$$

∴ u is a homogeneous function of degree 3.

To verify $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3u$

$$u = x^3 + y^3 + 3xy^2$$

Differentiating partially with respect to 'x' we get,

$$\frac{\partial u}{\partial x} = 3x^2 + 0 + 3y^2(1) = 3x^2 + 3y^2$$

$$x\frac{\partial u}{\partial x} = 3x^3 + 3xy^2 \qquad \ldots (1)$$

Differentiating 'u' partially with respect to 'y' we get,

$$\frac{\partial u}{\partial y} = 0 + 3y^2 + 6xy$$

$$y\frac{\partial u}{\partial y} = 3y^3 + 6xy^2 \qquad \ldots (2)$$

Adding (1) and (2),

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3x^3 + 3xy^2 + 3y^3 + 6xy^2 = 3x^3 + 3y^3 + 9xy^2 = 3(x^3 + y^3 + 3xy^2)$$

$$\therefore\ x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3u$$

Hence Euler's theorem is verified.

42. (a) $2x^2 + 7xy + 3y^2 + 5x + 5y + 2 = 0$

$a = 2; b = 3\ ;\ h = \frac{7}{2}\ ; g = \frac{5}{2}\ ; f = \frac{5}{2}; c = 2$

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = \begin{vmatrix} 2 & \frac{7}{2} & \frac{5}{2} \\ \frac{7}{2} & 3 & \frac{5}{2} \\ \frac{5}{2} & \frac{5}{2} & 2 \end{vmatrix} = 0$$

$2x^2 + 7xy + 3y^2 + 5x + 5y + 2 = (x + 3y + l)(2x + y + m)$

$$\left.\begin{aligned} 2l + m &= 5 \\ l + 3m &= 5 \end{aligned}\right\} \Rightarrow \begin{aligned} l &= 2 \\ m &= 1 \end{aligned}$$

$\therefore$ Separate equations are $x + 3y + 2 = 0$ and $2x + y + 1 = 1$

(OR)

(b) $\tan^{-1}\left(\frac{x+\not{1}+x-\not{1}}{1-(x+1)(x-1)}\right) = \tan^{-1}\left(\frac{4}{7}\right)$

$\Rightarrow \tan^{-1}\left(\frac{2x}{1-(x^2-1)}\right) = \tan^{-1}\left(\frac{4}{7}\right)$

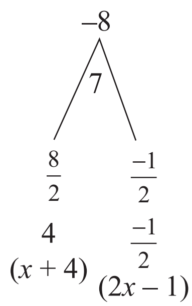
$\Rightarrow \tan^{-1}\left(\frac{2x}{1-x^2+1}\right) = \tan^{-1}\left(\frac{4}{7}\right)$

$\Rightarrow \tan^{-1}\left(\frac{2x}{2-x^2}\right) = \tan^{-1}\left(\frac{4}{7}\right)$

$\Rightarrow \left(\frac{2x}{2-x^2}\right) = \left(\frac{4}{7}\right)$

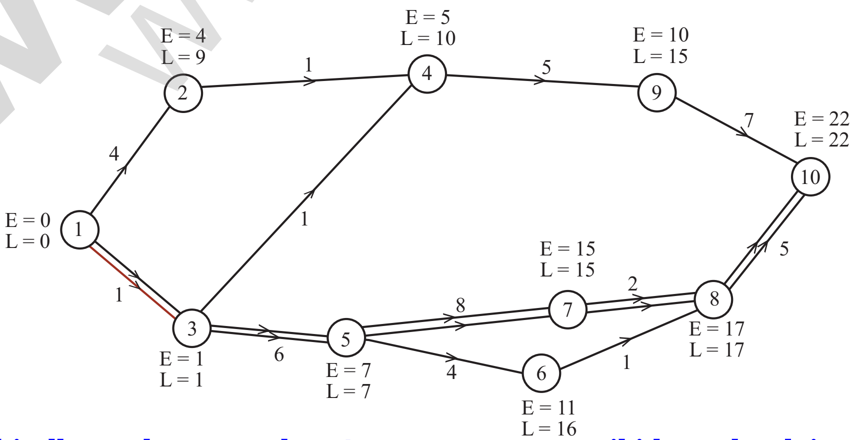
$\Rightarrow \frac{x}{2-x^2} = \frac{2}{7}$

$\Rightarrow 2x^2 + 7x - 4 = 0 \Rightarrow x = -4$ or $\frac{1}{2}$



Since $x = -4$ is not possible. $x = \frac{1}{2}$

43. (a)



