HIGHER SECONDARY FIRST YEAR PUBLIC EXAMINATION MARCH– 2024

MATHEMATICS – ANSWER KEY

PART-I

Note: i) Answer all the questions. $[20 \times 1 = 20]$

ii) Choose the most appropriate answer from the given four alternatives and write the option code and the corresponding answer

	TYPE-A	TYPE-B
1.	**(d)** $\frac{n(n+1)}{2}$	**(a)** $\frac{1}{4}$
2.	**(c)** 3	**(a)** $\frac{1}{2}a^2$
3.	**(a)** $\frac{1}{2}a^2$	**(a)** $P(A/B) \geq P(A)$
4.	**(b)** 2^n	**(c)** 10
5.	**(a)** $P(A/B) \geq P(A)$	**(a)** 9
6.	**(d)** 0	**(b)** $\sqrt{\tan x} + C$
7.	**(d)** 4	**(c)** 3
8.	**(c)** 3	**(c)** 3
9.	**(a)** $\frac{1}{4}$	**(c)** $[0, 9]$
10.	**(a)** 26	**(d)** $4\hat{\imath} + 5\hat{\jmath}$
11.	**(c)** 10	**(a)** 0
12.	**(a)** $-\frac{4}{15}$	**(a)** 0
13.	**(a)** 0	**(c)** $\cos x\, e^{\sin x}$
14.	**(c)** $\cos x\, e^{\sin x}$	**(b)** 2^n
15.	**(a)** 9	**(a)** $-\frac{4}{15}$
16.	**(d)** $4\hat{\imath} + 5\hat{\jmath}$	**(d)** $\log\left(\frac{a}{b}\right)$
17.	**(b)** $\sqrt{\tan x} + C$	**(d)** $\frac{n(n+1)}{2}$
18.	**(a)** 0	**(d)** 0
19.	**(c)** $[0, 9]$	**(a)** 26
20.	**(d)** $\log\left(\frac{a}{b}\right)$	**(d)** 4

PART-II

Note: $[7 \times 2 = 14]$

(i) Answer any **SEVEN** questions

(ii) Question number **30** is compulsory.

21. If $\frac{1}{7!} + \frac{1}{8!} = \frac{A}{9!}$ then find the value of A.

Solution:

$$\frac{1}{7!}+\frac{1}{8\times 7!}=\frac{A}{9\times 8\times 7!}$$
$$\frac{9}{8}=\frac{A}{9\times 8}$$
$$\Longrightarrow A=9\times 9\Longrightarrow \boxed{A=81}$$

22. Show that the lines are $3x+2y+9=0$ and $12x+8y-15=0$ are parallel lines.

Solution:

Let m_1 and m_2 be the slopes of given lines

Slope of $3x+2y+9=0$ is $m_1=-\frac{2}{3}$

Slope of $12x+8y-15=0$ is $m_2=-\frac{8}{12}=-\frac{2}{3}$

$$m_1=m_2$$

$\therefore$ The given lines are parallel

23. Compute $|A|$ if $A=\begin{bmatrix}3 & 4 & 1\\ 0 & -1 & 2\\ 5 & -2 & 6\end{bmatrix}$.

Solution:

$$\begin{aligned}|A| &= 3(-6+4)-4(0-10)+1(0+5)\\ &= 3(-2)-4(-10)+5\\ &= -6+40+5\\ &= 39\end{aligned}$$

24. If and are mutually exclusive events $P(A)=\frac{3}{8}$ and $P(B)=\frac{1}{8}$, then find $P(\bar{A}\cup\bar{B})$

Solution:

A and B are mutually exclusive events $\Longrightarrow P(A\cap B)=0$

$$\begin{aligned}P(\bar{A}\cup\bar{B}) &= P(\overline{A\cap B})\\ &= 1-P(A\cap B)\\ &= 1-0=1\end{aligned}$$

25. Find $|\vec{a}\times\vec{b}|$, where $\vec{a}=3\hat{\imath}+4\hat{\jmath}$ and $\vec{b}=\hat{\imath}+\hat{\jmath}+\hat{k}$.

Solution:

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 3 & 4 & 0\\ 1 & 1 & 1\end{vmatrix}=\hat{\imath}(4-0)-\hat{\jmath}(3-0)+\hat{k}(3-4)=4\hat{\imath}-3\hat{\jmath}-\hat{k}$$

$$|\vec{a}\times\vec{b}|=|4\hat{\imath}-3\hat{\jmath}-\hat{k}|=\sqrt{16+9+1}=\sqrt{26}$$

26. Find the domain of $\frac{1}{1-2\sin x}$.

Solution:

The function is defined for all $x\in\mathbb{R}$ except $1-2\sin x=0$.

