EXERCISE 1.8

Choose the correct or the most suitable answer from the given four alternatives:

1. If $|adj(adjA)|=|A|^9$, then the order of the square matrix A is
   (1)3 (2)4 (3)2 (4)5
2. If A is a 3 x 3 non-singular matrix such that $AA^T = A^TA$ and $B = A^{-1}A^T$, then BB^T =
   (1) A (2)B (3)I_3 (4)B^T
3. If A = $\begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$, B = adjA and C = 3A, then $\frac{|adjB|}{|C|}$ =
   (1)$\frac{1}{3}$ (2) $\frac{1}{9}$ (3)$\frac{1}{4}$ (4)1
4. If A $\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}$ = $\begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$, then A =
   (1) $\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix}$ (2) $\begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$ (3) $\begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$ (4) $\begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$
   (1) $\frac{2\pi}{3}$ (2) $\frac{3\pi}{4}$ (3) $\frac{5\pi}{6}$ (4) $\frac{\pi}{4}$
5. If A = $\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$, then $9I_2 - A$ =
   (1) A^{-1} (2) $\frac{A^{-1}}{2}$ (3) $3A^{-1}$ (4) $2A^{-1}$
6. If A = $\begin{bmatrix} 2 & 0 \\ 1 & 5 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 4 \\ 2 & 0 \end{bmatrix}$, then $|adj(AB)|$ =
   (1) -40 (2) – 80 (3) – 60 (4) – 20
7. If P = $\begin{bmatrix} 1 & x & 0 \\ 1 & 3 & 0 \\ 2 & 4 & -2 \end{bmatrix}$ is the adjoint of 3 x 3 matrix A and $|A|$ = 4, then x is
   (1) 15 (2) 12 (3) 14 (4) 11
8. If A = $\begin{bmatrix} 3 & 1 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & -1 \end{bmatrix}$ and A^{-1} = $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ then the value of a_{23} is
   (1) 0 (2) -2 (3) -3 (4) -1
9. If A , B and C are invertible matrices of some order, then which one of the following is not true?
   (1) adjA=$|A|A^{-1}$ (2) adj(AB) = (adjA)(adjB)
   (3) $\det A^{-1} = (\det A)^{-1}$ (4) $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$
10. If $(AB)^{-1}$ = $\begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix}$ and A^{-1} = $\begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$, then B^{-1} =
    (1) $\begin{bmatrix} 2 & -5 \\ -3 & 8 \end{bmatrix}$ (2) $\begin{bmatrix} 8 & 5 \\ 3 & 2 \end{bmatrix}$ (3) $\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ (4) $\begin{bmatrix} 8 & -5 \\ -3 & 2 \end{bmatrix}$
11. If A^TA^{-1} is symmetric , then A^2 =
    (1) A^{-1} (2) $(A^T)^2$ (3) A^T (4) $(A^{-1})^2$
12. If A is a non-singular matrix such that A^{-1} = $\begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$, then $(A^T)^{-1}$ =
    (1) $\begin{bmatrix} -5 & 3 \\ 2 & 1 \end{bmatrix}$ (2) $\begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$ (3) $\begin{bmatrix} -1 & -3 \\ 2 & 5 \end{bmatrix}$ (4) $\begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$
13. If A = $\begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ x & \frac{3}{5} \end{bmatrix}$ and $A^T = A^{-1}$, then the value of x is
    (1) $\frac{-4}{5}$ (2) $\frac{-3}{5}$ (3) $\frac{3}{5}$ (4) $\frac{4}{5}$
14. If A = $\begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}$ and AB = I_2 , then B =
    (1) $(\cos^2\frac{\theta}{2})A$ (2) $(\cos^2\frac{\theta}{2})A^T$ (3) $(\cos^2\theta)I$ (4) $(\sin^2\frac{\theta}{2})A$
15. If A = $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ and A(adjA) = $\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$, then k is
    (1) 0 (2) $\sin\theta$ (3) $\cos\theta$ (4) 1
16. If A = $\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that λA^{-1} = A , then λ is
    (1)17 (2) 14 (3) 19 (4) 21

17. If adjA = $\begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$ and adj B = $\begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}$ then adj(AB) is

(1) $\begin{bmatrix} -7 & -1 \\ 7 & -9 \end{bmatrix}$ (2) $\begin{bmatrix} -6 & 5 \\ -2 & -10 \end{bmatrix}$ (3) $\begin{bmatrix} -7 & 7 \\ -1 & -9 \end{bmatrix}$ (4) $\begin{bmatrix} -6 & -2 \\ 5 & -10 \end{bmatrix}$

18. The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \end{bmatrix}$ is

(1) 1 (2) 2 (3) 4 (4) 3

19. If $x^a y^b = e^m$, $x^c y^d = e^n$,$\Delta_1 = \begin{vmatrix} m & b \\ n & d \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} a & m \\ c & n \end{vmatrix}$, $\Delta_3 = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$ then the values of x and y are respectively,

(1) $e^{(\Delta_2/\Delta_1}, e^{(\Delta_3/\Delta_1)}$ (2) $\log(\Delta_1/\Delta_3), \log(\Delta_2/\Delta_3)$

(3) $\log(\Delta_2/\Delta_1), \log(\Delta_3/\Delta_1)$ (4) $e^{(\Delta_1/\Delta_3}, e^{(\Delta_2/\Delta_3)}$

20. Which of the following is/are correct?

(i) Adjoint of a symmetric matrix is also a symmetric matrix

(ii) Adjoint of a diagonal matrix is also a diagonal matrix.

(iii) If A is a square matrix of order n and λ is a scalar, then $adj\ (\lambda A) = \lambda^n\ adj\ A$.

(iv) $A(adj\ A) = (adj\ A)A = |A|I_n$

(1) Only (i) (2) (ii) and (iii) (3) (iii) and (iv) (4) (i), (ii) and (iv)

21. If $\rho(A) = \rho([A|B])$, then the system AX=B of linear equations is

(1) consistent and has a unique solution (2) consistent

(3) consistent and has infinitely many solution (4) inconsistent

22. If $0 \le \theta \le \pi$ and the system of equations $x + (\sin\theta)y - (\cos\theta)z = 0, (\cos\theta)x - y + z = 0, (\sin\theta)x + y - z = 0$ has a non-trivial solution then θ is

(1) $\frac{2\pi}{3}$ (2) $\frac{3\pi}{4}$ (3) $\frac{5\pi}{6}$ (4) $\frac{\pi}{4}$

23. The augmented matrix of a system of linear equations is $\begin{bmatrix} 1 & 2 & 7 & 3 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & \lambda - 7 & \mu + 5 \end{bmatrix}$. The system has infinitely many solutions if

(1) $\lambda = 7, \mu \ne -5$ (2) $\lambda = -7, \mu = 5$ (3) $\lambda \ne 7, \mu \ne -5$ (4) $\lambda = 7, \mu = -5$

24. Let $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and $4B = \begin{bmatrix} 3 & 1 & -4 \\ 1 & 3 & x \\ -1 & 1 & 3 \end{bmatrix}$.If B is the inverse of A. then the value of x is

(1) 2 (2) 4 (3) 3 (4) 1

25. If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, then $adj\ (adj\ A)$ is

(1) $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ (2) $\begin{bmatrix} 6 & -6 & 8 \\ 4 & -6 & 8 \\ 0 & -2 & 2 \end{bmatrix}$ (3) $\begin{bmatrix} -3 & 3 & -4 \\ -2 & 3 & -4 \\ 0 & 1 & -1 \end{bmatrix}$ (4) $\begin{bmatrix} 3 & -3 & 4 \\ 0 & -1 & 1 \\ 2 & -3 & 4 \end{bmatrix}$

EXERCISE 2.9

Choose the correct or the most suitable answer from the given four alternatives:

1. $i^n + i^{n+1} + i^{n+2} + i^{n+3}$ is

(1) 0 (2) 1 (3) -1 (4) i

2. The value of $+\sum_{i=1}^{13}(\ i^n + i^{n-1})$is

(1) $1 + i$ (2) i (3) 1 (4) 0

3. The area of the triangle formed by the complex numbers $z, i\,z$ and $z\ + i\,z$ in the Argand's diagram is

(1)$\frac{1}{2}|z|^2$ (2)$|z|^2$ (3)$\frac{3}{2}|z|^2$ (4)$2|z|^2$

4. The conjugate of a complex number is $\frac{1}{i-2}$. Then, the complex number is

(1) $\frac{1}{i+2}$ (2) $\frac{-1}{i+2}$ (3) $-\frac{1}{i-2}$ (4) $\frac{1}{i-2}$

