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MATHS

CHAPTER 1 & CHAPTER 2

Example 1.5

Find a matrix A if $\text{adj}(A) = \begin{bmatrix} 7 & 7 & -7 \\ -1 & 11 & 7 \\ 11 & 5 & 7 \end{bmatrix}$.

Solution

First, we find $|\text{adj}(A)| = \begin{vmatrix} 7 & 7 & -7 \\ -1 & 11 & 7 \\ 11 & 5 & 7 \end{vmatrix} = 7(77-35)-7(-7-77)-7(-5-121)=1764>0$.

So, we get

$$\begin{aligned} A &= \pm \frac{1}{\sqrt{|\text{adj } A|}} \text{adj}(\text{adj } A) = \pm \frac{1}{\sqrt{1764}} \begin{bmatrix} +77-35 & -(-7-77) & +(5-121) \\ -(49+35) & +(49+77) & -(35-77) \\ +(49+77) & -(49-7) & +(77+7) \end{bmatrix}^T \\ &= \pm \frac{1}{42} \begin{bmatrix} 42 & 84 & -126 \\ -84 & 126 & 42 \\ 126 & -42 & 84 \end{bmatrix}^T = \pm \begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ -3 & 1 & 2 \end{bmatrix}. \end{aligned}$$

Example 1.11

Prove that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal.

Solution

Let $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$. Then, $A^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}^T = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$.

So, we get

$$\begin{aligned} AA^T &= \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \\ &= \begin{bmatrix} \cos^2\theta + \sin^2\theta & \cos\theta\sin\theta - \sin\theta\cos\theta \\ \sin\theta\cos\theta - \cos\theta\sin\theta & \sin^2\theta + \cos^2\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2. \end{aligned}$$

Similarly, we get $A^T A = I_2$. Hence $AA^T = A^T A = I_2 \Rightarrow A$ is orthogonal.

Example 1.9

Verify $(AB)^{-1} = B^{-1}A^{-1}$ with $A = \begin{bmatrix} 0 & -3 \\ 1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} -2 & -3 \\ 0 & -1 \end{bmatrix}$.

Solution

$$\text{We get } AB = \begin{bmatrix} 0 & -3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} -2 & -3 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0+0 & 0+3 \\ -2+0 & -3-4 \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ -2 & -7 \end{bmatrix}$$

$$(AB)^{-1} = \frac{1}{(0+6)} \begin{bmatrix} -7 & -3 \\ 2 & 0 \end{bmatrix} = \frac{1}{6} \begin{bmatrix} -7 & -3 \\ 2 & 0 \end{bmatrix} \quad \dots (1)$$

$$A^{-1} = \frac{1}{(0+3)} \begin{bmatrix} 4 & 3 \\ -1 & 0 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 4 & 3 \\ -1 & 0 \end{bmatrix}$$

$$B^{-1} = \frac{1}{(2-0)} \begin{bmatrix} -1 & 3 \\ 0 & -2 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -1 & 3 \\ 0 & -2 \end{bmatrix}$$

$$B^{-1}A^{-1} = \frac{1}{2} \begin{bmatrix} -1 & 3 \\ 0 & -2 \end{bmatrix} \frac{1}{3} \begin{bmatrix} 4 & 3 \\ -1 & 0 \end{bmatrix} = \frac{1}{6} \begin{bmatrix} -7 & -3 \\ 2 & 0 \end{bmatrix} \quad \dots (2)$$

As the matrices in (1) and (2) are same, $(AB)^{-1} = B^{-1}A^{-1}$ is verified. ■

Example 1.14

Reduce the matrix $\begin{bmatrix} 0 & 3 & 1 & 6 \\ -1 & 0 & 2 & 5 \\ 4 & 2 & 0 & 0 \end{bmatrix}$ to a row-echelon form.

