HTM
12 - Std

HALF YEARLY EXAMINATION - 2023
MATHEMATICS

Time : 3.00 Hrs   Marks : 90

I Choose the correct answer.   20 X 1 = 20

1. $(AB)^{-1} = \begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$ then B^{-1} =

   a) $\begin{bmatrix} 2 & -5 \\ -3 & 8 \end{bmatrix}$ b) $\begin{bmatrix} 8 & 5 \\ 3 & 2 \end{bmatrix}$ c) $\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 8 & -5 \\ -3 & 2 \end{bmatrix}$

2. $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $\lambda.A^{-1} = A$ then λ is

   a) 17 b) 14 c) 19 d) 21

3. The value of $\sum_{n=1}^{13} \left(i^n + i^{n-1}\right)$ is

   a) 1 + i b) i c) 1 d) 0

4. $|Z| = 1$ then the value of $\frac{1+z}{1-z}$ is a) Z b) $\overline{Z}$ c) $\frac{1}{z}$ d) 1

5. If α, β and γ are the zeros of $x^3 + px^2 + qx + r$, then $\Sigma\frac{1}{\alpha}$ is

   a) $\frac{-q}{r}$ b) $\frac{-p}{r}$ c) $\frac{q}{r}$ d) $\frac{-q}{p}$

6. If $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right)$ is equal to

   a) $\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$ b) $\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)$ c) $\frac{1}{2}\tan^{-1}\left(\frac{3}{5}\right)$ d) $\tan^{-1}\left(\frac{1}{2}\right)$

7. The radius of the circle $3x^2 + by^2 + 4bx - 6by + b^2 = 0$ is

   a) 1 b) 3 c) $\sqrt{10}$ d) $\sqrt{11}$

8. The volume of the parallelepiped with its edges represented by the vectors $\hat{i} + \hat{j}$, $\hat{i} + 2\hat{j}$, $\hat{i} + \hat{j} + \pi\hat{k}$ is

   a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$ c) π d) $\frac{\pi}{4}$

9. The distance between the planes $x + 2y + 3z + 7 = 0$ and $2x + 4y + 6z + 7 = 0$ is a) $\frac{\sqrt{7}}{2\sqrt{2}}$ b) $\frac{7}{2}$ c) $\frac{\sqrt{7}}{2}$ d) $\frac{7}{2\sqrt{2}}$

10. A stone is thrown up vertically. The height it reaches at time t seconds is given by $x = 80t - 16t^2$. The stone reaches the maximum height in time t seconds is given by

a) 2 b) 2.5 c) 3 d) 3.5

11. The maximum value of the function x^2e^{-2x}, $x > 0$ is

a) $\frac{1}{e}$ b) $\frac{1}{2e}$ c) $\frac{1}{e^2}$ d) $\frac{4}{e^4}$

12. If $W(x, y) = x^y$, $x > 0$ then $\frac{\partial w}{\partial x}$ is equal to

a) $x^y \log x$ b) $y \log x$ c) yx^{y-1} d) $x \log y$

13. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x \cos x \, dx$ is

a) $\frac{3}{2}$ b) $\frac{1}{2}$ c) 0 d) $\frac{2}{3}$

14. The value of $\int_0^1 x(1-x)^{99} dx$ is

a) $\frac{1}{11000}$ b) $\frac{1}{10100}$ c) $\frac{1}{10010}$ d) $\frac{1}{10001}$

15. The order and degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{1/3} + x^{1/4} = 0$ are respectively a) 2, 3 b) 3, 3 c) 2, 6 d) 2, 4

16. The solution of the differential equation $\frac{dy}{dx} = 2xy$ is

a) $y = ce^{x^2} + c$ b) $y = 2x^2 + c$ c) $y = ce^{-x^2} + c$ d) $y = x^2 + c$

17. The random variable x has binomial distribution with n = 25 and p = 0.8 then the standard deviation of x is a) 6 b) 4 c) 3 d) 2

18. If $f(x) = \begin{cases} 2x, & 0 \le x \le a \\ 0, & other\ wise \end{cases}$ is a probability density function of a random variable, then the value of a is a) 1 b) 2 c) 3 d) 4

19. If a compound statement involves 3 simple statements then the number of rows in the truth table is

a) 9 b) 8 c) 6 d) 3

20. The operation * defined by $a * b = \frac{ab}{7}$ is not a binary operation on?

a) Q^+ b) Z c) R d) C

SECTION - B

II Answer any 7 questions. Q.No. 30 is compulsory. 7 x 2 = 14

21. If $adj(A) = \begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & 6 \\ -3 & 0 & 6 \end{bmatrix}$ then, find A^{-1}.

22. Obtain the Cartesian form of the locus of $z = x + iy$ in $|z + i| = |z - 1|$.

23. Find the principal value of $\tan^{-1}\left(\sqrt{3}\right)$.

24. Show that the three vectors $2\hat{i} + 3\hat{j} + \hat{k}$, $\hat{i} - 2\hat{j} + 2\hat{k}$ and $3\hat{i} + \hat{j} + 3\hat{k}$ are coplanar.

25. Find a polynomial equation of minimum degree with rational coefficients having $2-\sqrt{3}$ as a root.

26. Solve : $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$.

27. A random variable x has the following probability mass function

x	1	2	3	4	5	6
P (X = x)	k	2k	6k	5k	6k	10k

then find P (2 < x < 6).