That is, except $\sin x=\frac{1}{2}\Longrightarrow \sin x=\sin\frac{\pi}{6}$

That is, except $x=n\pi+(-1)^n\frac{\pi}{6}$, $n\in Z$.

Hence the domain is $\mathbb{R}-\left\{n\pi+(-1)^n\frac{\pi}{6}\right\}$, $n\in Z$

27. Show that $\tan(45°-A)=\frac{1-\tan A}{1+\tan A}$.

Solution:

$$\tan(45°-A)=\frac{\tan 45°-\tan A}{1+\tan 45°\tan A}=\frac{1-\tan A}{1+\tan A}$$

28. Find f'' if $f(x) = x\cos x$.

Solution:

$$f(x) = x\cos x$$
$$f'(x) = x(-\sin x) + \cos x\,(1)$$
$$= -x\sin x + \cos x$$
$$f''(x) = -\big(x(\cos x) + \sin x\,(1)\big) + (-\sin x)$$
$$= -x\cos x - \sin x - \sin x$$
$$f''(x) = -x\cos x - 2\sin x$$

29. Find the separate the equation of the straight lines $5x^2 + 6xy + y^2 = 0$.

Solution:

$$5x^2 + 6xy + y^2 = 0$$
$$5x^2 + 5xy + xy + y^2 = 0$$
$$5x(x+y) + y(x+y) = 0$$
$$(5x+y)(x+y) = 0$$
$$5x + y = 0\ ;\ x + y = 0$$

30. Find the number of ways of arranging the letters of the word INDIA.

Solution:

INDIA

Number of letters = 5

Number of I's = 2

Required number of arrangements $= \frac{5!}{2!} = 60$

PART-III

Note: **[$7 \times 3 = 21$]**

(i) Answer any **SEVEN** questions

(ii) Question number **40** is compulsory.

31. Simplify: $\frac{1}{3-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2}$.

Solution:

$$\frac{1}{3-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2}$$
$$= \frac{1}{3-\sqrt{8}} \times \frac{3+\sqrt{8}}{3+\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} \times \frac{\sqrt{8}+\sqrt{7}}{\sqrt{8}+\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} \times \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}}$$
$$- \frac{1}{\sqrt{6}-\sqrt{5}} \times \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}} + \frac{1}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2}$$
$$= \frac{3+\sqrt{8}}{9-8} - \frac{\sqrt{8}+\sqrt{7}}{8-7} + \frac{\sqrt{7}+\sqrt{6}}{7-6} - \frac{\sqrt{6}+\sqrt{5}}{6-5} + \frac{\sqrt{5}+2}{5-4}$$
$$= 3 + \sqrt{8} - \sqrt{8} - \sqrt{7} + \sqrt{7} + \sqrt{6} - \sqrt{6} - \sqrt{5} + \sqrt{5} + 2$$
$$= 5$$

32. Evaluate : $\int xe^x\,dx$

Solution:

$$\text{Let } I = \int xe^x\,dx$$
$$\text{Take } u = x\ ;\ dv = e^x dx$$
$$du = dx\ ;\ v = e^x$$

$$\text{w.k.t.} \int u\,dv = uv - \int v\,du$$
$$\int xe^x\,dx = xe^x - \int e^x\,dx$$
$$\int xe^x\,dx = xe^x - e^x + c = e^x(x-1) + c$$

33. Compute the sum of first n terms of the series: $6 + 66 + 666 + 6666 + \cdots$

Solution:

$$\text{Given } S_n = 6 + 66 + 666 + 6666 + \cdots$$
$$S_n = 6(1 + 11 + 111 + 1111 + \cdots)$$
$$S_n = \frac{6}{9}(9 + 99 + 999 + 9999 + \cdots n \text{ terms})$$
$$S_n = \frac{6}{9}\begin{pmatrix}(10-1) + (100-1) + (1000-1) \\ +(10000-1) + \cdots n \text{ terms}\end{pmatrix}$$
$$S_n = \frac{6}{9}\begin{pmatrix}(10-1) + (10^2-1) + (10^3-1) \\ +(10^4-1) + \cdots n \text{ terms}\end{pmatrix}$$
$$S_n = \frac{6}{9}\begin{pmatrix}(10 + 10^2 + 10^3 + 10^4 + \cdots n \text{ terms}) \\ -(1+1+1+1+\cdots n \text{ terms})\end{pmatrix}$$
$$[\text{In the above } 10 + 10^2 + 10^3 + \cdots n \text{ terms is in GP}$$
$$10 + 10^2 + 10^3 + \cdots n \text{ terms} = \frac{10(10^n-1)}{10-1} = \frac{10(10^n-1)}{9}]$$
$$S_n = \frac{6}{9}\left(\frac{10(10^n-1)}{9} - n\right)$$
$$S_n = \frac{60(10^n-1)}{81} - \frac{6n}{9}$$