5. If $z = \frac{(\sqrt{3}+i)^2(3i+4)^2}{(8+6i)^2}$then | z | is equal to

(1) 0 (2) 1 (3) 2 (4) 3

6. If z is a non zero complex number, such that $2iz^2 = \bar{z}$, then $|z|$ is

(1) 1/ 2 (2) 1 (3) 2 (4) 3

7. If $|z - 2 + i| \leq 2$, then the greatest value of $|z|$ is

(1) $\sqrt{3} - 2$ (2) $\sqrt{3} + 2$ (3) $\sqrt{5} - 2$ (4) $\sqrt{5} + 2$

8. If $\left|Z - \frac{3}{Z}\right| = 2$, then the least value of $|z|$ is

(1) 1 (2) 2 (3) 3 (4) 5

9. If $|z| = 1$, then the value of $\frac{1+z}{1+\bar{z}}$ is

(1) z (2) $\bar{z}$ (3) $\frac{1}{z}$ (4) 1

10. The solution of the equation $|z| - z = 1 + 2i$ is

(1) $\frac{3}{2} - 2i$ (2) $-\frac{3}{2} + 2i$ (3) $2 - \frac{3}{2}i$ (4) $2 + \frac{3}{2}i$

11. If $|z_1| = 1, |z_2| = 2, |z_3| = 3$ and $|9z_1z_2 + 4z_1z_3 + z_2z_3| = 12$, then the value $|z_1 + z_2 + z_3|$ is

(1) 1 (2) 2 (3) 3 (4) 4

12. If z is a complex number such that $z \in \mathbb{C}\backslash\mathbb{R}$ and $z + \frac{1}{z} \in \mathbb{R}$, then $|z|$ is

(1) 0 (2) 1 (3) 2 (4) 3

13. z_1, z_2 and z_3 are complex numbers such that $z_1 + z_2 + z_3 = 0$ and $|z_1| = |z_2| = |z_3| = 1$ then $z_1^2 + z_2^2 + z_3^2$ is

(1) 3 (2) 2 (3) 1 (4) 0

14. If $\frac{z-1}{z+1}$ is purely imaginary, then $|z|$ is

(1) $\frac{1}{2}$ (2) 1 (3) 2 (4) 3

15. If $z = x + iy$ is a complex number such that $|z + 2| = |z - 2|$, then the locus of z is

(1) real axis (2) imaginary axis (3) ellipse (4) circle

16. The principal argument of $\frac{3}{-1+i}$ is

(1) $\frac{-5\pi}{6}$ (2) $\frac{-2\pi}{3}$ (3) $\frac{-3\pi}{4}$ (4) $\frac{-\pi}{2}$

17. The principal argument of $(\sin 40^\circ + i\cos 40^\circ)^5$ is

(1) -110° (2) -70° (3) 70° (4) 110°

18. If $(1 + i)(1 + 2i)(1 + 3i)\cdots(1 + ni) = (x + iy)$, then $2 \cdot 5 \cdot 20 \cdots (1 + n^2)$

(1) 1 (2) i (3) $x^2 + y^2$ (4) $1 + n^2$

19. If $\omega \neq 1$ is a cubic root of unity and $(1 + \omega)^7 = A + B\omega$, then (A, B) equals

(1) (1,0) (2) (−1,1) (3) (0,1) (4) (1,1)

20. The principal argument of the complex number $\frac{(1+i\sqrt{3})^2}{4i(1-i\sqrt{3})}$ is

(1) $\frac{2\pi}{3}$ (2) $\frac{\pi}{6}$ (3) $\frac{5\pi}{6}$ (4) $\frac{\pi}{2}$

21. If α and β are the roots of $x^2 + x + 1 = 0$, then $\alpha^{2020} + \beta^{2020}$ is

(1) −2 (2) −1 (3) 1 (4) 2

22. The product of all four values of $\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^{\frac{3}{4}}$ is

(1) −2 (2) −1 (3) 1 (4) 2

23. If $\omega \neq 1$ is a cubic root of unity and $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k$, then k is equal to

(1) 1 (2) −1 (3) $\sqrt{3}i$ (4) $-\sqrt{3}i$

24. The value of $\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{10}$ is

(1) $cis\ \frac{2\pi}{3}$ (2) $cis\ \frac{4\pi}{3}$ (3) $-cis\ \frac{2\pi}{3}$ (4) $-cis\ \frac{4\pi}{3}$

25. If $= cis\ \frac{2\pi}{3}$, then the number of distinct roots of $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0.$

(1) 1 (2) 2 (3) 3 (4) 4

EXERCISE 3.7

Choose the correct or the most suitable answer from the given four alternatives:

1. A zero of $x^3 + 64$ is

(1) 0 (2) 4 (3) $4i$ (4) -4

2. If f and g are polynomials of degrees m and n respectively, and if $h(x) = (f o g)(x)$, then the degree of h is

(1) mn (2) $m+n$ (3) m^n (4) n^m

3. A polynomial equation in x of degree n always has

(1) n distinct roots (2) n real roots (3) n imaginary roots (4) at most one root

4. If α, β, and γ are the zeros of $x^3 + px^2 + qx + r$, then $\sum \frac{1}{\alpha}$ is

(1) $\frac{-q}{r}$ (2) $\frac{-p}{r}$ (3) $\frac{q}{r}$ (4) $-\frac{q}{p}$

5. According to the rational root theorem, which number is not possible rational zero of $4x^7 + 2x^4 - 10x^3 - 5$?

(1) -1 (2) $\frac{5}{4}$ (3) $\frac{4}{5}$ (4) 5

6. The polynomial $x^3 - kx^2 + 9x$ has three real zeros if and only if, k satisfies

(1) $|k| \le 6$ (2) $k = 0$ (3) $|k| > 6$ (4) $|k| \ge 6$

7. The number of real numbers in $[0, 2\pi]$ satisfying $sin^4 x - 2\,sin^2 x + 1$ is

(1) 2 (2) 4 (3) 1 (4) ∞

8. If $x^3 + 12x^2 + 10ax + 1999$ definitely has a positive zero, if and only if

(1) $a \ge 0$ (2) $a > 0$ (3) $a < 0$ (4) $a \le 0$

9. The polynomial $x^3 + 2x + 3$ has

(1) one negative and two imaginary zeros (2) one positive and two imaginary zeros

(3) three real zeros (4) no zeros

10. The number of positive zeros of the polynomial $\sum_{j=0}^{n} nC_r(-1)^r x^r$ is

(1) 0 (2) n (3) $< n$ (4) r

EXERCISE 4.6

Choose the correct or the most suitable answer from the given four alternatives.

1. The value of $sin^{-1}(\cos), 0 \le x \le \pi$ is

(1) $\pi - x$ (2) $x - \frac{\pi}{2}$ (3) $\frac{\pi}{2} - x$ (4) $x - \pi$

2. If $sin^{-1}x + sin^{-1}y = \frac{2\pi}{3}$; then $cos^{-1}x + cos^{-1}y$ is equal to

(1) $\frac{2\pi}{3}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{6}$ (4) π

3. $sin^{-1}\frac{3}{5} - cos^{-1}\frac{12}{13} + sec^{-1}\frac{5}{3} - cosec^{-1}\frac{13}{12}$ is equal to

(1) 2π (2) π (3) 0 (4) $tan^{-1}\frac{12}{65}$

4. If $sin^{-1}x = 2sin^{-1}\alpha$ has a solution, then

(1) $|\alpha| \le \frac{1}{\sqrt{2}}$ (2) $|\alpha| \ge \frac{1}{\sqrt{2}}\pi$ (3) $|\alpha| < \frac{1}{\sqrt{2}}$ (4) $|\alpha| > \frac{1}{\sqrt{2}}$

5. $sin^{-1}(cos\,x) = \frac{\pi}{2} - x$ is valid for

(1) $-\pi \le x \le 0$ (2) $0 \le x \le \pi$ (3) $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ (4) $-\frac{\pi}{4} \le x \le \frac{3\pi}{4}$

6. If $sin^{-1}x + sin^{-1}y + sin^{-1}z = \frac{3\pi}{2}$, the value of $x^{2017} + y^{2018} + z^{2019} - \frac{9}{x^{101} + y^{101} + z^{101}}$ is

(1) 0 (2) 1 (3) 2 (4) 3

7. If $cot^{-1}x = \frac{2\pi}{5}$ for some $x \in R$, the value of $tan^{-1}x$ is

(1) $-\frac{\pi}{10}$ (2) $\frac{\pi}{5}$ (3) $\frac{\pi}{10}$ (4) $-\frac{\pi}{5}$