28. Evaluate the limit : $\lim_{x\to 0} \frac{\sin mx}{x}$.

29. Find the equation of the hyperbola with vertices $(0, \pm 4)$ and $(0, \pm 6)$.

30. Show that the differential equation corresponding to y = A sin x (where A is an arbitrary constant) is $y = y^1 \tan x$.

SECTION - C

III Answer any 7 questions. Q.No. 40 is compulsory. 7 x 3 = 21

31. If $U = \log(x^3 + y^3 + z^3)$ then find $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$.

32. If $A = \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$ verify that $A(adjA) = (adjA)\,A = |A|\,I$.

33. Prove that the equation of the parabola with focus (4, 0) and directrix x = -4 is $y^2 = 16x$.

34. Which one of the points 10-8i, 11 + 6i is closest to 1 + i ?

35. Evaluate : $\lim_{x\to\infty} \left(\frac{2x^2 - 3}{x^2 - 5x + 3}\right)$.

36. A circular plate expands uniformly under the influence of heat. If its radius increases from 10.50 cm to 10.75cm, then find an approximate change in the area.

37. Show that p->q and q -> p are not equivalent.

38. Find the acute angle between the lines

$$\vec{r} = \left(4\hat{i} - \hat{j}\right) + t\left(\hat{i} + 2\hat{j} - 2\hat{k}\right),\ \vec{r} = \left(\hat{i} - 2\hat{j} + 4\hat{k}\right) + s\left(-\hat{i} - 2\hat{j} + 2\hat{k}\right).$$

39. Find the centre and radius of the circle $x^2 + y^2 + 6x - 4y + 4 = 0$.

40. Show that $\int_0^1 \frac{\sqrt{x}}{\sqrt{1-x}+\sqrt{x}}\,dx = \frac{1}{2}$.

SECTION - D

IV Answer all questions. **7 X 5 = 35**

41. a) solve the system of linear equations by Crammer's rule
$3x + 3y - z = 11$, $2x - y + 2z = 9$, $4x + 3y + 2z = 25$. ***(OR)***
b) A particle is fired straight up from the ground to reach a height of s feet in t seconds where $s(t) = 128t - 16t^2$.
i) Compute the maximum height of the particle reached.
ii) What is the velocity when the particle hits the ground?

42. a) Show that $\left(2+i\sqrt{3}\right)^{10} - \left(2-i\sqrt{3}\right)^{10}$ is purely imaginary. ***(OR)***
b) Find the area of the region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ using integration.

43. a) Find the parametric form of vector equation and Cartesian equation of the plane containing the line $\vec{r}=(\hat{i}-\hat{j}+3\hat{k})+t(2\hat{i}-\hat{j}+4\hat{k})$ and perpendicular to the plane $\vec{r}.(\hat{i}+2\hat{j}+\hat{k})=8$. **(OR)** b) Solve the equation
$6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$ if it is known that $\frac{1}{3}$ is a solution.

44. a) show that $\int_0^1 (\tan^{-1} x+\tan^{-1}(1-x))\,dx=\frac{\pi}{2}-\log 2$. ***(OR)*** b) Find the area of the region bounded by the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ and its latus rectum.

45. a) On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4m when it is 6m away from the point of projection. Finally it reaches the ground 12m away from the starting point. Show that the angle of projection is $\tan^{-1}\left(\frac{4}{3}\right)$. ***(OR)*** b) A random variable x has the following probability mass function.

x	1	2	3	4	5
F(x)	k^2	$2k^2$	$3k^2$	2k	3k

Find (i) the value of k (ii) P $(2 \le x \le 5)$ (iii) P $(3 < x)$

46. a) Using vector method, prove that cos (A - B) = Cos A cos B + sin A Sin B.
(OR)
b) Find two positive numbers whose product is 20 and their sum is minimum.

47. a) Solve the differential equation $\frac{dy}{dx}+\frac{y}{x}=\sin x$. ***(OR)***
b) Verify whether the following compound proposition is tautology or contradiction or contingency : $(p\to q) \leftrightarrow (\neg p\to q)$.

1	(a) $\begin{bmatrix} 2 & -5 \\ -3 & 8 \end{bmatrix}$
2	(c) 19
3	(a) 1+ *i*
4	(a) z
5	(a) -q/*r*
6	(d) $\tan^{-1}\left(\frac{1}{2}\right)$
7	(c) √10
8	(c) π
9	(a) √7 / 2√2
10	(b) 2.5
11	(c) $1/e^2$
12	(c) yx^{y-1}
13	(d) 2/3
14	(b) 1/10100
15	(a) 2, 3
16	(a)$y = Cex^2 + c$
17	(d) 2
18	(a) 1
19	(b) 8
20	(b)$\mathbb{Z}$
21	Given adj A $= \begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$ $\lvert \text{adj A} \rvert = \begin{vmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{vmatrix}$ $\lvert \text{adj A} \rvert = 0(12-0) + 2(36-18) + 0(0+6) = 36$ $A^{-1} = \pm \frac{1}{\sqrt{\lvert \text{adj A} \rvert}} (\text{adj A})$ $= \pm \frac{1}{\sqrt{36}} \begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$ $A^{-1} = \pm \frac{1}{6} \begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$