$E_1 = 0$
$E_2 = 0 + 4 = 4$
$E_3 = 0 + 1 = 1$
$E_4 = 4 + 1 = 5$
$E_5 = 1 + 6 = 7$
$E_6 = 7 + 4 = 11$
$E_7 = 8 + 7 = 15$
$E_8 = 15 + 2 = 17$
$E_9 = 5 + 5 = 10$
$E_{10} = 17 + 5 = 22$
$L_{10} = 22$
$L_9 = 22 - 7 = 15$
$L_8 = 22 - 5 = 17$
$L_7 = 17 - 2 = 15$
$L_6 = 17 - 1 = 16$
$L_5 = (16 - 4)$ or $(15 - 8)$
whichever is minimum $= 7$
$L_4 = 15 - 5 = 10$
$L_3 = (10 - 1)$ or $(7 - 6)$
whichever is minimum $= 1$
$L_2 = 10 - 1 = 9$
$L_1 = 0$

Activity	Duration	EST	EFT = EST + t_{ij}	LST = LFT – t_{ij}	LFT
1 – 2	4	0	4	9 – 4 = 5	9
1 – 3	1	0	1	1 – 1 = 0	1
2 – 4	1	4	5	10 – 1 = 9	10
3 – 4	1	1	2	10 – 1 = 9	10
3 – 5	6	1	7	7 – 6 = 1	7
4 – 9	5	5	10	15 – 5 = 10	15
5 – 6	4	7	11	16 – 4 = 12	16
5 – 7	8	7	15	15 – 8 = 7	15
6 – 8	1	11	12	17 – 1 = 16	17
7 – 8	2	15	17	17 – 2 = 15	17
8 – 10	5	17	22	22 – 5 = 17	22
9 – 10	7	10	17	22 – 7 = 15	22

Since EFT and LFT is same on 1 – 3, 3 – 5, 5 – 7 and 7 – 8 and 8 – 10 the critical path is 1 – 3 – 5 – 7 – 8 – 10 and the duration is 22 time units.

(OR)

(b)

CI	Frequency	Cumulative Frequency
10 - 20	12	12
20 - 30	19	31
30 - 40	5	36
40 - 50	10	46
50 - 60	9	55
60 - 70	6	61
70 - 80	6	67
	N = 67	

$Q_1 = \text{Size of } \left(\frac{N}{4}\right)^{th} \text{ value } = \text{size of } \left(\frac{67}{4}\right)^{th} \text{ value} = 16.75^{th} \text{ value}$

Thus, Q_1 lies in the class 20 – 30; and corresponding values are

$$L = 20, \frac{N}{4} = 16.75;\ pcf = 12, f = 19, c = 10$$

$$\therefore Q_1 = L + \left(\frac{\frac{N}{4} - pcf}{f}\right)c = 20 + \frac{16.75 - 12}{19} \times 10 = 20 + 2.5 = 22.5$$

$$Q_3 = \text{Size of } \left(\frac{3N}{4}\right)^{th} \text{ value } = 50.25^{th} \text{ Value}$$

Thus Q_3 lies in the class 50 – 60 and corresponding values are

$$L = 50, \frac{3N}{4} = 50.25, f = 9, pcf = 46, c = 10$$

$$\therefore Q_3 = L + \left(\frac{\frac{3N}{4} - pcf}{f}\right) \times c$$

$$Q_3 = 50 + \left[\frac{50.25 - 46}{9}\right] \times 10 = 54.72$$

$$QD = \frac{1}{2}(Q_3 - Q_1) = \frac{54.72 - 22.5}{2} = 16.11$$

$$QD = 16.11$$

44. (a) Let P(n) denote the statement:

$$1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\text{Put } n = 1$$

$$\therefore \text{LHS} = 1^2 = 1$$

$$\text{RHS} = \frac{1(1+1)(2+1)}{6} = 1$$

$$\text{LHS} = \text{RHS}$$

$\therefore$ P(1) is true.

Assume that P(k) is true.

$$P(k) = 1^2 + 2^2 + 3^2 + \ldots + k^2 = \frac{k(k+1)(2k+1)}{6}$$

Now, $1^2 + 2^2 + 3^2 + \ldots + K^2 + (k+1)^2 = P(k) + (k+1)^2$

$$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2$$

$$= \frac{k(k+1)(2k+1) + 6(k+1)^2}{6}$$

$$= \frac{(k+1)[k(2k+1) + 6(k+1)]}{6}$$

$$= \frac{(k+1)(2k^2 + 7k + 6)}{6}$$

$$= \frac{(k+1)(k+2)(2k+3)}{6}$$

∴ P(k + 1) is true whenever P(k) is true

∴ P(n) is true for all $n \in \mathbb{N}$

(OR)