34. Calculate $\lim\limits_{x\to3}\frac{x^2-6x+5}{x^3-8x+7}$.

Solution:

$$\lim_{x\to3}\frac{x^2-6x+5}{x^3-8x+7} = \frac{\lim\limits_{x\to3}(x^2-6x+5)}{\lim\limits_{x\to3}(x^3-8x+7)}$$
$$= \frac{(3)^2 - 6(3) + 5}{(3)^3 - 8(3) + 7}$$
$$= \frac{9-18+5}{27-24+7}$$
$$= -\frac{4}{10}$$
$$= -\frac{2}{5}$$

35. Differentiate the following with respect to x : $y = xe^x \log x$

Solution:

$$y = xe^x \log x$$
$$\frac{dy}{dx} = x\,e^x\left(\frac{1}{x}\right) + e^x.\log x\,(1) + x\log x\,(e^x)$$
$$= e^x + e^x\log x + x\,e^x\log x$$
$$= e^x(1 + \log x + x\log x)$$

36. If $^nP_r = 720$, and $^nC_r = 120$, find n, r.

Solution:

$$\text{Given } {}^nP_r = 720 \text{ and } {}^nC_r = 120$$
$$\text{w.k.t. } {}^nP_r = r! \times {}^nC_r$$
$$720 = r! \times 120$$
$$r! = 6 \Longrightarrow r! = 3! \Longrightarrow \boxed{r = 3}$$
$$^nP_r = 720$$
$$^nP_3 = 720$$
$$n(n-1)(n-2) = 72 \times 10$$
$$n(n-1)(n-2) = 10 \times 9 \times 8$$
$$\boxed{n = 10}$$

37. Find the value of cos 105°.

Solution:

$$\cos 105° = \cos(60° + 45°)$$
$$= \cos 60° \cos 45° - \sin 60° \sin 45°$$
$$= \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) = \frac{1-\sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{2}-\sqrt{6}}{4}$$

38. Write the values of f at −3, 5, 0 if $f(x) = \begin{cases} x^2 + x - 5 & \text{if } x \in (-\infty, 0) \\ x^2 + 3x - 2 & \text{if } x \in (3, \infty) \\ x^2 & \text{if } x \in (0, 2) \\ x^2 - 3 & \text{otherwise} \end{cases}$

Solution:

$$f(-3) = (-3)^2 - 3 - 5 = 9 - 8 = 1$$
$$f(5) = 5^2 + 3(5) - 2 = 25 + 15 - 2 = 38$$
$$f(0) = 0^2 - 3 = -3$$

39. Show that the vectors $2\hat{\imath} - \hat{\jmath} + \hat{k}, 3\hat{\imath} - 4\hat{\jmath} - 4\hat{k}, \hat{\imath} - 3\hat{\jmath} - 5\hat{k}$ form a right-angled triangle.

Solution:

$$\vec{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}, \vec{b} = 3\hat{\imath} - 4\hat{\jmath} - 4\hat{k}, \vec{c} = \hat{\imath} - 3\hat{\jmath} - 5\hat{k}$$
$$|\vec{a}| = |2\hat{\imath} - \hat{\jmath} + \hat{k}| = \sqrt{2^2 + (-1)^2 + 1^2} = \sqrt{6} \Longrightarrow |\vec{a}|^2 = 6$$
$$|\vec{b}| = |3\hat{\imath} - 4\hat{\jmath} - 4\hat{k}| = \sqrt{3^2 + (-4)^2 + (-4)^2} = \sqrt{41} \Longrightarrow |\vec{b}|^2 = 41$$
$$|\vec{c}| = |\hat{\imath} - 3\hat{\jmath} - 5\hat{k}| = \sqrt{1^2 + (-3)^2 + (-5)^2} = \sqrt{35} \Longrightarrow |\vec{c}|^2 = 35$$
$$|\vec{b}|^2 = |\vec{a}|^2 + |\vec{c}|^2$$