Solution

$$\begin{array}{c} \begin{bmatrix} 0 & 3 & 1 & 6 \\ -1 & 0 & 2 & 5 \\ 4 & 2 & 0 & 0 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} -1 & 0 & 2 & 5 \\ 0 & 3 & 1 & 6 \\ 4 & 2 & 0 & 0 \end{bmatrix} \xrightarrow{R_1 \rightarrow R_1 + 4R_2} \begin{bmatrix} -1 & 0 & 2 & 5 \\ 0 & 3 & 1 & 6 \\ 0 & 2 & 8 & 20 \end{bmatrix} \\ \xrightarrow{R_1 \rightarrow R_1 - \frac{2}{3}R_2} \begin{bmatrix} -1 & 0 & 2 & 5 \\ 0 & 3 & 1 & 6 \\ 0 & 0 & 22 & 16 \end{bmatrix} \xrightarrow{R_1 \rightarrow 3R_2} \begin{bmatrix} -1 & 0 & 2 & 5 \\ 0 & 3 & 1 & 6 \\ 0 & 0 & 22 & 48 \end{bmatrix}. \end{array}$$

Example 1.19

Show that the matrix $\begin{bmatrix} 3 & 1 & 4 \\ 2 & 0 & -1 \\ 5 & 2 & 1 \end{bmatrix}$ is non-singular and reduce it to the identity matrix by elementary row transformations.

Solution

Let $A = \begin{bmatrix} 3 & 1 & 4 \\ 2 & 0 & -1 \\ 5 & 2 & 1 \end{bmatrix}$. Then, $|A| = 3(0+2) - 1(2+5) + 4(4-0) = 6 - 7 + 16 = 15 \neq 0$. So, A is non-singular. Keeping the identity matrix as our goal, we perform the row operations sequentially on A as follows:

$$\begin{array}{c} \begin{bmatrix} 3 & 1 & 4 \\ 2 & 0 & -1 \\ 5 & 2 & 1 \end{bmatrix} \xrightarrow{R_1 \rightarrow \frac{1}{3}R_1} \begin{bmatrix} 1 & \frac{1}{3} & \frac{4}{3} \\ 2 & 0 & -1 \\ 5 & 2 & 1 \end{bmatrix} \xrightarrow{R_2 \rightarrow R_2 - 2R_1, R_3 \rightarrow R_3 - 5R_1} \begin{bmatrix} 1 & \frac{1}{3} & \frac{4}{3} \\ 0 & -\frac{2}{3} & -\frac{11}{3} \\ 0 & \frac{1}{3} & -\frac{17}{3} \end{bmatrix} \xrightarrow{R_3 \rightarrow \left(\frac{3}{2}\right)R_3} \begin{bmatrix} 1 & \frac{1}{3} & \frac{4}{3} \\ 0 & 1 & \frac{11}{2} \\ 0 & \frac{1}{3} & -\frac{17}{3} \end{bmatrix} \\ \xrightarrow{R_1 \rightarrow R_1 - \frac{1}{3}R_2, R_3 \rightarrow R_3 - \frac{1}{3}R_2} \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{11}{2} \\ 0 & 0 & -\frac{15}{2} \end{bmatrix} \xrightarrow{R_3 \rightarrow \left(-\frac{2}{15}\right)R_3} \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{11}{2} \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_1 \rightarrow R_1 + \frac{1}{2}R_2, R_2 \rightarrow R_2 - \frac{11}{2}R_1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \end{array}$$

Example 1.24

If $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, find the products AB and BA and hence solve the

system of equations $x-y+z=4, x-2y-2z=9, 2x+y+3z=1$.

Solution

We find $AB = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -4+4+8 & 4-8+4 & -4-8+12 \\ -7+1+6 & 7-2+3 & -7-2+9 \\ 5-3-2 & -5+6-1 & 5+6-3 \end{bmatrix} = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = 8I_3$

and $BA = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} = \begin{bmatrix} -4+7+5 & 4-1-3 & 4-3-1 \\ -4+14-10 & 4-2+6 & 4-6+2 \\ -8-7+15 & 8+1-9 & 8+3-3 \end{bmatrix} = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = 8I_3$.

So, we get $AB = BA = 8I_3$. That is, $\left(\frac{1}{8}A\right)B = B\left(\frac{1}{8}A\right) = I_3$. Hence, $B^{-1} = \frac{1}{8}A$.

