IMPORTANAT PROBLEMS

UNIT – 1 – ELECTROSTATICS

1. Calculate the number of electrons in one coulomb of negative charge.

Solution

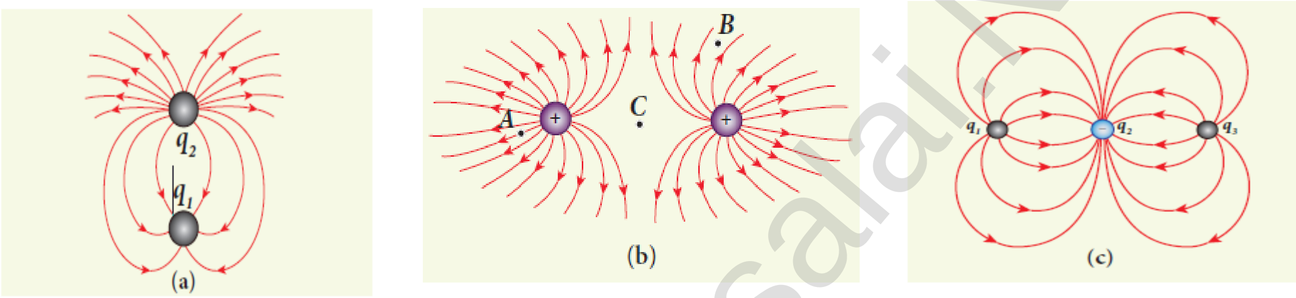
According to the quantisation of charge,

$$q = ne$$

Here q = 1C. So the number of electrons in 1 coulomb of charge is

$n = \frac{q}{e} = \frac{1C}{1.6 \times 10^{-19}} = 6.25 \times 10^{18}$ electrons.

2. The following pictures depict electric field lines for various charge configurations.



(i) In figure (a) identify the signs of two charges and find the ratio $\left|\frac{q_1}{q_2}\right|$

(ii) In figure (b), calculate the ratio of two positive charges and identify the strength of the electric field at three points A, B, and C

[iii] Figure (c) represents the electric field lines for three charges. If q_2 = –20 nC, then calculate the values of q_1 and q_3

Solution

(i) The electric field lines start at q_2 and end at q_1. In figure (a), q_2 is positive and q_1 is negative. The number of lines starting from q_2 is 18 and number of the lines ending at q_1 is 6. So q_2 has greater magnitude. The ratio of $\left|\frac{q_1}{q_2}\right| = \frac{N_1}{N_2} = \frac{6}{18} = \frac{1}{3}$. It implies that $|q_2| = 3|q_1|$.

(ii) In figure (b), the number of field lines emanating from both positive charges are equal (N=18). So the charges are equal. At point A, the electric field lines are denser compared to the lines at point B. So the electric field at point A is greater in magnitude compared to the field at point B. Further, no electric field line passes through C, which implies that the resultant electric field at C due to these two charges is zero.

(iii) In the figure (c), the electric field lines start at q_1 and q_3 and end at q_2. This implies that q_1 and q_3 are positive charges. The ratio of the number of field lines is $\left|\frac{q_1}{q_2}\right| = \frac{8}{16} = \left|\frac{q_2}{q_3}\right| = \frac{1}{2}$, implying that q_1 and q_3 are half of the magnitude of q_2.
So $q_1 = q_3 = +10nC$.

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3. **A sample of HCl gas is placed in a uniform electric field of magnitude 3×10^4 N C^{-1}. The dipole moment of each HCl molecule is 3.4×10^{-30} Cm. Calculate the maximum torque experienced by each HCl molecule.**

<u>Solution</u>

The maximum torque experienced by the dipole is when it is aligned perpendicular to the applied field.

$\tau_{max} = pE \sin 90^0 = 3.4 \times 10^{-30} \times 3 \times 10^4$

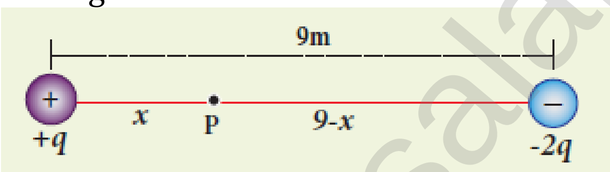
$\tau_{max} = 10.2 \times 10^{-26}$ Nm.

4. **Consider a point charge $+q$ placed at the origin and another point charge $-2q$ placed at a distance of 9 m from the charge $+q$. Determine the point between the two charges at which electric potential is zero.**

<u>Solution</u>

According to the superposition principle, the total electric potential at a point is equal to the sum of the potentials due to each charge at that point.

Consider the point at which the total potential zero is located at a distance x from the charge $+q$ as shown in the figure.



Since the total electric potential at P is zero,

$$V_{tot} = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{x} - \frac{2q}{(9-x)}\right) = 0 \ (or)$$

$$\frac{q}{x} = \frac{2q}{(9-x)} \ \text{(or)}$$

$$\frac{1}{x} = \frac{2}{(9-x)}$$

Hence, x = 3 m.

5. **A water molecule has an electric dipole moment of 6.3×10^{-30} Cm. A sample contains 10^{22} water molecules, with all the dipole moments aligned parallel to the external electric field of magnitude 3×10^5 N C^{-1}. How much work is required to rotate all the water molecules from $\theta = 0^0$ to 90^0 ?**

<u>Solution</u>

When the water molecules are aligned in the direction of the electric field, it has minimum potential energy. The work done to rotate the dipole from $\theta = 0^0$ to 90^0 is equal to the potential energy difference between these two configurations.

$W = \Delta U = U(90^0) - U(0^0)$

From the equation (1.51), we write $U = -pE\cos\theta$, Next we calculate the work done to rotate one water molecule from $\theta = 0^0$ to 90^0.

For one water molecule

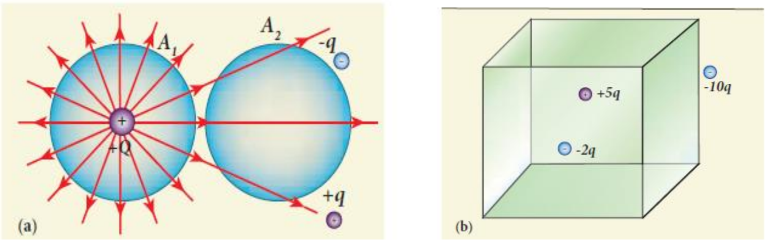
$W = -pE \cos 90^0 + pE \cos 90^0 = pE$

$W = 6.3 \times 10^{-30} \times 3 \times 10^5 = 18.9 \times 10^{-25}$J

For 10^{22} water molecules, the total work done is

$$W_{tot} = 18.9 \times 10^{-25} \times 10^{22} = 18.9 \times 10^{-3} J.$$

6.



(i) In figure (a), calculate the electric flux through the closed areas A_1 and A_2.

(ii) In figure (b), calculate the electric flux through the cube

Solution

(i) In figure (a), the area A_1 encloses the charge Q. So electric flux through this closed surface A_1 is $\frac{Q}{\epsilon_0}$. But the closed surface A_2 contains no charges inside, so electric flux through A_2 is zero.

(ii) In figure (b), the net charge inside the cube is $3q$ and the total electric flux in the cube is therefore $\varphi_E = \frac{3q}{\epsilon_0}$. Note that the charge $-10\,q$ lies outside the cube and it will not contribute the total flux through the surface of the cube.

7. Parallel plate capacitor has square plates of side 5 cm and separated by a distance of 1 mm. (a) Calculate the capacitance of this capacitor. (b) If a 10 V battery is connected to the capacitor, what is the charge stored in any one of the plates? (The value of $\epsilon_0 = 8.85 \times 10^{-12}\ N^{-1}\,m^{-2}\,C^2$)

Solution

(a) The capacitance of the capacitor is

$$C = \frac{\epsilon_0 A}{d} = \frac{8.85\times10^{-12}\times25\times10^{-4}}{1\times10^{-3}}$$

$$= 221.2 \times 10^{-13}\ F$$

$$C = 22.12 \times 10^{-12} F = 22.12\ F$$

(b) The charge stored in any one of the plates is Q = CV, Then

$$Q = 22.12 \times 10^{-12} \times 10 = 221.2 \times 10^{-12}\ C = 221.2\ pC.$$

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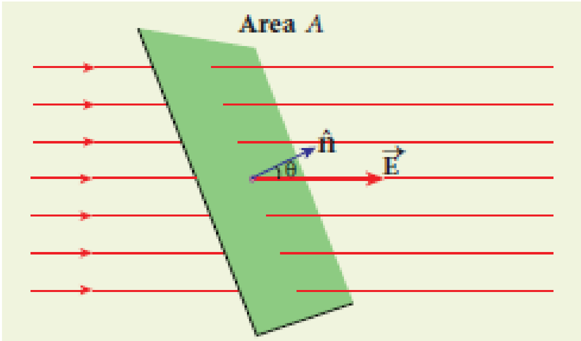
8. **Calculate the electric flux through the rectangle of sides 5 cm and 10 cm kept in the region of a uniform electric field 100 NC^{-1}. The angle θ is 60^{o}. If θ becomes zero, what is the electric flux?**



Solution

$\varphi_E = \vec{E}.\vec{A}$ = EA cos θ

= 100 x 5 x 10 x 10-4 x cos 600

φ_E = 0.25 Nm2C-1

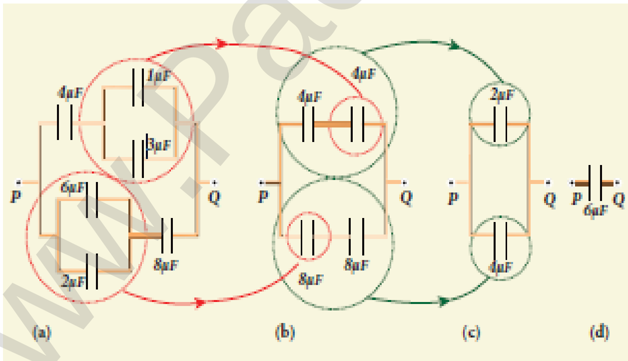
For θ = 0o,

$\varphi_E = \vec{E}.\vec{A}$ = EA

= 100 x 5 x 10 x 10-4

= 0.5 Nm2C-1

9. **Find the equivalent capacitance between P and Q for the configuration shown below in the figure (a).**



Solution

The capacitors 1 µF and 3 µF are connected in parallel and 6 µF and 2 µF are also separately connected in parallel. So these parallel combinations reduced to equivalent single capacitances in their respective positions, as shown in the figure (b).

C_{eq} = 1 + 3 = 4 µF

C_{eq} = 6 + 2 = 8 µF

From the figure (b), we infer that the two 4 µF capacitors are connected in series and the two 8 µF capacitors are connected in series. By using formula for the series, we can reduce to their equivalent capacitances as shown in figure (c).

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$\frac{1}{C_{eq}} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$ $\Rightarrow C_{eq} = 2\ \mu F$

And

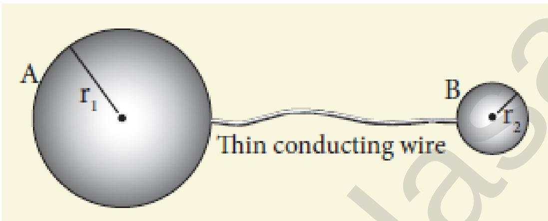
$\frac{1}{C_{eq}} = \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$ $\Rightarrow C_{eq} = 4\ \mu F$

From the figure (c), we infer that 2 µF and 4 µF are connected in parallel. So the equivalent capacitance is given in the figure (d).

$C_{eq} = 2+4 = 6\ \mu F$

Thus the combination of capacitances in figure (a) can be replaced by a single capacitance 6 µF.

10. Two conducting spheres of radius r1 = 8 cm and r2 = 2 cm are separated by a distance much larger than 8 cm and are connected by a thin conducting wire as shown in the figure. A total charge of Q = +100 nC is placed on one of the spheres. After a fraction of a second, the charge Q is redistributed and both the spheres attain electrostatic equilibrium.



(a) Calculate the charge and surface charge density on each sphere.
(b) Calculate the potential at the surface of each sphere.

Solution

(a) The electrostatic potential on the surface of the sphere A is VA = $\frac{1}{4\pi\epsilon_0}\frac{q_1}{r_1}$

(b) The electrostatic potential on the surface of the sphere B is VB = $\frac{1}{4\pi\epsilon_0}\frac{q_2}{r_2}$

Since $V_A = V_B$. We have

$\frac{q_1}{r_1} = \frac{q_2}{r_2} \Rightarrow q1 = \left(\frac{r_1}{r_2}\right) q_2$

But from the conservation of total charge, $Q = q_1 + q_2$, we get $q_1 = Q - q_2$. By substituting this in the above equation,

$Q - q_2 = \left(\frac{r_1}{r_2}\right) q_2$

so that $q_2 = Q\left(\frac{r_2}{r_1+r_2}\right)$

Therefore

$Q_2 = 100 \times 10\text{-}9 \times \left(\frac{2}{10}\right) = 20nC$

and $q_1 = Q - q_2 = 80nC$

The electric charge density on sphere A is $\sigma_1 = \frac{q_1}{4\pi r_1^2}$

The electric charge density on sphere B is $\sigma_2 = \frac{q_2}{4\pi r_2^2}$

Therefore,

$\sigma_1 = \frac{80 \times 10^{-9}}{4\pi \times 64 \times 10^{-4}} = 0.99 \times 10^{-6}\ Cm^{-2}$

and

$\sigma_2 = \frac{20 \times 10^{-9}}{4\pi \times 4 \times 10^{-4}} = 3.9 \times 10^{-6}\ Cm^{-2}$

Note that the surface charge density is greater on the smaller sphere compared to the larger sphere ($\sigma_2 \approx 4\sigma_1$) which confirms the result $\frac{\sigma_1}{\sigma_2} = \frac{r_1}{r_2}$

The potential on both spheres is the same. So we can calculate the potential on any one of the spheres.

$V_A = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_1} = \frac{9 \times 10^9 \times 80 \times 10^{-9}}{8 \times 10^{-2}} = 9$ kV.

11. Dielectric strength of air is 3×10^6 V m^{-1}. Suppose the radius of a hollow sphere in the Van de Graff generator is R = 0.5 m, calculate the maximum potential difference created by this Van de Graaff generator.

Solution

The electric field on the surface of the sphere is given by (by Gauss law)

$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^2}$

The potential on the surface of the hollow metallic sphere is given by

$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{R} = ER$

Since $V_{max} = E_{max} R$

Here $E_{max} = 3 \times 10^6$ Vm^{-1}. So the maximum potential difference created is given by

$V_{max} = 3 \times 10^6 \times 0.5$

$= 1.5 \times 10^6$ V (or) 1.5 million volt.

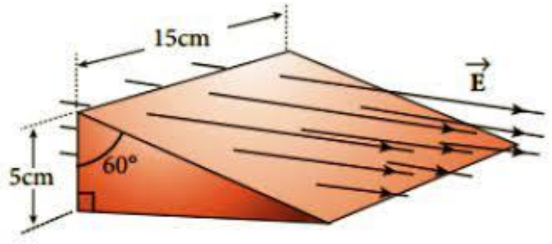
12. When two objects are rubbed with each other, approximately a charge of 50 nC can be produced in each object. Calculate the number of electrons that must be transferred to produce this charge.

Solution

Given charge produced q = 50 nC = 50×10^{-9} C

Number of electrons transferred $n = \frac{q}{e} = \frac{50 \times 10^{-9}}{1.6 \times 10^{-16}} = 31.25 \times 10^{10}$

13. A closed triangular box is kept in an electric field of magnitude E = 2 x10^3 NC^{-1} as shown in the figure. Calculate the electric flux through the
a) Vertical rectangular surface b) Slanted surface and c) Entire surface.



Solution

(a) Electric flux through the vertical rectangular surface

ϕ = EA cos 0^0 = 2 x 10^3 x (0.5 x 0.05) = 15 Nm^2 C^{-1}

(b) Electric flux through the slanted surface Q = EA cos 60^0 = $\frac{EA}{2}$

From figure, cos 60^0 = $\frac{PQ}{QR}$, $\frac{1}{2}$ = $\frac{0.05}{QR}$, QR = 0.1 m

Area of slanted surface = 0.15 x 0.1 = 15 x 10^{-3} m^2

Flux linked ϕ = $\frac{2\times10^{-3}\times15\times10^{-3}}{2}$ = 15 Nm^2 C^{-1}

This acts opposite to the flux due to vertical surface.

(c) Flux due to entire surface = Flux due to (vertical rectangular surface + slanted surface + two ends) = 15-15+0+0 = 0 = zero.