∴The vectors $2\hat{\imath} - \hat{\jmath} + \hat{k}, 3\hat{\imath} - 4\hat{\jmath} - 4\hat{k}, \hat{\imath} - 3\hat{\jmath} - 5\hat{k}$ form a right-angled triangle

40. Prove that $\frac{\cos 11° + \sin 11°}{\cos 11° - \sin 11°} = \tan 56°$.

Solution:

$$\frac{\cos 11° + \sin 11°}{\cos 11° - \sin 11°} = \frac{\frac{\cos 11°}{\cos 11°} + \frac{\sin 11°}{\cos 11°}}{\frac{\cos 11°}{\cos 11°} - \frac{\sin 11°}{\cos 11°}} = \frac{1 + \tan 11°}{1 - \tan 11°} = \frac{\tan 45° + \tan 11°}{1 - \tan 45° \tan 11°}$$
$$= \tan(45° + 11°) = \tan 56°$$

M.SANKAR M.Sc., B.Ed.,

PART-IV

ANSWER ALL QUESTIONS. $[7 \times 5 = 35]$

41. (a) Resolve into partial fractions: $\frac{2x}{(x^2+1)(x-1)}$.

Solution:

$$\text{Let } \frac{2x}{(x^2+1)(x-1)} = \frac{Ax+B}{x^2+1} + \frac{C}{x-1} \rightarrow ①$$

$$2x = (Ax+B)(x-1) + C(x^2+1)$$

Put $x = 1$

$$2(1) = 0 + C(1^2+1)$$
$$C = 1$$

Put $x = 0$

$$0 = (0+B)(0-1) + C(0+1)$$
$$0 = -B + C \Rightarrow 0 = -B + 1 \Rightarrow B = 1$$

Put $x = -1$

$$2(-1) = (A(-1)+B)(-1-1) + C((-1)^2+1)$$
$$-2 = (-A+B)(-2) + C(2)$$
$$-1 = A - B + C$$
$$-1 = A - 1 + 1$$
$$A = -1$$

Sub A, B and C values in ①

$$\therefore \frac{2x}{(x^2+1)(x-1)} = \frac{1-x}{x^2+1} + \frac{1}{x-1}$$

(OR)

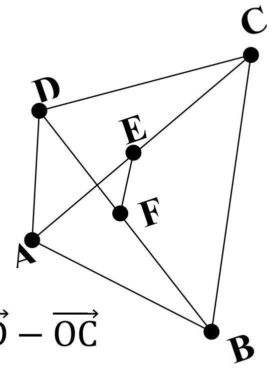
(b) If $y = e^{\tan^{-1}x}$, show that $(1+x^2)y'' + (2x-1)y' = 0$.

Solution:

$$y = e^{\tan^{-1}x}$$
$$y' = e^{\tan^{-1}x} \times \frac{1}{1+x^2}$$
$$(1+x^2)y' = e^{\tan^{-1}x}$$
$$(1+x^2)y' = y$$
$$(1+x^2)y'' + y'(2x) = y'$$
$$(1+x^2)y'' + 2xy' - y' = 0$$
$$(1+x^2)y'' + (2x-1)y' = 0$$

42. (a) If ABCD is a quadrilateral and E and F are the midpoints of AC and BD respectively, then prove that $\overrightarrow{AB} + \overrightarrow{AD} + \overrightarrow{CB} + \overrightarrow{CD} = 4\overrightarrow{EF}$.

Solution:



Let O be a origin.

Let $\overrightarrow{OA} = \vec{a}, \overrightarrow{OB} = \vec{b}, \overrightarrow{OC} = \vec{c}, \overrightarrow{OD} = \vec{d}$

E is a midpoint of AC, then $\overrightarrow{OE} = \frac{\vec{a}+\vec{c}}{2} \Rightarrow 2\overrightarrow{OE} = \vec{a} + \vec{c}$

F is a midpoint of BD, then $\overrightarrow{OF} = \frac{\vec{b}+\vec{d}}{2} \Rightarrow 2\overrightarrow{OF} = \vec{b} + \vec{d}$

$$\text{L.H.S} = \overrightarrow{AB} + \overrightarrow{AD} + \overrightarrow{CB} + \overrightarrow{CD}$$
$$= \overrightarrow{OB} - \overrightarrow{OA} + \overrightarrow{OD} - \overrightarrow{OA} + \overrightarrow{OB} - \overrightarrow{OC} + \overrightarrow{OD} - \overrightarrow{OC}$$
$$= \vec{b} - \vec{a} + \vec{d} - \vec{a} + \vec{b} - \vec{c} + \vec{d} - \vec{c}$$
$$= 2(\vec{b} + \vec{d}) - 2(\vec{a} + \vec{c})$$
$$= 2(2\overrightarrow{OF}) - 2(2\overrightarrow{OE}) = 4(\overrightarrow{OF} - \overrightarrow{OE}) = 4\overrightarrow{EF} = \text{R.H.S}$$