22	$z = x + iy$ $\|z + i\| = \|z - 1\|$ $\|x + iy + i\| = \|x + iy - 1\|$ $\|x + i(y+1)\| = \|x - 1 + iy\|$ $\sqrt{x^2 + (y+1)^2} = \sqrt{(x-1)^2 + y^2}$ Squaring on both sides $x^2 + (y+1)^2 = (x-1)^2 + y^2$ $x^2 + y^2 + 2y + 1 = x^2 + 2x + 1 + y^2$ $2y = -2x$ $2x + 2y = 0$ $2(x+y) = 0 \Rightarrow x + y = 0$
23	Let $\tan^{-1}(\sqrt{3}) = y$. Then, $\tan y = \sqrt{3}$. Thus, $y = \pi/3$. Since $\pi/3 \in (-\pi/2, \pi/2)$. Thus, the principal value of $\tan^{-1}(\sqrt{3})$ is $\pi/3$.
24	Here, $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}, \vec{b} = 2\hat{i} - \hat{j} + 2\hat{k}, \vec{c} = 3\hat{i} + \hat{j} - \hat{k}$ We know that $\vec{a}, \vec{b}, \vec{c}$ are coplanar if and only if $[\vec{a}, \vec{b}, \vec{c}] = 0$. Now, $[\vec{a}, \vec{b}, \vec{c}] = \begin{vmatrix} 1 & 2 & -3 \\ 2 & -1 & 2 \\ 3 & 1 & -1 \end{vmatrix} = 0$.
25	Since $2 - \sqrt{3}$ is a root and the coefficients are rational numbers, 2 + is also a root. A required polynomial equation is given by x^2 - (Sum of the roots) x + Product of the roots = 0 and hence $x^2 - 4x + 1 = 0$ is a required equation.
26	(i) $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$ The equation can be written as $\frac{dy}{\sqrt{1-y^2}} = \frac{dx}{\sqrt{1-x^2}}$ Taking Integration on both sides, we get $\int \frac{dy}{\sqrt{1-y^2}} = \int \frac{dx}{\sqrt{1-x^2}}$ $\sin^{-1} y = \sin^{-1} x + C$
27	Since the given function is a probability mass function, the total probability is one. That is $\sum_x f(x) = 1$. From the given data $k + 2k + 6k + 5k + 6k + 10k = 1$ $30k = 1$

$\Rightarrow k = 1/30$

Therefore the probability mass function is

x	1	2	3	4	5	6
$f(x)$	$\frac{1}{30}$	$\frac{2}{30}$	$\frac{6}{30}$	$\frac{5}{30}$	$\frac{6}{30}$	$\frac{10}{30}$

$$P(2 < X < 6) = f(3) + f(4) + f(5) = \frac{6}{30} + \frac{5}{30} + \frac{6}{30} = \frac{17}{30}$$

28 If we directly substitute $x = 0$ we get an indeterminate form 0/0 and hence we apply the l'Hôpital's rule to evaluate the limit as,

$$\lim_{x\to 0}\left(\frac{\sin mx}{x}\right) = \lim_{x\to 0}\left(\frac{m \times \cos mx}{1}\right)$$

$$= m$$

The next example tells that the limit does not exist.

29 From Fig. 5.38, the midpoint of line joining foci is the centre C(0,0).

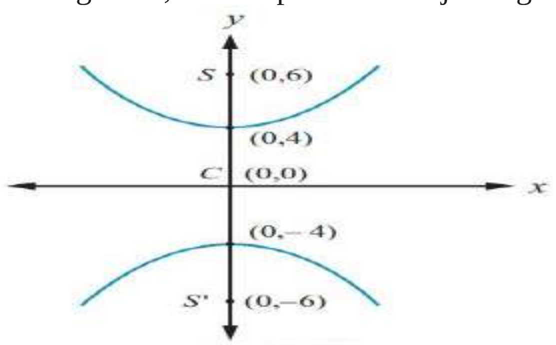


Fig.5.38

Transverse axis is y -axis

$AA' = 2a \Rightarrow 2a = 8$,

$SS' = 2c = 12$, $c = 6$

$a = 4$

$b^2 = c^2 - a^2 = 36 - 16 = 20$.

Hence the equation of the required hyperbola is = $\frac{y^2}{16} - \frac{x^2}{20} = 1.$

30 y =A sin x

y' =A cos x

y/y' = tan x

y = y' tan x

31 $U(x, y, z) = \log(x^3 + y^3 + z^3)$

$$\frac{\partial U}{\partial x} = \frac{3x^2}{x^3 + y^3 + z^3}$$

$$\frac{\partial U}{\partial y} = \frac{3y^2}{x^3 + y^3 + z^3}$$

$$\frac{\partial U}{\partial z} = \frac{3z^2}{x^3 + y^3 + z^3}$$

$$\frac{\partial U}{\partial x} + \frac{\partial U}{\partial y} + \frac{\partial U}{\partial z} = \frac{3(x^2 + y^2 + z^2)}{x^3 + y^3 + z^3}$$

