PHYSICS

SOLUTION BOOK

Objective Type Questions and Exercise Problems

12

This book has been brought out with the Guidance of

THE DIRECTORATE OF SCHOOL EDUCATION

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CONTENTS

Chapter 1

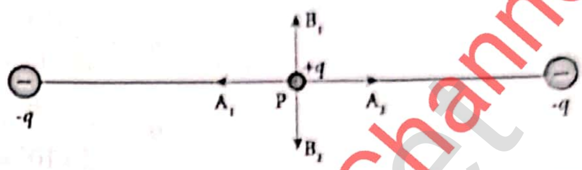
Electrostatics

Objective type Questions

1. Two identical point charges of magnitude $-q$ are fixed as shown in the figure below. A third charge $+q$ is placed midway between the two charges at the point P. Suppose this charge $+q$ is displaced a small distance from the point P in the directions indicated by the arrows, in which direction(s) will $+q$ be stable with respect to the displacement?

(a) A_1 and A_2 (b) B_1 and B_2

(c) both directions (d) No stable



Solution

When an object is slightly disturbed and the force is then removed, if it returns to its original position, then it is said to be in stable equilibrium.

[Option : (b)]

If q is taken to B_1, the resultant force on it due to the two $-q$ charges acts towards P and restores it to the initial position; the same is true for B_2. Therefore for B_1 and B_2 displacements, it is in stable equilibrium.

2. Which charge configuration produces a uniform electric field?

(a) point charge (b) infinite uniform line charge

(c) uniformly charged infinite plane (d) uniformly charged spherical shell

Solution

Uniformly charged infinite plane will produce a uniform electric field. $\left(\because\ E = \dfrac{\sigma}{2\varepsilon_0}\right)$.

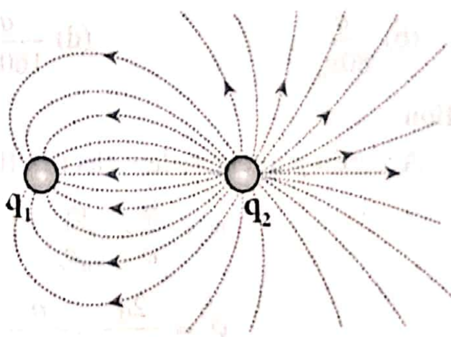
In all other options, $\bar{E}$ depends on r.

[Option : (c)]

3. What is the ratio of the charges $\left|\dfrac{q_1}{q_2}\right|$ for the following electric field line pattern?

(a) $\dfrac{1}{5}$ (b) $\dfrac{25}{11}$

(c) 5 (d) $\dfrac{11}{25}$



Solution

$$\frac{|q_1|}{|q_2|} = \frac{|N_1|}{|N_2|}$$

N_1 is the number of lines ending at q_1 from q_2.

N_2 is the number of lines leaving q_2.

$$\frac{|N_1|}{|N_2|} = \frac{11}{25}.$$

[Option : (d)]

4. An electric dipole is placed at an alignment angle of 30° with an electric field of $2\times10^5\ NC^{-1}$. It experiences a torque equal to $8\,Nm$. The charge on the dipole if the dipole length is $1cm$ is

(a) $4\,mC$ (b) $8\,mC$ (c) $5\,mC$ (d) $7\,mC$

Solution

$\theta = 30°$, $B = 2\times10^5\ NC^{-1}$, $\tau = 8\,Nm$, $2a = 1cm$

$$\tau = pE\sin\theta$$

$$\tau = q\times 2a\times E\sin\theta$$

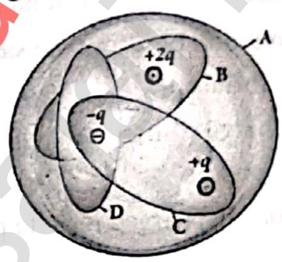
$$q = \frac{\tau}{2a\cdot E\cdot\sin\theta} = \frac{8}{0.01\times2\times10^5\times\sin 30°}$$

$$q = \frac{8}{2\times10^3\times\frac{1}{2}} = 8\times10^{-3}C = 8mC$$

[Option : (b)]

5. Four Gaussian surfaces are given below with charges inside each Gaussian surface. Rank the electric flux through each Gaussian surface in increasing order.