Writing the given system of equations in matrix form, we get

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}. \text{ That is, } B \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}.$$

$$\text{So, } \begin{bmatrix} x \\ y \\ z \end{bmatrix} = B^{-1} \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix} = \left(\frac{1}{8}A\right) \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix} = \frac{1}{8} \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix} = \frac{1}{8} \begin{bmatrix} -16+36+4 \\ -56+7+3 \\ 20-27-1 \end{bmatrix} = \frac{1}{8} \begin{bmatrix} 24 \\ -16 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}.$$

Hence, the solution is $(x=3, y=-2, z=-1)$. ■

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Example 1.27

Solve the following system of linear equations, by Gaussian elimination method :

$$4x+3y+6z=25, \quad x+5y+7z=13, \quad 2x+9y+z=1.$$

Solution

Transforming the augmented matrix to echelon form, we get

$$\left[\begin{array}{ccc|c} 4 & 3 & 6 & 25 \\ 1 & 5 & 7 & 13 \\ 2 & 9 & 1 & 1 \end{array} \right] \xrightarrow{\substack{R_2 \leftrightarrow R_3 \\ R_1 + R_2 \rightarrow R_1}} \left[\begin{array}{ccc|c} 1 & 5 & 7 & 13 \\ 4 & 3 & 6 & 25 \\ 2 & 9 & 1 & 1 \end{array} \right] \xrightarrow{\substack{R_1 \rightarrow R_1 - 4R_2 \\ R_3 - R_2 \rightarrow R_3}} \left[\begin{array}{ccc|c} 1 & 5 & 7 & 13 \\ 0 & -17 & -22 & -77 \\ 2 & 9 & 1 & 1 \end{array} \right] \xrightarrow{\substack{R_3 - 2R_1 \rightarrow R_3 \\ R_2 \rightarrow R_2 - 2R_1}} \left[\begin{array}{ccc|c} 1 & 5 & 7 & 13 \\ 0 & -17 & -22 & -77 \\ 0 & -1 & -13 & -25 \end{array} \right]$$

$$\xrightarrow{\substack{R_2 \rightarrow R_2 + (-1)R_3 \\ R_3 \rightarrow R_3 - (-1)R_2}} \left[\begin{array}{ccc|c} 1 & 5 & 7 & 13 \\ 0 & 17 & 22 & 27 \\ 0 & 1 & 13 & 25 \end{array} \right] \xrightarrow{\substack{R_1 \rightarrow R_1 - 17R_3 \\ R_2 \rightarrow R_2 - 17R_3}} \left[\begin{array}{ccc|c} 1 & 5 & 7 & 13 \\ 0 & 0 & 199 & 398 \\ 0 & 1 & 13 & 25 \end{array} \right].$$

The equivalent system is written by using the echelon form:

$$x+5y+7z=13, \quad \dots (1)$$

$$17y+22z=27, \quad \dots (2)$$

$$199z=398. \quad \dots (3)$$

$$\text{From (3), we get } z = \frac{398}{199} = 2.$$

$$\text{Substituting } z=2 \text{ in (2), we get } y = \frac{27-22 \times 2}{17} = \frac{-17}{17} = -1.$$

$$\text{Substituting } z=2, y=-1 \text{ in (1), we get } x=13-5 \times (-1)-7 \times 2=4.$$

$$\text{So, the solution is } (x=4, y=-1, z=2).$$

Note. The above method of going from the last equation to the first equation is called the method of back substitution.

4. A boy is walking along the path $y=ax^2+bx+c$ through the points $(-6,8), (-2,-12)$, and $(3,8)$. He wants to meet his friend at $P(7,60)$. Will he meet his friend? (Use Gaussian elimination method.)

Example 1.31

Test for consistency of the following system of linear equations and if possible solve:
 $x-y+z=-9, 2x-2y+2z=-18, 3x-3y+3z=27=0$.

Solution

Here the number of unknowns is 3.