Unit -2 – current electricity

1. Compute the current in the wire if a charge of 120 C is flowing through a copper wire in 1 minute.

Solution

The current (rate of flow of charge) in the wire is

$I = \frac{Q}{t} = \frac{120}{60} = 2$ A

2. If an electric field of magnitude 570 N C^{-1}, is applied in the copper wire, find the acceleration experienced by the electron.

Solution

E = 570 N C^{-1}, e = 1.6 × 10^{-19} C,
m = 9.11 × 10^{-31} kg and a = ?

$F = ma = eE$

$a = \frac{eE}{m} = \frac{570 \times 1.6 \times 10^{-19}}{9.11 \times 10^{-31}}$

$= \frac{912 \times 10^{-19} \times 10^{-31}}{9.11}$

= 1.001 x 10^{14} ms^{-2}

3. **A copper wire of cross-sectional area 0.5 mm^2 carries a current of 0.2 A. If the free electron density of copper is $8.4 \times 10^{28}\ m^{-3}$ then compute the drift velocity of free electrons.**

<u>Solution</u>

The relation between drift velocity of electrons and current in a wire of cross-sectional area A is

$$v_d = \frac{I}{ne\,A} = \frac{0.2}{8.4 \times 10^{28} \times 1.6 \times 10^{-19} \times 0.5 \times 10^{-6}}$$

$= 0.03 \times 10^{-3}\ m\ s^{-1}$.

4. **Determine the number of electrons flowing per second through a conductor, when a current of 32 A flows through it.**

<u>Solution</u>

I = 32 A , t = 1 s

Charge of an electron, e = 1.6×10^{-19} C

The number of electrons flowing per second, n =?

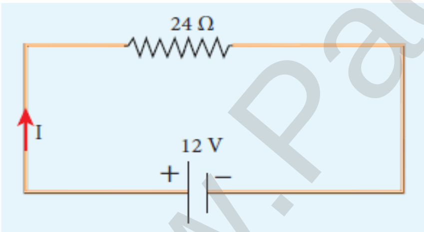
$$I = \frac{q}{t} = \frac{ne}{t}$$

$$n = \frac{It}{e}$$

$$n = \frac{32 \times 1}{1.6 \times 10^{-19} C}$$

$n = 20 \times 10^{19} = 2 \times 10^{20}$ electrons.

5. **A potential difference across 24 Ω resistor is 12 V. What is the current through the resistor?**



<u>Solution</u>

V = 12 V and R = 24 Ω

Current, I = ?

From Ohm's law, $I = \frac{V}{R} = \frac{12}{24}$ = 05.A

6. **The resistance of a wire is 20 Ω. What will be new resistance, if it is stretched uniformly 8 times its original length?**

<u>Solution</u>

R_1 = 20 Ω, R_2 = ?

Let the original length of the wire (l_1) be l.

New length, $l_2 = 8l_1$ (i.,e) $l_2 = 8l$

Original resistance, $R_1 = \rho \frac{l_1}{A_1}$

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New resistance $R_2 = \rho \frac{l_2}{A_2} = \frac{\rho(8l)}{A_2}$

Though the wire is stretched, its volume remains unchanged.

Initial volume = Final volume

$A_1 l_1 = A_2 l_2$, $A_1 l = A_2 (8l)$

$\frac{A_1}{A_2} = \frac{8l}{l} = 8$

By dividing equation for R_2 by equation for R_1, we get

$$\frac{R_2}{R_1} = \frac{\rho(8l)}{A_2} \times \frac{A_1}{\rho l}$$

$$\frac{R_2}{R_1} = \frac{A_1}{A_2} \times 8$$

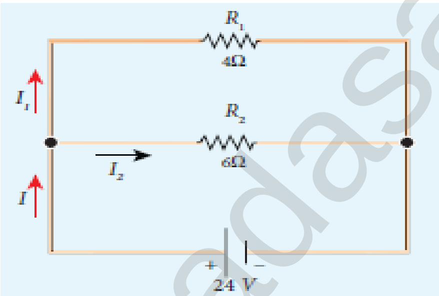
Substituting the value of $\frac{A_1}{A_2}$, we get

$$\frac{R_2}{R_1} = 8 \times 8 = 64$$

$$R_2 = 64 \times 20 = 1280\ \Omega$$

Hence, stretching the length of the wire has increased its resistance.

7. Calculate the equivalent resistance in the following circuit and also find the values of current *I*, *I₁* and *I₂* in the given circuit.



Solution

Since the resistances are connected in parallel, the equivalent resistance in the circuit is

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{4} + \frac{1}{6}$$

$$\frac{1}{R_p} = \frac{5}{12}\ \Omega \text{ or } R_p = \frac{12}{5}\ \Omega$$

The resistors are connected in parallel, the potential difference (voltage) across them is the same.

$$I_1 = \frac{V}{R_1} = \frac{24V}{4\Omega} = 6\text{ A}$$

$$I_2 = \frac{V}{R_2} = \frac{24}{6} = 4\text{ A}$$

The current I is the sum of the currents in the two branches. Then, $I = I_1 + I_2 = 6\text{ A} + 4\text{ A} = 10\text{ A}$

8. **Two resistors when connected in series and parallel, their equivalent resistances are 15 Ω and $\frac{56}{15}$ Ω respectively. Find the values of the resistances.**

Solution

$R_s = R_1 + R_2 = 15\ \Omega$ (1)

$R_p = \frac{R_1 R_2}{R_1 + R_2} = \frac{56}{15}\ \Omega$ (2)

From equation (1) substituting for $R_1 + R_2$ in equation (2)

$$\frac{R_1 R_2}{R_1 + R_2} = \frac{56}{15}\ \Omega$$

$\therefore R_1 R_2 = 56$

$R_2 = \frac{56}{R_1}\ \Omega$ (3)

Substituting for R_2 in equation (1) from equation (3)

$R_1 = \frac{56}{R_1} = 15$

Then, $\frac{R_1^2 + 56}{R_1} = 15$

$R_1^2 + 56 = 15R_1$

$R_1^2 - 15 R_1 + 56 = 0$

The above equation can be solved using factorisation.

$R_1 = 8\ \Omega$ (or) $R_1 = 7\ \Omega$

If $R_1 = 8\ \Omega$

Substituting in equation (1)

$8 + R_2 = 15$

$R_2 = 15 - 8 = 7\ \Omega$,

$R_2 = 7\ \Omega$ i.e , (when $R_1 = 8\ \Omega$; $R_2 = 7\ \Omega$)

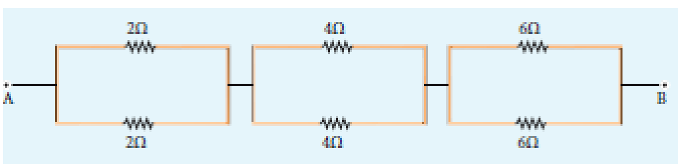
If $R_1 = 7\ \Omega$

Substituting in equation (1)

$7 + R_2 = 15$

$R_2 = 8\ \Omega$, i.e , (when $R_1 = 7\ \Omega$; $R_2 = 8\ \Omega$)

9. **Calculate the equivalent resistance between A and B in the given circuit.**



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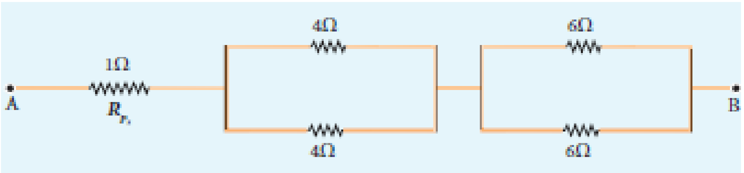
Solution

In all the sections, the resistors are connected in parallel.

Section 1

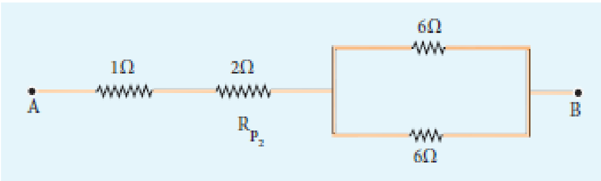
$$\frac{1}{R_{p_1}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_{p_1}} = \frac{1}{2} + \frac{1}{2} = \frac{2}{2} \quad R_{p_1} = 1\Omega$$



Section II

$$\frac{1}{R_{p_2}} = \frac{1}{4} + \frac{1}{4} = \frac{2}{4} \quad \frac{1}{R_{p_2}} = \frac{1}{2}, \quad R_{p_2} = 2\Omega$$



Section III

$$\frac{1}{R_{p_3}} = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} \quad \frac{1}{R_{p_3}} = \frac{1}{3}, \quad R_{p_3} = 3\Omega$$

Equivalent resistance is given by

$R = R_{p_1} + R_{p_2} + R_{p_3}$

$R = 1\ \Omega + 2\ \Omega + 3\ \Omega = 6\ \Omega$

The circuit becomes,

Equivalent resistance between A and B is

10. If the resistance of coil is 3 Ω at 20⁰ C and α = 0.004/⁰C then determine its resistance at 100⁰ C.

Solution

$R_0 = 3\ \Omega$, $T = 100^0$ C, $T_0 = 20^0$ C

$a = 0.004/^0$ C, $R_T = ?$

$R_T = R_0 (1 + a(T - T_0))$

$R_{100} = 3(1 + 0.004 \times 80)$

$R_{100} = 3.96\ \Omega$

11. Resistance of a material at 20^0 C and 40^0 C are 45 Ω and 85 Ω respectively. Find its temperature coefficient of resistivity.

<u>Solution</u>

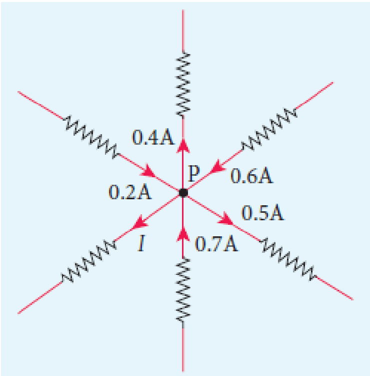
$T_o = 20^0$ C, T = 40^0 C, R_o= 45 Ω , R = 85 Ω

$$\alpha = \frac{1}{R_0}\frac{\Delta R}{\Delta T}$$

$$\alpha = \frac{1}{45}\left(\frac{85-45}{40-20}\right) = \frac{1}{45}(2)$$

$$\alpha = 0.044 \text{ per}^0 C$$

12. For the given circuit find the value of *I*.



<u>Solution</u>

Applying Kirchhoff 's rule to the point P in the circuit,

The arrows pointing towards P are positive and away from P are negative.

Therefore, 0.2A – 0.4A + 0.6A – 0.5A + 0.7A – *I* = 0

1.5A – 0.9A – *I* = 0

0.6A – I = 0

I = 0.6 A

13. In a Wheatstone's bridge P = 100 Ω, Q = 1000 Ω and R = 40 Ω. If the galvanometer shows zero deflection, determine the value of S.

<u>Solution</u>

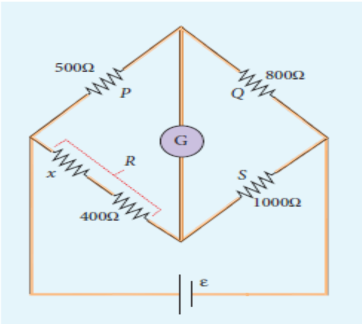
$$\frac{P}{Q} = \frac{R}{S}$$

$$S = \frac{Q}{P} \times R$$

$$S = \frac{1000}{100} \times 40 \quad S = 400\ \Omega$$

14. What is the value of x when the Wheatstone's network is balanced?

P = 500 Ω, Q = 800 Ω, R = x + 400, S = 1000 Ω



Solution

$\frac{P}{Q} = \frac{R}{S}$, when the network is balanced

$$\frac{500}{800} = \frac{x+400}{1000}$$

$$x + 400 = \frac{5}{8} \times 1000$$

$$x + 400 = 625$$

$$x = 625 - 400$$

$$x = 225\ \Omega$$

15. In a meter bridge experiment with a standard resistance of 15 Ω in the right gap, the ratio of balancing length is 3:2. Find the value of the other resistance.

Solution

Q = 15 Ω, $l_1:l_2$ = 3:2

$$\frac{l_1}{l_2} = \frac{3}{2}$$

$$\frac{P}{Q} = \frac{l_1}{l_2}$$

$$P = Q\frac{l_1}{l_2}$$

$$P = 15 \times \frac{3}{2} = 22.5\ \Omega$$

16. In a meter bridge experiment, the value of resistance in the resistance box connected in the right gap is 10 Ω. The balancing length is l_1 = 55 cm. Find the value of unknown resistance.

Solution

Q = 10 Ω

$$\frac{P}{Q} = \frac{l_1}{100-l_1} = \frac{l_1}{l_2}$$

$$P = Q \times \frac{l_1}{100 - l_1}$$

$$P = \frac{10 \times 55}{100 - 55}$$

$$P = \frac{550}{45} = 12.2\ \Omega$$

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17. Find the heat energy produced in a resistance of 10 Ω when 5 A current flows through it for 5 minutes.

Solution

R = 10 Ω, I = 5 A, t = 5 minutes = 5 × 60 s
$H = I^2 Rt$
= 5^2 × 10 × 5 × 60
= 25 × 10 × 300
= 25 × 3000
= 75000 J (or) 75 kJ

18. A Copper wire of 10^{-6} m^2 area of cross – section carries a current of 2A. If the number of free electrons per cubic meter in the wire is 8 x 10^{28}, Calculate the current density and average drift velocity of electrons.

Solution

Given A = 10^6 m^2, I = 2A, n = 8 x 10^{28}

Current density J = $\frac{I}{A}$ = $\frac{2}{10^{-6}}$ = 2 x 10^6 A m^{-2}

Drift velocity V_d = $\frac{1}{neA}$ = $\frac{2}{8\times10^{-6}\times1.6\times10^{-19}\times10^{-6}}$ = $\frac{2\times10^{-3}}{12.8}$ = 15.6 x 10^{-5} ms^{-1}

19. The resistance of a nichrome wire at 20^{0}C is 10Ω. If its temperature coefficient of resistivity of nichrome is 0.004 / ^{0}C, find the resistance of the wire at boiling point of water. Comment on the result.

Solution

Given R_0 = 10Ω, a = 0.004/^{0}C, t = 100^{0}C

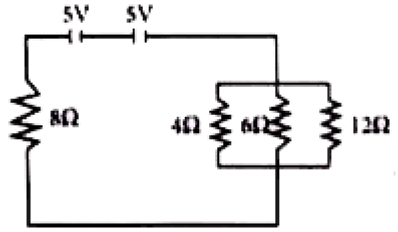
$R_T = R_0 [1 + \alpha(T - T_0)]$ = 10 [1+ 0.004(100-20)]

= 10 (1+0.32) = 10 x 1.32 = 13.2Ω

As the temperature increases the resistance of the wire also increases.

20. Two cells each of 5V are connected in series across a 8Ω resistor and three parallel resistors of 4Ω, 6Ω and 12Ω. Draw a circuit diagram for the above arrangement.
Calculate (i) The current drawn from the cell (ii) current through each resistor.

Solution



For the parallel combination $\frac{1}{R_p} = \frac{1}{4} + \frac{1}{6} + \frac{1}{12} = \frac{12}{24} = \frac{1}{2}$, R_p = 2Ω

Total resistance in the circuit R = 8 +2 = 10 Ω

(i) Current drawn from the cell I = $\frac{\xi}{R} = \frac{5+5}{10}$ = 1A

(ii) Voltage across the parallel resistors = IR = 1 x 2 = 2V

Current through 4 Ω is $\frac{2}{4}$ = 0.5 A, Current through6 Ω is $\frac{2}{6}$ = 0.33 A

Current through 12 Ω is $\frac{2}{12}$ = 0.17 A

21. In a potentiometer arrangement, a cell of emf 1.25 V gives a balance point at 35 cm length of the wire. If the cell is replaced by another cell and the balance point shifts to 63 cm, what is the emf of the second cell?