(OR)

(b) **Let** $f(x)=\begin{cases}\frac{\sin x}{x}+\cos x & ; \text{ if } x\neq 0\\ 2 & ; \text{ if } x=0\end{cases}$ **Show that f is continuous at x = 0.**

Solution:

$$f(x)=\begin{cases}\frac{\sin x}{x}+\cos x & ; \text{ if } x\neq 0\\ 2 & ; \text{ if } x=0\end{cases}$$

$$f(0)=2$$

$$\lim_{x\to 0} f(x)=\lim_{x\to 0}\left(\frac{\sin x}{x}+\cos x\right)=\lim_{x\to 0}\frac{\sin x}{x}+\lim_{x\to 0}\cos x=1+1=2$$

$$\therefore \lim_{x\to 0} f(x)=f(0)$$

Hence f(x) is continuous at x = 0

43. (a) **Prove that** $\log_{10}2+16\log_{10}\frac{16}{15}+12\log_{10}\frac{25}{24}+7\log_{10}\frac{81}{80}=1.$

Solution:

$$\begin{aligned}\text{L.H.S.}&=\log_{10}2+16\log_{10}\frac{16}{15}+12\log_{10}\frac{25}{24}+7\log_{10}\frac{81}{80}\\
&=\log_{10}2+16(\log_{10}16-\log_{10}15)+12(\log_{10}25-\log_{10}24)\\
&\quad+7(\log_{10}81-\log_{10}80)\\
&=\log_{10}2+16(\log_{10}2^4-\log_{10}3\times 5)\\
&\quad+12(\log_{10}5^2-\log_{10}2^3\times 3)+7(\log_{10}3^4-\log_{10}2^4\times 5)\\
&=\log_{10}2+16(4\log_{10}2-\log_{10}3-\log_{10}5)\\
&\quad+12(2\log_{10}5-3\log_{10}2-\log_{10}3)\\
&\quad+7(4\log_{10}3-4\log_{10}2-\log_{10}5)\\
&=\log_{10}2+64\log_{10}2-16\log_{10}3-16\log_{10}5+24\log_{10}5\\
&\quad-36\log_{10}2-12\log_{10}3+28\log_{10}3-28\log_{10}2\\
&\quad-7\log_{10}5\\
&=\log_{10}2+\log_{10}5\\
&=\log_{10}10=1=\text{R.H.S.}\end{aligned}$$

(OR)

(b) **There are two identical urns containing respectively 6 black and 4 red balls, 2 black and 2 red balls. An urn is chosen at random and a ball is drawn from it. (i) find the probability that the ball is black (ii) if the ball is black, what is the probability that it is from the first urn?**

Solution:

Let U_1, U_2 be the event of selecting urn1 and Urn2 respectively

B, R be the event of selecting Black and Red colour respectively

$$P(U_1)=\frac{1}{2}\ ;\ P(B/U_1)=\frac{6}{10}=\frac{3}{5}P(U_2)=\frac{1}{2}\ ;\ P(B/U_2)=\frac{2}{4}=\frac{1}{2}$$

i) P(selected ball is black) = P(B)

$$\begin{aligned}&=P(U_1)P(B/U_1)+P(U_2)P(B/U_2)\\
&=\left(\frac{1}{2}\right)\left(\frac{3}{5}\right)+\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\\
&=\frac{22}{40}=\frac{11}{20}\end{aligned}$$

ii) P(Selected black ball is drawn from first urn) $= P(U_1/B)$

$$= \frac{P(U_1)P(B/U_1)}{P(U_1)P(B/U_1) + P(U_2)P(B/U_2)}$$

$$= \frac{\left(\frac{1}{2}\right)\left(\frac{3}{5}\right)}{\left(\frac{1}{2}\right)\left(\frac{3}{5}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)} = \frac{\frac{3}{10}}{\frac{11}{20}} = \frac{6}{11}$$

44. (a) Evaluate: $\int \frac{3x+5}{x^2+4x+7}dx$.