32

$$A = \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 8 & -4 \\ -5 & 3 \end{vmatrix} = 24 - 20 = 4$$

$$\text{adj } A = \begin{bmatrix} +M_{11} & -M_{12} \\ -M_{21} & +M_{22} \end{bmatrix}^T$$

$$\text{adj } A = \begin{bmatrix} 3 & 5 \\ 4 & 8 \end{bmatrix}^T$$

$$\text{adj } A = \begin{bmatrix} 3 & 4 \\ 5 & 8 \end{bmatrix}$$

$$A(\text{adj } A) = \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix} \begin{bmatrix} 3 & 4 \\ 5 & 8 \end{bmatrix}$$

$$A(\text{adj } A) = \begin{bmatrix} 24 - 20 & 32 - 32 \\ -15 + 15 & -20 + 24 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$$(\text{adj } A)\, A = \begin{bmatrix} 3 & 4 \\ 5 & 8 \end{bmatrix} \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$$

$$(\text{adj } A)\, A = \begin{bmatrix} 24 - 20 & -12 + 12 \\ 40 - 40 & -20 + 24 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$$A(\text{adj } A) = (\text{adj } A)\, A = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

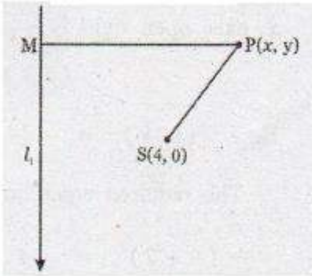
$$= 4 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{-----}(1)$$

$$|A| = \begin{vmatrix} 8 & -4 \\ -5 & 3 \end{vmatrix}$$

$$= 24 - 20 = 4$$

Equation (1) $\Rightarrow$

$$A(\text{adj } A) = (\text{adj } A)\, A = |A|\, I_2.$$

33	(i) focus (4, 0) and directrix $x = -4$ Let l be the straight line $x = -4$ and S(4, 0) be the fixed point. Let P(x, y) be a moving point such $\frac{SP}{PM} = 1 \Rightarrow SP^2 = PM^2$  SP - Distance between the points S(4, 0) and P(x, y) PM - Length of the perpendicular from P(x, y) to the line l whose equation is $x + 4 = 0$ $SP^2 = PM^2$ $(x-4)^2 + (y-0)^2 = \left(\left\lvert \frac{x+4}{\sqrt{1^2+0^2}} \right\rvert\right)^2$ $(x-4)^2 + y^2 = (x+4)^2$ $x^2 - 8x + 16 + y^2 = x^2 + 8x + 16$ $y^2 = 8x + 8x$ $y^2 = 16x$ which is the required equation of the parabola.
34	Let $z_1 = 10 - 8i$, $z_2 = 11 + 6i$, $z = 1 + i$ Distance between z and z_1 is $\lvert z - z_1 \rvert = \lvert (1+i) - (10-8i) \rvert$ $= \lvert 1 + i - 10 - 8i \rvert$ $= \lvert -9 + 9i \rvert$ $= \sqrt{(-9)^2 + 9^2}$ $= \sqrt{81 + 81} = \sqrt{2 \times 81}$ $= \sqrt{2 \times 9^2} = 9\sqrt{2}$ $\lvert z - z_1 \rvert = 9\sqrt{2} = 12.72$ Distance between z and z_2 is $\lvert z - z_2 \rvert = \lvert (1+i) - (11+6i) \rvert$ $= \lvert 1 + i - 11 - 6i \rvert$ $= \lvert -10 - 5i \rvert$ $= \sqrt{(-10)^2 + (-5)^2}$ $= \sqrt{100 + 25}$ $= \sqrt{125}$ $= \sqrt{5 \times 25} = 5\sqrt{5}$ $\lvert z - z_2 \rvert = 5\sqrt{5} = 11.18$ $\therefore \lvert z - z_1 \rvert > \lvert z - z_2 \rvert$ Hence the point $z_2 = 11 + 6i$ is closest to $z = 1 + i$.

35

$$\lim_{x\to\infty}\frac{x}{\log x}\quad\left[\frac{\infty}{\infty}\ \textit{Indeterminate form}\right.$$

Applying L' *Hôpital's* rule,

$$=\lim_{x\to\infty}\frac{1}{\frac{1}{x}}=\lim_{x\to\infty}x=\infty$$

36

Area of the circular plate $A = \pi r^2$

$$= \pi \times 10.5 \times 105$$
$$= 110.25\pi$$
$$dA = 2\pi r dr$$
$$= 2\pi \times 10.5 \times 0.25$$
$$= 5.25\pi$$

Approximate percentage change in the area

$$= \frac{dA}{A} \times 100$$
$$= \frac{5.25\pi}{110.25\pi} \times 100$$
$$= 0.04761 \times 100$$
$$= 4.76\ \%$$

37

p	q	$p\to q$	$q\to p$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

From the table, it is clear that

$$p\to q \not\equiv q\to p$$

38

(i) $\vec{r} = (4\hat{i}-\hat{j})+t(\hat{i}+2\hat{j}-2\hat{k})$,

$$\vec{r} = (\hat{i}-2\hat{j}+4\hat{k})+s(-\hat{i}-2\hat{j}+2\hat{k})$$
$$\vec{b} = \hat{i}+2\hat{j}-2\hat{k}$$
$$\vec{d} = -\hat{i}-2\hat{j}+2\hat{k}$$
$$\vec{b}\cdot\vec{d} = 1(-1)+2(-2)-2(2)$$
$$= -1-4-4=-9$$
$$|\vec{b}\cdot\vec{d}| = 9$$
$$|\vec{b}| = \sqrt{1+4+4} = \sqrt{9} = 3$$
$$|\vec{d}| = \sqrt{1+4+4} = \sqrt{9} = 3$$
$$\cos\theta = \frac{|\vec{b}\cdot\vec{d}|}{|\vec{b}||\vec{d}|} = \frac{9}{3\times 3} = 1$$
$$\cos\theta = 1$$
$$\theta = 0^\circ$$