Solution

$$\frac{\xi_2}{\xi_1} = \frac{l_2}{l_1} = \frac{\xi_2}{1.25} = \frac{63}{35}, \ \xi_2 = \frac{1.25 \times 63}{35} = 2.25 \text{ V}$$

Unit – 3 – Magnetism and magnetic effects of electric current

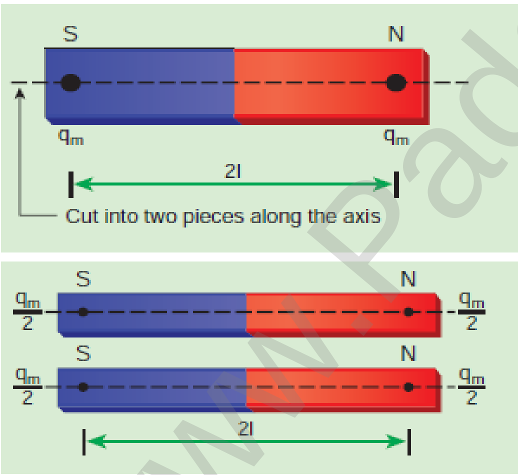
1. **Let the magnetic moment of a bar magnet be $\vec{p}_m$ whose magnetic length is $d = 2l$ and pole strength is q_m. Compute the magnetic moment of the bar magnet when it is cut into two pieces**

(a) along its length

(b) perpendicular to its length.

Solution

(a) a bar magnet cut into two pieces along its length:



When the bar magnet is cut along the axis into two pieces, new magnetic pole strength is

$q'_m = \frac{qm}{2}$ but magnetic length does not change. So, the magnetic moment is

$$p'_m = q'_m 2l$$

$$p'_m = \frac{qm}{2} 2l = \frac{1}{2}(q_m 2l) = \frac{1}{2} p_m$$

In vector notation, $\vec{p}'_m = \frac{1}{2}\vec{P}_m$

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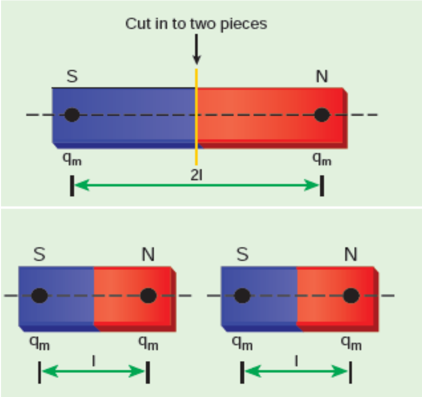
(b) a bar magnet cut into two pieces perpendicular to the axis.

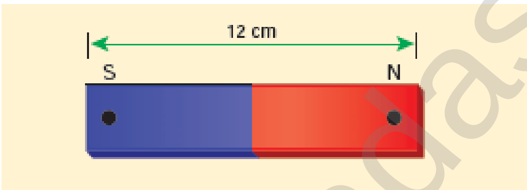


When the bar magnet is cut perpendicular to the axis into two pieces, magnetic pole strength will not change but magnetic length will be halved. So the magnetic moment is

$$p'_m = q_m \times \frac{1}{2}(2l) = \frac{1}{2}(q_m.\,2l) = \frac{1}{2}\,p_m$$

In vector notation, $\vec{p}'_m = \frac{1}{2}\,\vec{P}_m$

2. **Compute the magnetic length of a uniform bar magnet if the geometrical length of the magnet is 12 cm. Mark the positions of magnetic pole points.**



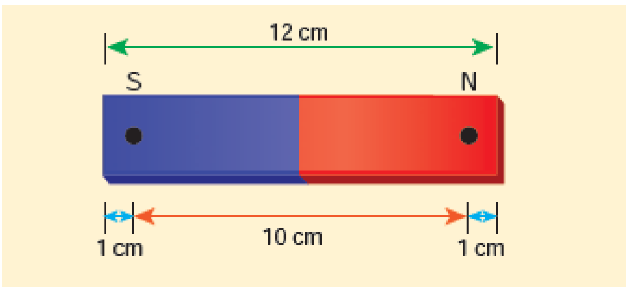
Solution

Geometric length of the bar magnet is 12 cm

Magnetic length = $\frac{5}{6}$ x (Geometric length)

$= \frac{5}{6} \times 12 = 10$ cm

In this figure, the dot implies the pole points.



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3. **The repulsive force between two magnetic poles in air is 9×10^{-3} N. If the two poles are equal in strength and are separated by a distance of 10 cm, calculate the pole strength of each pole.**

Solution

The magnitude of the force between two poles is given by

$$F = k\frac{q_{m_A} q_{m_B}}{r^2}$$

Given : $F = 9 \times 10^{-3}$ N, $r = 10$ cm $= 10 \times 10^{-2}$ m

Since $q_{m_A} = q_{m_B} = q_m$, we have

$$9 \times 10^{-3} = 10^{-7} \times \frac{q_m^2}{(10 \times 10^{-2})} \Rightarrow = q_m = 30 \text{ N T}^{-1}.$$

4. **Compute the intensity of magnetisation of the bar magnet whose mass, magnetic moment and density are 200 g, 2 A m^2 and 8 g cm^{-3}, respectively.**

Solution

Density of the magnet is

$$\text{Density} = \frac{Mass}{Volume} \Rightarrow \text{Volume} = \frac{Mass}{Density}$$

$$\text{Volume} = \frac{200 \times 10^{-3} kg}{(8 \times 10^{-3} kg) 10^{6}\, m^{-3}} = 25 \times 10^{-6} \text{ m}^3$$

Magnitude of magnetic moment $p_m = 2\text{Am}^2$

Intensity of magnetization,

$$M = \frac{Magenetic\ moment}{Voume} = \frac{2}{25 \times 10^{-6}}$$

$$M = 0.8 \times 10^{5} \text{ Am}^{-1}.$$

5. **Using the relation $\vec{B} = \mu_0 (\vec{H} + \vec{M})$, show that $x_m = \mu_r - 1$.**

Solution

$\vec{B} = \mu_0 (\vec{H} + \vec{M})$

But from equation (3.33), in vector form,

$\vec{M} = x_m \vec{H}$

Hence, $\vec{B} = \mu_0 (x_m + 1) \vec{H} \Rightarrow \vec{B} = \mu \vec{H}$

Where, $\mu = \mu_0 (x_m + 1) \Rightarrow x_m + 1 = \frac{\mu}{\mu_0} = \mu_r$

$\Rightarrow x_m = \mu_r - 1.$

6. **Two materials X and Y are magnetised whose values of intensity of magnetisation are 500 A m^{-1} and 2000 A m^{-1} respectively. If the magnetising field is 1000 A m^{-1}, then which one among these materials can be easily magnetized?**

Solution

The susceptibility of material X is

$$x_{m,X} = \left|\frac{\vec{M}}{\vec{H}}\right| = \frac{500}{1000} = 0.5$$

The susceptibility of material Y is

$$x_{m,Y} = \left|\frac{\vec{M}}{\vec{H}}\right| = \frac{2000}{1000} = 2$$

Since, susceptibility of material Y is greater than that of material X, which implies that material Y can be easily magnetized.

7. **A coil of a tangent galvanometer of diameter 0.24 m has 100 turns. If the horizontal component of Earth's magnetic field is 25 × 10^{-6} T then, calculate the current which gives a deflection of 60^0.**

Solution

The diameter of the coil is 0.24 m. Therefore, radius of the coil is 0.12 m.

Number of turns is 100 turns.

Earth's magnetic field is 25 × 10^{-6} T

Deflection is

$\theta = 60^0 \Rightarrow \tan 60^0 = \sqrt{3} = 1.732$

$$I = \frac{2RB_H}{\mu_0 N} \tan\theta$$

$$= \frac{2 \times 0.12 \times 25 \times 10^{-6}}{4 \times 10^{-7} \times 3.14 \times 100} \text{ x } 1.732 = 0.82 \text{ x } 10^{-1} A$$

I = 0.082 A

8. **Compute the magnitude of the magnetic field of a long, straight wire carrying a current of 1 A at distance of 1m from it. Compare it with Earth's magnetic field.**

Solution

Given that I = 1 A and radius r = 1 m

$$B_{\text{straightwire}} = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 1}{2\pi \times 1} = 2 \text{ x } 10^{-7} \text{ T}$$

But the Earth's magnetic field is $B_{Earth} \sim 10^{-5}$ T

So, $B_{straightwire}$ is one hundred times smaller than B_{Earth}.

9. **An electron moving perpendicular to a uniform magnetic field 0.500 T undergoes circular motion of radius 2.50 mm. What is the speed of electron?**

Solution

Charge of an electron $q = -1.60 \times 10^{-19}$ C $\Rightarrow |q| = 160 \times 10^{-19}$ C

Magnitude of magnetic field $B = 0.500$ T

Mass of the electron, $m = 9.11 \times 10^{-31}$ kg

Radius of the orbit, $r = 2.50$ mm $= 2.50 \times 10^{-3}$ m

Speed of the electron, $v = |q| \frac{rB}{m}$

$$v = 1.60 \times 10^{-19} \times \frac{2.50 \times 10^{-3} \times 0.500}{9.11 \times 10^{-31}}$$

$$v = 2.195 \times 10^{8} \text{ ms}^{-1}$$

10. **Suppose a cyclotron is operated to accelerate protons with a magnetic field of strength 1 T. Calculate the frequency in which the electric field between two Dees could be reversed.**

Solution

Magnetic field $B = 1$ T

Mass of the proton, $m_p = 1.67 \times 10^{-27}$ kg

Charge of the proton, $q = 1.60 \times 10^{-19}$ C

$$f = \frac{qB}{2\pi m_p} = \frac{(1.60 \times 10^{-19})(1)}{2(3.14)(1.67 \times 10^{-27})}$$

$$= 15.3 \times 10^{6} \text{ Hz} = 15.3 \text{ MHz}$$

11. **The resistance of a moving coil galvanometer is made twice its original value in order to increase current sensitivity by 50%. Find the percentage change in voltage sensitivity.**

Solution

Voltage sensitivity is $V_s = \frac{I_s}{R_g}$

When the resistance is doubled, then new resistance is $R_g' = 2R_g$

Increase in current sensitivity is $I_s' = \left(1 + \frac{50}{100}\right) I_s = \frac{3}{2} I_s$

The new voltage sensitivity is

$$V_s' = \frac{\frac{3}{2} I_s}{2R_g} = \frac{3}{4} V_s$$

Hence the voltage sensitivity decreases. The percentage decrease in voltage sensitivity is

$$\frac{V_s - V_s'}{V_s} \times 100\ \% = 25\%.$$

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12. A bar magnet having a magnetic moment $\vec{p}_m$ is cut into four pieces i.e., first cut in two places along the axis of the magnet and each piece is further cut into two pieces. Compute the magnetic moment of each piece.

Solution

(a) When the bar magnet is cut along its axis into two pieces, new magnetic pole strength is $q_m/2$. Magnetic length does not change.

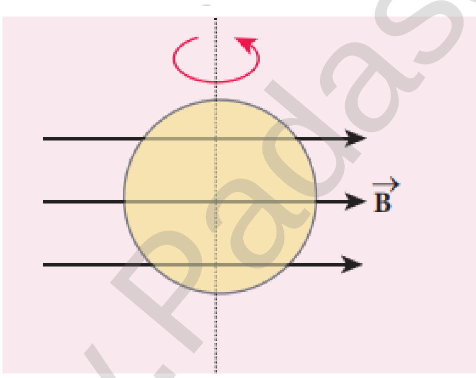
New Magnetic moment $\vec{p}_m = \left(\frac{qm}{2}\right) 2l = \frac{qm\ 2l}{2} = \frac{\vec{p}_m}{2}$

(b) When the piece are again cut into two pieces perpendicular to the axis, magnetic pole strength is $q_m/2$ and magnetic length is l.

$\therefore$ New magnetic moment $(\vec{p}_m)_{new} = \left(\frac{qm}{2}\right) l = \frac{qm\ (2l)}{4} = \frac{\vec{p}_m}{4}$

Unit – 4 – Electromagnetic induction and alternating current

1. **A circular loop of area $5 \times 10^{-2}\ m^2$ rotates in a uniform magnetic field of 0.2T. If the loop rotates about its diameter which is perpendicular to the magnetic field as shown in figure. Find the magnetic flux linked with the loop when its plane is (i) normal to the field (ii) inclined 60^o to the field and (iii) parallel to the field.**



Solution

$A = 5 \times 10^{-2}\ m^2$; $B = 0.2$ T

(i) $\theta = 0$;

$\varphi_B = BA \cos\theta = 0.2 \times 5 \times 10^{-2} \times \cos 0^0$

$\varphi_B = 1 \times 10^{-2}$ Wb

(ii) $\theta = 90° - 60° = 30°$;

$\varphi_B = BA \cos\theta = 0.2 \times 5 \times 10^{-2} \times \cos 30^0$

$\varphi_B = 1 \times 10^{-2} \times \frac{\sqrt{3}}{2} = 8.66 \times 10^{-3}$ Wb

(iii) $\theta = 90°$

$\varphi_B = BA \cos 90^0 = 0$

2. **A closed coil of 40 turns and of area 200 cm², is rotated in a magnetic field of flux density 2 Wb m⁻². It rotates from a position where its plane makes an angle of 30° with the field to a position perpendicular to the field in a time 0.2 s. Find the magnitude of the emf induced in the coil due to its rotation.**

Solution

N = 40 turns; $B = 2\ Wb\ m^{-2}$ $A = 200\ cm^2 = 200 \times 10^{-4}\ m^2$

Initial flux , $\varphi_i = BA\cos\theta$

$= 2 \times 200 \times 10^{-4} \times \cos 60^0$

Since $\theta = 90^0 - 30^0 = 60^0$

$\varphi_i = 2 \times 10^{-2}$ Wb

Final flux, $\varphi_f = BA\cos\theta$

$= 2 \times 200 \times 10^{-4} \times \cos 0^0$

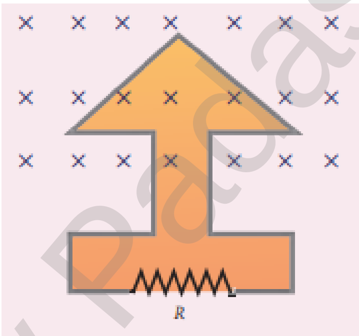
Since $\theta = 0^0$

$\varphi_f = 4 \times 10^{-2}$ Wb

The magnitude of the induced emf is

$$\varepsilon = N\frac{d\varphi_B}{dt} = \frac{40 \times (4\times10^{-2} - 2\times10^{2})}{0.2} = 4V$$

3. **The magnetic flux passes perpendicular to the plane of the circuit and is directed into the paper. If the magnetic flux varies with respect to time as per the following relation: $\Phi_B = +(2t^3 + 3t^2 + 8t + 5)$ mWb , what is the magnitude of the induced emf in the loop when t = 3 s? Find out the direction of current through the circuit.**



Solution

$\Phi_B = +(2t^3 + 3t^2 + 8t + 5)$ mWb; N = 1; t = 3s

(i) $\varepsilon = \frac{d(N\varphi_B)}{dt}$

$= \frac{d}{dt}(2t^3 + 3t^2 + 8t + 5) \times 10^{-3}$

$= (6t^2 + 6t + 8) \times 10^{-3}$ V

At t = 3s,

$\varepsilon = [(6 \times 9) + (6 \times 3) + 8] \times 10^{-3}$

$= 80 \times 10^{-3}$V $= 80$ mV

(ii) As time passes, the magnetic flux linked with the loop increases. According to Lenz's law, the direction of the induced current should be in a way so as to oppose the flux increase. So, the induced current flows in such a way to produce a magnetic field opposite to the given field. This magnetic field is perpendicularly outwards. Therefore, the induced current flows in anticlockwise direction.

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4. **A conducting rod of length 0.5 m falls freely from the top of a building of height 7.2 m at a place in Chennai where the horizontal component of Earth's magnetic field is 4.04 × 10^{-5} T. If the length of the rod is perpendicular to Earth's horizontal magnetic field, find the emf induced across the conductor when the rod is about to touch the ground. (Assume that the rod falls down with constant acceleration of 10 m s–2)**

Solution

l = 0.5 m; h = 7.2 m; u = 0 m s^{-1}; g = 10 m s^{-2}; BH = 4.04 × 10^{-5} T

The final velocity of the rod is

$\upsilon^2 = u^2 + 2\,gh = 0 + (2 \times 10 \times 7.2) = 144$

$\upsilon = 12 ms^1$

The magnitude of the induced emf when the rod is about to touch the ground is

$\varepsilon = B_H l\upsilon = 4.04 \times 10^{-5} \times 0.5 \times 12$

$= 242.4\mu V$

5. **A solenoid of 500 turns is wound on an iron core of relative permeability 800. The length and radius of the solenoid are 40 cm and 3 cm respectively. Calculate the average emf induced in the solenoid if the current in it changes from 0 to 3 A in 0.4 second.**

Solution

N = 500 turns; μr = 800 ; l = 40 cm = 0.4 m; r = 3 cm = 0.03 m; di = 3 – 0 = 3 A; dt = 0.4 s Self inductance,

$$L = \mu n^2 Al \left(\because \mu = \mu_0\mu_r; A = \pi r^2; n = \frac{N}{1}\right)$$

$$= \frac{\mu_0\mu_r N^2 \pi r^2}{l}$$

$$= \frac{4 \times 3.14 \times 10^{-7} \times 800 \times 500^2 \times 3.14 \times (3 \times 10^{12})^2}{0.4}$$

L = 1.77 H

Induced emf $\varepsilon = - L\frac{di}{dt}$

$$= - \frac{1.77 \times 3}{0.4}$$

$\varepsilon = -13.275$ V.