8. The domain of the function defined by $f(x) = sin^{-1}\sqrt{x-1}$ is

(1) $[1,2]$ (2) $[-1,1]$ (3) $[0,1]$ (4) $[-1,0]$

9. If $= \frac{1}{5}$, the value of $cos\,(cos^{-1}x + 2\,sin^{-1}x)$ is

(1) $-\sqrt{\frac{24}{25}}$ (2) $\sqrt{\frac{24}{25}}$ (3) $\frac{1}{5}$ (4) $-\frac{1}{5}$

10. $tan^{-1}\left(\frac{1}{4}\right) + tan^{-1}\left(\frac{2}{9}\right)$ is equal to

(1) $\frac{1}{2}\,cos^{-1}\left(\frac{3}{5}\right)$ (2) $\frac{1}{2}\,sin^{-1}\left(\frac{3}{5}\right)$ (3) $\frac{1}{2}\,tan^{-1}\left(\frac{3}{5}\right)$ (4) $tan^{-1}\left(\frac{1}{2}\right)$

11. If the function $(x) = sin^{-1}(x^2 - 3)$, then x belongs to

(1) $[-1,1]$ (2) $[\sqrt{2},2]$ (3) $[-2,-\sqrt{2}]\cup[\sqrt{2},2]$ (4) $[-2,-\sqrt{2}]$

12. If $cot^{-1}\ 2$ and $cot^{-1}\ 3$ are two angles of a triangle, then the third angle is

(1) $\frac{\pi}{4}$ (2) $\frac{3\pi}{4}$ (3) $\frac{\pi}{6}$ (4) $\frac{\pi}{3}$

13. $sin^{-1}\left(\tan\frac{\pi}{4}\right)-\ sin^{-1}\left(\sqrt{\frac{3}{x}}\right)$, Then x is a root of the equation

(1) $x^2-x-6=0$ (2) $x^2-x-12=0$ (3) $x^2+x-12=0$ (4) $x^2+x-6=0$

14. $sin^{-1}(2cos^{-1}x-1)+cos^{-1}(1-2sin^2x)=$

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{6}$

15. If $cot^{-1}\left(\sqrt{\sin\ \alpha}\right)tan^{-1}\left(\sqrt{\sin\alpha}\right)\ =\ u$, then $cos\ 2u$ is equal to

(1) $tan^2\ \alpha$ (2) 0 (3) -1 (4) $tan\ 2\alpha$

16. If $|x|\le 1$, then $2tan^{-1}x-sin^{-1}\left(\frac{2x}{1+x^2}\right)$ is equal to

(1) $tan^{-1}\ x$ (2) $sin^{-1}\ x$ (3) 0 (4) π

17. The equation $tan\ ^{-1}x-cot^{-1}x=\ tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ has

(1) no solution (2) unique solution (3) two solutions (4) infinite number of solutions

18. If $sin^{-1}x+\ cot^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{2}$, then x is equal to

(1) $\frac{1}{2}$ (2) $\frac{1}{\sqrt{5}}$ (3) $\frac{2}{\sqrt{5}}$ (4) $\frac{\sqrt{3}}{2}$

19. If $sin^{-1}\frac{x}{5}+\ cosec^{-1}\left(\frac{5}{4}\right)=\frac{\pi}{2}$, then x is equal to

(1) 4 (2) 5 (3) 2 (4) 3

20. $\sin(tan^{-1}x\ ), |\ x\ |\ <\ 1$ is equal to

(1) $\frac{x}{\sqrt{1-x^2}}$ (2) $\frac{1}{\sqrt{1-x^2}}$ (3) $\frac{1}{\sqrt{1+x^2}}$ (4) $\frac{x}{\sqrt{1+x^2}}$

EXERCISE 5.6

Choose the correct or the most suitable answer from the given four alternatives:

1. The equation of the circle passing through (1,5) and (4,1) and touching y -axis is $x^2+y^2-5x-6y+9+\lambda(4x+3y-19)=0$,where λ is equal to

(1) $0,-\frac{40}{9}$ (2) 0 (3) $\frac{40}{9}$ (4) $-\frac{40}{9}$

2. The eccentricity of the hyperbola whose latus rectum is 8 and conjugate axis is equal to half the distance between the foci is

(1) $\frac{4}{3}$ (2) $\frac{4}{\sqrt{3}}$ (3) $\frac{2}{\sqrt{3}}$ (4) $\frac{3}{2}$

3. The circle $x^2+y^2=4x+8y+5$ intersects the line $3x-4y=m$ at two distinct points if

(1) $15\ <\ m\ <65$ (2) $35\ <\ m\ <85$
(3) $-85\ <\ m\ <-35$ (4) $-\ 35<\ m\ <\ 15$

4. The length of the diameter of the circle which touches the x -axis at the point (1,0) and passes through the point (2,3) .

(1) $\frac{6}{5}$ (2) $\frac{5}{3}$ (3) $\frac{10}{3}$ (4) $\frac{3}{5}$

5. The radius of the circle $3x^2+by^2+4bx-6by+b^2$ is

(1) 1 (2) 3 (3) $\sqrt{10}$ (4) $\sqrt{11}$

6. The centre of the circle inscribed in a square formed by the lines $x^2-8x-12=0$ and $y^2-14y+45=0$ is

(1) (4,7) (2) (7,4) (3) (9,4) (4) (4,9)

7. The equation of the normal to the circle $x^2+y^2\ -2x-2y+1=0$ which is parallel to the line $2x+4y=3$ is

(1) $x+2y=3$ (2) $x+2y+3=0$ (3) $2x+4y+3=0$ (4) $x-2y+3=0$

8. If $P\ (x,y)$ be any point on $16x^2+25y^2=400$ with foci $F_1(3,0)$ and $F_2(-3,0)$ then PF_1+PF_2 is

(1) 8 (2) 6 (3) 10 (4) 12

9. The radius of the circle passing through the point (6,2) two of whose diameter are $x+y\ =6$ and $x+2y\ =4$ is

(1) 10 (2) $2\sqrt{5}$ (3) 6 (4) 4

10. The area of quadrilateral formed with foci of the hyperbolas $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$ is
    (1) $4(a^2 + b^2)$ (2) $2(a^2 + b^2)$ (3) $(a^2 + b^2)$ (4) $\frac{1}{2}(a^2 + b^2)$
11. If the normals of the parabola $y^2 = 4x$ drawn at the end points of its latus rectum are tangents to the circle $(x-3)^2 + (y+2)^2 = r^2$, then the value of r^2 is
    (1) 2 (2) 3 (3) 1 (4) 4
12. If $x + y = k$ is a normal to the parabola $y^2 = 12x$, then the value of k is
    (1) 3 (2) -1 (3) 1 (4) 9
13. The ellipse $E_1 : \frac{x^2}{9} + \frac{y^2}{4} = 1$ is inscribed in a rectangle R whose sides are parallel to the coordinate axes. Another ellipse E_2 passing through the point(0,4) circumscribes the rectangle R. The eccentricity of the ellipse is
    (1) $\frac{\sqrt{2}}{2}$ (2) $\frac{\sqrt{3}}{2}$ (3) $\frac{1}{2}$ (4) $\frac{3}{4}$1.
14. The equation of the circle tangents are drawn to the hyperbola $\frac{x^2}{9} + \frac{y^2}{4} = 1$ parallel to the straight line$2x - y = 1$. One of the points of contact of tangents on the hyperbola is
    (1) $\left(\frac{9}{2\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$ (2) $\left(\frac{-9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ (3) $\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ (4) $(3\sqrt{3}, -2\sqrt{2})$
15. The equation of the circle passing through the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ having centre at (0,3) is
    (1) $x^2 + y^2 - 6y - 7 = 0$ (2) $x^2 + y^2 - 6y + 7 = 0$
    (3) $x^2 + y^2 - 6y - 5 = 0$ (4) $x^2 + y^2 - 6y + 5 = 0$
16. Let C be the circle with centre at (1,1) and radius =1. If T is the circle centered at $(0, y)$ passing through the origin and touching the circleC externally, then the radius of T is equal to
    (1) $\frac{\sqrt{3}}{\sqrt{2}}$ (2) $\frac{\sqrt{3}}{2}$ (3) $\frac{1}{2}$ (4) $\frac{3}{4}$
17. Consider an ellipse whose centre is of the origin and its major axis is along x-axis. If its eccentricity is $\frac{3}{5}$ and the distance between its foci is 6, then the area of the quadrilateral inscribed in the ellipse with diagonals as major and minor axis of the ellipse is
    (1) 8 (2) 32 (3) 80 (4) 40
18. Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$is
    (1) $2ab$ (2) ab (3) $\sqrt{ab}$ (4) $\frac{a}{b}$
19. An ellipse has OB as semi minor axes, F and F′ its foci and the angle FBF′ is a right angle. Then the eccentricity of the ellipse is
    (1) $\frac{1}{\sqrt{2}}$ (2) $\frac{1}{2}$ (3) $\frac{1}{4}$ (4) $\frac{1}{\sqrt{3}}$
20. The eccentricity of the ellipse $(x-3)^2 + (y-4)^2 = \frac{y^2}{9}$ is
    (1) $\frac{\sqrt{3}}{2}$ (2) $\frac{1}{3}$ (3) $\frac{1}{3\sqrt{2}}$ (4) $\frac{1}{\sqrt{3}}$
21. If the two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles then the locus of P is
    (1) $2x + 1 = 0$ (2) $x = -1$ (3) $2x - 1 = 0$ (4) $x = 1$
22. The circle passing through $(1, -2)$ and touching the axis of x at (3,0) passing through the point
    (1) $(-5,2)$ (2) $(2,-5)$ (3) $(5,2)$ (4) $(-2,5)$
23. The locus of a point whose distance from $(-2,0)$ is $\frac{2}{3}$ times its distance from the line $x = \frac{-9}{2}$ is
    (1) a parabola (2) a hyperbola (3) an ellipse (4) a circle
24. The values of m for which the line $y = mx + 2\sqrt{5}$ touches the hyperbola are the roots of $16x^2 - 9y^2 = 144$, then the value of $(a + b)$ is
    (1) 2 (2) 4 (3) 0 (4) -2
25. If the coordinates at one end of a diameter of the circle $x^2 + y^2 - 8x - 4y + c$ are (11,2), the coordinates of the other end are
    (1) $(-5,2)$ (2) $(2,-5)$ (3) $(5,-2)$ (4) $(-2,5)$