Solution:

$$\text{Let } I = \int \frac{3x+5}{x^2+4x+7}dx$$

$$3x + 5 = A\frac{d}{dx}(x^2 + 4x + 7) + B$$

$$3x + 5 = A(2x + 4) + B$$

Comparing x term	comparing constent
$3 = 2A$	$5 = 4A + B$
$A = \frac{3}{2}$	$5 = 4\left(\frac{3}{2}\right) + B \Rightarrow B = -1$

$$I = \int \frac{\frac{3}{2}(2x+4) - 1}{x^2+4x+7}dx$$

$$= \frac{3}{2}\int \frac{2x+4}{x^2+4x+7}dx - \int \frac{1}{x^2+4x+7}dx$$

$$= \frac{3}{2}\int \frac{2x+4}{x^2+4x+7}dx - \int \frac{1}{(x+2)^2 + (\sqrt{3})^2}dx$$

$$\int \frac{3x+5}{x^2+4x+7}dx = \frac{3}{2}\log|x^2 + 4x + 7| - \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x+2}{\sqrt{3}}\right) + c$$

(OR)

(b) Show that $\cot\left(7\frac{1}{2}^\circ\right) = \sqrt{2} + \sqrt{3} + \sqrt{4} + \sqrt{6}$.

Solution:

$$\cot\frac{\theta}{2} = \frac{1+\cos\theta}{\sin\theta}$$

$$\cot\left(7\frac{1}{2}^\circ\right) = \cot\left(\frac{15}{2}^\circ\right) = \frac{1+\cos 15^\circ}{\sin 15^\circ}$$

$$= \frac{1 + \frac{\sqrt{3}+1}{2\sqrt{2}}}{\frac{\sqrt{3}-1}{2\sqrt{2}}} = \frac{\frac{2\sqrt{2}+\sqrt{3}+1}{2\sqrt{2}}}{\frac{\sqrt{3}-1}{2\sqrt{2}}} = \frac{2\sqrt{2}+\sqrt{3}+1}{\sqrt{3}-1}$$

$$= \frac{2\sqrt{2}+\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{2\sqrt{6}+2\sqrt{2}+3+\sqrt{3}+\sqrt{3}+1}{3-1}$$

$$= \sqrt{6} + \sqrt{2} + 2 + \sqrt{3}$$

$$= \sqrt{2} + \sqrt{3} + \sqrt{4} + \sqrt{6}$$

M.SANKAR M.Sc., B.Ed.,

45. (a) Prove that $\sqrt[3]{x^3+6}-\sqrt[3]{x^3+3}$ is approximately equal to $\frac{1}{x^2}$ when x is sufficiently large.

Solution:

$$\sqrt[3]{x^3+6}-\sqrt[3]{x^3+3}=(x^3+6)^{\frac{1}{3}}-(x^3+3)^{\frac{1}{3}}$$

$$\text{x is sufficiently large} \Longrightarrow \frac{1}{x} \text{ is smaller}$$

$$\sqrt[3]{x^3+6}-\sqrt[3]{x^3+3}=(x^3)^{\frac{1}{3}}\left(1+\frac{6}{x^3}\right)^{\frac{1}{3}}-(x^3)^{\frac{1}{3}}\left(1+\frac{3}{x^3}\right)^{\frac{1}{3}}$$

$$=x\left(1+\frac{6}{x^3}\right)^{\frac{1}{3}}-x\left(1+\frac{3}{x^3}\right)^{\frac{1}{3}}$$

$$=x\left(1+\left(\frac{1}{3}\right)\left(\frac{6}{x^3}\right)+\frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2!}\left(\frac{6}{x^3}\right)^2+\cdots\right)-x\left(1+\left(\frac{1}{3}\right)\left(\frac{3}{x^3}\right)+\frac{\frac{1}{3}\left(\frac{1}{3}-1\right)}{2!}\left(\frac{3}{x^3}\right)^2+\cdots\right)$$

$$=x\left(1+\frac{2}{x^3}-\frac{1}{9}\left(\frac{6}{x^3}\right)^2+\cdots\right)-x\left(1+\frac{1}{x^3}-\frac{1}{9}\left(\frac{3}{x^3}\right)^2+\cdots\right)$$

$$=x+\frac{2}{x^2}-\frac{36}{9}\left(\frac{1}{x^5}\right)+\cdots-x-\frac{1}{x^2}+\frac{1}{9}\left(\frac{9}{x^5}\right)-\cdots$$

$$\approx\frac{2-1}{x^2}$$

$$\sqrt[3]{x^3+6}-\sqrt[3]{x^3+3}\approx\frac{1}{x^2}$$

(OR)

(b) If $f: R \to R$ is defined by $f(x)=2x-3$, prove that f is a bijection and find its inverse.

