IMPORTANT 2 MARK QUESTIONS

1.APPLICATIONS OF MATRICES AND DETERMINANTS

1. Find the adjoint of the following:

   (i) $\begin{bmatrix} -3 & 4 \\ 6 & 2 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$ (iii) $\frac{1}{3}\begin{bmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}$
2. Find the inverse (if it exists) of the following:

   (i) $\begin{bmatrix} -2 & 4 \\ 1 & -3 \end{bmatrix}$ (ii) $\begin{bmatrix} 5 & 1 & 1 \\ 1 & 5 & 1 \\ 1 & 1 & 5 \end{bmatrix}$ (iii) $\begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$
3. If $A = \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$, verify that $A(\text{adj } A) = (\text{adj } A)A = |A|I_2$.
4. If $\text{adj }(A) = \begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$, find A^{-1}.
5. Find the matrix A for which $A\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix}$.
6. Find the rank of the following matrices by minor method:(i) $\begin{bmatrix} 2 & -4 \\ -1 & 2 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & -2 & -1 & 0 \\ 3 & -6 & -3 & 1 \end{bmatrix}$
7. Solve the following system of linear equations by matrix inversion method:(i) $2x + 5y = -2, x + 2y = -3$.
8. Solve the following systems of linear equations by Cramer's rule:

   (i) $5x - 2y + 16 = 0, x + 3y - 7 = 0$ (ii) $\frac{3}{x} + 2y = 12,\ \frac{2}{x} + 3y = 13$
9. If $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$, verify that $A(\text{adj } A) = (\text{adj } A)A = |A|I_3$.
10. If A is a non-singular matrix of odd order, prove that $|\text{adj } A|$ is positive.
11. If A is symmetric, prove that adj A is also symmetric.
12. Prove that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal.
13. Reduce the matrix $\begin{bmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{bmatrix}$ to a row-echelon form.
14. Solve the following system of linear equations, using matrix inversion method: $5x + 2y = 3, 3x + 2y = 5$.
15. Solve, by Cramer's rule, the system of equations $x_1 - x_2 = 3, 2x_1 + 3x_2 + 4x_3 = 17,\ x_2 + 2x_3 = 7$.

2.COMPLEX NUMBERS

1. Simplify the following: 1. $i^{59} + \frac{1}{i^{59}}$ 2. $\sum_{n=1}^{10} i^{n+50}$
2. Simplify the following (i) i^7 (ii) i^{1729} (iii) $i^{-1924} + i^{2018}$ (iv) $\sum_{n=1}^{102} i^n$ (v) $ii^2i^3 \cdots i^{40}$
3. Evaluate the following if $z = 5 - 2i$ and $w = -1 + 3i$ (i) $z^2 + 2zw + w^2$.
4. If $z_1 = 3,\ z_2 = -7i$, and $z_3 = 5 + 4i$, show that (i) $z_1(z_2 + z_3) = z_1z_2 + z_1z_3$ (ii) $(z_1 + z_2)z_3 = z_1z_3 + z_2z_3$.
5. If $z_1 = 2 + 5i,\ z_2 = -3 - 4i$, and $z_3 = 1 + i$, find the additive and multiplicative inverse of $Z_1,\ Z_2$, and Z_3.
6. Simplify $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3$. into rectangular form.
7. If $\frac{z+3}{z-5i} = \frac{1+4i}{2}$, find the complex number z in the rectangular form.
8. If $z = x + iy$, find the following in rectangular form. i) Re $(i\bar{z})$ (ii) Im $(3z + 4\bar{z} - 4i)$
9. If $z_1 = 2 - i$ and $z_2 = -4 + 3i$, find the inverse of z_1z_2 and $\frac{z_1}{z_2}$.
10. Prove the following properties: (i) z is real if and only if $z = \bar{z}$ (ii) Re $(z) = \frac{z+\bar{z}}{2}$ and Im $(z) = \frac{z-\bar{z}}{2i}$
11. Find the following (i) $\left|\frac{2+i}{-1+2i}\right|$ (ii) $\left|\overline{(1+i)}(2+3i)(4i-3)\right|$ (iii) $\left|\frac{i(2+i)^3}{(1+i)^2}\right|$
12. Find the square root of $6 - 8i$.
13. Find the modulus of the complex numbers(i) $\frac{2i}{3+4i}$ (ii) $\frac{2-i}{1+i} + \frac{1-2i}{1-i}$ (iii) $(1-i)^{10}$ (iv) $2i(3-4i)(4-3i)$.
14. Which one of the points $10 - 8i, 11 + 6i$ is closest to $1 + i$.

15. If $|z| = 3$, show that $7 \le |z + 6 - 8i| \le 13$.
16. If $|z| = 1$, show that $2 \le |z^2 - 3| \le 4$.
17. Find the square roots of (i) $4 + 3i$ (ii) $-6 + 8i$ (iii) $-5 - 12i$.
18. Show that $|3z - 5 + i| = 4$ represents a circle, and, find its centre and radius.
19. Obtain the Cartesian form of the locus of $z = x + iy$ in each of the following cases: (i) $[\text{Re}\,(iz)]^2 = 3$.
20. Obtain the Cartesian equation for the locus of $z = x + iy$ in each of the following cases: (i) $|z - 4| = 16$.
21. Write in polar form of the following complex numbers (i) $2 + i2\sqrt{3}$ (ii) $3 - i\sqrt{3}$ (iii) $-2 - i2$
22. If $\omega \ne 1$ is a cube root of unity, show that $\frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2} + \frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2} = -1$.