(b) (i) $\sin y = x \sin(a+y)$

$$x = \frac{\sin y}{\sin(a+y)}$$

Differentiating with respect to y,

$$\frac{dx}{dy} = \frac{\sin(a+y)\cos y - \sin y \cdot \cos(a+y)}{\sin^2(a+y)} = \frac{\sin(a+y-y)}{\sin^2(a+y)} = \frac{\sin a}{\sin^2(a+y)}$$

$$\therefore \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$$

45. (a)

Rank in Commerce (R_X)	Rank in Accountancy (R_Y)	$d = R_X - R_Y$	d^2
6	4	2	4
4	1	3	9
3	6	– 3	9
1	7	– 6	36
2	5	– 3	9
7	8	– 1	1
9	10	– 1	1
8	9	– 1	1
10	3	7	49
5	2	3	9
			$\Sigma d^2 = 128$

Rank Correlation is given by

$$\rho = 1 - \frac{6\Sigma d^2}{N(N^2-1)} = 1 - \frac{6(128)}{10(100-1)} = 1 - \frac{768}{990} = 1 - 0.7758$$

$$\rho = 0.224$$

(OR)

(b) **Step 1: For 20% stocks :**

F.V. = ₹100

Income = $\frac{20}{100} \times 10000 =$ ₹ 2,000 ... (1)

Investment = ₹10,000

Face value = ₹100

Market value = ₹100 + 42 – 2 = 140

Number of shares = $\frac{\text{investments}}{\text{FV}} = \frac{10,000}{100} = 100$

Sales proceeds = 100 × 140 = 14,000

Step 2: For 15% stocks :

M.V. = ₹100 – 22 + 2 = 80

Number of shares = $\frac{\text{investments}}{\text{FV}} = \frac{14000}{80} = 175$

Income = $175 \times \frac{15}{100} \times 100 =$ ₹2625

Step 3: Change of income :

Change of income = ₹2625 – ₹ 2000 = ₹ 625

46. (a) Let the required numbers be x, y and z.

Given $x + y + z = 20$
$2x + y - z = 23$
$3x + y + z = 46$

These equations can be written in matrix form as $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 20 \\ 23 \\ 46 \end{bmatrix} \Rightarrow AX = B$

Where A = $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 1 & 1 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$, B= $\begin{bmatrix} 20 \\ 23 \\ 46 \end{bmatrix}$

$$|A| = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 1 & 1 \end{bmatrix} = 1\begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} - 1\begin{vmatrix} 2 & -1 \\ 3 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix}$$

$= 1(1+1) - 1(2+3) + 1(2-3) = 2-5-1 = 2-6 = -4 \neq 0 \Rightarrow A^{-1}$ exists.

$$\text{adj } A = \begin{bmatrix} +\begin{vmatrix}1 & -1\\1 & 1\end{vmatrix} & -\begin{vmatrix}2 & -1\\3 & 1\end{vmatrix} & +\begin{vmatrix}2 & 1\\3 & 1\end{vmatrix} \\ -\begin{vmatrix}1 & 1\\1 & 1\end{vmatrix} & +\begin{vmatrix}1 & 1\\3 & 1\end{vmatrix} & -\begin{vmatrix}1 & 1\\3 & 1\end{vmatrix} \\ +\begin{vmatrix}1 & 1\\1 & -1\end{vmatrix} & -\begin{vmatrix}1 & 1\\2 & -1\end{vmatrix} & +\begin{vmatrix}1 & 1\\2 & 1\end{vmatrix}\end{bmatrix}^T = \begin{bmatrix} +(1+1) & -(2+3) & +(2-3) \\ -(1-1) & +(1-3) & -(1-3) \\ +(-1-1) & -(-1-2) & +(1-2)\end{bmatrix}^T$$

$$= \begin{bmatrix} 2 & -5 & -1 \\ 0 & -2 & 2 \\ -2 & 3 & -1\end{bmatrix}^T = \begin{bmatrix} 2 & 0 & -2 \\ -5 & -2 & 3 \\ -1 & 2 & -1\end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}\text{adj } A = \frac{-1}{4}\begin{bmatrix} 2 & 0 & -2 \\ -5 & -2 & 3 \\ -1 & 2 & -1\end{bmatrix}$$

$$X = A^{-1}B = \frac{-1}{4}\begin{bmatrix} 2 & 0 & -2 \\ -5 & -2 & 3 \\ -1 & 2 & -1\end{bmatrix}\begin{bmatrix}20\\23\\46\end{bmatrix} = \frac{-1}{4}\begin{bmatrix}40+0-92\\-100-46+138\\-20+46-46\end{bmatrix} = \frac{-1}{4}\begin{bmatrix}-52\\-8\\-20\end{bmatrix}$$

$X = \begin{bmatrix}13\\2\\5\end{bmatrix}$ $\therefore$ The required numbers are 13, 2 and 5.