6. **The self-inductance of an air-core solenoid is 4.8 mH. If its core is replaced by iron core, then its self-inductance becomes 1.8 H. Find out the relative permeability of iron.**

Solution

$L_{iar} = 4.8 \times 10^{-3}$ H

$L_{iron} = 1.8$ H

$L_{air} = \mu_0 n^2 Al = 4.8 \times 10^{-3}$ H

$L_{iron} = \mu n^2 Al = \mu_0\mu_r, n^2 Al = 1.8$ H

$\because \mu_r = \frac{L_{iron}}{L_{air}} = \frac{1.8}{4.8 \times 10^{-3}} = 375$

7. **The current flowing in the first coil changes from 2 A to 10 A in 0.4 s. Find the mutual inductance between two coils if an emf of 60 mV is induced in the second coil. Also determine the magnitude of induced emf in the second coil if the current in the first coil is changed from 4 A to 16 A in 0.03 s. Consider only the magnitude of induced emf.**

Solution

Case (i): $di_1 = 10 - 2 = 8$ A; $dt = 0.4$ s; $\varepsilon_2 = 60 \times 10^{-3}$ V

Case (ii): $di_1 = 16 - 4 = 12$ A; $dt = 0.03$ s

(i) Mutual inductance between the coils.

$$M = \frac{\varepsilon_2}{di_1/dt} = \frac{60 \times 10^{-3} \times 0.4}{8}$$

$M = 3 \times 10^{-3}$H

(ii) Induced emf in the second coil due to the rate of change of current in the first coil is

$$\varepsilon_2 = M \frac{di_1}{dt} = \frac{3 \times 10^{-3} \times 12}{0.03}$$

$\varepsilon_2 = 1.2$ V

8. **A circular metal of area 0.03 m² rotates in a uniform magnetic field of 0.4 T. The axis of rotation passes through the centre and perpendicular to its plane and is also parallel to the field. If the disc completes 20 revolutions in one second and the resistance of the disc is 4 Ω, calculate the induced emf between the axis and the rim and induced current flowing in the disc.**

Solution

$A = 0.03$ m² ; $B = 0.4$ T; $f = 20$ rps; $R = 4\ \Omega$

Area swept out by the disc in unit time = Area of the disc × frequency

$$\frac{dA}{dt} = 0.03 \times 20$$

$= 0.6$ m²s⁻¹

The magnitude of the induced emf,

$$\varepsilon = \frac{d\varphi_B}{dt} = \frac{d(BA)}{dt} = B\frac{dA}{dt}$$

$$\varepsilon = \frac{0.4 \times 0.6}{1} = 0.24 \text{ V}$$

Induced current, $I = \frac{\varepsilon}{R} = \frac{0.24}{4} = 0.06$A

9. **A rectangular coil of area 70 cm² having 600 turns rotates about an axis perpendicular to a magnetic field of 0.4 Wb m⁻². If the coil completes 500 revolutions in a minute, calculate the instantaneous emf when the plane of the coil is (i) perpendicular to the field (ii) parallel to the field and (iii) inclined at 60° with the field.**

Solution

$A = 70 \times 10^{-4}$ m² ; $N = 600$ turType equation here.ns $B = 0.4$ Wbm⁻²; $f = 500$ rpm

The instantaneous emf is

$\varepsilon = \varepsilon_m \sin\omega t$

Since $\varepsilon_m = N\varphi_m\omega = N\,(BA)\,(2\pi f)$

$$\varepsilon = NBA \times 2\pi f \times \sin\omega t$$

(i) When ωt = 0^o,

$\varepsilon = \varepsilon_m \sin 0 = 0$

(ii) When ωt = 90^o

$\varepsilon = \varepsilon_m \sin 90^0$ = NBA x $2\pi f$ x 1

= 600 x 0.4 x 70 x 10^{-4} x 2 x $\frac{22}{7}$ x $\left(\frac{500}{60}\right)$

= 88 V

(iii) When ωt = 90° – 60° = 30^0

$\varepsilon = \varepsilon_m \sin 30^0$ = 88 x $\frac{1}{2}$ = 44 V

10. **An electric power of 2 MW is transmitted to a place through transmission lines of total resistance R = 40 Ω, at two different voltages. One is lower voltage (10 kV) and the other is higher (100 kV). Let us now calculate and compare power losses in these two cases.**

<u>Solution</u>

Case (i): P = 2 MW; R = 40 Ω; V = 10 kV Power, P = VI

∵ current, I = $\frac{P}{V}$

$= \frac{2\times10^6}{10\times10^3}$ = 200 A

Power loss = Heat produced = I^2R

= $(200)^2$ x 40 = 1.6 x 10^6W

% of power loss = $\frac{1.6\times10^6}{2\times10^6}$ x 100%

= 0.8 x 100 % = 8 %

Case (ii): P = 2 MW; R = 40 Ω; V = 100 kV

∵ current, I = $\frac{P}{V}$

$= \frac{2\times10^6}{100\times10^3}$ = 20 A

Power loss = I^2R

= $(20)^2$ x 40 = 0.016 x 10^6W

% of power loss = $\frac{0.016\times10^6}{2\times10^6}$ x 100%

= 0.008 x 100% = 0.8%

Thus it is clear that when an electric power is transmitted at higher voltage, the power loss is reduced to a large extent

11. **An ideal transformer has 460 and 40,000 turns in the primary and secondary coils respectively. Find the voltage developed per turn of the secondary if the transformer is connected to a 230 V AC mains. The secondary is given to a load of resistance 104 Ω. Calculate the power delivered to the load.**

Solution

N_P = 460 turns; N_S = 40,000 turns V_P = 230 V; R_S = 10^4 Ω

(i) Secondary voltage,

$V_s = \frac{V_p N_s}{N_p} = \frac{230 \times 40{,}000}{460}$ = 20,000 V

Secondary voltage per turn $\frac{V_s}{N_s} = \frac{20{,}000}{40{,}000}$ = 0.5 V

(ii) Power delivered

$= V_2 I_s = \frac{V_s^2}{R_s} = \frac{20{,}000 \times 20{,}000}{10^4}$ = 40 kW

12. **An inverter is common electrical device which we use in our homes. When there is no power in our house, inverter gives AC power to run a few electronic appliances like fan or light. An inverter has inbuilt step-up transformer which converts 12 V AC to 240 V AC. The primary coil has 100 turns and the inverter delivers 50 mA to the external circuit. Find the number of turns in the secondary and the primary current.**

Solution

V_p = 12 V; V_s = 240 V; I_s = 50mA; N_p = 100 turn

$\frac{V_s}{V_p} = \frac{N_s}{N_p} = \frac{I_p}{I_s} = K$

$Transformation\ ration, K = \frac{240}{120} = 20$

The number of turns in the secondary

$N_s = N_p \times K = 100 \times 20 = 2000$

Primary current,

$I_p = K \times I_s = 20 \times 50\text{ mA} = 1\text{ A}$

13. **Write down the equation for a sinusoidal voltage of 50 Hz and its peak value is 20 V. Draw the corresponding voltage versus time graph.**

Solution

$f = 50\ Hz;\ V_m = 20\ V$

Instantaneous voltage, $\upsilon = V_m \sin \omega t$

$= V_m \sin 2\pi f t$

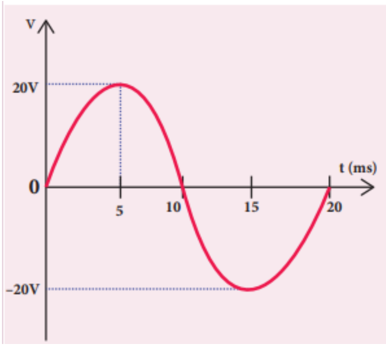
$= 20 \sin (2\pi \times 50)\ t = 20 \sin(100 \times 3.14)\ t$

$\upsilon = 20 \sin 314t$

Time for one cycle, $T = \frac{1}{f} = \frac{1}{50} = 0.02$ s

$= 20 \times 10^{-3}$s = 20 ms

The wave form is given below.



14. **The equation for an alternating current is given by I = 77 sin 314t. Find the peak current, frequency, time period and instantaneous value of current at t = 2 ms.**

<u>Solution</u>

I = 77 sin 314 t ; t = 2 m s = 2×10^{-3} s The general equation of an alternating current is I = I_m sinωt . On comparison,

(i) Peak current, I_m =77 A

(ii) Frequency, $f = \frac{\omega}{2\pi} = \frac{314}{2\times3.14}$ = 50 Hz

(iii) Time period, T = $\frac{1}{f} = \frac{1}{50}$ = 0.02s

(iv) At t = 2m s, instantaneous current,

I = 77 sin(314 x 2 x 10^{-3})

= 77 sin $\left(314 \text{ x } 2 \text{ x } 10 - 3 \text{ x } \frac{180^0}{3.14}\right)$

= 77 sin 36^0 = 77 x 0.5878

= 45.26 A

15. **A 400 mH coil of negligible resistance is connected to an AC circuit in which an effective current of 6 mA is flowing. Find out the voltage across the coil if the frequency is 1000 Hz.**

<u>Solution</u>

L = 400 × 10^{-3} H; I_{eff} = 6 × 10^{-3}A ; f = 1000 H

Inductive reactance, X_L = Lω = L x $2\pi f$

= 2 x 3.14 x 1000 x 0.4

= 2512 Ω

Voltage across L

V = I X_L = 6 x 10^{-3} x 2512

V = 15.072 V(RMS)

16. **A capacitor of capacitance $\frac{10^2}{\pi}$ μF is connected across a 220 V, 50 Hz A.C. mains. Calculate the capacitive reactance, RMS value of current and write down the equations of voltage and current**

<u>Solution</u>

C = $\frac{10^2}{\pi}$ x 10^{-6} F, V_{RMS} = 220 V; $f = 50\ Hz$

(i) Capacitive reactance,

$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

$= \frac{1}{2\times\pi\times50\times\frac{10^{-4}}{\pi}}$

(i) RMS value of current

$I_{RMS} = \frac{V_{RMS}}{X_C} = \frac{220}{100}$ = 2.2A

(i) $V_m = 220 \text{ x } \sqrt{2}$ = 311 V

I_m = 2.2 x $\sqrt{2}$ = 3.1 A

Therefore,

υ = 311 sin 314t

I = 3.1 sin (314t + $\pi/2$)

17. **A 500 μH inductor, $\frac{80}{\pi^2}$ pF capacitor and a 628 Ω resistor are connected to form a series RLC circuit. Calculate the resonant frequency and Q-factor of this circuit at resonance.**

Solution

L = 500 x 10^{-6} H; C = $\frac{80}{\pi^2}$ x 10^{-12}F; R = 628Ω

(i) Resonant frequency is

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{500\times10^{-6}\times\frac{80}{\pi^2}\times10^{-12}}}$$

$$= \frac{1}{2\pi\sqrt{40{,}000\times10^{-18}}} = \frac{10{,}000\times10^{3}}{4} = 2500 \text{ KHz}$$

(ii) Q – Factor

$$= \frac{\omega_r L}{R} = \frac{2\times3.14\times2500\times10^{3}\times500\times10^{-6}}{628}$$

Q = 12.5

18. **Find the impedance of a series RLC circuit if the inductive reactance, capacitive reactance and resistance are 184 Ω, 144 Ω and 30 Ω respectively. Also calculate the phase angle between voltage and current.**

Solution

X_L = 184 Ω; X_C = 144Ω

R = 30Ω

(i) The impedance is

Z = $\sqrt{R^2 + (X_L - X_C)^2}$

= $\sqrt{30^2 + (184 - 144)^2}$

= $\sqrt{900 + 1600}$

Z = 50 Ω

(ii) Phase angle ϕ between voltage and current is

tan ϕ = $\frac{X_L - X_C}{R}$ = $\frac{184-144}{30}$ = 1.33

ϕ = 53.1^0

Since the phase angle is positive, voltage leads current by 53.1° for this inductive circuit.

19. **Find the instantaneous value of alternating voltage υ = 10sin($3\pi \times 10^4 t$) volt at i) 0 s ii) 50 μs iii) 75 μs.**

Solution

The given equation is υ = 10sin($3\pi \times 10^4 t$)

(i) At t = 0 s,

υ = 10sin0^0 = 0V

(ii) At t = 50 µs

$\upsilon = 10\sin(3\pi \times 10^4 \times 50 \times 10^{-6})$

$= 10 \sin\left(150\pi \times 10^{-2} \times \frac{180^0}{\pi}\right)$

= 10 sin (270^0) = 10 x -1

= -10 V

(iii) At t = 75 µs

$\upsilon = 10\sin(3\pi \times 10^4 \times 75 \times 10^{-6})$

$= 10 \sin\left(225\pi \times 10^{-2} \times \frac{180^0}{\pi}\right)$

= 10 sin (405^0) = 10 sin 45^0

= 10 x $1/\sqrt{2}$ = 7.07 V

20. **A series RLC circuit which resonates at 400 kHz has 80 µH inductor, 2000 pF capacitor and 50 Ω resistor. Calculate (i) Q-factor of the circuit (ii) the new value of capacitance when the value of inductance is doubled and (iii) the new Q-factor.**

Solution

L = 80 x 1^{-6}; C = 2000 x 10^{-12}F; R = 50Ω ; f_r = 400 x 10^3 Hz.

(i) Q – factor, $Q_1 = \frac{1}{R}\sqrt{\frac{L}{C}}$

$= \frac{1}{50}\sqrt{\frac{80\times10^{-6}}{2000\times10^{-12}}} = 4$

(ii) When $L_2 = 2L$

= 2 x 80 x 10^{-6} H

= 160 x 10^{-6} H

$C_2 = \frac{1}{4\pi^2 f_r^2 L_2}$

$= \frac{1}{4 \times 3.14^2 \times (400\times10^3)^2 \times 160\times10^{-6}}$

≃ = 1000 x 10^{-12} F

C_2 = 1000 pF

(iii) $Q_2 = \frac{1}{R} = \sqrt{\frac{L_2}{C_2}} = \frac{1}{50}\sqrt{\frac{160\times10^{-6}}{1000\times10^{-12}}}$

$= \frac{1}{50}\sqrt{\frac{16\times10^{-5}}{10^{-9}}} = \frac{4\times10^2}{50} = 8$

21. **A capacitor of capacitance $\frac{10^{-4}}{\pi}$F, an inductor of inductance $\frac{2}{\pi}$ H and a resistor of resistance 100 Ω are connected to form a series RLC circuit. When an AC supply of 220 V, 50 Hz is applied to the circuit, determine (i) the impedance of the circuit (ii) the peak value of current flowing in the circuit (iii) the power factor of the circuit and (iv) the power factor of the circuit at resonance.**

Solution

$L = \frac{2}{\pi}$ H; $C = \frac{10^{-4}}{\pi}$ F; R = 100Ω

V_{RMS} = 220 V; f = 50Hz

$X_L = 2\pi f L = 2\pi \times 50 \times 2/\pi = 200\Omega$

$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times \frac{10^{-4}}{\pi}}$

(i) Impedance, $Z = \sqrt{R^2 (X_L - X_C)^2}$

$= \sqrt{100^2 (200-100)^2} = 141.4\ \Omega$

(ii) Peak value of current,

$I_m = \frac{V_m}{Z} = \frac{\sqrt{2} V_{RMS}}{Z}$

$= \frac{\sqrt{2} \times 220}{141.4} = 2.2$ A

(iii) Power factor of the circuit,

$\cos\phi = \frac{R}{Z} = \frac{100}{141.4} = 0.707$

(iv) Power factor at resonance,

$\cos\phi = \frac{R}{Z} = \frac{R}{R} = 1$

22. **A square coil of side 30 cm with 500 turns is kept in a uniform magnetic field of 0.4T. The plane of the coil is inclined at an angle of 30^0 to the field. Calculate the magnetic flux through the coil.**

Solution

A = 0.3 x 0.3 = 9 x 10^2 m^2, N = 500, B = 0.4 T, $\theta = 90^0 - 30^0 = 60^0$

ϕ = NAB cos θ = 500 x 9 x 10^{-2} x 0.4 x cos 60^0 = 18 x $\frac{1}{2}$ = 9 Wb.