3.THEORY OF EQUATIONS

1. If α and β are the roots of the quadratic equation $2x^2 - 7x + 13 = 0$, construct a quadratic equation whose roots are α^2 and β^2.
2. Find the sum of the squares of the roots of $ax^4 + bx^3 + cx^2 + dx + e = 0, a \ne 0$.
3. A 12 metre tall tree was broken into two parts. It was found that the height of the part which was left standing was the cube root of the length of the part that was cut away. Formulate this into a mathematical problem to find the height of the part which was left standing.
4. Find a polynomial equation of minimum degree with rational coefficients, having $2 - \sqrt{3}$ as a root.
5. Find a polynomial equation of minimum degree with rational coefficients, having $2i + 3$ as a root.
6. Find a polynomial equation of minimum degree with rational coefficients, having $\sqrt{5} - \sqrt{3}$ as a root.
7. Solve the cubic equation : $2x^3 - x^2 - 18x + 9 = 0$. if sum of two of its roots vanishes.
8. Solve the equation: $x^4 - 14x^2 + 45 = 0$.
9. Show that the polynomial $9x^9 + 2x^5 - x^4 - 7x^2 + 2$ has at least six imaginary roots.
10. Discuss the nature of the roots of the following polynomials: (i) $x^{2018} + 1947x^{1950} + 15x^8 + 26x^6 + 2019$ (ii) $x^5 - 19x^4 + 2x^3 + 5x^2 + 11$.
11. Discuss the maximum possible number of positive and negative roots of the polynomial equation $9x^9 - 4x^8 + 4x^7 - 3x^6 + 2x^5 + x^3 + 7x^2 + 7x + 2 = 0$.
12. Show that the equation $x^9 - 5x^5 + 4x^4 + 2x^2 + 1 = 0$ has atleast 6 imaginary solutions.
13. Determine the number of positive and negative roots of the equation $x^9 - 5x^8 - 14x^7 = 0$.

11.PROBABILITY DISTRIBUTIONS

1. Suppose two coins are tossed once. If X denotes the number of tails, (i) write down the sample space (ii) find the inverse image of 1 (iii) the values of the random variable and number of elements in its inverse images.
2. Suppose X is the number of tails occurred when three fair coins are tossed once simultaneously. Find the values of the random variable X and number of points in its inverse images.
3. In a pack of 52 playing cards, two cards are drawn at random simultaneously. If the number of black cards drawn is a random variable, find the values of the random variable and number of points in its inverse images.
4. Three fair coins are tossed simultaneously. Find the probability mass function for number of heads occurred.
5. The probability density function of X is given by $f(x) = \begin{cases} kxe^{-2x} & \text{for } x > 0 \\ 0 & \text{for } x \le 0 \end{cases}$. Find the value of k.
6. If μ and σ^2 are the mean and variance of the discrete random variable X, and $E(X + 3) = 10$ and $E(X + 3)^2 = 116$, find μ and σ^2.
7. Compute $P(X = k)$ for the binomial distribution, $B(n, p)$ where (i) $n = 6, p = \frac{1}{3}, k = 3$ (ii) $n = 10, p = \frac{1}{5}, k = 4$ (iii) $n = 9, p = \frac{1}{2}, k = 7$

 12. DISCRETE MATHEMATICS

1. Verify the (i) closure property, (ii) commutative property, (iii) associative property (iv) existence of identity and (v) existence of inverse for the arithmetic operation $+$ on $\mathbb{Z}_o$ = the set of all odd integers.
2. Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ be any two boolean matrices of the same type. Find $A \vee B$ and $A \wedge B$.
3. Determine whether $*$ is a binary operation on the sets given below.
   (i) $a * b = a \cdot |b|$ on $\mathbb{R}$ (ii) $a * b = \min(a, b)$ on $A = \{1,2,3,4,5\}$ (iii) $(a * b) = a\sqrt{b}$ is binary on $\mathbb{R}$.
4. On $\mathbb{Z}$, define $*$ by $(m * n) = m^n + n^m : \forall m, n \in \mathbb{Z}$. Is $*$ binary on $\mathbb{Z}$?
5. Let $*$ be defined on $\mathbb{R}$ by $(a * b) = a + b + ab - 7$. Is $*$ binary on $\mathbb{R}$? If so, find $3 * \left(\frac{-7}{15}\right)$.
6. Let $A = \{a + \sqrt{5}b : a, b \in \mathbb{Z}\}$. Check whether the usual multiplication is a binary operation on A.
7. Let p : Jupiter is a planet and q : India is an island be any two simple statements. Give verbal sentence describing each of the following statements.
   (i) $\neg p$ (ii) $p \wedge \neg q$ (iii) $\neg p \vee q$ (iv) $p \rightarrow \neg q$ (v) $p \leftrightarrow q$
8. Write each of the following sentences in symbolic form using statement variables p and q.
   (i) 19 is not a prime number and all the angles of a triangle are equal.
   (ii) 19 is a prime number or all the angles of a triangle are not equal
   (iii) 19 is a prime number and all the angles of a triangle are equal
   (iv) 19 is not a prime number
9. Determine the truth value of each of the following statements
   (i) If $6 + 2 = 5$, then the milk is white.
   (ii) China is in Europe or $\sqrt{3}$ is an integer
   (iii) It is not true that $5 + 5 = 9$ or Earth is a planet
   (iv) 11 is a prime number and all the sides of a rectangle are equal
10. Which one of the following sentences is a proposition?
    (i) $4 + 7 = 12$ (ii) What are you doing?
    (iii) $3^n \leq 81, n \in \mathbb{N}$ (iv) Peacock is our national bird
    (v) How tall this mountain is!
11. Write the converse, inverse, and contrapositive of each of the following implication.
    (i) If x and y are numbers such that $x = y$, then $x^2 = y^2$
    (ii) If a quadrilateral is a square then it is a rectangle
12. Construct the truth table for the following statements.
    (i) $\neg p \wedge \neg q$ (ii) $\neg(p \wedge \neg q)$ (iii) $(p \vee q) \vee \neg q$ (iv) $(\neg p \rightarrow r) \wedge (p \leftrightarrow q)$