The matrix form of the system is $AX=B$, where

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & -3 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} -9 \\ -18 \\ -27 \end{bmatrix}.$$

Applying elementary row operations on the augmented matrix $[A|B]$, we get

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -9 \\ 2 & -2 & 2 & -18 \\ 3 & -3 & 3 & -27 \end{array} \right] \xrightarrow{\substack{R_2 \rightarrow R_2 - 2R_1 \\ R_3 \rightarrow R_3 - 3R_1}} \left[\begin{array}{ccc|c} 1 & -1 & 1 & -9 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right].$$

$$\text{So, } \rho(A)=\rho([A|B])=1 < 3.$$

From the echelon form, we get the equivalent equations $x-y+z=-9, 0=0, 0=0$.

The equivalent system has one non-trivial equation and three unknowns.

Taking $y=s, z=t$ arbitrarily, we get $x-s+t=-9$; or $x=-9+s-t$.

So, the solution is $(x=-9+s-t, y=s, z=t)$, where s and t are parameters.

The above solution set is a two-parameter family of solutions.

Here, the given system of equations is consistent and has infinitely many solutions which form a two parameter family of solutions.

Example 1.36

Solve the system: $x+3y-2z=0, 2x-y+4z=0, x-11y+14z=0$.

Solution

Here the number of unknowns is 3.

Transforming into echelon form (Gaussian elimination method), the augmented matrix becomes

$$\left[\begin{array}{ccc|c} 1 & 3 & -2 & 0 \\ 2 & -1 & 4 & 0 \\ 1 & -11 & 14 & 0 \end{array} \right] \xrightarrow{\substack{R_2 \rightarrow R_2 - 2R_1 \\ R_3 \rightarrow R_3 - R_1}} \left[\begin{array}{ccc|c} 1 & 3 & -2 & 0 \\ 0 & -7 & 8 & 0 \\ 0 & -14 & 16 & 0 \end{array} \right] \xrightarrow{\substack{R_3 \rightarrow R_3 - (-2)R_2 \\ R_2 \rightarrow R_2 - R_3}} \left[\begin{array}{ccc|c} 1 & 3 & -2 & 0 \\ 0 & 7 & -8 & 0 \\ 0 & 0 & -8 & 0 \end{array} \right] \xrightarrow{R_3 \rightarrow R_3 - R_2} \left[\begin{array}{ccc|c} 1 & 3 & -2 & 0 \\ 0 & 7 & -8 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right].$$

$$\text{So, } \rho(A)=\rho([A|O])=2 < 3 = \text{Number of unknowns.}$$

Hence, the system has a one parameter family of solutions.

Writing the equations using the echelon form, we get

$$x+3y-2z=0, \quad 7y-8z=0, \quad 0=0.$$

Taking $z=t$, where t is an arbitrary real number, we get by back substitution,

Example 1.40

If the system of equations $px+by+cz=0, ax+gy+cz=0, ax+hy+rz=0$ has a non-trivial solution and $p=a, q=b, r=c$, prove that $\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} = 2$.

Solution

Assume that the system $px+by+cz=0, ax+gy+cz=0, ax+hy+rz=0$ has a non-trivial solution.

$$\text{So, we have } \begin{vmatrix} p & b & c \\ a & g & c \\ a & h & r \end{vmatrix} = 0. \text{ Applying } R_2 \rightarrow R_2 - R_1 \text{ and } R_3 \rightarrow R_3 - R_1 \text{ in the above equation,}$$

we get

$$\begin{vmatrix} p & b & c \\ a-p & q-b & 0 \\ a-p & 0 & r-c \end{vmatrix} = 0. \text{ That is, } \begin{vmatrix} p & b & c \\ -(p-a) & q-b & 0 \\ -(p-a) & 0 & r-c \end{vmatrix} = 0.$$

$$\text{Since } p=a, q=b, r=c, \text{ we get } (p-a)(q-b)(r-c) \begin{vmatrix} p & b & c \\ p-a & q-b & r-c \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = 0.$$

$$\text{So, we have } \begin{vmatrix} p & b & c \\ p-a & q-b & r-c \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = 0.$$

$$\text{Expanding the determinant, we get } \frac{p}{p-a} + \frac{b}{q-b} + \frac{c}{r-c} = 0.$$

$$\text{That is, } \frac{p}{p-a} + \frac{q-(q-b)}{q-b} + \frac{r-(r-c)}{r-c} = 0 \Rightarrow \frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} = 2.$$