(a) $D<C<B<A$
(b) $A<B=C<D$
(c) $C<A=B<D$
(d) $D>C>B>A$



Solution

According to Gauss law the flux in the Gaussian surface

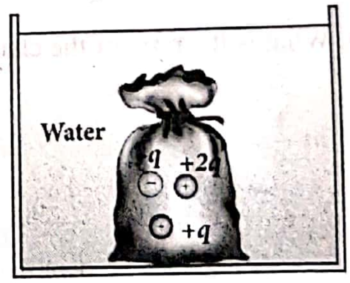
(a) A is $\frac{2q}{\epsilon_0}$ (b) B is $\frac{q}{\epsilon_0}$ (c) C is 0 (d) D is $\frac{-q}{\epsilon_0}$.

$\because D<C<B<A$ **[Option : (a)]**

6. The total electric flux for the following closed surface which is kept inside water

(a) $\frac{80q}{\varepsilon_0}$ (b) $\frac{q}{40\varepsilon_0}$

(b) $\frac{q}{80\varepsilon_0}$ (d) $\frac{q}{160\varepsilon_0}$



$[\varepsilon_r$ for water $=80]$

Solution

According to Gauss law electrix flux in a medium

$$\phi = \frac{q}{\varepsilon} = \frac{q}{\varepsilon_0\varepsilon_r}$$

$$\phi = \frac{2q}{80\varepsilon_0} = \frac{q}{40\varepsilon_0}$$

[Option : (b)]

7. Two identical conducting balls having unequal positive charges q_1 and q_2 are seperated by a centre to center distance r. If they are made to touch each other and then separated to the same distance, the force between them will be

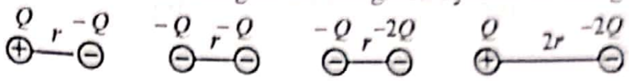
(a) Less than before (b) same as before (c) more than before (d) zero

Solution

When the balls touch each other, they share the charges equally $\left(\text{i.e., } q_1' = q_2' = \dfrac{q_1+q_2}{2}\right)$.

$\Rightarrow F' \propto \left(\dfrac{q_1+q_2}{2}\right)^2$; $\left(\dfrac{q_1+q_2}{2}\right)^2$ is always $> q_1q_2$. $\therefore F'$ is always $> F$. **[Option : (c)]**

8. Rank the electrostatic potential energies for the given system of charges in increasing order.



(1) (2) (3) (4)

(a) $1=4<2<3$ (b) $2=4<3<1$ (c) $2=3<1<4$ (d) $3<1<2<4$

Solution Potential energy $U = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1q_2}{r}$

(1) $U_1 \propto \dfrac{-Q^2}{r}$

(2) $U_2 \propto \dfrac{+Q^2}{r}$

(3) $U_3 \propto \dfrac{2Q^2}{r}$

(4) $U_4 \propto \dfrac{-Q^2}{r}$ $\left(\because \dfrac{Q\times 2Q}{2r} = \dfrac{-Q^2}{r}\right)$

$\therefore\ 1 = 4 < 2 < 3$ **[Option : (a)]**

9. An electric field $\vec{E} = 10x\hat{i}$ exists in a certain region of space. Then the potential difference $V = V_o - V_A$, where V_o is the potential at the origin and V_A is the potential at $x = 2m$ is