EXERCISE 6.10

Choose the correct or the most suitable answer from the given four alternatives:

1. If $\vec{a}$ and $\vec{b}$ are parallel vectors, then $[\vec{a},\vec{c},\vec{b}]$ is equal to

(1) 2 (2) −1 (3) 1 (4) 0

2. If a vector $\vec{\alpha}$ lies in the plane of $\vec{\beta}$ and $\vec{\gamma}$, then

(1) $[\vec{\alpha},\vec{\beta},\vec{\gamma}] = 1$ (2) $[\vec{\alpha},\vec{\beta},\vec{\gamma}] = -1$ (3) $[\vec{\alpha},\vec{\beta},\vec{\gamma}] = 0$ (4) $[[\vec{\alpha},\vec{\beta},\vec{\gamma}] = 2$

3. If $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{a} = 0$, then the value of $[\vec{a},\vec{b},\vec{c}]$ is

(1) $|\vec{a}|\,\vec{b}||\vec{c}|$ (2) $\frac{1}{3}|\vec{a}\,||\vec{b}||\vec{c}|$ (3) 1 (4) −1

4. If $\vec{a},\vec{b},\vec{c}$ are three unit vectors such that $\vec{a}$ is perpendicular to $\vec{b}$, and is parallel to $\vec{c}$ then $\vec{a}\times(\vec{b}\times\vec{c})$ is equal to

(1) $\vec{a}$ (2) $\vec{b}$ (3) $\vec{c}$ (4) $\vec{0}$

5. If $[\vec{a},\vec{b},\vec{c}] = 1$ then the value of $\frac{\vec{a}\cdot(\vec{b}\times\vec{c})}{(\vec{c}\times\vec{a})\cdot\vec{b}} + \frac{\vec{b}\cdot(\vec{c}\times\vec{a})}{(\vec{a}\times\vec{b})\cdot\vec{c}} + \frac{\vec{c}\cdot(\vec{a}\times\vec{b})}{(\vec{c}\times\vec{b})\cdot\vec{a}}$ is

(1) 1 (2) −1 (3) 2 (4) 3

6. The volume of the parallelepiped with its edges represented by the vectors $\hat{\imath}+\hat{\jmath}, \hat{\imath}+2\hat{\jmath}, \hat{\imath}+\hat{\jmath}+\pi\hat{k}$ is

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$ (3) π (4) $\frac{\pi}{4}$

7. If $\vec{a}$ and $\vec{b}$ are unit vectors such that $[\vec{a},\vec{b},\vec{a}\times\vec{b}] = \frac{\pi}{4}$, then the angle between $\vec{a}$ and $\vec{b}$ is

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{2}$

8. If $\vec{a} = \hat{\imath}+\hat{\jmath}+\hat{k}, \vec{b} = \hat{\imath}+\hat{\jmath}, \vec{c} = \hat{\imath}$ and $(\vec{a}\times\vec{b})\times\vec{c} = \lambda\vec{a}+\mu\vec{b}$ then the value of $\lambda+\mu$ is

(1) 0 (2) 1 (3) 6 (4) 3

9. If $\vec{a},\vec{b},\vec{c}$ are non-coplanar, non-zero vectors such that $[\vec{a},\vec{b},\vec{c}] = 3$, then $\{[\vec{a}\times\vec{b},\vec{b}\times\vec{c},\vec{c}\times\vec{a}]\}^2$ is equal to

(1) 81 (2) 9 (3) 27 (4)18

10. If $\vec{a},\vec{b},\vec{c}$ are three non-coplanar vectors such that $\vec{a}\cdot(\vec{b}\times\vec{c}) = \frac{\vec{b}+\vec{c}}{\sqrt{2}}$, then the angle between $\vec{a}$ and $\vec{b}$ is

(1) $\frac{\pi}{2}$ (2) $\frac{3\pi}{4}$ (3) $\frac{\pi}{4}$ (4) π

11. If the volume of the parallelepiped with $\vec{a}\times\vec{b}, \vec{b}\times\vec{c}, \vec{c}\times\vec{a}$ as coterminous edges is 8 cubic units, then the volume of the parallelepiped with $(\vec{a}\times\vec{b})\times(\vec{b}\times\vec{c}), (\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a})$ and $(\vec{c}\times\vec{a})\times(\vec{a}\times\vec{b})$ as coterminous edges is,

(1) 8 cubic units (2) 512 cubic units (3) 64 cubic units (4) 24 cubic units

12. Consider the vectors $\vec{a},\vec{b},\vec{c},\vec{d}$ such that $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}) = \vec{o}$. Let P_1 and P_2 be the planes determined by the pairs of vectors , $\vec{a},\vec{b}$ and $\vec{c},\vec{d}$ respectively. Then the angle between P_1 and P_2 is

(1) 0° (2) 45° (3) 60° (4) 90°

13. If $\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\times\vec{b})\times\vec{c}$ where $\vec{a},\vec{b},\vec{c}$ are any three vectors such that $\vec{b}\cdot\vec{c}\neq 0$ and $\vec{a}\cdot\vec{b}\neq 0$, then $\vec{a}$ and $\vec{c}$ are

(1) perpendicular (2) parallel

(3) inclined at an angle $\frac{\pi}{3}$ (4) inclined at an angle $\frac{\pi}{6}$

14. If $\vec{a} = 2\hat{\imath}+3\hat{\jmath}-\hat{k}, \vec{b} = \hat{\imath}+2\hat{\jmath}-5\hat{k}, \vec{c} = 3\hat{\imath}+5\hat{\jmath}-\hat{k}$ then a vector perpendicular to $\vec{a}$ and lies in the plane containing $\vec{b}$ and $\vec{c}$ is

(1) $-17\hat{\imath}+21\hat{\jmath}-97\hat{k}$ (2) $-17\hat{\imath}+21\hat{\jmath}-123\hat{k}$

(3) $-17\hat{\imath}+21\hat{\jmath}+97\hat{k}$ (4) $-17\hat{\imath}-21\hat{\jmath}-97\hat{k}$

15. The angle between the lines $\frac{x-2}{3} = \frac{y+1}{-2} = z = 2$, and $\frac{x-1}{1} = \frac{2y+3}{3} = \frac{z+5}{2}$ is

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{2}$

16. If the line $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ lies in the plane $x+3y-\alpha z+\beta = 0$, then (α,β) is

(1) (−5,5) (2) (−6,7) (3) (5, −5) (4) (6,−7)

17. The angle between the line $\vec{r} = (\hat{\imath}+2\hat{\jmath}-3\hat{k}) + t(2\hat{\imath}+\hat{\jmath}-2\hat{k})$ and the plane $\vec{r}\cdot(\hat{\imath}+\hat{\jmath})+4 = 0$ is