Example 2.2

Find the value of the real numbers x and y , if the complex number $(2+i)x+(1-i)y+2i-3$ and $x+(-1+2i)y+1+i$ are equal

Solution

$$\text{Let } z_1 = (2+i)x+(1-i)y+2i-3 = (2x+y-3)+(x-y+2)i \text{ and}$$

$$z_2 = x+(-1+2i)y+1+i = (x-y+1)+(2y+1)i.$$

Given that $z_1 = z_2$.

$$\text{Therefore } (2x+y-3)+(x-y+2)i = (x-y+1)+(2y+1)i.$$

Equating real and imaginary parts separately, gives

$$\begin{aligned} 2x+y-3 &= x-y+1 & \Rightarrow x+2y=4, \\ x-y+2 &= 2y+1 & \Rightarrow x-3y=-1. \end{aligned}$$

Solving the above equations, gives

$$x=2 \text{ and } y=1.$$

Example 2.5

If $\frac{z+3}{z-5i} = \frac{1+4i}{2}$, find the complex number z in the rectangular form

Solution

$$\text{We have } \frac{z+3}{z-5i} = \frac{1+4i}{2}$$

$$\Rightarrow 2(z+3) = (1+4i)(z-5i)$$

$$\Rightarrow 2z+6 = (1+4i)z+20-5i$$

$$\Rightarrow (2-1-4i)z = 20-5i-6$$

$$\Rightarrow z = \frac{14-5i}{1-4i} = \frac{(14-5i)(1+4i)}{(1-4i)(1+4i)} = \frac{34+5i}{17} = 2+3i.$$

Example 2.7

Find z^{-1} , if $z=(2+3i)(1-i)$.

Solution

$$\text{We have } z = (2+3i)(1-i) = (2+3)+(3-2)i = 5+i$$

$$\Rightarrow z^{-1} = \frac{1}{z} = \frac{1}{5+i}.$$

Multiplying the numerator and denominator by the conjugate of the denominator, we get

$$z^{-1} = \frac{(5-i)}{(5+i)(5-i)} = \frac{5-i}{5^2+1^2} = \frac{5}{26}-i\frac{1}{26}$$

$$\Rightarrow z^{-1} = \frac{5}{26}-i\frac{1}{26}.$$

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Example 2.12

If z_1 , z_2 , and z_3 are complex numbers such that $|z_1| = |z_2| = |z_3| = |z_1 + z_2 + z_3| = 1$,

find the value of $\left| \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} \right|$.

Solution

Since, $|z_1| = |z_2| = |z_3| = 1$,

$$|z_1|^2 = 1 \Rightarrow z_1 \bar{z}_1 = 1, |z_2|^2 = 1 \Rightarrow z_2 \bar{z}_2 = 1, \text{ and } |z_3|^2 = 1 \Rightarrow z_3 \bar{z}_3 = 1$$

Therefore, $\bar{z}_1 = \frac{1}{z_1}$, $\bar{z}_2 = \frac{1}{z_2}$, and $\bar{z}_3 = \frac{1}{z_3}$ and hence

$$\begin{aligned} \left| \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} \right| &= \left| \bar{z}_1 + \bar{z}_2 + \bar{z}_3 \right| \\ &= \overline{|z_1 + z_2 + z_3|} = |z_1 + z_2 + z_3| = 1. \end{aligned}$$

Example 2.16

Show that the equation $z^2 = \bar{z}$ has four solutions.

Solution

We have,

$$z^2 = \bar{z}.$$

$$\Rightarrow |z|^2 = |z|$$

$$\Rightarrow |z|(|z| - 1) = 0,$$

$$\Rightarrow |z| = 0, \text{ or } |z| = 1.$$

$$|z| = 0 \Rightarrow z = 0 \text{ is a solution, } |z| = 1 \Rightarrow z\bar{z} = 1 \Rightarrow \bar{z} = \frac{1}{z}.$$

$$\text{Given } z^2 = \bar{z} \Rightarrow z^2 = \frac{1}{z} \Rightarrow z^3 = 1.$$

It has 3 non-zero solutions. Hence including zero solution, there are four solutions. ■

Example 2.17

Find the square root of $6 - 8i$.