Solution:

$$f(x)=2x-3$$

one – to – one:

$$f(x)=f(y)\Longrightarrow 2x-3=2y-3\Longrightarrow 2x=2y\Longrightarrow x=y$$

$$\therefore \text{f is one} - \text{to} - \text{one function}$$

onto:

$$\forall\, y\in\mathbb{R} \text{ such that } x=\frac{y+3}{2}$$

$$f(x)=f\left(\frac{y+3}{2}\right)=2\left(\frac{y+3}{2}\right)-3=y+3-3\Longrightarrow f(x)=y$$

$$\therefore \text{f is onto}$$

Hence, f is bijection function

Inverse:

$$y=f(x)\Longrightarrow x=f^{-1}(y)\quad\to\quad (1)$$

$$y=f(x)=2x-3\Longrightarrow y+3=2x\Longrightarrow x=\frac{y+3}{2}\quad\to\quad (2)$$

From (1) and (2), we get

$$f^{-1}(y)=\frac{y+3}{2}$$

Replace y by x

$$f^{-1}(x)=\frac{x+3}{2}$$

46. (a) State and prove Napier's Formula.

In ΔABC, we have

$$\text{(i) } \tan\frac{A-B}{2} = \frac{a-b}{a+b}\cot\frac{C}{2}$$
$$\text{(ii) } \tan\frac{B-C}{2} = \frac{b-c}{b+c}\cot\frac{A}{2}$$
$$\text{(iii) } \tan\frac{C-A}{2} = \frac{c-a}{c+a}\cot\frac{B}{2}$$

Proof:

$$\text{w.k.t. } a = 2R\sin A, b = 2R\sin B, c = 2R\sin C$$
$$\frac{a-b}{a+b}\cot\frac{C}{2} = \frac{2R\sin A - 2R\sin B}{2R\sin A + 2R\sin B}\cot\frac{C}{2}$$
$$= \frac{\sin A - \sin B}{\sin A + \sin B}\cot\frac{C}{2}$$
$$= \frac{2\cos\frac{A+B}{2}\sin\frac{A-B}{2}}{2\sin\frac{A+B}{2}\cos\frac{A-B}{2}}\cot\frac{C}{2}$$
$$= \cot\frac{A+B}{2}\tan\frac{A-B}{2}\cot\frac{C}{2}$$
$$= \cot\left(90^\circ - \frac{C}{2}\right)\tan\frac{A-B}{2}\cot\frac{C}{2}$$
$$= \tan\frac{C}{2}\tan\frac{A-B}{2}\frac{1}{\tan\frac{C}{2}}$$
$$= \tan\frac{A-B}{2}$$

Similarly, we can prove the other two results.

(OR)

(b) If the equation $\lambda x^2 - 10xy + 12y^2 + 5x - 16y - 3 = 0$ represents a pair of straight lines, find (i) the value of λ and the separate equations of the lines (ii) point of intersection of the lines (iii) angle between the lines

Solution:

(i) general equation is $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

Comparing the given equation $\lambda x^2 - 10xy + 12y^2 + 5x - 16y - 3 = 0$

$$a = \lambda, b = 12, c = -3, h = -5, g = \frac{5}{2}, f = -8$$

condition for pair of straight lines

$$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$
$$\lambda(12)(-3) + 2(-8)\left(\frac{5}{2}\right)(-5) - \lambda(-8)^2 - 12\left(\frac{5}{2}\right)^2 - (-3)(-5)^2 = 0$$
$$-36\lambda + 200 - 64\lambda - 75 + 75 = 0, \Longrightarrow \lambda = 2$$

The equation is $2x^2 - 10xy + 12y^2 + 5x - 16y - 3 = 0$

$$2x^2 - 10xy + 12y^2 \equiv (x-2y)(2x-6y)$$
$$2x^2 - 10xy + 12y^2 + 5x - 16y - 3 \equiv (x - 2y + c_1)(2x - 6y + c_2)$$

Equating like terms, we get

$$2c_1 + c_2 = 5, \quad 3c_1 + c_2 = 8, \quad c_1c_2 = -3$$

Solving first two equations, we get $c_1 = 3, c_2 = -1$

The separate equations of the lines are $x - 2y + 3 = 0$ and $2x - 6y - 1 = 0$

(ii) Point of intersection of the lines is given by

$$\left(\frac{hf - bg}{ab - h^2}, \frac{hg - af}{ab - h^2}\right) = \left(-10, -\frac{7}{2}\right)$$