23. **A straight metal wire crosses a magnetic field of flux 4 mWb in a time 0.4 s. Find the magnitude of the emf induced in the wire.**

Solution

$d\phi = 4 \times 10^{-3}$ *Wb, dt = 0.4 s*

Emf induced $e = \frac{d\phi}{dt} = \frac{4\times10^{-3}}{0.4}$ = 10 x 10^{-3} V = 10 mV

24. **The magnetic flux passing through a coil perpendicular to its plane is a function of time and is given by $\varphi_B = (2t^3 + 4t^2 + 8t + 8)$ Wb. If the resistance of the coil is 5Ω, determine the induced current through the coil at a time t = 3 second.**

Solution

$\varphi_B = 2t^3 + 4t^2 + 8t + 8$, R = 5Ω, t = 3 s

induced emf $e = \frac{d\varphi_B}{dt} = (6t^2 + 8t + 8)$

when t = 3 s, e = - (6 x 9 + 8 x 3 + 8) = -86 V = 86 V

induced current $I = \frac{e}{R} = \frac{86}{5}$ = 17.2 A

25. **A closely would circular coil of radius 0.02 m is placed perpendicular to the magnetic field. When the magnetic field is changed from 8000 T to 2000 T in 6 s, an emf 44 V is induced. Calculate the number of turns in the coil.**

Solution

r = 0.02m, dB = 8000 T – 2000 T = 6000 T, dt = 6s, e = 44 V

Induced emf e = $NA\frac{dB}{dt}$, 44 = N x 3.14 x $(0.02)^2$ x $\frac{6000}{6}$, N = 35 turns.

26. **A rectangular coil of area 6 cm^2 having 3500 turns is kept in a uniform magnetic field of 0.4 T. Initially, the plane of the coil is perpendicular to the field and is then rotated through an angle of 180^0 in 1 second. If the resistance of the coil is 35Ω, find the amount of charge flowing through the coil.**

Solution

A = 6 x 10^{-4} m^2, N = 3500, B = 0.4 T, $\theta_1 = 0^0$, $\theta_2 = 180^0$

R = 35Ω, t = 1s

ϕ = N B A ($Cos0^0$ – cos 180^0) = 3500 x 0.4 x 6 x 10^{-4} (1+1) = 1.68 Wb

e = $\frac{d\phi}{dt}$ = $\frac{1.68}{1}$ = 1.68 volt, i = $\frac{e}{R} = \frac{Q}{t}$, Q = $\frac{1.68 \times 1}{35}$ = 48 x 10^{-3} C

27. **An induced current of 2.5 mA flows through a single conductor of resistance 100Ω. Find out the rate at which the magnetic flux is cut by the conductor.**

Solution

i = 2.5 x 10^{-3} A, R = 100Ω, e = iR = 2.5 x 10^{-3} x 100 = 0.25 V

e = $\frac{d\phi}{dt}$ = 0.25 = 250 x 10^{-3} = 250 m Wbs^1

28. **A fan of metal blades of length 0.4 m rotates normal to a magnetic field of 4 x 10^{-3}T. If the induced emf between the centre and edge of the blade is 0.02V, determine the rate of rotation of the blade.**

Solution

l = 0.4 m, B = 4 x 10^{-3} T, e = 0.02 V, f =?

e = $\frac{\phi}{T}$ = $B\pi l^2$f, f = $\frac{e}{B\pi l^2} = \frac{0.02}{4 \times 10^{-3} \times 3.14 \times 0.4^2} = \frac{20}{2.0096}$ = 9.95 Hz

29. **Determine the self – inductance of 4000 turn air – core solenoid of length 2 m and diameter 0.04m.**

Solution

N = 4000, l = 2m, r = 0.02m, L = ?

L = $\frac{\mu_0 N^2 A}{l} = \frac{4 \times 3.14 \times 10^{-7} \times 4000^2 \times 3.14 \times 0.02^2}{2}$ = 12.62 x 10^{-3} H = 12.62 mH.

30. **A coil of 200 turns carries a current of 4 A. if the magnetic flux through the coil is 6 x 10^{-5} Wb, find the magnetic energy stored in the medium surrounding the coil.**

Solution

N = 200, I = 4 A, ϕ = 6 x 10^{-5} Wb

$U_B = \frac{1}{2} L I^2 = \frac{1}{2}\left(\frac{N\phi}{l}\right) I^2 = \frac{1}{2} N \phi I = \frac{1}{2}$ x 200 x 6 x 10^{-5} x 4 = 0.024 J.

31. **A 50 cm long solenoid has 400 turns per cm. The diameter of the solenoid is 0.04 m. Find the magnetic flux linked with each turn when it carries a current of 1A.**

<u>Solution</u>

$n = 2 \times 10^4$, I = 1A, $r = 2 \times 10^{-2}$ m

$\phi = \mu_0$, n A I = 4 x 3.14 x 10^{-7} x 4 x10^4 x 3.14 x $(2 \times 10^{-2})^2$ x 1 = 0.63 x 10^{-4} Wb

32. **A coil of 200 turns carries a current of 0.4 A. If the magnetic flux of 4 m Wb is linked with each turn of the coil, find the inductance of the coil.**

<u>Solution</u>

N = 200, I = 0.4 A, $\phi = 4 \times 10^{-3}$ Wb

Self inductance of the coil $L = \frac{N\phi}{I} = \frac{200 \times 4 \times 10^{-3}}{0.4} = 2$ H

33. **A step = down transformer connected to main supply of 220 V is used to operate 11V, 88 W lamp. Calculate (i) voltage transformation ratio and (ii) current in the primary.**

<u>Solution</u>

(i) Voltage transformation ratio $K = \frac{V_S}{V_P} = \frac{11}{220} = \frac{1}{20}$

Power $P = V_P I_P$, $88 = 220 I_P$, $I_P = \frac{88}{220} = 0.4$ A

34. **A 200 V / 120 V step – down transformer of 90 % efficiency is connected to an induction stove of resistance 40Ω . Find the current drawn by the primary of the transformer.**

<u>Solution</u>

$V_P = 200$ V, $V_S = 120$ V, $\eta = 0.9$, $R_s = 40$ Ω, I_P = ?

$\eta = \frac{V_S I_S}{V_p I_p}$, $0.9 = \frac{120 I_S}{200 I_P}$, $\frac{I_S}{I_p} = \frac{0.9 \times 200}{120} = 1.5$, $I_s = 1.5\ I_p$

$I_s = \frac{V_S}{R_S}$, $\frac{120}{40} = 1.5\ I_P$, $I_p = \frac{120}{40 \times 1.5} = 2$A

35. **The 300 turn primary of a transformer has resistance 0.82 Ω and the resistance of its secondary of 1200 turns is 6.2 Ω. Find the voltage across the primary if the power output from the secondary at 1600 V is 32 kW. Calculate the power losses in both coils when the transformer efficiency is 80%.**

<u>Solution</u>

$N_P = 300$, $R_P = 0.82Ω$, $N_s = 1200$, $R_s = 6.2$ Ω

$V_s I_s = 32 \times 10^3$ W, $\eta = 0.8$, $V_s = 1600$ V

$V_s I_s = 32 \times 10^3$ W, $1600 I_s = 32 \times 10^3$, $I_s = 20$ A

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(i) Power loss in secondary coil I_s^2 R$_s$ = 20 x 20 x 6.2 = 2.48 kW

$\eta = \frac{V_S I_S}{V_p I_p}$, 0.8 $= \frac{32\times10^3}{V_p I_p}$, $V_p I_p = \frac{32\times10^3}{0.8}$ = 4 x 10^4 W

$\frac{V_p}{\text{Vs}} = \frac{N_p}{\text{Ns}}$, $\frac{V_p}{1600} = \frac{300}{1200}$, V$_P$ = 400 V

V$_P$ I$_P$ = 4 x 10^4, 400 I$_P$ = 4 x 10^4, I$_P$ = 100 A

(ii) Power loss in the primary coil = I_p^2 R$_p$ = 100^2 x 0.82 = 8.2 kW

36. **Calculate the instantaneous value at 60^0, average value and RMS value of an alternating current whose peak value is 20 A.**

Solution

$\theta = \omega$t = 60^0, I$_m$ = 20 A

(i) i = I$_m$ sin ωt = 20 sin 60^0 = 20 x $\frac{\sqrt{3}}{2}$ = 10 x 1.732 = 17.32 A

(ii) I$_{ave}$ = $\frac{2I_m}{\pi} = \frac{2\times20}{3.14}$ = 12.74 A

(iii) I$_{RMS}$ = $\frac{I_m}{\sqrt{2}} = \frac{20}{1.414}$ = 14.14 A

Unit – 5 – Electromagnetic waves

1. **The relative magnetic permeability of the medium is 2.5 and the relative electrical permittivity of the medium is 2.25. Compute the refractive index of the medium.**

Solution

Dielectric constant (relative permittivity of the medium), ε_r = 2.25

Magnetic permeability, μ_r = 2.5

Refractive index of the medium,

n $= \sqrt{\varepsilon_r \mu_r} = \sqrt{2.25 \times 2.5}$ = 2.37

2. **Compute the speed of the electromagnetic wave in a medium if the amplitude of electric and magnetic fields are 3 × 10^4 N C^{-1} and 2 × 10^{-4} T, respectively.**

Solution

The amplitude of the electric field, E$_o$ = 3 × 10^4 N C^{-1} The amplitude of the magnetic field, B$_o$ = 2 × 10^{-4} T. Therefore, speed of the electromagnetic wave in that medium is

$\upsilon = \frac{3\times10^4}{2\times10^{-4}}$ = 1.5 x 10^8 ms^{-1}

3. A magnetron in a microwave oven emits electromagnetic waves (em waves) with frequency f = 2450 MHz. What magnetic field strength is required for electrons to move in circular paths with this frequency?

Solution

Frequency of the electromagnetic waves given, f = 2450 MHz

The corresponding angular frequency is

$\omega = 2\pi f = 2 \times 3.14 \times 2450 \times 10^6$

$= 15{,}386 \times 10^6$ Hz

$= 1.54 \times 10^{10}\ s^{-1}$

The required magnetic field, $B = \frac{m_e \omega}{|q|}$

Mass of the electron, me = 9.11×10^{-31} kg

Charge of the electron,

$q = -1.60 \times 10^{-19}$ C

$\Rightarrow |q| = 1.60 \times 10^{-19}$ C

$B = \frac{(9.11 \times 10^{-31})(1.54 \times 10^{10})}{(1.60 \times 10^{-19})} = 8.7683 \times 10^{-2}$ T

$= 0.08768$ T

This magnetic field can be easily produced with a permanent magnet. So, electromagnetic waves of frequency 2450 MHz can be used for heating and cooking food because they are strongly absorbed by water molecules.

4. A transmitter consists of LC circuit with an inductance of 1μH and capacitance of 1 μF. What is the wavelength of the electromagnetic waves it emits?

Solution

Frequency $f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2 \times 3.14\sqrt{10^{-6} \times 10^{-6}}} = \frac{1}{6.28 \times 10^{-6}}$

Wavelength $\lambda = \frac{c}{f} = 3 \times 10^8 \times 6.28 \times 10^{-6} = 18.84 \times 10^2$ m

5. If the relative permeability and relative permittivity of the medium is 1.0 and 2.25 respectively find, the speed of the electromagnetic wave in this medium.

Solution

$v = \frac{c}{n} = \frac{c}{\sqrt{\varepsilon_r \mu_r}} = \frac{3 \times 10^8}{\sqrt{2.25 \times 1}} = \frac{3 \times 10^8}{1.5} = 2 \times 10^8\ ms^{-1}$.

UNIT – 6 – RAY OPTICS

1. **Pure water has refractive index 1.33. What is the speed of light through it?**

   Solution:

   $n = \frac{c}{v}$; $v = \frac{c}{n}$

   $v = \frac{3 \times 10^8}{1.33} = 2.26 \times 10^8\ ms^{-1}$.

   Light travels with a speed of 2.26×108ms-1 through pure water.

2. **Light travels from air into a glass slab of thickness 50 cm and refractive index 1.5.**
   (a) What is the speed of light in the glass slab?
   (b) What is the time taken by the light to travel through the glass slab?
   (c) What is the optical path of the glass slab?

   Solution:

   Given, thickness of glass slab, d = 50 cm = 0.5 m, refractive index, n = 1.5
   refractive index, $n = \frac{c}{v}$

   (a) speed of light in the glass slab is,

   $v = \frac{c}{n} = \frac{3 \times 10^8}{1.5} = 2 \times 10^8\ ms^{-1}$

   (b) time taken by light to travel through the glass slab is,

   $t = \frac{d}{v} = \frac{0.5}{2 \times 10^8} = 2.5 \times 10^{-9}\ s$

   (c) optical path,

   d' = nd = 1.5 x 0.5 = 0.75 m = 75 cm

   Light would have travelled an additional 25 cm (75 cm – 50 cm) in vacuum at the same time had there been no glass slab in its path.

3. **Light travelling through transparent oil enters in to glass of refractive index 1.5. If the refractive index of glass with respect to the oil is 1.25, what is the refractive index of the oil?**

   Solution:

   Given, $n_{go} = 1.25$ and $n_g = 1.5$
   Refractive index of glass with respect to oil,

   $n_{go} = \frac{n_g}{n_o}$

   Rewriting for refractive index of oil,

$n_o = \frac{n_g}{n_{go}} = \frac{1.5}{1.25} = 1.2$

The refractive index of oil is, $n_o = 1.2$

4. What is the radius of the illumination when seen above from inside a swimming pool from a depth of 10 m on a sunny day? What is the total angle of view? [Given, refractive index of water is 4/3]

Solution:

Given, n = 4/3, d = 10 m.

Radius of illumination, $R = \frac{d}{\sqrt{n^2-1}}$

$R = \frac{10}{\sqrt{(4/3)^2-1}} \text{ x } \frac{10 \times 3}{\sqrt{16-9}}$

$R = \frac{30}{\sqrt{7}} = 11.32 \text{ m}$

To find the critical angle,

$i_c = \sin^{-1}\left(\frac{1}{n}\right)$

$i_c = \sin^{-1}\left(\frac{1}{4/3}\right) = \sin^{-1}\left(\frac{3}{4}\right) = 48.6^0$

The total angle of view of the cone is,

$2i_c = 2 \text{ x } 4.86^0 = 97.2.$

5. The thickness of a glass slab is 0.25 m. It has a refractive index of 1.5. A ray of light is incident on the surface of the slab at an angle of 60^0. Find the lateral displacement of the light when it emerges from the other side of the glass slab.

Solution:

Given, thickness of the slab, t = 0.25 m, refractive index, n = 1.5, angle of incidence, $i = 60^0$.

Using Snell's law, 1 sin i = n sin r

$\sin r = \frac{\sin i}{n} = \frac{\sin 60^0}{1.5} = 0.58$

$r = \sin^{-1}(0.58) = 35.25^0 = 35^0\ 15'0'$

Lateral displacement is, $L = t\left(\frac{\sin(i-r)}{\cos(r)}\right)$

$L = (0.25) \text{ x } \left(\frac{\sin(60-35.25)}{\cos(35.25)}\right) = 0.1281 \text{ m}$

The lateral displacement is, L =12 81 . cm

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6. **A monochromatic light is incident on an equilateral prism at an angle 30^0 and is emergent at an angle of 75^0. What is the angle of deviation produced by the prism?**

<u>Solution:</u>

Since, the prism is equilateral, $A = 60^0$;

Given, $i_1 = 30^0$; $i_2 = 75^0$

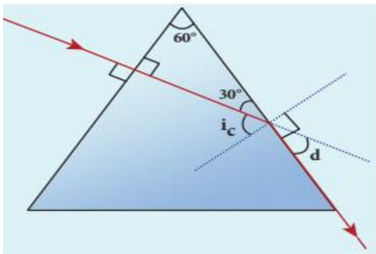
Equation for angle of deviation, $d = i_1 + i_2 - A$

Substituting the values, $d = 30° + 75° - 60° = 45°$

The angle of deviation produced, $d = 45^0$

7. **Light ray falls at normal incidence on the first face and emerges gracing the second face for an equilateral prism.**
(a) What is the angle of deviation produced?
(b) What is the refractive index of the material of the prism?

<u>Solution:</u>



The given situation is shown in the figure.