Solution

We compute $|6 - 8i| = \sqrt{6^2 + (-8)^2} = 10$

and applying the formula for square root, we get

$$\begin{aligned} \sqrt{6 - 8i} &= \pm \left(\sqrt{\frac{10+6}{2}} - i \sqrt{\frac{10-6}{2}} \right) \quad (\because b \text{ is negative, } \frac{b}{|b|} = -1) \\ &= \pm \left(\sqrt{8} - i\sqrt{2} \right) \\ &= \pm \left(2\sqrt{2} - i\sqrt{2} \right). \end{aligned}$$

Example 2.23

Represent the complex number (i) $-1 - i$ (ii) $1 + i\sqrt{3}$ in polar form.

Solution

(i)

$$\text{Let } -1 - i = r(\cos\theta + i\sin\theta)$$

$$\text{We have } r = \sqrt{x^2 + y^2} = \sqrt{1^2 + 1^2} = \sqrt{1+1} = \sqrt{2}$$

$$\alpha = \tan^{-1} \left| \frac{y}{x} \right| = \tan^{-1} 1 = \frac{\pi}{4}.$$

Since the complex number $-1 - i$ lies in the third quadrant, it has the principal value,

$$\theta = \alpha - \pi = \frac{\pi}{4} - \pi = -\frac{3\pi}{4}$$

$$\text{Therefore, } -1 - i = \sqrt{2} \left(\cos \left(-\frac{3\pi}{4} \right) + i \sin \left(-\frac{3\pi}{4} \right) \right)$$

$$= \sqrt{2} \left(\cos \frac{3\pi}{4} - i \sin \frac{3\pi}{4} \right).$$

$$-1 - i = \sqrt{2} \left(\cos \left(\frac{3\pi}{4} + 2k\pi \right) - i \sin \left(\frac{3\pi}{4} + 2k\pi \right) \right), k \in \mathbb{Z}.$$

Example 2.24

Find the principal argument $\operatorname{Arg} z$, when $z = \frac{-2}{1+i\sqrt{3}}$.

Solution

$$\arg z = \arg \frac{-2}{1+i\sqrt{3}}$$

$$= \arg(-2) - \arg(1+i\sqrt{3}) \quad (\because \arg \left(\frac{z_1}{z_2} \right) = \arg z_1 - \arg z_2)$$

$$= \left(\pi - \tan^{-1} \left(\frac{0}{2} \right) \right) - \tan^{-1} \left(\frac{\sqrt{3}}{1} \right)$$

$$= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

This implies that one of the values of $\arg z$ is $\frac{2\pi}{3}$.

Since $\frac{2\pi}{3}$ lies between $-\pi$ and π , the principal argument $\operatorname{Arg} z$ is $\frac{2\pi}{3}$. ■

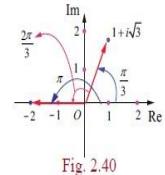


Fig. 2.40

Example 2.26

Find the quotient $\frac{2 \left(\cos \frac{9\pi}{4} + i \sin \frac{9\pi}{4} \right)}{4 \left(\cos \left(\frac{-3\pi}{2} \right) + i \sin \left(\frac{-3\pi}{2} \right) \right)}$ in rectangular form.

Solution

$$\frac{2 \left(\cos \frac{9\pi}{4} + i \sin \frac{9\pi}{4} \right)}{4 \left(\cos \left(\frac{-3\pi}{2} \right) + i \sin \left(\frac{-3\pi}{2} \right) \right)}$$

$$= \frac{1}{2} \left(\cos \left(\frac{9\pi}{4} - \left(-\frac{3\pi}{2} \right) \right) + i \sin \left(\frac{9\pi}{4} - \left(-\frac{3\pi}{2} \right) \right) \right)$$

$$= \frac{1}{2} \left(\cos \left(\frac{9\pi}{4} + \frac{3\pi}{2} \right) + i \sin \left(\frac{9\pi}{4} + \frac{3\pi}{2} \right) \right)$$