IMPORTANT 3 MARK QUESTIONS.

1.APPLICATIONS OF MATRICES AND DETERMINANTS

16. If $A = \frac{1}{9}\begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$, prove that $A^{-1} = A^T$.
17. If $A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & -3 \\ 5 & 2 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1}A^{-1}$.
18. If adj $(A) = \begin{bmatrix} 2 & -4 & 2 \\ -3 & 12 & -7 \\ -2 & 0 & 2 \end{bmatrix}$, find A.
19. $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, show that $A^T A^{-1} = \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$.
20. Given $A = \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}, B = \begin{bmatrix} 3 & -2 \\ 1 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$, find a matrix X such that $AXB = C$.

21. Find the rank of the following matrices by minor method: (i) $\begin{bmatrix} 0 & 1 & 2 & 1 \\ 0 & 2 & 4 & 3 \\ 8 & 1 & 0 & 2 \end{bmatrix}$
22. Find the rank of the following matrices by row reduction method:
    (i) $\begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & 3 & 4 \\ 5 & -1 & 7 & 11 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 1 & -2 & 3 \\ 1 & -1 & 1 \end{bmatrix}$ (iii) $\begin{bmatrix} 3 & -8 & 5 & 2 \\ 2 & -5 & 1 & 4 \\ -1 & 2 & 3 & -2 \end{bmatrix}$
23. Find the inverse of each of the following by Gauss-Jordan method: (i) $\begin{bmatrix} 2 & -1 \\ 5 & -2 \end{bmatrix}$
24. Solve the following system of linear equations by matrix inversion method: (i) $2x - y = 8,\ 3x + 2y = -2$
    (ii) $2x + 3y - z = 9,\ x + y + z = 9,\ 3x - y - z = -1$
    (iii) $x + y + z - 2 = 0,\quad 6x - 4y + 5z - 31 = 0,\quad 5x + 2y + 2z = 13$
25. A man is appointed in a job with a monthly salary of certain amount and a fixed amount of annual increment. If his salary was ₹19,800 per month at the end of the first month after 3 years of service and ₹23,400 per month at the end of the first month after 9 years of service, find his starting salary and his annual increment. (Use matrix inversion method to solve the problem.)
26. Four men and 4 women can finish a piece of work jointly in 3 days while 2 men and 5 women can finish the same work jointly in 4 days. Find the time taken by one man alone and that of one woman alone to finish the same work by using matrix inversion method.
27. In a competitive examination, one mark is awarded for every correct answer while $\frac{1}{4}$ mark is deducted for every wrong answer. A student answered 100 questions and got 80 marks. How many questions did he answer correctly ? (Use Cramer's rule to solve the problem).
28. A chemist has one solution which is 50% acid and another solution which is 25% acid. How much each should be mixed to make 10 litres of a 40% acid solution? (Use Cramer's rule to solve the problem).
29. Find the inverse of the matrix $\begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix}$.
30. Verify the property $(A^T)^{-1} = (A^{-1})^T$ with $A = \begin{bmatrix} 2 & 9 \\ 1 & 7 \end{bmatrix}$.
31. Show that the matrix $\begin{bmatrix} 3 & 1 & 4 \\ 2 & 0 & -1 \\ 5 & 2 & 1 \end{bmatrix}$ is non-singular and reduce it to the identity matrix by elementary row transformations.