(a) 10J (b) −20J (c) +20J (d) −10J

Solution

$$\vec{E} = 10x\hat{i}$$

$$E = -\frac{dV}{dx}$$

$$dV = -E dx$$

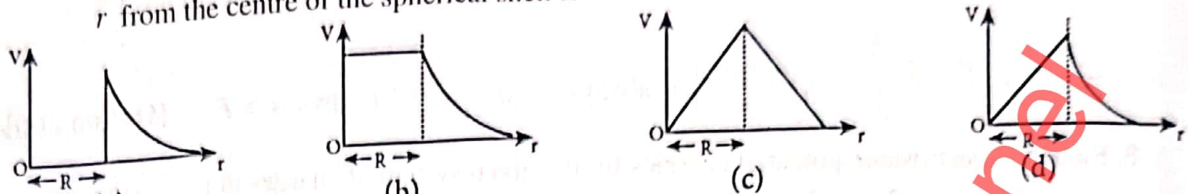
$$\int_{V_o}^{V_A} dV = V_A - V_o = -\int_0^2 E\,dx$$

$$= -\int_0^2 (10x\hat{i})\cdot(dx\hat{i})$$

$$V_A - V_o = -\int_0^2 10x\,dx = -10\left(\frac{x^2}{2}\right)_0^2 = -20\,\text{V}$$

$\Rightarrow V_o - V_A = 20\,\text{V}$. **[Option : (c)]**

10. A thin conducting spherical shell of radius R has a charge Q which is uniformly distributed on its surface. The correct plot representing the variation of electric potential with distance r from the centre of the spherical shell is



(a) (b) (c) (d)

Solution

Potential inside a conducting spherical shell is constant, outside it varies as $\frac{1}{r}$

[Option : (b)]

11. Two points A and B are maintained at a potential of $7\,V$ and $-4\,V$ respectively. The work done in moving 50 electrons from A to B is

(a) $8.80\times10^{-17}\,J$ (b) $-8.80\times10^{-17}\,J$ (c) $4.40\times10^{-17}\,J$ (d) $5.80\times10^{-17}\,J$

Solution

P.D. between A and B $= V_B - V_A = -4-(7) = -11\,V$

Work done in shifting 50 electrons from A to B, $W = Vq$

$$= -50\times1.6\times10^{-19}\times-11 \quad (\because\ q = ne)$$

$$= 8.80\times10^{-17}\,J.$$

[Option : (a)]

12. If voltage applied on a capacitor is increased from V to $2\,V$, choose the correct conclusion.

(a) Q remains the same, C is doubled (b) Q is doubled, C is doubled

(c) C remains same, Q doubled (d) Both Q and C remain same

Solution

If $V = 2V$, $Q = 2Q$ $(\because Q = CV)$

[As V doubles, Q is doubled]

$$C = \frac{Q}{V} \ ; \ C' = \frac{2Q}{2V}$$

$$C' = C \quad \text{(i.e., } C \text{ remains same)}$$

[Option : (c)]

13. A parallel plate capacitor stores a charge Q at a voltage V. Suppose the area of the parallel plate capacitor and the distance between the plates are each doubled then which is the quantity that will change?

(a) Capacitance (b) Charge (c) Voltage (d) Energy density

Solution

If A and d are doubled, then C will not change $\left(\because\ C' = \frac{\varepsilon_0(2A)}{2d} = \frac{\varepsilon_0 A}{d} = C\right)$ as C is constant V is constant, Q will also be constant.

$$u_E = \frac{U}{\text{volume}} \ ; \text{ if } A' = 2A \quad d' = 2d$$

$$\text{then } V' = 2A\times2d = 4Ad = 4V = u'_E = \frac{U}{4V} < u_E$$

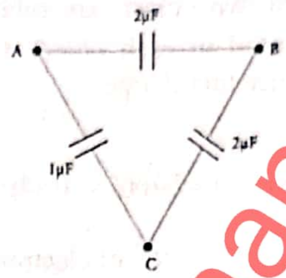
Volume is increased, so energy density will decrease.