(1) 0° (2) 30° (3) 45° (4) 90°

18. The coordinates of the point where the line $\vec{r} = (6\hat{\imath}-\hat{\jmath}-3\hat{k}) + t(-\hat{\imath}+4\hat{k})$ meets the plane $\vec{r}\cdot(\hat{\imath}+\hat{\jmath}-\hat{k}) = 3$ are

(1) (2,1,0) (2) $(7,-1,-7)$ (3) $(1,2,-6)$ (4) $(5,-1,1)$

19. Distance from the origin to the plane $3x-6y+2z+7=0$ is

(1) 0 (2) 1 (3) 2 (4) 3

20. The distance between the planes $x+2y+3z+7=0$ and $2x+4y+6z+7=0$ is

(1) $\frac{\sqrt{7}}{2\sqrt{2}}$ (2) $\frac{7}{2}$ (3) $\frac{\sqrt{7}}{2}$ (4) $\frac{7}{2\sqrt{2}}$

21. If the direction cosines of a line are $\frac{1}{c},\frac{1}{c},\frac{1}{c}$ then

(1) $c = \pm 3$ (2) $c = \pm\sqrt{3}$ (3) $c > 0$ (4) $0 < c < 1$

22. The vector equation $\vec{r} = (\hat{\imath}-2\hat{\jmath}-\hat{k}) + t(6\hat{\imath}-\hat{k})$ represents a straight line passing through the points

(1) $(0,6,-1)$ and $(1,-2,-1)$ (2) $(0,6,-1)$ and $(-1,-4,-2)$
(3) $(1,-2,-1)$ and $(1,4,-2)$ (4) $(1,-2,-1)$ and $(0,-6,1)$

23. If the distance of the point (1,1,1) from the origin is half of its distance from the plane $x+y+z+k=0$, then the values of k are

(1) ± 3 (2) ± 6 (3) $-3,9$ (4) $3,-9$

24. If the planes $\vec{r}\cdot(2\hat{\imath}-\lambda\hat{\jmath}+\hat{k}) = 3$ and $\vec{r}\cdot(4\hat{\imath}+\hat{\jmath}-\mu\hat{k}) = 5$ are parallel, then the value of λ and μ are

(1) $\frac{1}{2},-2$ (2) $-\frac{1}{2},2$ (3) $-\frac{1}{2},-2$ (4) $\frac{1}{2},2$

25. If the length of the perpendicular from the origin to the plane $2x+3y+\lambda z=1$, $\lambda > 0$ is $\frac{1}{5}$, then the value of λ is

(1) $2\sqrt{3}$ (2) $3\sqrt{2}$ (3) 0 (4) 1

EXERCISE 7.10

Choose the Correct or the most suitable answer from the given four alternatives:

1. The volume of a sphere is increasing in volume at the rate of 3π cm³/sec. The rate of change of its radius when radius is $\frac{1}{2}$ cm

(1) 3 cm/s (2) 2 cm/s (3) 1 cm/s (4) $\frac{1}{2}$ cm/s

2. A balloon rises straight up at 10 m/s. An observer is 40 m away from the spot where the balloon left the ground. The rate of change of the balloon's angle of elevation in radian per second when the balloon is 30 metres above the ground.

(1) $\frac{3}{25}$ radians/sec (2) $\frac{4}{25}$ radians/sec (3) $\frac{1}{5}$ radians/sec (4) $\frac{1}{3}$ radians/sec

3. The position of a particle moving along a horizontal line of any time t is given by $s(t) = 3t^2 - 2t - 8$. The time at which the particle is at rest is

(1) t = 0 (2) t = $\frac{1}{3}$ (3) t = 1 (4) t = 3

4. A stone is thrown up vertically. The height it reaches at time t seconds is given by $x = 80t - 16t^2$. The stone reaches the maximum height in time t seconds is given by

(1) 2 (2) 2.5 (3) 3 (4) 3.5

5. The point on the curve $6y = x^3 + 2$ at which y-coordinate changes 8 times as fast as x-coordinate is

(1) (4,11) (2) (4,-11) (3) (-4,11) (4) (-4,-11)

6. The abscissa of the point on the curve $f(x) = \sqrt{8-2x}$ at which the slope of the tangent is -0.25?

(1) -8 (2) -4 (3) -2 (4) 0

7. The slope of the line normal to the curve $f(x) = 2\cos 4x$ at $x = \frac{\pi}{2}$ is

(1) $-4\sqrt{3}$ (2) -4 (3) $\frac{\sqrt{3}}{12}$ (4) $4\sqrt{3}$

8. The tangent to the curve $y^2-xy+9=0$ is vertical when

(1) y = 0 (2) $y = \pm\sqrt{3}$ (3) $y = \frac{1}{2}$ (4) $y = \pm 3$

9. Angle between $y^2 = x$ and $x^2 = y$ at the origin is

(1) $\tan^{-1}\frac{3}{4}$ (2) $\tan^{-1}\frac{4}{3}$ (3) $\frac{\pi}{2}$ (4) $\frac{\pi}{4}$

10. The value of the limit $\lim_{x\to 0}\left(\cot x - \frac{1}{x}\right)$ is

(1) 0 (2) 1 (3) 2 (4) ∞

11. The function $\sin^4 x + \cos^4 x$ is increasing in the interval

(1) $\left[\frac{5\pi}{8}, \frac{3\pi}{4}\right]$ (2) $\left[\frac{\pi}{2}, \frac{5\pi}{8}\right]$ (3) $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$ (4) $\left[0, \frac{\pi}{4}\right]$

12. The number given by the Rolle's theorem for the function $x^3 - 3x^2$, $x \in [0,3]$ is
    (1) 1 (2) $\sqrt{2}$ (3) $\frac{3}{2}$ (4) 2
13. The number given by the Mean value theorem for the function $\frac{1}{x}$, $x \in [1,9]$ is
    (1) 2 (2) 2.5 (3) 3 (4) 3.5
14. The minimum value of the function $|3 - x| + 9$ is
    (1) 0 (2) 3 (3) 6 (4) 9
15. The maximum slope of the tangent to the curve $y = e^x \sin x$, $x \in [0, 2\pi]$ is at
    (1) $x = \frac{\pi}{4}$ (2) $x = \frac{\pi}{2}$ (3) $x = \pi$ (4) $x = \frac{3\pi}{2}$
16. The maximum value of the function $x^2 e^{-2x}$, $x > 0$ is
    (1) $\frac{1}{e}$ (2) $\frac{1}{2e}$ (3) $\frac{1}{e^2}$ (4) $\frac{4}{e^4}$
17. One of the closest points on the curve $x^2 - y^2 = 4$ to the point (6,0) is
    (1) (2,0) (2) $(\sqrt{5}, 1)$ (3) $(3, \sqrt{5})$ (4) $(\sqrt{13}, -\sqrt{3})$
18. The maximum value of the product of two positive numbers, when their sum of the squares 200, is
    (1) 100 (2) $25\sqrt{7}$ (3) 28 (4) $24\sqrt{14}$
19. The curve $y = ax^4 + bx^2$ with $ab > 0$
    (1) has no horizontal tangent (2) is concave up
    (3) is concave down (4) has no points of inflection
20. The point of inflection of the curve $y = (x-1)^3$ is
    (1) (0,0) (2) (0,1) (3) (1,0) (4) (1,1)

EXERCISE 8.8

Choose the correct or the most suitable answer from the given four alternatives :

1. A circular template has a radius of 10 cm. The measurement of radius has an approximate error of 0.02 cm. Then the percentage error in calculating area of this template is
   (1) 0.2% (2) 0.4% (3) 0.04% (4) 0.08%
2. The percentage error of fifth root of 31 is approximately how many times the percentage error in 31?
   (1) $\frac{1}{31}$ (2) $\frac{1}{5}$ (3) 5 (4) 31
3. If $U(x, y) = e^{x^2+y^2}$, then $\frac{\partial u}{\partial x}$ is equal to
   (1) $e^{x^2+y^2}$ (2) $2xu$ (3) $x^2 u$ (4) $y^2 u$
4. If $V(x, y) = \log(e^x + e^y)$, then $\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y}$ is equal to
   (1) $e^x + e^y$ (2) $\frac{1}{e^x + e^y}$ (3) 2 (4) 1
5. If $(x, y) = x^2$, then $\frac{\partial w}{\partial x}$ is equal to
   (1) $x^y \log x$ (2) $y \log x$ (3) yx^{y-1} (4) $x \log y$
6. If $f(x, y) = e^{xy}$, then $\frac{\partial^2 f}{\partial x \partial y}$ is equal to
   (1) xye^{xy} (2) $(1 + xy)e^{xy}$ (3) $(1 + y)e^{xy}$ (4) $(1 + x)e^{xy}$
7. If we measure the side of a cube to be 4 cm with an error of 0.1 cm, then the error in our calculation of the volume is
   (1) 0.4 cu.cm (2) 0.45 cu.cm (3) 2 cu.cm (4) 4.8 cu.cm
8. The change in the surface area $S = 6x^2$ of a cube when the edge length varies from x_0 to $x_0 + dx$ is
   (1) $12x_0 + dx$ (2) $12x_0\,dx$ (3) $6x_0\,dx$ (4) $6x_0 + dx$
9. The approximate change in the volume V of a cube of side x metres caused by increasing the side by 1% is
   (1) $0.3xdx\,m^3$ (2) $0.03x\,m^3$ (3) $0.03x^2\,m^3$ (4) $0.03x^3\,m^3$