2.COMPLEX NUMBERS

1. Find the value of the real numbers x and y, if the complex number $(2+i)x + (1-i)y + 2i - 3$ and $x + (-1+2i)y + 1 + i$ are equal.
2. Given the complex number $z = 2 + 3i$, represent the complex numbers in Argand diagram.
   (i) z, iz, and $z + iz$ (ii) $z, -iz$, and $z - iz$.
3. Find the values of the real numbers x and y, if the complex numbers $(3-i)x - (2-i)y + 2i + 5$ and $2x + (-1+2i)y + 3 + 2i$ are equal.
4. The complex numbers u, v, and w are related by $\frac{1}{u} = \frac{1}{v} + \frac{1}{w}$. If $v = 3 - 4i$ and $w = 4 + 3i$, find u in rectangular form.
5. State and prove Triangular Inequality.
6. If Z is imaginary if and only iff $z = -\bar{z}$.
7. Find the least value of the positive integer n for which $(\sqrt{3} + i)^n$ (i) real (ii) purely imaginary.
8. If z_1, z_2, and z_3 are complex numbers such that $|z_1| = |z_2| = |z_3| = |z_1 + z_2 + z_3| = 1$, find the value of $\left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right|$.
9. If $|z| = 2$ show that $3 \le |z + 3 + 4i| \le 7$

10. Show that the points $1, -\frac{1}{2} + i\frac{\sqrt{3}}{2}$, and $-\frac{1}{2} - i\frac{\sqrt{3}}{2}$ are the vertices of an equilateral triangle.

11. Show that the equation $z^2 = \bar{z}$ has four solutions.
12. If the area of the triangle formed by the vertices z, iz, and $z + iz$ is 50 square units, find the value of $|z|$.
13. If $z = x + iy$ is a complex number such that $\left|\frac{z-4i}{z+4i}\right| = 1$ show that the locus of z is real axis.
14. Obtain the Cartesian form of the locus of $z = x + iy$ in each of the following cases: (i) $\text{Im}\,[(1-i)z + 1] = 0$
15. Show that the following equations represent a circle, and, find its centre and radius. i) $|3z - 6 + 12i| = 8$.
16. Obtain the Cartesian equation for the locus of $z = x + iy$ in each of the foiiowing (i) $|z-4|^2 - |z-1|^2 = 16$.
17. Find the quotient $\frac{2\left(\cos\frac{9\pi}{4} + i\sin\frac{9\pi}{4}\right)}{4\left(\cos\left(\frac{-3\pi}{2}\right) + i\sin\left(\frac{-3\pi}{2}\right)\right)}$ in rectangular form.
18. Find the rectangular form of the complex numbers (i) $\frac{\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}}{2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)}$.
19. If $(x_1 + iy_1)(x_2 + iy_2)(x_3 + iy_3)\cdots(x_n + iy_n) = a + ib$, show that
    (i) $(x_1^2 + y_1^2)(x_2^2 + {y_2}^2)(x_3^2 + y_3^2)\cdots({x_n}^2 + y_n^2) = a^2 + b^2$ (ii) $\sum_{r=1}^{n}\tan^{-1}\left(\frac{y_r}{x_r}\right) = \tan^{-1}\left(\frac{b}{a}\right) + 2k\pi, k \in \mathbb{Z}$.
20. If $\frac{1+z}{1-z} = \cos 2\theta + i\sin 2\theta$, show that $z = i\tan\theta$.
21. If $\cos\alpha + \cos\beta + \cos\gamma = \sin\alpha + \sin\beta + \sin\gamma = 0$, show that
    (i) $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha+\beta+\gamma)$ and (ii) $\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha+\beta+\gamma)$.
22. If $z = (\cos\theta + i\sin\theta)$, show that $z^n + \frac{1}{z^n} = 2\cos n\theta$ and $z^n - \frac{1}{z^n} = 2i\sin n\theta$.
23. Simplify $\left(\frac{1+\cos 2\theta + i\sin}{1+\cos 2\theta - is}\right)^{30}$
24. Find the value of $\left(\frac{1+\sin\frac{\pi}{10} + i\cos\frac{\pi}{10}}{1+\sin\frac{\pi}{10} - i\cos\frac{\pi}{10}}\right)^{10}$.
25. Solve the equation $z^3 + 27 = 0$.
26. If $\omega \neq 1$ is a cube root of unity, show that
    (i) $(1-\omega+\omega^2)^6 + (1+\omega-\omega^2)^6 = 128$. (ii) $(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)\cdots\left(1+\omega^{2^{11}}\right) = 1$.

3.THEORY OF EQUATIONS

1. Find the condition that the roots of cubic equation $x^3 + ax^2 + bx + c = 0$ are in the ratio $p:q:r$.
2. If p is real, discuss the nature of the roots of the equation $4x^2 + 4px + p + 2 = 0$, in terms of p.
3. If the sides of a cubic box are increased by 1,2,3 units respectively to form a cuboid, then the volume is increased by 52 cubic units. Find the volume of the cuboid.
4. Construct a cubic equation with roots (i) 1,2, and 3 (ii) 1,1 , and -2(iii) $2, \frac{1}{2}$ and 1 .
5. Solve the equation $x^3 - 9x^2 + 14x + 24 = 0$ if it is given that two of its roots are in the ratio $3:2$.
6. If p and q are the roots of the equation $lx^2 + nx + n = 0$, show that $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0$.
7. If the equations $x^2 + px + q = 0$ and $x^2 + p'x + q' = 0$ have a common root, show that it must be equal to $\frac{pq'-p'q}{q-q'}$ or $\frac{q-q'}{p'-p}$
8. Form a polynomial equation with integer coefficients with $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ as a root.
9. If k is real, discuss the nature of the roots of the polynomial equation $2x^2 + kx + k = 0$, in terms of k.
10. Find a polynomial equation of minimum degree with rational coefficients, having $2 + \sqrt{3}i$ as a root.
11. Solve the equation $x^3 - 3x^2 - 33x + 35 = 0$.
12. Solve the equation $2x^3 + 11x^2 - 9x - 18 = 0$.