[Option : (d)]

14. Three capacitors are connected in triangle as shown in the figure. The equivalent capacitance between the points A and C is

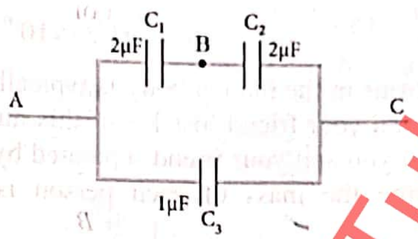
(a) $1\mu F$ (b) $2\mu F$

(c) $3\mu F$ (d) $\frac{1}{4}\mu F$



Solution

Equivalent diagram is



C_1 & C_2 are in series

$$\therefore \frac{1}{C_s} = \frac{1}{C_1}+\frac{1}{C_2}=\frac{1}{2}+\frac{1}{2}=1;\ C_s = 1\mu F$$

C_s & C_3 are in parallel.

$$\therefore C_p = C_s + C_3 = 1+1 = 2\mu F.$$ **[Option : (b)]**

15. Two metallic spheres of radii 1 cm and 3 cm are given charges of $-1\times10^{2}C$ and $5\times10^{-2}C$ respectively. If these are connected by a conducting wire, the final charge on the bigger sphere is

(a) $3\times10^{-2}C$ (b) $4\times10^{-2}C$ (c) $1\times10^{-2}C$ (d) $2\times10^{-2}C$

Solution

Since the spheres are connected by a conducting wire, their potential will be equal $\left(\frac{KQ}{r}=\text{constant}\right)$. Let the charges be Q_1' and Q_2'.

$$\Rightarrow Q_1'+Q_2' = -1\times10^{-2}+5\times10^{-2}=4\times10^{-2}\text{C}$$

$$\frac{KQ_1'}{1}=\frac{KQ_2'}{3} \Rightarrow 3Q_1'=Q_2'$$

$$Q_1'+Q_2' = -1\times10^{-2}+5\times10^{-2}=4\times10^{-2}$$

$$Q_1'+3Q_1' = 4\times10^{-2}\ ;\ 4Q_1' = 4\times10^{-2};\ Q_1'=1\times10^{-2}\text{C}\ ;\ \therefore\ Q_2'=3\times10^{-2}\text{C}.$$

[Option : (a)]

Exercise Problems

1. When two objects are rubbed with each other, approximately a charge of 50nC can be produced in each object. Calculate the number of electrons that must be transferred to produce this charge.

Solution

Given : $q = 50\text{nC} = 50\times10^{-9}\text{C}$, Electronic charge, $e = 1.6\times10^{-19}\text{C}$

No. of electrons transferred, $n = \dfrac{q}{e}$

$$n = \frac{50\times10^{-9}}{1.6\times10^{-19}}$$

$$n = 31.25\times10^{10}$$

2. The total number of electrons in the human body is typically in the order of 10^{28}. Suppose, due to some reason, you and your friend lost 1% of this number of electrons. Calculate the electrostatic force between you and your friend separated by a distance of 1m. Compare this with your weight. Assume the mass of each person is 60 kg and use point charge approximation

Given :

Total no. of electrons in the human body $= 10^{28}$

Total no. of electrons lost by me and my friend $= 10^{28}\times\dfrac{1}{100} = 10^{26}$.

Solution

$q_1 = q_2 = ne = 10^{26}\times1.6\times10^{-19}$

Since they lose electrons, they are positively charged

Electrostatic force, $F_e = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1q_2}{r^2} = \dfrac{9\times10^9\left[10^{26}\times1.6\times10^{-19}\right]^2}{1^2}$

$; \ 23\times10^{23}\text{N}$

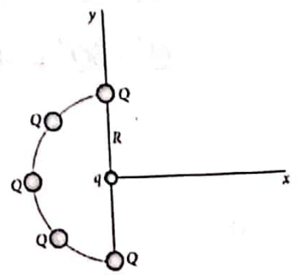
Mass of each person $m_1 = m_2 = 60\,\text{kg}$

Weight of each person, $W = mg = 60\times9.8\,\text{N}$

$W = 588\,\text{N}$

$$\frac{F_e}{W} = \frac{23\times10^{23}}{588} = 3.9\times10^{21}.$$

3. Five identical charges Q are placed equidistant on a semicircle as shown in the figure. Another point charge q is kept at the center of the circle of radius R. Calculate the electrostatic force experienced by the charge q.