39

$x^2 + y^2 + 6x - 4y + 4 = 0$

Centre is $(-g, -f) = (-3, 2)$

Radius $= \sqrt{g^2+f^2-c} = \sqrt{(3)^2+(-2)^2-4} = \sqrt{9+4-4} = \sqrt{9} = 3$

40

Let $I = \int_0^1 \frac{\sqrt{x}}{\sqrt{1-x}+\sqrt{x}}\, dx \longrightarrow (1)$

Applying the formula $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

$$I = \int_0^1 \frac{\sqrt{1+0-x}}{\sqrt{1-(1+0-x)}+\sqrt{1+0-x}}\, dx$$

$$= \int_0^1 \frac{\sqrt{1-x}}{\sqrt{1-1-0+x}+\sqrt{1-x}}\, dx$$

$$I = \int_0^1 \frac{\sqrt{1-x}}{\sqrt{x}+\sqrt{1-x}}\, dx \longrightarrow (2)$$

Add equ (1) & (2)

$$2I = \int_0^1 \frac{\sqrt{x}+\sqrt{1-x}}{\sqrt{1-x}+\sqrt{x}}\, dx$$

$$= \int_0^1 dx$$

$$= [x]_0^1$$

$$= 1 - 0$$

$$2I = 1$$

$$I = \frac{1}{2}.$$

41 (a)

The given equations are

$3x + 3y - z = 11$ -----(1)

$2x - y + 2z = 9$ -----(2)

$4x + 3y + 2z = 25$ -----(3)

$$\Delta = \begin{vmatrix} 3 & 3 & -1 \\ 2 & -1 & 2 \\ 4 & 3 & 2 \end{vmatrix}$$

$= 3(-2-6) - 3(4-8) - 1(6+4)$

$= 3 \times -8 - 3 \times -4 - 1 \times 10$

$\Delta = -24 + 12 - 10 = -22$

$$\Delta_1 = \begin{vmatrix} 11 & 3 & -1 \\ 9 & -1 & 2 \\ 25 & 3 & 2 \end{vmatrix}$$

$= 11(-2-6) - 3(18-50) - 1(27+25)$

$= -88 + 96 - 52 = 96 - 140$

$\Delta_1 = -44$

$$\Delta_2 = \begin{vmatrix} 3 & 11 & -1 \\ 2 & 9 & 2 \\ 4 & 25 & 2 \end{vmatrix}$$

$= 3(18-50) - 11(4-8) - 1(50-36)$

$= 3 \times -32 - 11 \times -4 - 1 \times 14$

$= -96 + 44 - 14$

$\Delta_2 = -66$

$$\Delta_3 = \begin{vmatrix} 3 & 3 & 11 \\ 2 & -1 & 9 \\ 4 & 3 & 25 \end{vmatrix}$$

$$= 3[(-25-27) - 3(50-36) + 11(6+4)]$$

$$= 3 \times -52 - 3 \times 14 + 11 \times 10$$

$$\Delta_3 = -156 - 42 + 110 = -88$$

$$x = \frac{\Delta_1}{\Delta} = \frac{-44}{-22} = 2$$

$$y = \frac{\Delta_2}{\Delta} = \frac{-66}{-22} = 3$$

$$z = \frac{-88}{-22} = 4$$

$\therefore$ The solution is $x = 2,\ y = 3,\ z = 4$.

(or)

(b) (i) At the maximum height, the velocity $v(t)$ of the particle is zero.
Now, we find the velocity of the particle at time t.
$v(t) = ds/dt = 128 - 32t$
$v(t) = 0 \Rightarrow 128 - 32t = 0 \Rightarrow t = 4$.
After 4 seconds, the particle reaches the maximum height.
The height at $t = 4$ is $s(4) = 128(4) - 16(4)^2 = 256$ ft.
(ii) When the particle hits the ground then $s = 0$.
$s = 0 \Rightarrow 128t - 16t^2 = 0$
$\Rightarrow t = 0, 8$ seconds.
The particle hits the ground at $t = 8$ seconds.
The velocity when it hits the ground is $v(8) = -128$ ft/s.

42 (a)

Let $z = (2+i\sqrt{3})^{10} - (2-i\sqrt{3})^{10}$

$$\bar{z} = \overline{(2+i\sqrt{3})^{10} - (2-i\sqrt{3})^{10}}$$

$$= \overline{(2+i\sqrt{3})^{10}} - \overline{(2-i\sqrt{3})^{10}}$$

$$= \left[\overline{(2+i\sqrt{3})}\right]^{10} - \left[\overline{(2-i\sqrt{3})}\right]^{10}$$

$$= (2-i\sqrt{3})^{10} - (2+i\sqrt{3})^{10}$$

$$= -\left[(2+i\sqrt{3})^{10} - (2-i\sqrt{3})^{10}\right]$$

$$\bar{z} = -z \Rightarrow z = -\bar{z}$$

z is purely imaginary

$\therefore z = (2+i\sqrt{3})^{10} - (2-i\sqrt{3})^{10}$ is purely imaginary.