13. Solve the equation $9x^3 - 36x^2 + 44x - 16 = 0$ if the roots form an arithmetic progression.
14. Solve the equation $3x^3 - 26x^2 + 52x - 24 = 0$ if its roots form a geometric progression.
15. Solve the cubic equations : (i) $2x^3 - 9x^2 + 10x = 3$, (ii) $8x^3 - 2x^2 - 7x + 3 = 0$.
16. Solve the equation $(x-2)(x-7)(x-3)(x+2) + 19 = 0$.
17. Solve the equation $7x^3 - 43x^2 = 43x - 7$.
18. Solve the following equations (i) $\sin^2 x - 5\sin x + 4 = 0$ (ii) $12x^3 + 8x = 29x^2 - 4$
19. Examine for the rational roots of (i) $2x^3 - x^2 - 1 = 0$ (ii) $x^8 - 3x + 1 = 0$.
20. Solve : $2\sqrt{\frac{x}{a}} + 3\sqrt{\frac{a}{x}} = \frac{b}{a} + \frac{6a}{b}$.
21. Discuss the maximum possible number of positive and negative zeros of the polynomials $x^2 - 5x + 6$ and $x^2 - 5x + 16$. Also draw rough sketch of the graphs.
22. Find the exact number of real zeros and imaginary of the polynomial $x^9 + 9x^7 + 7x^5 + 5x^3 + 3x$.

11.PROBABILITY DISTRIBUTIONS

1. Suppose a pair of unbiased dice is rolled once. If X denotes the total score of two dice, write down (i) the sample space (ii) the values taken by the random variable X, (iii) the inverse image of 10 , and (iv) the number of elements in inverse image of X.
2. An urn contains 2 white balls and 3 red balls. A sample of 3 balls are chosen at random from the urn. If X denotes the number of red balls chosen, find the values taken by the random variable X and its number of inverse images.
3. An urn contains 5 mangoes and 4 apples. Three fruits are taken at random. If the number of apples taken is a random variable, then find the values of the random variable and number of points in its inverse images.
4. Two balls are chosen randomly from an urn containing 6 red and 8 black balls. Suppose that we win ₹ 15 for each red ball selected and we lose ₹ 10 for each black ball selected. If X denotes the winning amount, find the values of X and number of points in its inverse images.
5. Find the probability mass function and cumulative distribution function of number of girl child in families with 4 children, assuming equal probabilities for boys and girls.
6. If X is the random variable with distribution function $F(x)$ given by,
$$F(x) = \begin{cases} 0, & x < 0 \\ x, & 0 \le x < 1 \\ 1, & 1 \le x \end{cases}$$
then find (i) the probability density function $f(x)$ (ii) $P(0.2 \le X \le 0.7)$.
7. The probability density function of X is $f(x) = \begin{cases} x & 0 < x < 1 \\ 2 - x & 1 \le x < 2 \\ 0 & \text{otherwise} \end{cases}$.
Find (i) $P(0.2 \le X < 0.6)$ (ii) $P(1.2 \le X < 1.8)$ (iii) $P(0.5 \le X < 1.5)$
8. Two balls are drawn in succession without replacement from an urn containing four red balls and three black balls. Let X be the possible outcomes drawing red balls. Find the probability mass function and mean for X.
9. Four fair coins are tossed once. Find the probability mass function, mean and variance for number of heads occurred.
10. A lottery with 600 tickets gives one prize of ₹200, four prizes of ₹ 100 , and six prizes of ₹ 50. If the ticket costs is ₹2, find the expected winning amount of a ticket.
11. Using binomial distribution find the mean and variance of X for the following experiments
(i) A fair coin is tossed 100 times, and X denote the number of heads.
(ii) A fair die is tossed 240 times, and X denote the number of times that four appeared.

12. DISCRETE MATHEMATICS

1. Verify (i) closure property (ii) commutative property, and (iii) associative property of the following operation on the given set. $(a * b) = a^b, \forall a, b \in \mathbb{N}$ (exponentiation property)

2. Fill in the following table so that the binary operation $*$ on $A = \{a, b, c\}$ is commutative.

$*$	a	b	c
a	b		
b	c	b	a
c	a		c

3. Let $A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix}, B = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}, C = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$ be any three boolean matrices of the same type.
Find (i) $A \vee B$ (ii) $A \wedge B$ (iii) $(A \vee B) \wedge C$ (iv) $(A \wedge B) \vee C$.

4. Consider the binary operation $*$ defined on the set $A = \{a, b, c, d\}$ by the following table:

$*$	a	b	c	d
a	a	c	b	d
b	d	a	b	c
c	c	d	a	a
d	d	b	a	c

Is it commutative and associative?