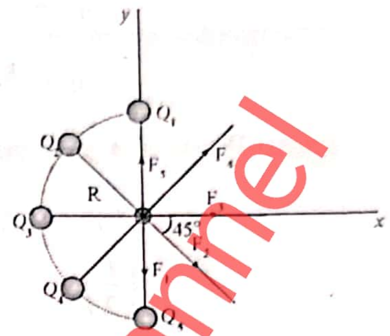
Solution

Let the forces acting on the charge q due to the charges Q_1, Q_2, Q_3, Q_4 and Q_5 be F_1, F_2, F_3, F_4 and F_5 respectively.

From the figure $F_1 = F_5$ and act in opposite direction. Hence they cancel each other.

Similarly, vertical components $F_2 \sin 45° = F_4 \sin 45°$.

$\Rightarrow$ these forces also cancel each others.



$$\bar{F} = \left(F_3 + F_2 \cos 45° + F_4 \cos 45°\right)\hat{i}$$

$$= \left(\frac{kQq}{R^2} + \frac{kQq}{R^2} \times \frac{1}{\sqrt{2}} + \frac{kQq}{R^2} \cdot \frac{1}{\sqrt{2}}\right)\hat{i}$$

$$= \frac{kQq}{R^2}\left[1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right]\hat{i}$$

$$= \frac{kQq}{R^2}\left[1 + \frac{2}{\sqrt{2}}\right]\hat{i}$$

$$= \frac{kQq}{R^2}\left(1 + \sqrt{2}\right)\hat{i}$$

$$\bar{F} = \frac{1}{4\pi \in_0} \frac{Qq}{R^2}\left(1+\sqrt{2}\right)\hat{i} \; N \quad \left[\because k = \frac{1}{4\pi \in_0}\right]$$

4. Suppose a charge $+q$ on Earth's surface and another $+q$ charge is placed on the surface of the Moon. (a) Calculate the value of q required to balance the gravitational attraction between Earth and Moon (b) Suppose the distance between the Moon and Earth is halved, would the charge q change? (Take $M_E = 5.9 \times 10^{24}$ kg, $M_M = 7.9 \times 10^{22}$ kg)

Given

$$M_E = 5.9 \times 10^{24} \text{ kg}$$

$$M_M = 7.9 \times 10^{22} \text{ kg}$$

$$G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$$

Solution

(i) $$F_g = \frac{GM_E M_M}{r^2}$$

$$F_e = \frac{kq^2}{r^2}$$

If $F_e = F_g$ then

$$\frac{kq^2}{r^2} = \frac{GM_E M_M}{r^2}$$

$$q^2 = \frac{GM_E M_M}{k}$$

$$q^2 = \frac{6.67\times10^{-11}\times5.9\times10^{24}\times7.9\times10^{22}}{9\times10^{9}} ; \frac{310.9}{9}\times10^{26} = 34.54\times10^{26}$$

$$q = 5.87\times10^{13}\text{C}$$

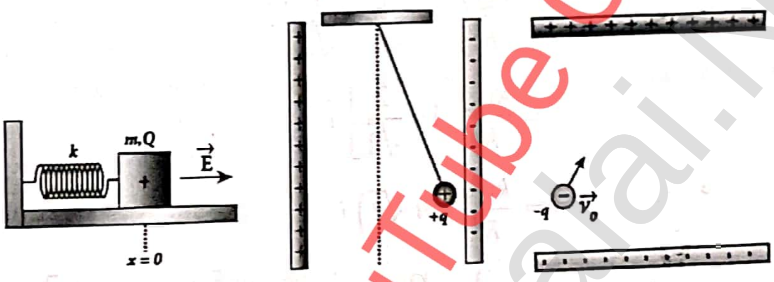
(ii) When $r \to \frac{r}{2}$

$$\frac{kq^2}{\left(\frac{r}{2}\right)^2} = \frac{GM_EM_M}{\left(\frac{r}{2}\right)^2}$$

$$q^2 = \frac{GM_EM_M}{k}$$

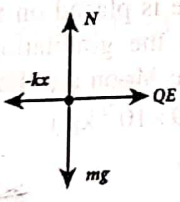
q remains the same.