(or)

(b) The ellipse is symmetric about both major and minor axes. It is sketched as in Fig.9.16. So, viewing in the positive direction of y -axis, the required area A is four times the area of the region bounded by the portion of the ellipse in the first quadrant

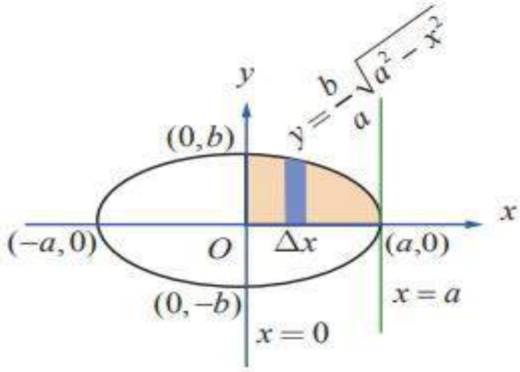


Fig. 9.16

$\left(y=\frac{b}{a}\sqrt{a^2-x^2}, 0<x<a\right)$, x-axis, $x=0$ and $x=a$.

Hence, by taking vertical strips, we get

$$A = 4\int_0^a y\,dx = 4\int_0^a \frac{b}{a}\sqrt{a^2-x^2}\,dx$$

$$= \frac{4b}{a}\left[\frac{x\sqrt{a^2-x^2}}{2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]_0^a = \frac{4b}{a}\times\frac{\pi a^2}{4} = \pi ab$$

43 (a)

$\vec{a} = \hat{i}-\hat{j}+3\hat{k}$, $\vec{b} = 2\hat{i}-\hat{j}+4\hat{k}$, $\vec{c} = \hat{i}+2\hat{j}+\hat{k}$

Parametric form of vector equation

$$\vec{r} = \vec{a}+s\vec{b}+t\vec{c}$$

$$\vec{r} = \left(\hat{i}-\hat{j}+3\hat{k}\right)+s\left(2\hat{i}-\hat{i}+4\hat{k}\right)+t\left(\hat{i}+2\hat{j}+\hat{k}\right)$$

$$\vec{b}\times\vec{c} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -1 & 4\\ 1 & 2 & 1\end{vmatrix} = \hat{i}(-1-8)-\hat{j}(2-4)+\hat{k}(4+1)$$

$$= -9\hat{i}+2\hat{j}+5\hat{k}$$

$$= -\left(9\hat{i}-2\hat{j}-5\hat{k}\right)$$

Non-parametric form of Vector equation:

$$(\vec{r}-\vec{a})\cdot(\vec{b}\times\vec{c}) = \vec{0}$$

$$\left[\vec{r}-\left(\hat{i}-\hat{j}+3\hat{k}\right)\right]\cdot\left(-\left(9\hat{i}-2\hat{j}-5\hat{k}\right)\right) = \vec{0}$$

$$\vec{r}\cdot\left(+9\hat{i}-2\hat{j}-5\hat{k}\right) = +9+2-15$$

$$\left[\vec{r}\cdot\left(9\hat{i}-2\hat{j}-5\hat{k}\right)\right] = -4$$

Cartesian equation

$$9x-2y-5z = -4$$

$$9x-2y-5z+4 = 0$$

(or)

(b) The equation of the given polynomial is

$6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$ -----(1)

Given that $\frac{1}{3}$ is a solution of (1).

$\therefore$ $x - \frac{1}{3}$ is a factor of (1)

ie. $3x - 1$ is a factor of (1)

To find the other roots of (1),

divide equation (1) by $3x - 1$.

$$
\begin{array}{r|l}
 & 2x^3 - x^2 - 13x - 6 \\
\hline
3x - 1 & 6x^4 - 5x^3 - 38x^2 - 5x + 6 \\
 & 6x^4 - 2x^3 \\
\hline
 & -3x^3 - 38x^2 \\
 & -3x^3 + x^2 \\
\hline
 & -39x^2 - 5x \\
 & -39x^2 + 13x \\
\hline
 & -18x + 6 \\
 & -18x + 6 \\
\hline
 & 0
\end{array}
$$

To find the other roots we will solve the equation.

$P(x) = 2x^3 - x^2 - 13x - 6 = 0$

By comparing the cofficients,

we see that $1, -1$ are not the roots of $P(x)$.

$P(2) = 2 \times 2^3 - 2^2 - 13 \times 2 - 6$

$= 2 \times 8 - 4 - 26 - 6$

$= 16 - 36 \neq 0$

$P(-2) = 2(-2)^3 - (-2)^2 - 13 \times -2 - 6$

$= 2 \times -8 - 4 + 26 - 6$

$= -16 - 4 + 26 - 6$

$= -26 + 26 = 0$

$\therefore$ -2 is a root of $P(x)$

Hence $x + 2$ is a factor of $P(x)$.