Given, $A = 60^0$; $i_1 = 0^0$; $i_2 = 90^0$

(a) Equation for angle of deviation,

$d = i_1 + i_2 - A$

Substituting the values,

$d = 0^0 + 90^0 - 60^0 = 30^0$

The angle of deviation produced is, $d = 30^0$

(b) The light inside the prism must be falling on the second face at critical angle as it graces the boundary. $i_c = 90° - 30° = 60°$

Equation for critical angle is, $\sin i_c = \frac{1}{n}$

$n = \frac{1}{\sin i_c}$; $n = \frac{1}{\sin 60^0} = \frac{1}{\sqrt{3}/2} = \frac{2}{\sqrt{3}} = 1.15$

The refractive index of the material of the prism is, $n = 1.15$

8. **The angle of minimum deviation for an equilateral prism is 37^0. Find the refractive index of the material of the prism.**

<u>Solution:</u>

Given, $A = 60°$; $D = 37°$

Equation for refractive index is,

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$$n = \frac{\sin\left(\frac{A+D}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

Substituting the values,

$$n = \frac{\sin\left(\frac{60^0+30^0}{2}\right)}{\sin\left(\frac{60^0}{2}\right)} = \frac{\sin(48.5^0)}{\sin(30^0)} = \frac{0.75}{0.5} = 1.5$$

The refractive index of the material of the prism is, n=1.5

9. Find the dispersive power of a prism if the refractive indices of flint glass for red, green and violet colours are 1.613, 1.620 and 1.632 respectively.

Solution:

Given, n_v = 1.632; n_R =1.613; n_G = 1.620

Equation for dispersive power is,

$$\omega = \frac{(n_V - n_R)}{(n_G - 1)}$$

Substituting the values,

$$\omega = \frac{1.632 - 1.613}{1.620 - 1} = \frac{0.019}{0.620} = 0.0306$$

The dispersive power of the prism is, ω = 0.0306.

10. Find the ratio of intensities of lights with wavelengths 500 nm and 300 nm which undergo Rayleigh scattering.

Solution:

Given λ_1 = 500 nm, λ_2 = 300 nm

$$\frac{I_1}{I_2} = \frac{\lambda_2^{\,4}}{\lambda_1^{\,4}} = \frac{(300)^4}{(500)^4} = \frac{81}{625},\ I_1 : I_2 = 81 : 625$$

UNIT – 7 – WAY OPTICS

1. **The wavelength of light from sodium source in vacuum is 5893Å.What are its (a) wavelength, (b) speed and (c) frequency when this light travels in water which has a refractive index of 1.33.**

Solution:

The refractive index of vacuum, n_1 = 1

The wavelength in vacuum, λ_1 = 5893 Å.

The speed in vacuum, $c = v_1 = 3 \times 10^8\ ms^{-1}$.

The refractive index of water, n_2 = 1.33

The wavelength of light in water, λ_2

The speed of light in water, v_2

(a) The equation relating the wavelength and refractive index is,

$\frac{\lambda_1}{\lambda_2} = \frac{n_2}{n_1}$

Rewriting, $\lambda_2 = \frac{n_1}{n_2} \times \lambda_1$

Substituting the values,

$\lambda_2 = \frac{1}{1.33} \times 5893$ Å

$\lambda_2 = 4431$ Å

(b) The equation relating the speed and refractive index is,

$$\frac{v_1}{v_2} = \frac{n_2}{n_1}$$

Rewriting $v_2 = \frac{n_1}{n_2} \times v_1$

Substituting the values,

$v_2 = \frac{1}{1.33} \times 3 \times 10^8$

$v_2 = 2.256 \times 10^8\ ms^{-1}$.

(c) Frequency of light in vacuum is

$\nu_1 = \frac{c}{\lambda_1}$

Substituting the values,

$\nu_1 = \frac{3 \times 10^8}{5893 \times 10^{-10}} = 5.091 \times 10^{14}$ Hz.

Frequency of light in water is, $\nu_2 = \frac{v}{\lambda_2}$

Substituting the values,

$\nu_2 = \frac{2.256 \times 10^8 ms^{-1}}{4431 \times 10^{-10}} = 5.091 \times 10^{14}$ Hz

The results show that the frequency remains same in all media.

2. **Two light sources with amplitudes 5 units and 3 units respectively interfere with each other. Calculate the ratio of maximum and minimum intensities.**

Solution:

Amplitudes, $a_1 = 5$, $a_2 = 3$

Resultant amplitude,

$A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2 cos\ \emptyset}$

Resultant amplitude is maximum when,

$\emptyset = 0, \cos 0 = 1, A_{max} = \sqrt{a_1^2 + a_2^2 + 2a_1a_2}$

$A_{max} = \sqrt{(a_1 + a_2)^2} = \sqrt{(5+3)^2} = \sqrt{(8)^2}$

$= 8$ units

Resultant amplitude is minimum when,

$\emptyset = \pi, \cos \pi = -1, \sqrt{a_1^2 + a_2^2 - 2a_1a_2}$

$A_{max} = \sqrt{(a_1 - a_2)^2} = \sqrt{(5-3)^2} = \sqrt{(2)^2}$

= 2 units

$I \propto A^2$

$\frac{I_{max}}{I_{min}} = \frac{(A_{max})^2}{(A_{min})^2}$

Substituting,

$\frac{I_{max}}{I_{min}} = \frac{(8)^2}{(2)^2} = \frac{64}{4}$ = 16 (or) $I_{max} : I_{min}$ = 16 : 1

3. **Two light sources of equal amplitudes interface with each other. Calculate the ratio of maximum and minimum intensities.**

Solution:

Let the amplitude be a.

The intensity is, $I \propto 4a^2 \cos^2 (\emptyset / 2)$

Or $I = 4I_0 \cos^2 (\emptyset / 2)$

Resultant intensity is maximum when

$\emptyset = 0$, $\cos 0 = 1$, $I_{max} \propto 4a^2$

Resultant amplitude is minimum when,

$\emptyset = \pi$, $\cos (\pi/2) = 0$, $I_{min} = 0$

$I_{max} : I_{min} = 4a^2 : 0$

4. **Two light sources have intensity of light as I_0. What is the resultant intensity at a point where the two light waves have a phase difference of $\pi/3$?**

Solution:

Let the intensities be I_0.

The resultant intensity is, $I = 4 I_0 \cos^2 (\emptyset/2)$

Resultant intensity when, $\emptyset = \pi/3$, is

$I = 4 I_0 \cos^2 (\pi/6)$

$I = 4 I_0 (\sqrt{3}/2)^2 = 3I_0$.

5. **The wavelength of a light is 450 nm. How much phase it will differ for a path of 3 mm?**

Solution

Wavelength, λ= 450 nm = 450 x 10-9 m

Path difference, $\delta = 3\ mm$ = 3 x 10-3 m

Relation between phase difference and path difference, $\emptyset = \frac{2\pi}{\lambda} \times \delta$

Substituting,

$$\phi = \frac{2\pi}{450\times10^{-9}}\times3\times10^{-3} = \frac{\pi}{75}\times10^{6}$$

$$\phi = \frac{\pi}{75}\times10^{6}\,\text{rad} = 4.19\times10^{4}\,\text{rad.}$$

6. **Calculate the distance upto which ray optics is a good approximation for light of wave length 500 nm falls on an aperture of width 0.5 mm.**

Solution:

$a = 0.5 \text{ mm} = 0.5 \times 10^{-3} \text{ m} = 5 \times 10^{-4} \text{ m}$

$\lambda = 500 \text{ nm} = 500 \times 10^{-9} \text{m}; \ z = ?$

Equation for Fresnel's distance is, $z = \frac{a^2}{2\lambda}$

Substituting,

$$z = \frac{\left(5 \times 10^{-4}\right)^2}{2 \times 500 \times 10^{-9}} = \frac{25 \times 10^{-8}}{1 \times 10^{-6}} = 0.25 \text{ m}$$

$z = 0.25 \text{ m} = 25 \text{ cm}$

7. **A diffraction grating consists of 4000 slits per centimeter. It is illuminated by a monochromatic light. The second order diffraction maximum is produced at an angle of 30°. What is the wavelength of the light used?**

Solution

Number of lines = 4000 cm^{-1} ; m = 2;

$\theta = 30°$; $\lambda = ?$

Number of lines per unit length,

$N = \frac{4000}{1 \times 10^{-2}} = 4 \text{ x } 10^5 \text{ ms}^{-1}$

Equation for diffraction maximum for grating is, $\sin \theta = Nm\lambda$

After rewriting, $\lambda = \frac{\sin \emptyset}{Nm}$

Substituting,

$$\lambda = \frac{\sin 30^\circ}{4 \times 10^5 \times 2} = \frac{0.5}{4 \times 10^5 \times 2}$$

$$= \frac{1}{2 \times 4 \times 10^5 \times 2} = \frac{1}{16 \times 10^5}$$

$\lambda = 6250 \times 10^{-10} \text{m} = 6250 \text{ Å}$

8. **A monochromatic light of wavelength of 500 nm strikes a grating and produces fourth order maximum at an angle of 30°. Find the number of slits per centimeter.**

Solution:

$\lambda = 500 \text{ nm} = 500 \times 10^{-9} \text{ m}$; m = 4;

$\theta = 30°$; number of lines per cm = ?

Equation for diffraction maximum for grating is, $\sin \theta = Nm\lambda$

Rewriting, N $= \frac{\sin \theta}{m\lambda}$

Substituting, $N = \frac{0.5}{4 \times 500 \times 10^{-9}}$

$= 2.5 \times 10^5 \text{ m}^{-1}$

$= 2.5 \times 10^3 \text{ cm}^{-1}$

9. **The optical telescope in the Vainu Bappu observatory at Kavalur has an objective lens of diameter 2.3 m. What is its angular resolution if the wavelength of light used is 589 nm?**

Solution:

$a = 2.3$ m; $\lambda = 589$ nm $= 589\times10^{-9}$ m; $\theta = ?$

The equation for angular resolution is,

$$\theta = \frac{1.22\,\lambda}{a}$$

Substituting

$$\theta = \frac{1.22\times589\times10^{-9}}{2.3} = 3.124\times10^{-7}$$

$$\theta = 3.124\times10^{-7}\,\text{rad} \quad \text{(or)} \quad \theta = 0.0011'$$

Note: The angular resolution of human eye is approximately, 3×10^{-4} rad $\approx 1.03'$.

10. **Two polaroids are kept with their transmission axes inclined at 30^0. Unpolarised light of intensity I falls on the first polaroid. Find out the intensity of light emerging from the second polaroid.**

Solution:

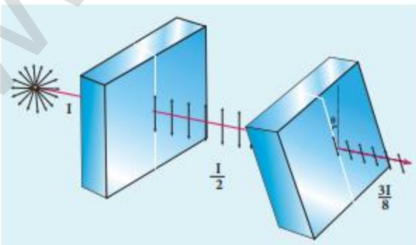
As the intensity of the unpolarised light falling on the first polaroid is I, the intensity of polarized light emerging from it will be, $I_0 = \left(\frac{I}{2}\right)$. Let I' be the intensity of light emerging from the second polaroid.

Malus' law, $I' = I_0 \cos^2\theta$

Substituting,

$$I' = \left(\frac{I}{2}\right)\cos^2(30^\circ) = \left(\frac{I}{2}\right)\left(\frac{\sqrt{3}}{2}\right)^2 = I\frac{3}{8}$$

$$I' = \left(\frac{3}{8}\right)I$$



11. **Two polaroids are kept crossed (transmission axes at 90^0) to each other.**
(a) What will be the intensity of the light coming out from the second polaroid when an unpolarised light of intensity I falls on the first polaroid?
(b) What will be the intensity of light coming out from the second polaroid if a third polaroid is kept in between at 45^0 inclination to both of them.

Solution:

(a) As the intensity of the unpolarised light falling on the first polaroid is I, the intensity of polarized light emerging from it will be $I_0 = \left(\frac{I}{2}\right)$. Let I\` be the intensity of light emerging from the second polaroid.

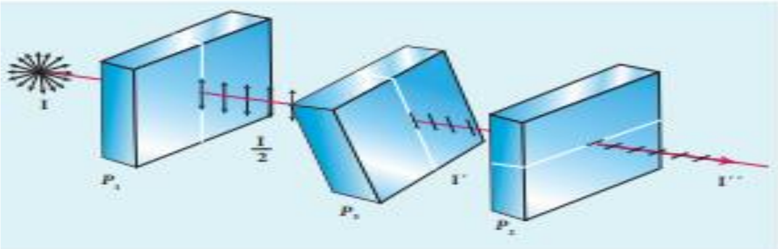
Malus' law, $I` = I_0 \cos^2\theta$

Here θ is 90^0 as the transmission axes are perpendicular to each other.

Substituting,

$I` = \left(\frac{I}{2}\right)\cos^2(90^0) = 0$

No light comes out from the second polaroid.



(b) Let the first polaroid be P_1 and the second polaroid be P_2. They are oriented at 90^0. The third polaroid P_3 is introduced between them at 45^0. Let I′ be the intensity of light emerging from P_3.

Angle between P_1 and P_3 is 45^0. The intensity of light coming out from P_3 is,

$I' = I_0 \cos^2\theta$

Substituting,

$I' = \left(\frac{1}{2}\right)\cos^2(45^0) = \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{I}{4`} = I` = \frac{I}{4}$

Finally, the light has to pass through P_2. Angle between P_3 and P_2 is 45^0. Let I″ is the intensity of light coming out from P_2 $I'' = I'\cos^2\theta$.

Here, I' is the intensity of polarized light existing between P_3 and P_2. $I' = \frac{I}{4}$.

Substituting,

$I'' = \left(\frac{1}{4}\right)\cos^2(45^0) = \left(\frac{1}{4}\right)\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{I}{8}$

$I'' = \frac{I}{8}$.

12. **Find the polarizing angles for (i) glass of refractive index 1.5 and (ii) water of refractive index 1.33.**

Solution:

Brewster's law, $\tan i_p = n$

For glass, $\tan i_p = 1.5$; $i_p = \tan^{-1} 1.5$; $i_p = 56.3^0$

For water, $\tan i_p = 1.33$; $i_p = \tan^{-1} 1.33$; $i_p = 53.1^0$.

13. What is the angle at which a glass plate of refractive index 1.65 is to be kept with respect to the horizontal surface so that an unpolarised light travelling horizontal after reflection from the glass plate is found to be plane polarised?

Solution:

n = 1.65

Brewster's law, tan $i_p = n$

tan $i_p = 1.65$; $i_p = \tan^{-1} 1.65$; $i_p = 58.8^0$

The inclination with the horizontal surface is, $(90^0 - 58.8^0) = 31.2^0$

14. The ratio of maximum and minimum intensities in an interference pattern is 36:1. What is the ratio of amplitudes of the two interfering waves?

Solution:

$$\frac{I_{max}}{I_{min}} = \frac{36}{1} = \frac{(a_1+a_2)^2}{(a_1-a_2)^2}, 6 = \frac{a_1+a_2}{a_1-a_2}$$

$6a_1 - 6a_2 = a_1 + a_2, 5a_1 = 7a_2, \frac{a_1}{a_2} = \frac{7}{5}$, $a_1 : a_2 = 7 : 5$

15. Light of wavelength 600 nm that falls on a pair of slits producing interference pattern on a screen in which the bright fringes are separated by 7.2 mm. What must be the wavelength of another light which produces bright fringes separated by 8.1mm with the same distance?

Solution:

$$\beta_1 = \frac{D}{d}\lambda_1, \beta_2 = \frac{D}{d}\lambda_2$$

$$\frac{\lambda_2}{\lambda_1} = \frac{\beta_2}{\beta_1}, \frac{\lambda_2}{600} = \frac{8.1}{7.2}, \lambda_2 = \frac{8.1 \times 600}{7.2} = 675 \text{ nm}$$

16. Light of wavelength 5000 A^0 produces diffraction pattern of the single slit of width 2.5 μm. What is the maximum order of diffraction possible?

Solution:

$$A\sin\theta = n\,\lambda, n = \frac{2.5 \times 10^{-6} \times \sin 90^0}{5 \times 10^{-7}} = \frac{25}{5} = 5$$

17. I_0 is the intensity of light existing between two cross polaroids kept with their axes perpendicular to each other. A third polaroid is introduced between them. What must be the angle between the axes of first and the newly introduced polaroid to get the maximum light from the whole arrangement?

Solution:

I_o is the intensity of light existing between two cross polaroids kept with their axes perpendicular to each other.

θ = angle between first and 3rd polaroid.