5. Verify whether the following compound propositions are tautologies or contradictions or contingency
(i) $(p \wedge q) \wedge \neg(p \vee q)$ (ii) $((p \vee q) \wedge \neg p) \rightarrow q$

6. Show that (i) $\neg(p \wedge q) \equiv \neg p \vee \neg q$ (ii) $\neg(p \rightarrow q) \equiv p \wedge \neg q$.

7. Prove that $q \rightarrow p \equiv \neg p \rightarrow \neg q$.

8. Show that $p \rightarrow q$ and $q \rightarrow p$ are not equivalent.

9. Show that $\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$.

IMPORTANT 5 MARK QUESTIONS

5.TWO DIMENSIONAL ANALYTICAL GEOMETRY-II

1. Find the equation of the circle passing through the points $(1,1), (2,-1)$, and $(3,2)$.

2. Find the equation of the circle through the points $(1,0), (-1,0)$, and $(0,1)$.

3. For the ellipse $4x^2 + y^2 + 24x - 2y + 21 = 0$, find the centre, vertices, and the foci. Also prove that the length of latus rectum is 2 .

4. Find the vertex, focus, equation of directrix and length of the latus rectum of the following:
(i) $x^2 - 2x + 8y + 17 = 0$ (ii) $y^2 - 4y - 8x + 12 = 0$

5. Identify the type of conic and find centre, foci, vertices, and directrices of each of the following :
(i) $18x^2 + 12y^2 - 144x + 48y + 120 = 0$ (ii) $9x^2 - y^2 - 36x - 6y + 18 = 0$

6. Show that the line $x - y + 4 = 0$ is a tangent to the ellipse $x^2 + 3y^2 = 12$. Also find the coordinates of the point of contact.

7. A semielliptical archway over a one-way road has a height of 3 m and a width of 12 m. The truck has a width of $3m$ and a height of $2.7m$. Will the truck clear the opening of the archway?

8. A room 34 m long is constructed to be a whispering gallery. The room has an elliptical ceiling, as shown in Fig. 5.64. If the maximum height of the ceiling is 8 m, determine where the foci are located.

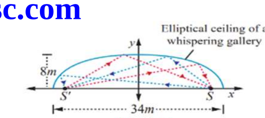


Fig. 5.64

9. If the equation of the ellipse is $\frac{(x-11)^2}{484}+\frac{y^2}{64}=1$ (x and y are measured in centimeters) where to the nearest centimeter, should the patient's kidney stone be placed so that the reflected sound hits the kidney stone?

10. A bridge has a parabolic arch that is 10 m high in the centre and 30 m wide at the bottom. Find the height of the arch $6m$ from the centre, on either sides.

11. A tunnel through a mountain for a four lane highway is to have a elliptical opening. The total width of the highway (not the opening) is to be 16 m, and the height at the edge of the road must be sufficient for a truck $4m$ high to clear if the highest point of the opening is to be $5m$ approximately . How wide must the opening be?

12. At a water fountain, water attains a maximum height of 4 m at horizontal distance of 0.5 m from its origin. If the path of water is a parabola, find the height of water at a horizontal distance of 0.75 m from the point of origin.

13. An engineer designs a satellite dish with a parabolic cross section. The dish is 5 m wide at the opening, and the focus is placed 1.2 m from the vertex
(a) Position a coordinate system with the origin at the vertex and the x-axis on the parabola's axis of symmetry and find an equation of the parabola.
(b) Find the depth of the satellite dish at the vertex.

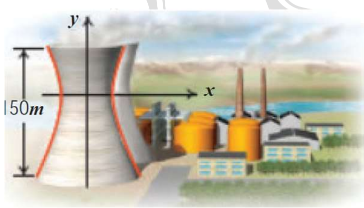
14. A rod of length 1.2 m moves with its ends always touching the coordinate axes. The locus of a point P on the rod, which is 0.3 m from the end in contact with x-axis is an ellipse. Find the eccentricity.

15. Assume that water issuing from the end of a horizontal pipe, 7.5 m above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position 2.5 m below the line of the pipe, the flow of water has curved outward $3m$ beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground?

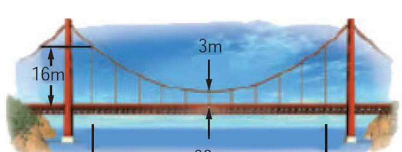
16. On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of $4m$ when it is $6m$ away from the point of projection. Finally it reaches the ground 12 m away from the starting point. Find the angle of projection.

17. Points A and B are 10 km apart and it is determined from the sound of an explosion heard at those points at different times that the location of the explosion is 6 km closer to A than B. Show that the location of the explosion is restricted to a particular curve and find an equation of it.

18. Cross section of a Nuclear cooling tower is in the shape of a hyperbola with equation $\frac{x^2}{30^2}-\frac{y^2}{44^2}=1$. The tower is 150 m tall and the distance from the top of the tower to the centre of the hyperbola is half the distance from the base of the tower to the centre of the hyperbola. Find the diameter of the top and base of the tower.