To find the remaining roots divide $P(x)$ by $x + 2$.

$$
\begin{array}{r|l}
 & 2x^2 - 5x - 3 \\
\hline
x+2 & 2x^3 - x^2 - 13x - 6 \\
 & 2x^3 + 4x^2 \\
\hline
 & \quad -5x^2 - 13x \\
 & \quad -5x^2 - 10x \\
\hline
 & \qquad -3x - 6 \\
 & \qquad -3x - 6 \\
\hline
 & \qquad\quad 0
\end{array}
$$

The remaining roots are obtained by solving the equation.

$$2x^2 - 5x - 3 = 0$$

$$2x^2 - 6x + x - 3 = 0$$

$$2x(x-3) + 1(x-3) = 0$$

$$(2x+1)(x-3) = 0$$

$$2x + 1 = 0 \quad \text{or} \quad x - 3 = 0$$

$$x = -\frac{1}{2} \quad \text{or} \quad x = 3$$

$\therefore$ The required solution are $\frac{1}{3}, -2, -\frac{1}{2}, 3$

44 (a)

$$I = \int_0^1 \left(\tan^{-1} x + \tan^{-1}(1-x)\right) dx$$

$$= \int_0^1 \tan^{-1} x \, dx + \int_0^1 \tan^{-1}(1-x)\, dx$$

$$= \int_0^1 \tan^{-1} x \, dx + \int_0^1 \tan^{-1}(1-(1-x))\, dx, \text{ since } \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$

$$= \int_0^1 \tan^{-1} x \, dx + \int_0^1 \tan^{-1} x \, dx$$

$$= 2\int_0^1 \tan^{-1} x \, dx$$

$$= \left[2\int u dv\right]_0^1, \text{ where } u = \tan^{-1} x \text{ and } dv = dx$$

$$= 2\left[uv - \int v du\right]_0^1, \text{ applying integration by parts}$$

$$= 2\left(x \tan^{-1} x - \int x \frac{dx}{1+x^2}\right)_0^1 = 2\left(x\tan^{-1} x - \frac{1}{2}\log\left(1+x^2\right)\right)_0^1 = \frac{\pi}{2} - \log 2$$

(or)

(b) The ellipse is symmetric about both major and minor axes. It is sketched as in Fig.9.16. So, viewing in the positive direction of y-axis, the required area A is four times the area of the region bounded by the portion of the ellipse in the first quadrant

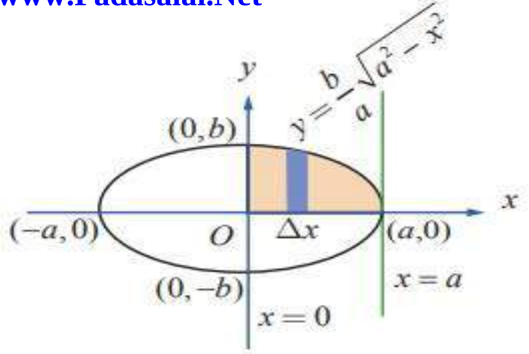


Fig. 9.16

$\left(y=\frac{b}{a}\sqrt{a^2-x^2}, 0<x<a\right)$, x-axis, $x=0$ and $x=a$.

Hence, by taking vertical strips, we get

$$A = 4\int_0^a y\,dx = 4\int_0^a \frac{b}{a}\sqrt{a^2-x^2}\,dx$$

$$= \frac{4b}{a}\left[\frac{x\sqrt{a^2-x^2}}{2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]_0^a = \frac{4b}{a}\times\frac{\pi a^2}{4} = \pi ab$$

Let $a^2 = 25$; $b^2 = 16$

$a = 5$; $b = 4$

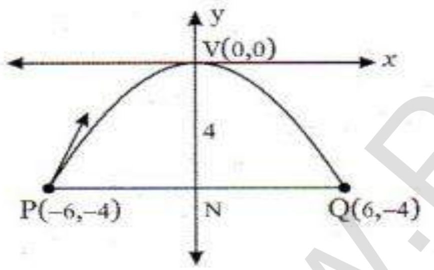
$A = \pi(5)(4) = 20\pi$

45 (a)

Given the rocket cracker is projected in a parabolic path.

It reaches a maximum height of 4 m when it is 6 m away from the point of projection.

Finally it reaches the ground 12 m away from the starting point.



Let P be the starting point and Q be the final point at which it reaches the ground.

By the given data PQ = 12 m

Choose the vertex of the parabolic path be the origin and axis along y axis.

The parabola is open downward.

∴ The coordinates of P , Q are P (−6 , −4) and Q (6 , −4).

VN = the maximum height = 4 m

The equation of the parabola is $x^2 = -4ay$ ----- (1)

It passes through the point (6 , −4)

∴ $6^2 = -4a(-4) \Rightarrow 36 = 16a \Rightarrow 4a = \frac{36}{4} = 9$

Substituting 4a = 9 in the equation of the parabola we get

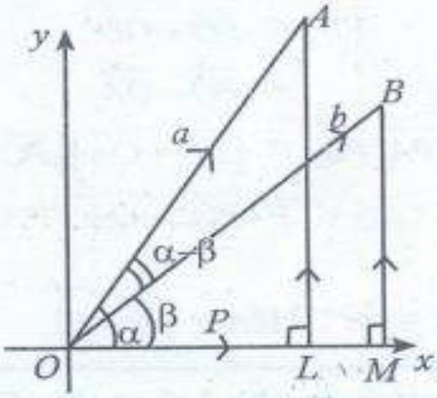
(1) ⇒ $x^2 = -9y$ ----- (2)

