XII-06

Mathematics

Applications of Vector Algebra

Time: 1.15 Marks: 50

Part - A

Answer any five questions **5 x 2 = 10**

1. Find the Vector equation and Cartesian equation of the plane passing through the point with position vector $2\hat{i}+6\hat{j}+3\hat{k}$ **and normal to the vector** $\hat{i}+3\hat{j}+5\hat{k}$

2. The volume of the parallelepiped whose coterminus edges are $7\hat{i}+\lambda\hat{j}-3\hat{k},\ \hat{i}+2\hat{j}-\hat{k},-3\hat{i}+7\hat{j}+5\hat{k}$ **is 90 cubic units. Find the value of** λ

3. If $\hat{a},\hat{b},\hat{c}$ **are three unit vectors such that** $\hat{b}$ **and** $\hat{c}$ **are non-parallel and** $\hat{a}\times(\hat{b}\times\hat{c})=\frac{1}{2}\hat{b}$**, find the angle between** $\hat{a}$ **and** $\hat{c}$

4. For any vector $\hat{a}$ **prove that** $\hat{i}\times(\hat{a}\times\hat{i})+\hat{j}\times(\hat{a}\times\hat{j})+\hat{k}\times(\hat{a}\times\hat{k})=2\hat{a}$

5. Find the non-parametric form of vector equation and cartesian equations of the straight line passing through the point with position vector $4\hat{i}+3\hat{j}-7\hat{k}$ **and parallel to the vector** $2\hat{i}-6\hat{j}+7\hat{k}$

6. Show that the points $(2,3,4),(-1,4,5)$ **and** $(8,1,2)$ **are collinear.**

Part - B

Answer any five questions **5 x 3 = 15**

7. With usual notations, in any triangle ABC, prove the following by vector method.

$a=b\cos C+c\cos B$

8. Prove by vector method that the diagonals of a rhombus bisect each other at right angles.

9. Show that the lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{3}$ **and** $\frac{x-1}{-3}=\frac{y-4}{2}=\frac{z-5}{1}$ **are coplanar. Also find the plane containing these lines.**

10. Prove that $\left[\vec{a}\times\vec{b},\vec{b}\times\vec{c},\vec{c}\times\vec{a}\right]=\left[\vec{a},\vec{b},\vec{c}\right]^2$

11. Find the angle between the line $\vec{r}=(2\hat{i}-\hat{j}+\hat{k})+t(\hat{i}+2\hat{j}-2\hat{k})$ **and the plane** $\vec{r}.(6\hat{i}+3\hat{j}+2\hat{k})=8$

12. Find the image of the point whose position vector is $\hat{i}+2\hat{j}+3\hat{k}$ **in the plane** $\vec{r}.(\hat{i}+2\hat{j}+4\hat{k})=38$

Part - C

Answer any five questions **5 x 5 = 25**

13. Using vector methods, prove that $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$

14. Prove by vector method that the perpendiculars from the vertices to the opposite sides of a triangle are concurrent.

15. $\vec{a}=2\hat{i}+3\hat{j}-\hat{k},\ \vec{b}=3\hat{i}+5\hat{j}+2\hat{k},\vec{c}=-\hat{i}-2\hat{j}+3\hat{k}$**, verify that** $(\vec{a}\times\vec{b})\times\vec{c}=(\vec{a}.\vec{c})\vec{b}-(\vec{b}.\vec{c})\vec{a}$

16. Show that the lines $\vec{r}=(6\hat{i}+\hat{j}+2\hat{k})+s(\hat{i}+2\hat{j}-3\hat{k})$ **are** $\vec{r}=(3\hat{i}+2\hat{j}-2\hat{k})+t(2\hat{i}+4\hat{j}-5\hat{k})$ **are skew lines and hence find the shortest distance between them.**

17. Find the vector parametric, vector non-parametric and Cartesian form of the equation of the plane passing through the points $(-1,2,0),(2,2,-1)$ **and parallel to the straight line** $\frac{x-1}{1}=\frac{2y+1}{2}=\frac{z+1}{-1}$

18. Find the non-parametric and Cartesian equations of the plane passing through the point $(4,2,4)$ **and is perpendicular in the planes** $2x+5y+4z+1=0$ **and** $4x+7y+6z+2=0$

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