19. Parabolic cable of a 60 m portion of the roadbed of a suspension bridge are positioned as shown below. Vertical Cables are to be spaced every 6 m along this portion of the roadbed. Calculate the lengths of first two of these vertical cables from the vertex.



APPLICATIONS OF VECTOR ALGEBRA

1. By vector method, prove that $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$.
2. Prove by vector method that $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$.
3. Prove by vector method that the perpendiculars (attitudes) from the vertices to the opposite sides of a triangle are concurrent.
4. Using vector method, prove that $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$.
5. Prove by vector method that $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$.
6. If $\vec{a}=\hat{\imath}-\hat{\jmath}, \vec{b}=\hat{\imath}-\hat{\jmath}-4\hat{k}, \vec{c}=3\hat{\jmath}-\hat{k}$ and $\vec{d}=2\hat{\imath}+5\hat{\jmath}+\hat{k}$, verify that
   (i) $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=[\vec{a},\vec{b},\vec{d}]\vec{c}-[\vec{a},\vec{b},\vec{c}]\vec{d}$ (ii) $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=[\vec{a},\vec{c},\vec{d}]\vec{b}-[\vec{b},\vec{c},\vec{d}]\vec{a}$
7. If $\vec{a}=2\hat{\imath}+3\hat{\jmath}-\hat{k}, \vec{b}=3\hat{\imath}+5\hat{\jmath}+2\hat{k}, \vec{c}=-\hat{\imath}-2\hat{\jmath}+3\hat{k}$, verify that
   (i) $(\vec{a}\times\vec{b})\times\vec{c}=(\vec{a}\cdot\vec{c})\vec{b}-(\vec{b}\cdot\vec{c})\vec{a}$ (ii) $\vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c}$
8. Show that the lines $\vec{r}=(6\hat{\imath}+\hat{\jmath}+2\hat{k})+s(\hat{\imath}+2\hat{\jmath}-3\hat{k})$ and $\vec{r}=(3\hat{\imath}+2\hat{\jmath}-2\hat{k})+t(2\hat{\imath}+4\hat{\jmath}-5\hat{k})$ are skew lines and hence find the shortest distance between them.
9. Show that the lines $\frac{x-3}{3}=\frac{y-3}{-1}$, $z-1=0$ and $\frac{x-6}{2}=\frac{z-1}{3}$, $y-2=0$ intersect. Also find the point of intersection.
10. Show that the straight lines $x+1=2y=-12z$ and $x=y+2=6z-6$ are skew and hence find the shortest distance between them.
11. Find the parametric form of vector equation of the straight line passing through $(-1,2,1)$ and parallel to the straight line $\vec{r}=(2\hat{\imath}+3\hat{\jmath}-\hat{k})+t(\hat{\imath}-2\hat{\jmath}+\hat{k})$ and hence find the shortest distance between the lines.
12. Find the foot of the perpendicular drawn from the point $(5,4,2)$ to the line $\frac{x+1}{2}=\frac{y-3}{3}=\frac{z-1}{-1}$. Also, find the equation of the perpendicular.
13. Find the non-parametric form of vector equation, and Cartesian equation of the plane passing through the point $(0,1,-5)$ and parallel to the straight lines $\vec{r}=(\hat{\imath}+2\hat{\jmath}-4\hat{k})+s(2\hat{\imath}+3\hat{\jmath}+6\hat{k})$ and
   $\vec{r}=\left(\hat{\imath}-3\hat{\jmath}+5\hat{k}\right)+t(\hat{\imath}+\hat{\jmath}-\hat{k})$.
14. Find the vector parametric, vector non-parametric and Cartesian form of the equation of the plane passing through the points $(-1,2,0), (2,2-1)$ and parallel to the straight line $\frac{x-1}{1}=\frac{2y+1}{2}=\frac{z+1}{-1}$.
15. Find the non-parametric form of vector equation, and Cartesian equation of the plane passing through the point $(2,3,6)$ and parallel to the straight lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-3}{1}$ and $\frac{x+3}{2}=\frac{y-3}{-5}=\frac{z+1}{-3}$
16. Find the non-parametric form of vector equation, and Cartesian equations of the plane passing through the points $(2,2,1), (9,3,6)$ and perpendicular to the plane $2x+6y+6z=9$.
17. Find parametric form of vector equation and Cartesian equations of the plane passing through the points $(2,2,1), (1,-2,3)$ and parallel to the straight line passing through the points $(2,1,-3)$ and $(-1,5,-8)$.
18. Find the non-parametric form of vector equation and cartesian equation of the plane passing through the point $(1,-2,4)$ and perpendicular to the plane $x+2y-3z=11$ and parallel to the line $\frac{x+7}{3}=\frac{y+3}{-1}=\frac{z}{1}$.
19. Find the parametric form of vector equation, and Cartesian equations of the plane containing the line
   $\vec{r}=(\hat{\imath}-\hat{\jmath}+3\hat{k})+t(2\hat{\imath}-\hat{\jmath}+4\hat{k})$ and perpendicular to plane $\vec{r}\cdot(\hat{\imath}+2\hat{\jmath}+\hat{k})=8$.
20. Find the parametric vector, non-parametric vector and Cartesian form of the equations of the plane passing through the three non-collinear points $(3,6,-2), (-1,-2,6)$, and $(6,4,-2)$.
21. Find the non-parametric form of vector equation, and Cartesian equations of the plane
   $\vec{r}=(6\hat{\imath}-\hat{\jmath}+\hat{k})+s(-\hat{\imath}+2\hat{\jmath}+\hat{k})+t(-5\hat{\imath}-4\hat{\jmath}-5\hat{k})$.