Maximum light coming out from third polaroid is $I_0 /2$

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Malu's law is $I' = I_0 \cos^2\theta$; $\frac{I_0}{2} = I_0 \cos^2\theta = \frac{1}{2}$, $\cos\theta = \frac{1}{\sqrt{2}}$, $\theta = 45^0$.

18. **The reflected light is found to be plane polarized when an unpolarized light falls on a denser medium at 60^0 with the normal. Find the angle of refraction and critical angle of incidence for total internal reflection in the denser to rarer medium reflection.**

Solution:

Angle of refraction $r = 90^0 - I_p = 90^0 - 60^0 = 30^0$

Critical angle $i_c = \sin^{-1}\left(\frac{1}{n}\right) = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right) = \sin^{-1}\left(\frac{1}{1.732}\right) = \sin^{-1}(0.5774) = 35.15^0$

Unit – 8 – Dual Nature of Radiation and Matter

1. **A radiation of wavelength 300 nm is incident on a silver surface. Will photoelectrons be observed? [work function of silver = 4.7 eV]**

Solution:

Energy of the incident photon is

$E = h\nu = \frac{hc}{\lambda}$ (in joules)

$E = \frac{hc}{\lambda e}$ (in eV)

Substituting the known values, we get

$E = \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{300 \times 10^{-9} \times 1.6 \times 10^{-19}}$

$E = 4.14$ eV.

The work function of silver = 4.7 eV. Since the energy of the incident photon is less than the work function of silver, photoelectrons are not observed in this case.

2. **When light of wavelength 2200Å falls on Cu, photo electrons are emitted from it. Find (i) the threshold wavelength and (ii) the stopping potential. Given: the work function for Cu is $\phi_0 = 4.65$ eV.**

Solution:

i) The threshold wavelength is given by

$\lambda_0 = \frac{hc}{\phi_0} = \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{4.65 \times 1.6 \times 10^{-19}}$

$= 2672$ Å

ii) Energy of the photon of wavelength 2200 Å is

$E = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{2200 \times 10^{-10}}$

$= 9.035 \times 10^{-19}$ J = 5.65 eV.

We know that kinetic energy of fastest photo electron is

$K_{max} = h\nu = \phi_0 = 5.65 - 4.65 = 1$ eV

From equation (8.3), $K_{max} = eV_0$

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$V_0 = \frac{K_{max}}{e} = \frac{1 \times 1.6 \times 10^{-19}}{1.6 \times 10^{-19}}$

Therefore, stopping potential = 1V.

3. The work function of potassium is 2.30 eV. UV light of wavelength 3000 Å and intensity 2 Wm^{-2} is incident on the potassium surface. i) Determine the maximum kinetic energy of the photo electrons ii) If 40% of incident photons produce photo electrons, how many electrons are emitted per second if the area of the potassium surface is 2 cm^2 ?

Solution:

i) The energy of the incident photon is

$E = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{3000 \times 10^{-10}}$

$E = 6.626 \times 10^{-19}$ J = 4.14 eV.

Maximum KE of the photoelectrons is

$K_{max} = hv = \phi_0 = 4.14 - 2.30 = 1.84$ eV

ii) The number of photons reaching the surface per second is

$n_p = \frac{I}{E} \times A$

$= \frac{2}{6.626 \times 10^{-19}} \times 2 \times 10^{-4}$

$= 6.04 \times 10^{14}$ photons / sec

The rate of emission of photoelectron is

$= (0.40)\, n_p = 0.4 \times 6.04 \times 10^{14}$

$= 2.416 \times 10^{14}$ photoelectrons / sec

4. Calculate the momentum and the de Broglie wavelength in the following cases:

i) an electron with kinetic energy 2 eV.

ii) a bullet of 50 g fired from rifle with a speed of 200 m/s

iii) A 4000 kg car moving along the highways at 50 m/s

Hence show that the wave nature of matter is important at the atomic level but is not really relevant at macroscopic level.

Solution:

i) Momentum of the electron is

$P = \sqrt{2mK} = \sqrt{2 \times 9.1 \times 10^{-31} \times 2 \times 1.6 \times 10^{-19}}$

$= 7.63 \times 10^{25}$ kg ms^{-1}.

It de Broglie wavelength is

$\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34}}{7.63 \times 10^{-25}} = 0.868 \times 10^{-9}$ m

$= 8.68$ Å

ii) Momentum of the bullet is

P = mv = 0.050 x 200 = 10 kgms^{-1}.

Its de Broglie wavelength is

$$\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34}}{10} = 6.626 \text{ x } 10^{-35} \text{ m}$$

iii) Momentum of the car is

P = mv = 4000 x 50 = 2 x 10^5 kgms^{-1}

Its de Broglie wavelength is

$$\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34}}{2 \times 10^5} = 3.313 \text{ x } 10^{-39} \text{ m}$$

From these calculations, we notice that electron has significant value of de Broglie wavelength (≈10^{-9} mwhich can be measured from diffraction studies) but moving bullet and car have negligibly small de Broglie wavelengths associated with them (≈10^{-33} m and 10^{-39} m respectively, which are not measurable by any experiment). This implies that the wave nature of matter is important at the atomic level but it is not really relevant at the macroscopic level.

5. Find the de Broglie wavelength associated with an alpha particle which is accelerated through a potential difference of 400V. Given that the mass of the proton is 1.67×10^{-27} kg.

Solution:

An alpha particle contains 2 protons and 2 neutrons. Therefore, the mass M of the alpha particle is 4 times that of a proton (m_p) (or a neutron) and its charge q is twice that of a proton (+e).

The de Broglie wavelength associated with it is

$$\lambda = \frac{h}{\sqrt{2MqV}} = \frac{h}{\sqrt{2\times(4m_p)\times(2e)\times V}}$$

$$= \frac{6.626 \times 10^{-34}}{\sqrt{2 \times 4 \times 1.67 \times 10^{-27} \times 2 \times 1.6 \times 10^{-19} \times 400}}$$

$$= \frac{6.626\times10^{-34}}{4\times20\times10^{-23}\sqrt{1.67\times1.6}} = 0.00507 \text{ Å}.$$

6. Calculate the cut-off wavelength and cut-off frequency of x-rays from an x –ray tube of accelerating potential 20,000 V.

Solution:

The cut-off wavelength of the x-rays in the continuous spectrum is given by

$$\lambda_0 = \frac{12400}{V} \text{ Å} = \frac{12400}{20000} \text{ Å} = 0.62 \text{ Å}$$

The corresponding frequency is

$$\upsilon_0 = \frac{c}{\lambda_0} = \frac{3 \times 10^8}{0.62 \times 10^{-10}} = 4.84 \times 10^{18} Hz$$

7. **How many photons per second emanate from a 50 mW laser of 640 nm?**

<u>Solution:</u>

$$n = \frac{P}{E} = \frac{p\,\lambda}{hc} = \frac{50 \times 10^{-3} \times 640 \times 10^{-9}}{6.626 \times 10^{-34} \times 3 \times 10^{8}} = 1.61 \times 10^{17}\ s^{-1}$$

8. **Calculate the maximum kinetic energy and maximum velocity of the photoelectrons emitted when the stopping potential is 81 V for the photoelectric emission experiment.**

<u>Solution:</u>

$$K_{max} = eV_0 = 1.6 \times 10^{-19} \times 81 = 1.3 \times 10^{-17}\ J$$

$$\frac{1}{2} mv^2_{max} = 1.3 \times 10^{-17},\ V_{max} = \sqrt{\frac{2 \times 1.3 \times 10^{-17}}{9.1 \times 10^{-31}}} = 5.3 \times 10^{6}\ ms^{-1}$$

9. **How many photons of frequency 10^{14} Hz will make up 19.86 J of energy?**

<u>Solution:</u>

$$Nh\upsilon = 19.86,\ n = \frac{19.86}{6.626 \times 10^{-34} \times 10^{14}} = 3 \times 10^{20}$$

10. **What should be the velocity of the electron so that its momentum equals that of 4000 Å wavelength photon?**

<u>Solution:</u>

$$m\upsilon = \frac{h}{\lambda},\ \upsilon = \frac{h}{m\lambda} = \frac{6.626 \times 10^{-34}}{9.1 \times 10^{-31} \times 4 \times 10^{-7}} = 1818\ ms^{-1}$$

11. **UV light of wavelength 1800 Å is incident on a lithium surface whose threshold wavelength is 4965 Å. Determine the maximum energy of the electron emitted.**

<u>Solution:</u>

$$K_{max} = \frac{hc}{\lambda} - \frac{hc}{\lambda_0} = \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{1.6 \times 10^{-19}} \left(\frac{1}{1.8 \times 10^{-7}} - \frac{1}{4.965 \times 10^{-7}}\right)$$

$$= 12.42\ (0.556 - 0.2) = 12.42 \times 0.356 = 4.4\ eV$$

12. **Calculate the de Broglie wavelength of a proton whose kinetic energy is equal to 81.9×10^{-15} J. (Given: mass of proton is 1836 times that of electron)**

<u>Solution:</u>

$$\lambda = \frac{h}{\sqrt{2mK}} = \frac{6.626 \times 10^{-34}}{\sqrt{2 \times 1836 \times 9.1 \times 10^{-31} \times 81.9 \times 10^{-15}}} = \frac{6.626 \times 10^{-11}}{1654} = 4 \times 10^{-14} m$$

13. **A deuteron and an alpha particle are accelerated with the same potential. Which one of the two has (i) greater value of de Broglie wavelength associated with it and (ii) less kinetic energy? Explain.**

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Solution:

(i) Mass of an $\propto$ - particle = 2 times the mass of a deuteron. $m_{\propto} = 2m_d$

$$\lambda_d = \frac{h}{\sqrt{2em_dV}}, \lambda_\alpha = \frac{h}{\sqrt{2(2e)(2m_d)V}} = \frac{\lambda_d}{2}, \lambda_d = \lambda_\alpha$$

(ii) $\lambda_d = \frac{h}{\sqrt{2m_dK_d}}, \lambda_\alpha = \frac{h}{\sqrt{2(2m_d)K_\alpha}}$

$$\frac{\lambda_d^2}{\lambda_\alpha^2} = \frac{2K_\alpha}{K_d}, \frac{(2\lambda_\alpha)^2}{\lambda_\alpha^2} = 4 = \frac{2K_\alpha}{K_d}, K_d = \frac{K_\alpha}{2}$$

14. An electron is accelerated through a potential difference of 81 V. What is the de Broglie wavelength associated with it? To which part of electromagnetic spectrum does this wavelength correspond?

Solution:

$$\lambda = \frac{12.27}{\sqrt{V}} \text{ Å} = \frac{12.27}{\sqrt{81}} \text{ Å} = \frac{12.27}{9} \text{ Å} = 1.36 \text{ Å}$$

UNIT – 9 – ATOMIC AND NUCLEAR PHYSICS

1. The radius of the 5_{th} orbit of hydrogen atom is 13.25 Å. Calculate the de broglie wavelength of the electron orbitting in the 5_{th} orbit.

Solution:

$2\pi r = n\lambda$

$2 \times 3.14 \times 13.25\text{Å} = 5 \times \lambda$

$\therefore \lambda = 16.64\text{Å}$

2. Find the (i) angular momentum (ii) velocity of the electron revolving in the 5_{th} orbit of hydrogen atom.

($h = 6.6 \times 10^{-34}$ Js, $m = 9.1 \times 10^{-31}$ kg)

Solution

(i) Angular momentum is given by

$l = nh = \frac{nh}{2\pi}$

$= \frac{5 \times 6.6 \times 10^{-34}}{2 \times 3.14} = 5.25 \times 10^{-34}$ kgm^2 s^{-1}

(ii) Velocity is given by

Velocity $v = \frac{l}{mr}$

$= \frac{(5.25 \times 10^{-34}\ kgm^2 s^{-1})}{(9.1 \times 10^{-31} kg)(13.25 \times 10^{-10} m)}$

$v = 4.4 \times 10^5$ ms^{-1}

3. **Calculate the radius of $^{197}_{79}$Au nucleus.**

Solution

According to the equation (9.19),

R = 1.2 x 10^{-15} x $(197)^{\frac{1}{3}}$ = 6.97 x 10^{-15} m

Or R = 6.97 F.

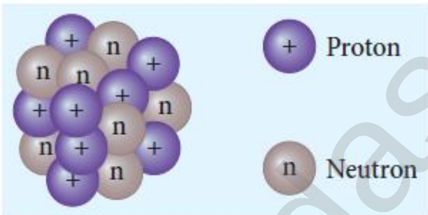
4. **Calculate the density of the nucleus with mass number *A*.**

Solution

From equation (9.19), the radius of the nuclecus, R = $R_0\, A^{\frac{1}{3}}$. Then the volume of the nucleus

$$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R_0^3 A$$

By ignoring the mass difference between the proton and neutron, the total mass of the nucleus having mass number *A* is equal to *A.m* where *m* is mass of the proton and is equal to 1.6726 x 10^{-27} kg.



Nuclear density

$$\rho = \frac{mass\ of\ the\ nuclei}{volume\ of\ the\ nuclei} = \frac{A.m}{\frac{4}{3}\pi R_0^3 A} = \frac{m}{\frac{4}{3}\pi R_0^3}$$

The above expression shows that the nuclear density is independent of the mass number *A*. In other words, all the nuclei ($Z > 10$) have the same density and it is an important characteristic property of all nuclei.

We can calculate the numerical value of this density by substituting the corresponding values.

$$\rho = \frac{1.67 \times 10^{-27}}{\frac{4}{3}\pi \times (1.2 \times 10^{-15})^3} = 2.3 \times 10^{17}\ \text{kg m}^{-3}.$$

It implies that nucleons are extremely tightly packed or compressed state in the Nucleus and compare this density with the density of water which is 10^3 kg m^{-3}.

5. **Compute the binding energy of $^{4}_{2}$He nucleus using the following data: Atomic mass of Helium atom, M_A (He) = 4.00260u and that of hydrogen atom, m_H = 1.00785u.**

Solution

Binding energy $BE = [Zm_H + Nm_H - M_A] c^2$

For helium nucleus, $Z = 2$, $N = A-Z = 4-2 = 2$

Mass defect

Δm = [(2 x 1.00785 u) + (2 x 1.008665 u) – 4.00260 u] Δm = 0.03043u

B.E = 0.03043u x c^2

B.E = 0.03043u x 93 MeV = 28.33 MeV

$[\because luc^2 = 931\ meV]$

The binding energy of the $^4_2$He nucleus is 28.33 MeV.

6. Compute the binding energy per nucleon of $^4_2$He nucleus.

Solution

Binding energy $BE = [Zm_H + Nm_H - M_A] c^2$

For helium nucleus, $Z = 2$, $N = A-Z = 4-2 = 2$

Mass defect

Δm = [(2 x 1.00785 u) + (2 x 1.008665 u) – 4.00260 u] Δm = 0.03043u

B.E = 0.03043u x c^2

B.E = 0.03043u x 93 MeV = 28.33 MeV

$[\because luc^2 = 931\ meV]$

The binding energy of the $^4_2$He nucleus is 28.33 MeV.

Binding energy per nucleon = $\overline{BE}$ = 28.33 MeV ≃ 7 MeV.

7. Calculate the number of nuclei of carbon-14 undecayed after 22,920 years if the initial number of carbon-14 atoms is 10,000. The half-life of carbon-14 is 5730 years.

Solution

To get the time interval in terms of half-life,

$n = \frac{t}{T_{1/2}} = \frac{22{,}920 yr}{5730\ yr} = 4$

The number of nuclei remaining undecayed after 22,920 years,

$N = \left(\frac{1}{2}\right)^n N_0 = \left(\frac{1}{2}\right)^4$ x 10,000

N = 625.

8. Calculate the amount of energy released when 1 kg of $^{235}_{92}$U undergoes fission reaction.

Solution

235 g of $^{235}_{92}$U has 6. 02 x 10^{23} atoms. In one gram of $^{235}_{92}$U, the number of atoms is equal to $\frac{6.02\ \times 10^{23}}{235}$ = 2.56 x 10^{21}

So the number of atoms present in 1 kg of $^{235}_{92}$U =2.56 x 10^{21} x 1000 = 2.56 x 10^{24}

Each $^{235}_{92}$U nucleus releases 200 MeV of energy during the fission. The total energy released by 1kg of $^{235}_{92}$U is

Q = 2.56 x 10^{24} x 200 MeV = 5.12x 10^{26} MeV

In terms of joules,

Q = 5.12 x 10^{26} x 1.6 x 10^{-13}J = 8.192 x 10^{13}J.