(OR)

(b) $P(A_1)$ = P(that the machine A_1 produces screws) $= \frac{1000}{6000} = \frac{1}{6}$

$P(A_2)$ = P(that the machine A_2 produces screws) $= \frac{2000}{6000} = \frac{1}{3}$

$P(A_3)$ = P(that the machine A_3 produces screws) $= \frac{3000}{6000} = \frac{1}{2}$

Let B be the event that the chosen screw is defective

$\therefore P(B/A_1)$ = P(that defective screw from the machine A_1) = 0.01

$P(B/A_2)$ = P(that defective screw from the machine A_2) = 0.015 and

$P(B/A_3)$ = P(that defective screw from the machine A_3) = 0.02

We have to find $P(A_1/B)$

Hence by Baye's theorem, we get

$$P(A_1/B) = \frac{P(A_1)P(B/A_1)}{P(A_1)P(B/A_1)+P(A_2)P(B/A_2)+P(A_3)P(B/A_3)}$$

$$= \frac{\left(\frac{1}{6}\right)(0.01)}{\left(\frac{1}{6}\right)(0.01)+\left(\frac{1}{3}\right)(0.015)+\left(\frac{1}{2}\right)(0.02)}$$

$$= \frac{0.01}{0.01+0.03+0.06} = \frac{0.01}{0.1} = \frac{1}{10}$$

47. (a) Let the equation of the circle be $x^2 + y^2 + 2gx + 2fy + c = 0$

It passes through the points (1, 0), (–1, 0) and (0, 1)

$$(1, 0) \Rightarrow 1 + 0 + 2g + c = 0$$

$$2g + c = -1 \quad \ldots (1)$$

$$(-1, 0) \Rightarrow 1 + 0 - 2g + c = 0$$

$$-2g + c = -1 \quad \ldots (2)$$

$$(0, -1) \Rightarrow 0 + 1 + 0 + 2f + c = 0$$

$$2f + c = -1 \quad \ldots (3)$$

$$\text{Now solving } (1) + (2) \Rightarrow 2c = -2$$

$$c = -1$$

Substitute in equation (1) we get

$$2g - 1 = -1$$

$$2g = 0 \Rightarrow g = 0$$

Substitute in equation (3) we get

$$2f - 1 = -1$$

$$2f = -1 + 1 \Rightarrow 2f = 0 \Rightarrow f = 0$$

So we get $g = 0, f = 0$ and $c = -1$

Therefore, the required equation of the circle is $x^2 + y^2 - 1 = 0$

(OR)

(b) $$\frac{x-2}{(x+2)(x-1)^2} = \frac{A}{x+2}+\frac{B}{x-1}+\frac{C}{(x-1)^2}$$

$$\Rightarrow \frac{x-2}{(x+2)(x-1)^2} = \frac{A(x-1)^2+B(x+2)(x-1)+C(x+2)}{(x+2)(x-1)^2}$$

$$\Rightarrow x-2 = A(x-1)^2 + B(x+2)(x-1) + C(x+2) \quad \dots (1)$$

Substituting x = 1 in (1) we get,

$$1-2 = C(1+2) \Rightarrow -1 = 3C \Rightarrow C = \frac{-1}{3}$$

Putting $x = -2$ in (1) we get,

$$-2-2 = A(-3)^2 \Rightarrow -4 = 9A \Rightarrow A = \frac{-4}{9}$$

Putting $x = 0$ in (1) we get,

$$-2 = A - 2B + 2C$$

$$\Rightarrow -2 = \frac{-4}{9} - 2B - \frac{2}{3} \quad \left[\because A = \frac{-4}{9}, C = \frac{-1}{3}\right]$$

$$\Rightarrow 2B = \frac{-4}{9} - \frac{2}{3} + 2$$

$$\Rightarrow 2B = \frac{-4-6+18}{9} = \frac{8}{9}$$

$$\Rightarrow B = \frac{4}{9}$$

$$\therefore \frac{x-2}{(x+2)(x-1)^2} = \frac{-\frac{4}{9}}{x+2} + \frac{\frac{4}{9}}{x-1} + \frac{-\frac{1}{3}}{(x-1)^2}$$

$$= \frac{-4}{9(x+2)} + \frac{4}{9(x-1)} - \frac{1}{3(x-1)^2}$$

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