(iii) Angle between the lines is given by

$$\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a + b}\right| = \frac{2\sqrt{25 - 24}}{2 + 12} = \frac{1}{7}$$

$$\theta = \tan^{-1}\left(\frac{1}{7}\right)$$

47. (a) Show that $\begin{vmatrix} \mathbf{b+c} & \mathbf{a} & \mathbf{a^2} \\ \mathbf{c+a} & \mathbf{b} & \mathbf{b^2} \\ \mathbf{a+b} & \mathbf{c} & \mathbf{c^2} \end{vmatrix} = (\mathbf{a+b+c})(\mathbf{a-b})(\mathbf{b-c})(\mathbf{c-a})$

Solution:

$$\text{Let } |A| = \begin{vmatrix} b+c & a & a^2 \\ c+a & b & b^2 \\ a+b & c & c^2 \end{vmatrix}$$

Put $a = b$

$$|A| = \begin{vmatrix} b+c & b & b^2 \\ c+b & b & b^2 \\ b+b & c & c^2 \end{vmatrix}_{\because R_1 \equiv R_2}$$

$$|A| = 0$$

$\therefore (a-b)^{2-1} = (a-b)$ is a factor of $|A|$

Similarly, $(b-c)$ and $(c-a)$ are factor of $|A|$

$$d.d = 4\,;\ f.d = 3$$

$$m = d.d - f.d = 4 - 3 = 1$$

other factor is $k(a+b+c)$

$$\begin{vmatrix} b+c & a & a^2 \\ c+a & b & b^2 \\ a+b & c & c^2 \end{vmatrix} = k(a+b+c)(a-b)(b-c)(c-a) \rightarrow ①$$

put $a = 0\,;\ b = 1\,;\ c = 2$

$$\begin{vmatrix} 1+2 & 0 & 0^2 \\ 2+0 & 1 & 1^2 \\ 0+1 & 2 & 2^2 \end{vmatrix} = k(0+1+2)(0-1)(1-2)(2-0)$$

$$\begin{vmatrix} 3 & 0 & 0 \\ 2 & 1 & 1 \\ 1 & 2 & 4 \end{vmatrix} = 6k$$

$$3(4-2) - 0 + 0 = 6k$$

$$6 = 6k$$

$$k = 1$$

Sub $k = 1$ in ① we get

$$\begin{vmatrix} b+c & a & a^2 \\ c+a & b & b^2 \\ a+b & c & c^2 \end{vmatrix} = (a+b+c)(a-b)(b-c)(c-a)$$

(OR)

(b) Prove that $3^{2n+2} - 8n - 9$ is divisible by 8 for all $n \geq 1$.

Solution:

$$P(n) := 3^{2n+2} - 8n - 9 \text{ is divisible by } 8$$

$$\text{Put } n = 1$$

$$P(1) := 3^{2(1)+2} - 8(1) - 9 = 81 - 17 = 64 \text{ is a multiple of } 8$$

$$P(1) \text{ is divisible by } 8$$

$$P(1) \text{ is true}$$

Let us assume that the statement is true for $n = k$

$$\text{i.e.,}\quad P(k) := 3^{2k+2} - 8k - 9 \text{ is divisible by } 8$$

$$\frac{3^{2k+2} - 8k - 9}{8} = m \Longrightarrow 3^{2k+2} - 8k - 9 = 8m$$

$$3^{2k+2} = 8m + 8k + 9$$

To prove that the statement is true for $n = k + 1$

$$\begin{aligned} \text{i.e.,}\quad P(k+1) &= 3^{2(k+1)+2} - 8(k+1) - 9 \\ &= 3^{2k+2}3^{2} - 8k - 8 - 9 \\ &= [8m + 8k + 9]9 - 8k - 17 \\ &= 72m + 72k + 81 - 8k - 17 \\ &= 72m + 64k + 64 \\ &= 8[9m + 8k + 8] \text{ is a multiple of } 8 \end{aligned}$$

$P(k+1)$ is divisible by 8

$$\Longrightarrow P(k+1) \text{ is true}$$

$$\therefore P(k+1) \text{ is true whenever } p(k) \text{ is true}$$

Hence by the principle of mathematical induction for $n \geq 1$

$$3^{2n+2} - 8n - 9 \text{ is divisible by } 8.$$

Prepared by
M. SANKAR M.Sc., B.Ed.
POST GRADUATE TEACHER
BHARATHIDASAN MAT. HR. SEC. SCHOOL
Tiruvallur