10. If $g(x,y) = 3x^2 - 5y + 2y^2, x(t) = e^t$ and $(t) = \cos t$, then $\frac{dg}{dt}$ is equal to

(1) $6e^{2t} + 5\sin t - 4\cos t \sin t$ (2) $6e^{2t} - 5\sin t + 4\cos t \sin t$

(3) $3e^{2t} + 5\sin t + 4\cos t \sin t$ (4) $3e^{2t} - 5\sin t + 4\cos t \sin t$

11. If $f(x) = \frac{x}{x+1}$, then its differential is given by

(1) $-\frac{1}{(x+1)^2}dx$ (2) $\frac{1}{(x+1)^2}dx$ (3) $\frac{1}{x+1}dx$ (4) $-\frac{1}{x+1}dx$

12. If $(x,y) = x^2 + 3xy + y - 2019$, then $\frac{\partial u}{\partial x}_{(4,-5)}$ is equal to

(1) −4 (2) −3 (3) −7 (4) 13

13. Linear approximation for $g(x) = \cos x$ at $x = \frac{\pi}{2}$ is

(1) $x + \frac{\pi}{2}$ (2) $-x + \frac{\pi}{2}$ (3) $x - \frac{\pi}{2}$ (4) $-x - \frac{\pi}{2}$

14. If $W(x,y,z) = x^2(y-z) + y^2(z-x) + z^2(x-y)$, then $\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z}$ is

(1) $xy + yz + zx$ (2) $x(y+z)$ (3) $y(z+x)$ (4) 0

15. If $(x,y,z) = xy + yz + zx$, then $f_x - f_z$ is equal to

(1) $z - x$ (2) $y - z$ (3) $x - z$ (4) $y - z$

EXERCISE 9.10

Choose the Correct or the most suitable answer from the given four alternatives:

1. The value of $\int_0^{\frac{2}{3}} \frac{dx}{\sqrt{4-9x^2}}$ is

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{2}$ (3) $\frac{\pi}{4}$ (4) π

2. The value of $\int_{-1}^{2} |x|\, dx$ is

(1) $\frac{1}{2}$ (2) $\frac{3}{2}$ (3) $\frac{5}{2}$ (4) $\frac{7}{2}$

3. For any value of $n \in \mathbb{Z}$, $\int_0^{\pi} e^{\cos^2 x} \cos^3[(2x+1)x]\, dx$ is

(1) $\frac{\pi}{2}$ (2) π (3) 0 (4) 2

4. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x \cos x\, dx$ is

(1) $\frac{3}{2}$ (2) $\frac{1}{2}$ (3) 0 (4) $\frac{2}{3}$

5. The value of $\int_{-4}^{4} \left[\tan^{-1}\left(\frac{x^2}{x^4+1}\right) + \tan^{-1}\left(\frac{x^4+1}{x^2}\right)\right] dx$ is

(1) π (2) 2π (3) 3π (4) 4π

6. The value of $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \left(\frac{2x^7 - 3x^5 + 7x^3 - x + 1}{\cos^2 x}\right) dx$ is

(1) 4 (2) 3 (3) 2 (4) 0

7. If $f(x) = \int_0^x t\cos t\, dt$, then $\frac{df}{dx}$ is

(1) $\cos x - x\sin x$ (2) $\sin x + x\cos x$ (3) $x\cos x$ (4) $x\sin x$

8. The area between $y^2 = 4x$ and its latus rectum is

(1) $\frac{2}{3}$ (2) $\frac{4}{3}$ (3) $\frac{8}{3}$ (4) $\frac{5}{3}$

9. The value of $\int_0^1 x(1-x)^{99}\, dx$ is

(1) $\frac{1}{11000}$ (2) $\frac{1}{101000}$ (3) $\frac{1}{10010}$ (4) $\frac{1}{10001}$

10. The value of $\int_0^{\pi} \frac{dx}{1+5^{\cos x}}$ is

(1) $\frac{\pi}{2}$ (2) π (3) $\frac{3\pi}{2}$ (4) 2π

11. If $\frac{\Gamma(n+2)}{\Gamma(n)} = 90$, then n is

(1) 10 (2) 5 (3) 8 (4) 9

12. The value of $\int_0^{\frac{\pi}{6}} \cos^3 3x\, dx$ is

(1) $\frac{2}{3}$ (2) $\frac{2}{9}$ (3) $\frac{1}{9}$ (4) $\frac{1}{3}$

13. The value of $\int_0^{\pi} \sin^4 x\, dx$ is

(1) $\frac{3\pi}{10}$ (2) $\frac{3\pi}{8}$ (3) $\frac{3\pi}{4}$ (4) $\frac{3\pi}{2}$

14. The value of $\int_0^{\infty} e^{-3x}x^2\, dx$ is

(1) $\frac{7}{27}$ (2) $\frac{5}{27}$ (3) $\frac{4}{27}$ (4) $\frac{2}{27}$

15. If $\int_0^{a} \frac{1}{4+x^2}\, dx = \frac{\pi}{8}$, then a is

(1) 4 (2) 1 (3) 3 (4) 2

16. The volume of solid of revolution of the region bounded by $y^2 = x(a-x)$ about $x - axis$ is

(1) πa^3 (2) $\frac{\pi a^3}{4}$ (3) $\frac{\pi a^3}{5}$ (4) $\frac{\pi a^3}{6}$

17. If $f(x) = \int_1^{x} \frac{e^{\sin u}}{u}\, du, x > 1$ and $\int_1^{3} \frac{e^{\sin x^2}}{x}\, dx = \frac{1}{2}\,[f(a) - f(1)]$, then one of the possible value of a is

(1) 3 (2) 6 (3) 9 (4) 5

18. The value of $\int_0^{1} (\sin^{-1} x)^2\, dx$ is

(1) $\frac{\pi^2}{4} - 1$ (2) $\frac{\pi^2}{4} + 2$ (3) $\frac{\pi^2}{4} + 1$ (4) $\frac{\pi^2}{4} - 2$

19. The value of $\int_0^{a} \left(\sqrt{a^2 - x^2}\right)^3\, dx$ is

(1) $\frac{\pi a^3}{16}$ (2) $\frac{3\pi a^4}{16}$ (3) $\frac{3\pi a^2}{8}$ (4) $\frac{3\pi a^4}{8}$

20. If $\int_0^{x} f(t)\, dt = x + \int_x^{1} t\, f(t)\, dt$, then the value of $f(1)$ is

(1) $\frac{1}{2}$ (2) 2 (3) 1 (4) $\frac{3}{4}$

EXERCISE 10.9

Choose the Correct or the most suitable answer from the given four alternatives:

1. If y = f$(x^2 + 2)$ *and* $f'(3) = 5$, then $\frac{dy}{dx}$ at x = 1 is

(1)5 (2)25 (3)15 (4)10

2. If y = $\frac{1}{a-z}$, then $\frac{dz}{dy}$ is

(1)$(a-z)^2$ (2) $-(z-a)^2$ (3) $(z+a)^2$ (4)$-(z+a)^2$

3. If x = a sin θ and y = b cos θ, then $\frac{d^2y}{dx^2}$ is

(1)$\frac{a}{b^2}\sec^2\theta$ (2)$-\frac{b}{a}\sec^2\theta$ (3)$-\frac{b}{a^2}\sec^3\theta$ (4)$-\frac{b^2}{a^2}\sec^3\theta$

4. If f(x) = $\begin{Bmatrix} x+1 & when\ x < 2 \\ 2x-1 & when\ x \geq 2 \end{Bmatrix}$, then f'(2) is

(1)0 (2)1 (3)2 (4)does not exists

5. If f(x) = $\begin{Bmatrix} x+2 & -1 < x < 3 \\ 5 & x = 3 \\ 8-x & x > 3 \end{Bmatrix}$, then at x = 3, f'(x) is

(1)1 (2)-1 (3)0 (4)does not exists

6. If f(x) = $x^2 - 3x$, then the points at which f(x) = f ' (x) are

(1) both positive integers (2)both negative integers
(3)both irrational (4)one rational and another irrational