		To find the Slope at (– 6 , – 4) Differentiating equation (1) with respect to x we get $2x = -9 \cdot \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = -\frac{2x}{9}$ At $(-6,-4)$, $\frac{dy}{dx} = \frac{-2 \times -6}{9} = \frac{12}{9} = \frac{4}{3}$ $\tan\theta = \frac{4}{3} \Rightarrow \theta = \tan^{-1}\left(\frac{4}{3}\right)$ $\therefore$ The angle of projection is $\tan^{-1}\left(\frac{4}{3}\right)$
	(or)	
	(b)	**(i)** Given f(x) in a probability mass function $\sum_x f(x) = 1$ $k^2 + 2k^2 + 3k^2 + 2k + 3k = 1$ $6k^2 + 5k = 1$ $6k^2 + 5k - 1 = 0$ $(k + 1)(6k - 1) = 0$ $k = \frac{1}{6}$ ($k \neq -1$ neglecting negative terms) Probability mass function x: 1, 2, 3, 4, 5 f(x): $\frac{1}{36}$, $\frac{2}{36}$, $\frac{3}{36}$, $\frac{2}{6}$, $\frac{3}{6}$ **(ii)** $P(2 \leq X < 5)$ $P(2 \leq X < 5) = P(X = 2) + P(X = 3) + P(X = 4)$ $= \frac{2}{36} + \frac{3}{36} + \frac{2}{6}$ $= \frac{2+3+12}{36} = \frac{17}{36}$ **(iii)** $P(X > 3) = P(X = 4) + P(X = 5)$ $= \frac{2}{6} + \frac{3}{6}$ $= \frac{5}{6}$

46 (a)

Solution:

Let $\vec{a} = \overrightarrow{OA}$, $\vec{b} = \overrightarrow{OB}$ be the unit vectors and which make angles α and β respectively with positive x-axis where A and B are as in diagram.



Draw AL and BM perpendicular to the X axis, then

$$\overline{OL} = \overline{OA}\cos\alpha$$
$$|\overline{OL}| = |\overline{OA}|\cos\alpha = \cos\alpha$$
$$|\overline{LA}| = |\overline{OA}|\sin\alpha = \sin\alpha$$
$$\overrightarrow{OL} = |\overline{OL}|\hat{i} = \cos\alpha\ \hat{i}$$
$$\overrightarrow{LA} = \sin\alpha\ (+\hat{j})$$
$$\vec{a} = \overrightarrow{OA} = \overrightarrow{OL} + \overrightarrow{LA}$$
$$= \cos\alpha\ \hat{i} + \sin\alpha\ \hat{j} \quad \ldots(1)$$

Similarly $\vec{b} = \cos\beta\ \hat{i} + \sin\beta\ \hat{j} \quad \ldots(2)$

The angle between $\vec{a}$ and $\vec{b}$ is $\alpha - \beta$ and so

$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos(\alpha - \beta) = \cos(\alpha - \beta) \quad \ldots(3)$

From (1) and (2)

$\vec{a}\cdot\vec{b} = (\cos\alpha\hat{i} + \sin\alpha\hat{j}).(\cos\beta\hat{i} + \sin\beta\hat{j})$

$= \cos\alpha\cos\beta + \sin\alpha\sin\beta \quad \ldots(4)$

From (3) and (4)

$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

Put α = A ; β= B

$\cos(A - B) = \cos A \cos B + \sin A \sin B$

(or)

	(b)	The given differential equation can be written as $\frac{dy}{dx} + \left(\frac{1}{x}\right) y = \sin x$ This is of the form $\frac{dy}{dx} + Py = Q$ where $P = \frac{1}{x}$ $Q = \sin x$ Thus, the given differential equation is linear. $I.F = e^{\int Pdx} = e^{\int \frac{1}{x} dx} = e^{\log x} = x$ So, the required solution is given by $y \times I.F = \int (Q \times I.F)\ dx + c$ $yx = \int \sin x \times x\, dx + c$ $= x(-\cos x) - (1)(-\sin x) + c$ $yx = -x\cos x + \sin x + c$ $yx + x\cos x = \sin x + c$ $(y + \cos x)x = \sin x + c$ is a required solution.
47	(a)	The given differential equation can be written as $\frac{dy}{dx} + \left(\frac{1}{x}\right) y = \sin x$ This is of the form $\frac{dy}{dx} + Py = Q$ where $P = \frac{1}{x}$ $Q = \sin x$ Thus, the given differential equation is linear. $I.F = e^{\int Pdx} = e^{\int \frac{1}{x} dx} = e^{\log x} = x$ So, the required solution is given by $y \times I.F = \int (Q \times I.F)\ dx + c$ $yx = \int \sin x \times x\, dx + c$ $= x(-\cos x) - (1)(-\sin x) + c$ $yx = -x\cos x + \sin x + c$ $yx + x\cos x = \sin x + c$ $(y + \cos x)x = \sin x + c$ is a required solution.
		(or)
	(b)	**(iii)** $(p \to q) \leftrightarrow (\neg p \to q)$

p	q	$\neg p$	$p \to q$	$\neg p \to q$	$(p \to q) \leftrightarrow (\neg p \to q)$
T	T	F	T	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	T	F	F

The entries in the last column are a combination of T and F.

∴ The given statement is a contingency.

- E. VIKRAM, M.Sc., B. Ed(Maths),
PG Assistant (SMC),
Govt. Girls Hr. Sec. School,
Periya Kozhapallur,
Peranamallur post,
TVM District.