In terms of kilowatt hour,

Q = $\frac{8.192 \times 10^{13}}{3.6 \times 10^{6}} = 2.27 \times 10^{7} kWh.$

9. Calculate the mass defect and the binding energy per nucleon of the $^{108}_{47}Ag$ nucleus. (atomic mass of Ag = 107. 905949)

<u>Solution</u>

Δm = Zm_H + Nm_n – M = 47 x 1.007825 + 61 x 1.008665 – 107.905949

= 47.367775 + 61.528565 – 107. 905949 = 0.99391 u,

$\overline{BE} = \frac{BE}{A} = \frac{0.990391 \times 931}{108}$ MeV = 8.5 MeV

10. Half lives of two radioactive elements A and B are 20 minutes and 40 minutes respectively. Initially, the samples have equal number of nuclei. Calculate the ratio of decayed numbers of A and B nuclei after 80 minutes.

<u>Solution</u>

(i) For sample A: $n_A = \frac{t}{T} = \frac{80}{20}$ = 4

Fraction remaining undecayed $N_A = \frac{N_0}{2^4} = \frac{N_0}{16}$

Fraction decayed = $\frac{15}{16}$ N_0............... (1)

(ii) For sample B : $N_B = \frac{t}{T} = \frac{80}{40}$ = 2

Fraction remaining undecayed $n_B = \frac{N_0}{2^2} = \frac{N_0}{4}$

∴ Fraction decayed = $\frac{3}{4}$ N_0............... (2)

Ratio of decayed numbers = $\frac{15N_0}{16}$ x $\frac{4}{3N_0}$ = $\frac{5}{4}$ = 5 : 4

11. Calculate the time required for 60% of a sample of radon undergo decay. Given $T_{1/2}$ of radon = 3.8 days.

<u>Solution:</u>

$\lambda = \frac{0.6931}{3.8}$ = 0.1824. Amount of sample present undecayed = $\frac{40}{100}$ N_0

N = $N_0 e^{-\lambda t}$, $\frac{4}{10}$ N_0 = $N_0 e^{-\lambda t}$, $\frac{4}{10} = \frac{1}{e^{\lambda t}}$, $e^{\lambda t} = \frac{4}{10}$ = 2.5

λt = 2.3026 $\log_{10}$ 2.5 = 2.3026 x 0.3979, t = $\frac{0.916}{0.1824}$ = 5.022 days

12. Assuming that energy released by the fission of a single $^{235}_{92}$*U nucleus* is 200 MeV, calculate the number of fission per second required to produce 1 watt power.

Solution:

$200 \times 10^6 \times 1.6 \times 10^{-19}$ J = 3.2×10^{-11} J is produced per fission.

∴ Number of fissions per second required to produce

1 watt power = $\frac{1}{3.2\times10^{-11}} = \frac{10\times10^{10}}{3.2}$ = 3.125×10^{10}

13. Show that the mass of radium ($^{226}_{88}Ra$) with an activity of 1 curie is almost a gram. Given $T_{1/2}$ = 1600 years.

Solution:

$\lambda = \frac{0.6931}{T_{1/2}} = \frac{0.6931}{1600\times365\times24\times60\times60}$ = 0.134×10^{-10}

1 curie = 3.7×10^{10} Becquerel

$\frac{dN}{dt} = \lambda N$, $N = \frac{1}{\lambda}\frac{dN}{dt} = \frac{1\times3.7\times10^{10}}{0.134\times10^{-10}}$ = 2.76×10^{21}

Mass of 6.023×10^{23} atoms of Radium = 226 gram

∴ mass of 2.76×10^{21} atoms of radium = $\frac{226\times2.76\times10^{21}}{6.023\times10^{23}}$ = 1.03 gram ≃ 1 gram

14. Charcoal pieces of tree is found from an archaeological site. The carbon – 14 content of this charcoal is only 17.5% that of equivalent sample of carbon from a living tree. What is the age of tree?

Solution:

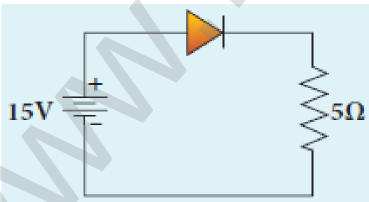
$\lambda = \frac{0.6931}{5730}$ = 1.21×10^{-4} year^{-1}

$N = N_0 e^{-\lambda t}$, $\frac{N}{N_0} = \frac{17.5}{100} = \frac{1}{e^{\lambda t}}$, $e^{\lambda t} = \frac{100}{17.5}$ = 5.714

$\lambda t = 2.302 \times \log_{10} 5.714 = 2.302 \times 0.7570$, $t = \frac{1.743}{1.21\times10^{-4}}$ = 1.44×10^4 year.

UNIT – 10 ELECTRONICS AND COMMUNICATION

1. An ideal diode and a 5 Ω resistor are connected in series with a 15 V power supply as shown in figure below. Calculate the current that flows through the diode.



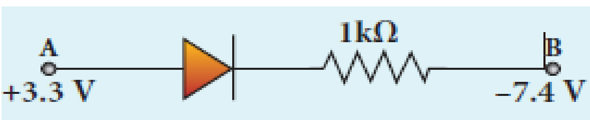
Solution

The diode is forward biased and it is an ideal one. Hence, it acts like a closed switch with no barrier voltage. Therefore, current that flows through the diode can be calculated using Ohm's law.

V = IR

$I = \frac{V}{R} = \frac{15}{5}$ = 3 A

2. **A silicon diode is connected with 1kΩ resistor as shown. Find the value of current flowing through *AB.***



Solution

The P.D. between A and B is given by

$V = [V_A - V_B] - V_b(Si)$

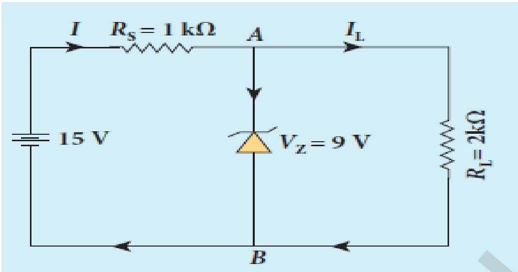
$= [3.3 - (-7.4)] - 0.7$

$= 10.7 - 0.7 = 10\ V$

The value of current flowing through *AB* can be obtained using Ohm's law.

$I = \frac{V}{R} = \frac{10}{1 \times 10^3} = 10^{-2}\ A = 10\ mA$

3. **Find the current through the Zener diode when the load resistance is 2 kΩ. Use diode approximation.**



Solution

Voltage across AB, $V_Z = 9V$

Voltage drop across $R_S = 15 - 9 = 6V$

Therefore current through the resistor R_S,

$I = \frac{6}{1 \times 10^3} = 6\ mA$

Voltage across the load resistor, $V_{AB} = 9V$

Current through load resistor,

$I_L = \frac{V_{AB}}{R_L} = \frac{9}{2 \times 10^3} = 4.5\ mA$

The current through the Zener diode,

$I_Z = I - I_L = 6\ mA - 4.5\ mA = 1.5\ mA.$

4. **Determine the wavelength of light emitted from LED which is made up of GaAsP semiconductor whose forbidden energy gap is 1.875 eV. Mention the colour of the light emitted (Take *h* = 6.6 × 10^{-34} Js).**

Solution

$E_g = \frac{hc}{\lambda}$

Therefore,

$\lambda = \frac{hc}{E_g} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{1.875 \times 1.6 \times 10^{-19}} = 660\ nm$

The wavelength 660 nm corresponds to red colour light.

5. In a transistor connected in the common base configuration, $\propto$ =095., A_E=1mA. Calculate the values of I_C and I_B.

Solution

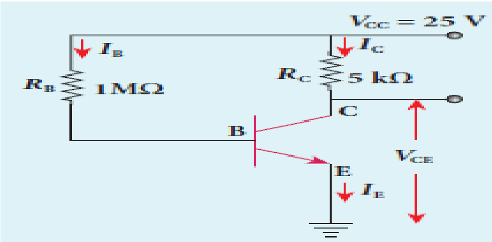
$$\propto = \frac{I_C}{I_E}$$

$I_C = \propto I_E = 0.95 \times 1 = 0.95$ mA

$I_E = I_B + I_C$

$\therefore I_B = I_E - I_C = 1 - 0.95 = 0.05$ mA

6. The current gain of a common emitter transistor circuit shown in figure is 120. Draw the DC load line and mark the *Q* point on it. (V_{BE} to be ignored).



Solution

$\beta = 120$

Base current, $I_B = \frac{25\ V}{1\ M\Omega} = \frac{25}{1 \times 10^6} = 25\mu A$

We know that

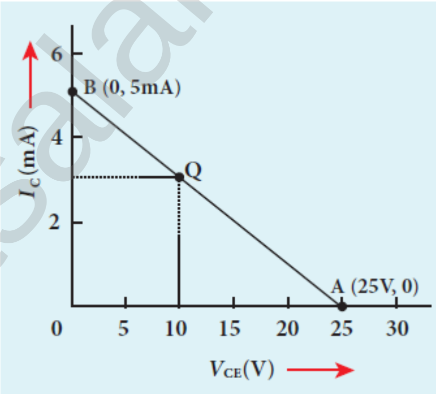
$\beta = \frac{I_C}{I_B}$ *(or)*

$I_C = \beta\, I_B = 120 \times 25\ \mu A$

$= 3000\ \mu A = 3$ mA

$V_{CE} = V_{CC} = I_C R_C$

$= 25 - (3mA \times 5k) = 10V$



7. Calculate the range of the variable capacitor that is to be used in a tuned-collector oscillator which has a fixed inductance of 150 μH. The frequency band is from 500 kHz to 1500 kHz.

Solution

Resonant frequency,

$$f = \frac{1}{2\pi\sqrt{LC}}$$

On simplifying, we get

$$C = \frac{1}{4\pi^2 f^2 L}$$

i) When frequency = 500 kHz,

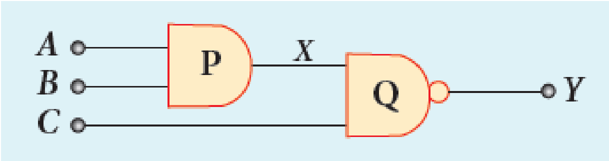
$$C = \frac{1}{4 \times 3.14^2 \times (1500 \times 10^3)^2 \times 150 \times 10^{-6}} = 676 \text{ pF}$$

ii) When frequency = 1500 kHz,

$$C = \frac{1}{4 \times 3.14^2 \times (1500 \times 10^3)^2 \times 150 \times 10^{-6}} = 75 \text{ pF}$$

Therefore, the capacitor range is from 75 to 676 pF.

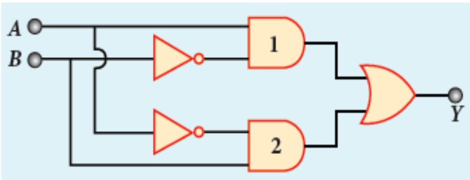
8. **What is the output *Y* in the following circuit, when all the three inputs *A*, *B*, and *C* are first 0 and then 1?**



Solution

A	B	C	X = A – B	Y = $\overline{X.C}$
0	0	0	0	1
1	1	1	1	0

9. **In the combination of the following gates, write the Boolean equation for output Y in terms of inputs A and B.**



Solution

The output at the 1st AND gate: $\overline{A}B$

The output at the 2nd AND gate: $A\overline{B}$

The output at the OR gate: Y = A. $\overline{B}$ + $\overline{A}$. B

10. **Prove the Boolean identity *AC + ABC = AC* and give its circuit description.**

Solution

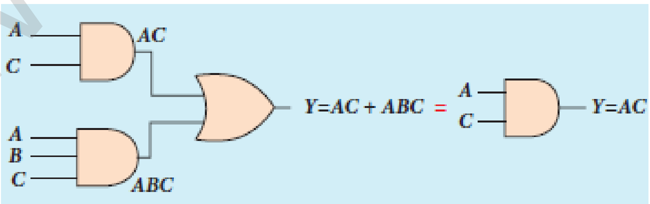
Step 1: $AC\,(1 + B) = AC.1$ [OR law-2]

Step 2: $AC\,.\,1 = AC$ [AND law – 2]

Therefore, $AC + ABC = AC$

Thus the Boolean identity is proved.

Circuit description:



11. **A transmitting antenna has a height of 40 m and the height of the receiving antenna is 30 m. What is the maximum distance between them for line-of-sight communication? The radius of the earth is 6.4×10^6 m.**

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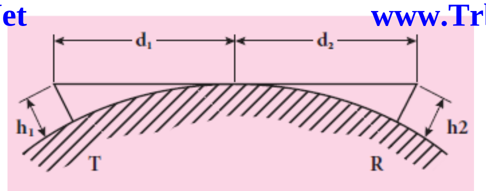


Solution

The total distance *d* between the transmitting and receiving antennas will be the sum of the individual distances of coverage.

$d = d_1 + d_2$

$= \sqrt{2Rh_1} + \sqrt{2Rh_2}$

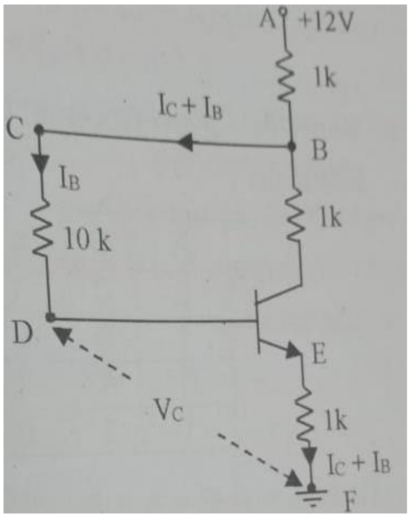
$= \sqrt{2R}\,(\sqrt{h_1} + \sqrt{h_2})$

$= \sqrt{2 \times 6.4 \times 10^6} \times (\sqrt{40} + \sqrt{30})$

$= 16 \times 10^2 \sqrt{5} \times (6.32 + 5.48)$

$= 42217$ m $= 42.217$ km

12. A transistor of $\alpha = 0.99$ and $V_{BE} = 0.7$ V, is connected in the common emitter configuration as shown in the figure. If the transistor is in saturation region, find the value of collector current.



Solution

$\beta = \frac{\alpha}{1-\alpha} = \frac{0.99}{1-0.99} = 99$

$\beta = \frac{I_C}{I_B}$, $99 = \frac{I_C}{I_B}$, $I_C = 99\, I_B$

Applying Kirchoffs voltage law for A B C D E F

$V_A - (I_C + I_B)\, 10^3 - I_B\,(10 \times 10^3) - V_{BE} - (I_C + I_B) \times 10^3 = 0$

$\frac{12}{10^3} - (99I_B + I_B) - 10I_B - 100I_B = 0$

$\frac{11.3}{10^3} - 100I_B - 10I_B = 100I_B = 0$

$210I_B = \frac{11.3}{10^3}$, $I_B = \frac{11.3}{210} \times 10^{-3} = 0.54 \times 10^{-3}$ A

$I_C = 99\, I_B = 99 \times 0.54 \times 10^{-3}$ A $= 5.25$ mA

13. **Prove the following Boolean expression using the laws and theorems of Boolean algebra**

(i) $(A + B)(A + \bar{B}) = A$ (ii) $A(\bar{A} + B) = AB$ (iii) $(A + B)(A + C) = A + BC$

Solution

(i) $(A + B)(A + \bar{B}) = AA + A\bar{B} + BA + B\bar{B} = A + A\bar{B} + BA + 0$

$= A(1 + B + \bar{B}) = A(1 + 1) = A(1) = A$

(ii) $A(\bar{A} + B) = A\bar{A} + AB = 0 + AB = AB$

(iii) $(A + B)(A + C) = AA + AC + AB + BC = A + AC + AB + BC$
$= A(1 + C + B) + BC = A + BC.$

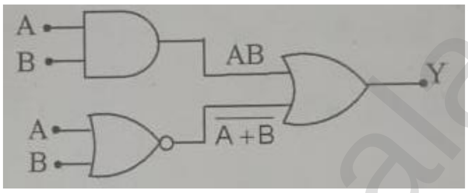
14. Verify the given Boolean equation $A + \bar{A}B = A + B$ using truth table.

<u>Solution</u>

A	B	$\bar{A}$	$\bar{A}B$	$A + \bar{A}B$	$A + B$
0	0	1	0	0	0
1	0	0	0	1	1
0	1	1	1	1	1
1	1	0	0	1	1

Thus $A + \bar{A}B = A + B$

15. Write down Boolean equation for the output Y of the given circuit and give its truth table.



Output $Y = AB + (\overline{A + B})$

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