7. $\frac{d}{dx}(e^{x+5\log x})$ is

(1) $e^x.x^4(x+5)$ (2) $e^x.x(x+5)$ (3)$e^x + \frac{5}{x}$ (4) $e^x - \frac{5}{x}$

8. If f(x) = x+2 , then f'(f(x)) at x = 4 is

(1)8 (2)1 (3)4 (4)5

9. It is given that f'(a) exists , then $\lim_{x\to a} \frac{xf(a)-af(x)}{x-a}$ is

(1)f(a)-af'(a) (2)f'(a) (3)-f'(a) (4)f(a)+af'(a)

10. The derivative of f(x) = x $|x|$ at x = -3 is

(1)6 (2)-6 (3)does not exist (4)0

11. $\frac{d}{dx}\left(\frac{2}{\pi}sinx°\right)$ is

(1) $\frac{\pi}{180}cosx°$ (2) $\frac{1}{90}cosx°$ (3) $\frac{\pi}{90}cosx°$ (4) $\frac{2}{\pi}cosx°$

12. If pv = 81, then $\frac{dp}{dv}$ at v=9 is

(1)1 (2)-1 (3)2 (4)-2

13. If y = mx+c and f(0) = f'(0)= 1, then f(2) is

(1)1 (2)2 (3)3 (4)-3

14. The differential coefficient of $\log_{10} x$ with respect to $\log_x 10$ is

(1)1 (2)$-(\log_{10} x)^2$ (3) $(\log_x 10)^2$ (4)$\frac{x^2}{100}$

15. If f(x) = $\begin{cases} 2a - x & for -a < x < a \\ 3x - 2a & for\ x \geq a \end{cases}$, then which one of the following is true?

(1) f(x) is not differentiable at x= a (2) f(x) is discontinuous at x = a

(3)f(x) is continuous for all x in R (4) f(x) is differentiable for all x≥a

16. If y=$\frac{1}{4}u^4$, u=$\frac{2}{3}x^3 + 5$, then $\frac{dy}{dx}$ is

(1)$\frac{1}{27}x^2(2x^3 + 15)^3$ (2) $\frac{2}{27}x(2x^3 + 5)^3$

(3) $\frac{2}{27}x^2(2x^3 + 15)^3$ (4) $\frac{-2}{27}x(2x^3 + 5)^3$

17. x = $\frac{1-t^2}{1+t^2}$; y=$\frac{2t}{1+t^2}$ then $\frac{dy}{dx}$ is

(1)$-\frac{y}{x}$ (2) $\frac{y}{x}$ (3)$-\frac{x}{y}$ (4) $\frac{x}{y}$

18. If f(x) = $\begin{cases} ax^2 - b & -1 < x < 1 \\ \frac{1}{|x|} & elsewhere \end{cases}$ is differentiable at x = 1, then

(1)$a = \frac{1}{2}, b = \frac{-3}{2}$ (2) $a = \frac{-1}{2}, b = \frac{3}{2}$

(3) $a = \frac{-1}{2}, b = \frac{-3}{2}$ (4) $a = \frac{1}{2}, b = \frac{3}{2}$

19. If the derivative of (ax-5)e^{3x} at x = 0 is -13 , then the value of a is

(1)8 (2)-2 (3)5 (4)2

20. If f(x) = $\begin{cases} x - 5 & if\ x \leq 1 \\ 4x^2 - 9 & if 1 < x < 2 \\ 3x + 4 & if\ x \geq 2 \end{cases}$, then the right hand derivative of f(x) at x= 2 is

(1)0 (2)2 (3)3 (4)4

21. If y= cos(sinx^2), then $\frac{dy}{dx}$ at x = $\sqrt{\frac{\pi}{2}}$ is

(1)-2 (2)2 (3)-2$\sqrt{\frac{\pi}{2}}$ (4)0

22. If f(x) = x$tan^{-1}x$, then f'(1) is

(1)1+$\frac{\pi}{4}$ (2)$\frac{1}{2} + \frac{\pi}{4}$ (3) $\frac{1}{2} - \frac{\pi}{4}$ (4)2

23. If y= $\frac{(1-x)^2}{x^2}$, then $\frac{dy}{dx}$ is

(1)$\frac{2}{x^2} + \frac{2}{x^3}$ (2)$-\frac{2}{x^2} + \frac{2}{x^3}$ (3)$-\frac{2}{x^2} - \frac{2}{x^3}$ (4) $\frac{2}{x^2} - \frac{2}{x^3}$

24. If g(x) = $(x^2 + 2x + 3)$f(x) and f(0) = 5 and $\lim_{x\to 0}\frac{f(x)-5}{x} = 4$, then g'(0) is

(1)20 (2)14 (3)18 (4)12

25. The number of points in R in which the function f(x) = $|x - 1| + |x - 3| + sinx$ is not differentiable is

(1)2 (2)2 (3)1 (4)4s

EXERCISE 11.6

Choose the Correct or the most suitable answer from the given four alternatives:

1. Let X be random variable with probability density function $f(x) = \begin{cases} \frac{2}{x^3} & x \geq 1 \\ 0 & x < 1 \end{cases}$ Which of the following statement is correct

(1) both mean and variance exist (2) mean exists but variance does not exist
(3) both mean and variance do not exist (4) variance exists but Mean does not exist.

2. A rod of length $2l$ is broken into two pieces at random. The probability density function of the shorter of the two pieces is $f(x) = \begin{cases} \frac{1}{l} & 0 < x < l \\ 0 & l \le x \le 2l \end{cases}$ The mean and variance of the shorter of the two pieces are respectively

(1) $\frac{l}{2}, \frac{l^2}{3}$ (2) $\frac{l}{2}, \frac{l^2}{6}$ (3) $l, \frac{l^2}{12}$ (4) $\frac{l}{2}, \frac{l^2}{12}$

3. Consider a game where the player tosses a six-sided fair die. If the face that comes up is 6, the player wins $Rs36$, otherwise he loses $Rs\ k^2$, where k is the face that comes up $k = \{1, 2, 3, 4, 5\}$. The expected amount to win at this game in Rs is

(1) $\frac{19}{6}$ (2) $-\frac{19}{6}$ (3) $\frac{3}{2}$ (4) $-\frac{3}{2}$

4. A pair of dice numbered 1, 2, 3, 4, 5, 6 of a six-sided die and 1, 2, 3, 4 of a four-sided die is rolled and the sum is determined. Let the random variable X denote this sum. Then the number of elements in the inverse image of 7 is

(1) 1 (2) 2 (3) 3 (4) 4

5. A random variable X has binomial distribution with $n = 25$ and $p = 0.8$ then standard deviation of X is

(1) 6 (2) 4 (3) 3 (4) 2

6. Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed n times. Then the possible values of X are

(1) $i + 2n, i = 0,1,2 \ldots n$ (2) $2i - n, i = 0,1,2 \ldots n$
(3) $n - i, i = 0,1,2 \ldots n$ (4) $2i + 2n, i = 0,1,2 \ldots n$

7. If the function $f(x) = \frac{1}{12}$ for $a < x < b$ represents a probability density function of a continuous random variable X, then which of the following cannot be the value of a and b?

(1) 0 *and* 12 (2) 5 *and* 17 (3) 7 *and* 19 (4) 16 *and* 24

8. Four buses carrying 160 students from the same school arrive at a football stadium. The buses carry, respectively, 42, 36, 34, and 48 students. One of the students is randomly selected. Let X denote the number of students that were on the bus carrying the randomly selected student. One of the 4 bus drivers is also randomly selected. Let Y denote the number of students on that bus. Then E[X] and E[Y] respectively are

(1) 50 ,40 (2) 40, 50 (3) 40, 75, 40 (4) 41,41

9. Two coins are to be flipped. The first coin will land on heads with probability 0.6, the second with Probability 0.5. Assume that the results of the flips are independent, and let X equal the total number of heads that result. The value of E[X] is

(1) 0.11 (2) 1.1 (3) 11 (4) 1

10. On a multiple-choice exam with 3 possible destructives for each of the 5 questions, the probability that a student will get 4 or more correct answers just by guessing is