22. Show that the straight lines $\vec{r} = (5\hat{i} + 7\hat{j} - 3\hat{k}) + s(4\hat{i} + 4\hat{j} - 5\hat{k})$ and
$\vec{r} = (8\hat{i} + 4\hat{j} + 5\hat{k}) + t(7\hat{i} + \hat{j} + 3\hat{k})$ are coplanar. Find the vector equation of the plane in which they lie.

23. Show that the lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{3}$ and $\frac{x-1}{-3} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar. Also, find the plane containing these lines.

24. Find the equation of the plane passing through the intersection of the planes $2x + 3y - z + 7 = 0$ and $x + y - 2z + 5 = 0$ and is perpendicular to the plane $x + y - 3z - 5 = 0$.

25. Find the image of the point whose position vector is $\hat{i} + 2\hat{j} + 3\hat{k}$ in the plane $\vec{r} \cdot (\hat{i} + 2\hat{j} + 4\hat{k}) = 38$.

26. Find the equation of the plane passing through the line of intersection of the planes $x + 2y + 3z = 2$ and $x - y + z = 3$, and at a distance $\frac{2}{\sqrt{3}}$ from the point $(3,1,-1)$.

27. Find the point of intersection of the line $x - 1 = \frac{y}{2} = z + 1$ with the plane $2x - y + 2z = 2$. Also, find the angle between the line and the plane.

12.DISCRETE MATHEMATICS

1. Verify the (i) closure property, (ii) commutative property, (iii) associative property (iv) existence of identity and (v) existence of inverse for the arithmetic operation $+$ on $\mathbb{Z}_e$ = the set of all even integers.

2. Verify (i) closure property, (ii) commutative property, (iii) associative property,
(iv) existence of identity, and (v) existence of inverse for following operation on the given set.
$$m * n = m + n - mn; m, n \in \mathbb{Z}$$

3. Verify (i) closure property, (ii) commutative property, (iii) associative property,
(iv) existence of identity, and (v) existence of inverse for the operation $+_5$ on $\mathbb{Z}_5$ using table corresponding to addition modulo 5.

4. Verify (i) closure property, (ii) commutative property, (iii) associative property,
(iv) existence of identity, and (v) existence of inverse for the operation $\times_{11}$ on a subset $A = \{1,3,4,5,9\}$ of the set of remainders $\{0,1,2,3,4,5,6,7,8,9,10\}$.

5. (i) Define an operation $*$ on $\mathbb{Q}$ as follows: $a * b = \left(\frac{a+b}{2}\right); a, b \in \mathbb{Q}$. Examine the closure, commutative, and associative properties satisfied by $*$ on $\mathbb{Q}$.
(ii) Define an operation $*$ on $\mathbb{Q}$ as follows: $a * b = \left(\frac{a+b}{2}\right); a, b \in \mathbb{Q}$. Examine the existence of identity and the existence of inverse for the operation $*$ on $\mathbb{Q}$.

6. (i) Let $M = \left\{\begin{pmatrix} x & x \\ x & x \end{pmatrix} : x \in R - \{0\}\right\}$ and let $*$ be the matrix multiplication. Determine whether M is closed under $*$. If so, examine the commutative and associative properties satisfied by $*$ on M.
(ii) Let $M = \left\{\begin{pmatrix} x & x \\ x & x \end{pmatrix} : x \in R - \{0\}\right\}$ and let $*$ be the matrix multiplication. Determine whether M is closed under $*$. If so, examine the existence of identity, existence of inverse properties for the operation $*$ on M.

7. (i) Let A be $\mathbb{Q} \setminus \{1\}$. Define $*$ on A by $x * y = x + y - xy$. Is $*$ binary on A ? If so, examine the commutative and associative properties satisfied by $*$ on A.
(ii) Let A be $\mathbb{Q} \setminus \{1\}$. Define $*$ on A by $x * y = x + y - xy$. Is * binary on A ? If so, examine the existence of identity, existence of inverse properties for the operation $*$ on A

8. Using the equivalence property, show that $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$.
9. Verify whether the following compound propositions are tautologies or contradictions or contingency
(i) $(p \to q) \leftrightarrow (\neg p \to q)$ (ii) $((p \to q) \wedge (q \to r)) \to (p \to r)$

10. Check whether the statement $p \to (q \to p)$ is a tautology or a contradiction without using the truth table.

11. Using truth table check whether the statements $\neg(p \vee q) \vee (\neg p \wedge q)$ and $\neg p$ are logically equivalent.

12. Prove $p \to (q \to r) \equiv (p \wedge q) \to r$ without using truth table.

13. Prove that $p \to (\neg q \vee r) \equiv \neg p \vee (\neg q \vee r)$ using truth table.