$$= \frac{1}{2} \left(\cos \left(\frac{15\pi}{4} \right) + i \sin \left(\frac{15\pi}{4} \right) \right) = \frac{1}{2} \left(\cos \left(4\pi - \frac{\pi}{4} \right) + i \sin \left(4\pi - \frac{\pi}{4} \right) \right)$$

$$= \frac{1}{2} \left(\cos \left(\frac{\pi}{4} \right) - i \sin \left(\frac{\pi}{4} \right) \right) = \frac{1}{2} \left(\frac{1}{\sqrt{2}} - i \frac{1}{\sqrt{2}} \right)$$

$$\frac{2 \left(\cos \frac{9\pi}{4} + i \sin \frac{9\pi}{4} \right)}{4 \left(\cos \left(\frac{-3\pi}{2} \right) + i \sin \left(\frac{-3\pi}{2} \right) \right)} = \frac{1}{2\sqrt{2}} - i \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4} - i \frac{\sqrt{2}}{4}. \text{ Which is in rectangular form.}$$

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Example 2.34

Solve the equation $z^3 + 8i = 0$, where $z \in \mathbb{C}$.

Solution

Let $z^3 + 8i = 0$. Then, we get

$$z^3 = -8i$$

$$= 8(-i) = 8\left(\cos\left(-\frac{\pi}{2} + 2k\pi\right) + i\sin\left(-\frac{\pi}{2} + 2k\pi\right)\right), k \in \mathbb{Z}.$$

$$\text{Therefore, } z = \sqrt[3]{8}\left(\cos\left(\frac{-\pi + 4k\pi}{6}\right) + i\sin\left(\frac{-\pi + 4k\pi}{6}\right)\right), k = 0, 1, 2.$$

Taking $k = 0, 1, 2$, we get,

$$k=0, \quad z = 2\left(\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right) = 2\left(\frac{\sqrt{3}}{2} - i\frac{1}{2}\right) = \sqrt{3} - i$$

$$k=1, \quad z = 2\left(\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right) = 2(0 + i) = 0 + 2i = 2i$$

$$k=2, \quad z = 2\left(\cos\left(\frac{7\pi}{6}\right) + i\sin\left(\frac{7\pi}{6}\right)\right) = 2\left(\cos\left(\pi + \frac{\pi}{6}\right) + i\sin\left(\pi + \frac{\pi}{6}\right)\right) \\ = 2\left(-\cos\left(\frac{\pi}{6}\right) - i\sin\left(\frac{\pi}{6}\right)\right) = 2\left(-\frac{\sqrt{3}}{2} - i\frac{1}{2}\right) = -\sqrt{3} - i.$$

The values of z are $\sqrt{3} - i$, $2i$, and $-\sqrt{3} - i$.

Example 2.35

Find all cube roots of $\sqrt{3} + i$.

Solution

We have to find $(\sqrt{3} + i)^{\frac{1}{3}}$. Let $z = (\sqrt{3} + i)^{\frac{1}{3}}$. Then, $z^3 = \sqrt{3} + i = r(\cos\theta + i\sin\theta)$.

Then, $r = \sqrt{3+1} = 2$, and $\theta = \theta = \frac{\pi}{6}$ ($\because \sqrt{3} + i$ lies in the first quadrant)

$$\text{Therefore, } z^3 = \sqrt{3} + i = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$$

$$\Rightarrow z = \sqrt[3]{2} \left(\cos\left(\frac{\pi + 12k\pi}{18}\right) + i\sin\left(\frac{\pi + 12k\pi}{18}\right)\right), k = 0, 1, 2.$$

Taking $k = 0, 1, 2$, we get

$$k=0, \quad z = 2^{\frac{1}{3}}\left(\cos\frac{\pi}{18} + i\sin\frac{\pi}{18}\right);$$

$$k=1, \quad z = 2^{\frac{1}{3}}\left(\cos\frac{13\pi}{18} + i\sin\frac{13\pi}{18}\right);$$

$$k=2, \quad z = 2^{\frac{1}{3}}\left(\cos\frac{25\pi}{18} + i\sin\frac{25\pi}{18}\right) = 2^{\frac{1}{3}}\left(-\cos\frac{7\pi}{18} - i\sin\frac{7\pi}{18}\right).$$



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