(1) $\frac{11}{243}$ (2) $\frac{3}{8}$ (3) $\frac{1}{243}$ (4) $\frac{2}{243}$

11. If $P\{X = 0\} = 1 - P\{X = 1\}$. If $E[X] = 3Var(X)$, then $P\{X = 0\}$.

(1) $\frac{2}{3}$ (2) $\frac{2}{5}$ (3) $\frac{1}{5}$ (4) $\frac{1}{3}$

12. If X is a binomial random variable with expected value 6 and variance 2.4, Then $P\{X = 5\}$ is

(1) $\binom{10}{5}\left(\frac{3}{5}\right)^6\left(\frac{2}{5}\right)^4$ (2) $\binom{10}{5}\left(\frac{3}{5}\right)^{10}$ (3) $\binom{10}{5}\left(\frac{3}{5}\right)^4\left(\frac{2}{5}\right)^6$ (4) $\binom{10}{5}\left(\frac{3}{5}\right)^5\left(\frac{2}{5}\right)^5$

13. The random variable X has the probability density function $f(x) = \begin{cases} ax + b, & 0 < x < 1 \\ 0, & otherwise \end{cases}$ and $E(X) = \frac{7}{12}$, then a and b are respectively

(1) 1 *and* $\frac{1}{2}$ (2) $\frac{1}{2}$ *and* 1 (3) 2 *and* 1 (4) 1 *and* 2

14. Suppose that X takes on one of the values 0, 1, *and* 2. If for some constant k , $P(X = i) = k\,P(X = i - 1)$ for $i = 1,2$ and $p(x = 0) = 1/7$, Then the value of k is

(1) 1 (2) 2 (3) 3 (4) 4

15. Which of the following is a discrete random variable?

I. The number of cars crossing a particular signal in a day.
II. The number of customers in a queue to buy train tickets at a moment.

III. The time taken to complete a telephone call.
(1) *I and II* (2) *II only* (3) *III only* (4) *II and III*

16. If $f(x) = \begin{cases} 2x & 0 < x < 1 \\ 0 & otherwise \end{cases}$ otherwise is a probability density function of a random variable, then the value of a is
(1) 1 (2) 2 (3) 3 (4) 4

17. The probability function of a random variable is defined as:

x	-2	-1	0	1	2
F(x)	k	2k	3k	4k	5

Then $E(X)$ is equal to:
(1) $\frac{1}{15}$ (2) $\frac{1}{10}$ (3) $\frac{1}{3}$ (4) $\frac{2}{3}$

18. Let X have a Bernoulli distribution with mean 0.4, then the variance of $(2X-3)$ is
(1) 0.24 2) 0.48 (3) 0.6 (4) 0.96

19. If in 6 trials, X is a binomial variate which follows the relation $9P(X = 4) = P(X = 2)$, then the probability of success is
(1) 0.125 (2) 0.25 (3) 0.375 (4) 0.75

20. A computer salesperson knows from his past experience that he sells computers to one in every twenty customers who enter the showroom. What is the probability that he will sell a computer to exactly two of the next three customers?
(1) $\frac{57}{20^3}$ (2) $\frac{57}{20^2}$ (3) $\frac{19^3}{20^3}$ (4) $\frac{57}{20}$

EXERCISE 12.3

Choose the correct or the most suitable answer from the given four alternatives.

1. A binary operation on a set S is a function from
(1) $S \to S$ (2) $(S \times S) \to S$ (3) $S \to (S \times S)$ (4) $(S \times S) \to (S \times S)$

2. Subtraction is not a binary operation in
(1) $\mathbb{R}$ (2) $\mathbb{Z}$ (3) $\mathbb{N}$ (4) $\mathbb{Q}$

3. Which one of the following is a binary operation on $\mathbb{N}$?
(1) Subtraction (2) Multiplication (3) Division (4) All the above .

4. In the set $\mathbb{R}$ of real numbers '* ' is defined as follows. Which one of the following is not a binary operation on $\mathbb{R}$? (1) $a * b = min(a.b)$ (2) $a * b = max(a, b)$
(3) $a * b = a$ (4) $a * b = a^b$

5. The operation * defined by $a * b = \frac{ab}{7}$ is not a binary operation on
(1) $\mathbb{Q}^+$ (2) $\mathbb{Z}$ (3) $\mathbb{R}$ (4) $\mathbb{C}$

6. In the set $\mathbb{Q}$ define $\odot\ b = a + b + ab$. For what value of y, $3 \odot (y \odot 5) = 7$?
(1) $y = \frac{2}{3}$ (2) $y = -\frac{2}{3}$ (3) $y = -\frac{3}{2}$ (4) $y = 4$

7. If $a * b = \sqrt{a^2 + b^2}$ on the real numbers then * is
(1) commutative but not associative (2) associative but not commutative
(3) both commutative and associative (4) neither commutative nor associative

8. Which one of the following statements has the truth valueT ?
(1) $\sin x$ is an even function.
(2) Every square matrix is non-singular
(3) The product of complex number and its conjugate is purely imaginary
(4) $\sqrt{5}$ is an irrational number

9. Which one of the following statements has truth value F ?
(1) Chennai is in India or $\sqrt{2}$ is an integer
(2) Chennai is in India or $\sqrt{2}$ is an irrational number
(3) Chennai is in China or $\sqrt{2}$ is an integer
(4) Chennai is in China or $\sqrt{2}$ is an irrational number

10. If a compound statement involves 3 simple statements, then the number of rows in the truth table is
(1) 9 (2) 8 (3) 6 (4) 3

11. Which one is the inverse of the statement $(p \vee q) \rightarrow (p \wedge q)$?

(1) $(p \wedge q) \rightarrow (p \vee q)$ (2) $\neg (p \vee q) \rightarrow (p \wedge q)$

(3) $(\neg p \vee \neg q) \rightarrow (\neg p \wedge \neg q)$ (4) $(\neg p \wedge \neg q) \rightarrow (\neg p \vee \neg q)$

12. Which one is the contrapositive of the statement $(p \vee q) \rightarrow r$?

(1) $\neg r \rightarrow (\neg p \wedge \neg q)$ (2) $\neg r \rightarrow (p \vee q)$

(3) $r \rightarrow (p \wedge q)$ (4) $p \rightarrow (q \vee r)$

13. The truth table for $(p \wedge q) \vee \neg q$ is given below

p	q	$(p \wedge q) \vee \neg q$
T	T	a
T	F	b
F	T	c
F	F	d

Which one of the following is true?

(1) T T T T (2) T F T T (3) T T F T (4) T F F F

14. In the last column of the truth table for $\neg (p \vee \neg q)$ the number of final outcomes of the truth value $'F'$ are

(1) 1 (2) 2 (3) 3 (4) 4

15. Which one of the following is incorrect? For any two propositions p and q , we have

(1) $\neg (p \vee q) \equiv \neg p \wedge \neg q$ (2) $\neg (p \wedge q) \equiv \neg p \vee \neg q$

(3) $\neg (p \vee q) \equiv \neg p \vee \neg q$ (4) $\neg (\neg p) \equiv p$

16.

p	q	$(p \wedge q) \vee \neg p$
T	T	a
T	F	b
F	T	c
F	F	d

Which one of the following is correct for the truth value of $(p \wedge q) \vee \neg p$?

(1) T T T T (2) F T T T (3) F F T T (4) T T T F

17. The dual of $\neg (p \vee q) \vee [(p \vee (p \wedge \neg r)]$ is

(1) $\neg (p \wedge q) \vee [(p \vee (p \wedge \neg r)]$ (2) $(p \wedge q) \wedge [(p \wedge (p \vee \neg r)]$

(3) $\neg (p \wedge q) \wedge [(p \wedge (p \wedge \neg r)]$ (4) $\neg (p \wedge q) \wedge [(p \wedge (p \vee \neg r)]$

18. The proposition $p \wedge (\neg p \vee q)$ is

(1) a tautology (2) a contradiction

(3) logically equivalent to $p \wedge q$ (4) logically equivalent to $p \vee q$

19. Determine the truth value of each of the following statements:

(a) $4 + 2 = 5$ *and* $6 + 3 = 9$ (b) $3 + 2 = 5$ *and* $6 + 1 = 7$

(c) $4 + 5 = 9$ *and* $1 + 2 = 4$ (d) $3 + 2 = 5$ *and* $4 + 7 = 11$

(1) F T F T (2) T F T F (3) T T F F (4) F F T T

20. Which one of the following is not true?

(1) Negation of a negation of a statement is the statement itself.

(2) If the last column of the truth table contains only T then it is a tautology.

(3) If the last column of its truth table contains only F then it is a contradiction

(4) If p and q are any two statements then $p \leftrightarrow q$ is a tautology.

MURALIRAJA .M.Sc.,B.Ed.,
DEPARTMENT OF MATHEMATICS
SENTHIL MATRIC HR.SEC. SCHOOL
ADTHIYAMANKOTTAI-DHARMAPURI
CELL NO. 9786760672