5. Draw the free body diagram for the following charges as shown in the figure (a), (b) and (c).

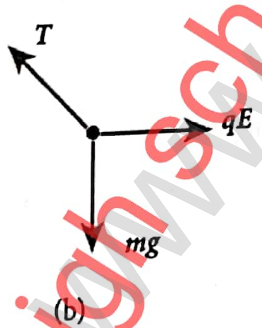


(a) (b) (c)

Solution



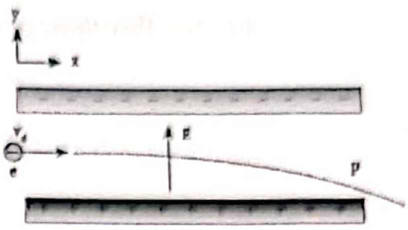
(i) Electrostatic force $(F_e = QE)$
(ii) Weight of the body (mg)
(iii) Restoring force $(-kx)$
(iv) Normal reaction (N)



(i) Electrostatic force $(F_e = qE)$
(ii) Weight of the bob (mg)
(iii) Tension (T)

(i) Force due to electric field $F_e = qE$
(ii) Downward gravitational force (mg)

6. Consider an electron travelling with a speed v_o and entering into a uniform electric field $\vec{E}$ which is perpendicular to $\vec{v}_o$ as shown in the Figure. Ignoring gravity, obtain the electron's acceleration, velocity and position as functions of time.



Solution

(a) Acceleration of the electron $a = \dfrac{F}{m}$

Electrostatic force $\vec{F} = -eE\hat{j}$

$$\vec{a} = \frac{\vec{F}}{m} = \left(\frac{-e\vec{E}}{m}\right)\hat{j}$$

(b) Let the initial velocity of the electron be $u = v_o$

$$\vec{v} = \vec{u} + \vec{a}t$$

$$\vec{a} = \frac{-e\vec{E}}{m}\hat{j} \quad ; \quad \vec{u} = v_o\hat{i}$$

$$\vec{v} = v_o\hat{i} - \frac{eE}{m}t\hat{j}$$

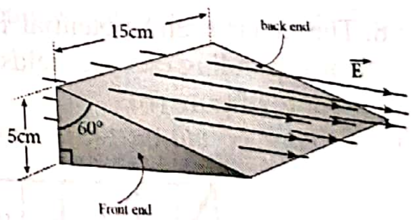
(c) Position of electron

$$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$

$$\vec{s} = \vec{r} \quad ; \quad \vec{u} = v_o\hat{i}$$

$$\vec{r} = v_o t\hat{i} - \frac{1}{2}\frac{eE}{m}t^2\hat{j}$$

7. A closed triangular box is kept in an electric field of magnitude $E = 2\times10^3\,NC^{-1}$ as shown in the figure. Calculate the electric flux through the (a) vertical rectangular surface (b) slanted surface and (c) enire surface.



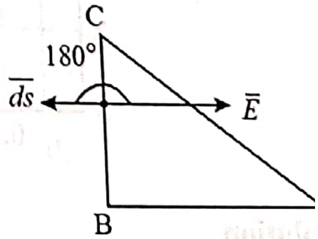
Solution

$$E = 2\times10^3\,\mathrm{NC^{-1}}$$

(i) Electric flux through vertical rectangular surface

$$\phi = \int E ds\cos\theta \quad \text{(flux lines are leaving the surface)}$$

Here $\theta = 180°$



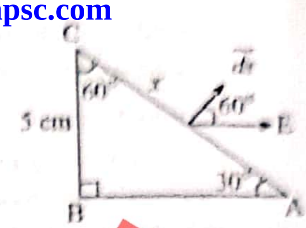
$$\therefore \phi = -\int Eds = -E\int ds$$

$$\int ds = s = l\times b = 15\times10^{-2}\times5\times10^{-2}$$

$$= 75\times10^{-4}\,\mathrm{m}^2$$

$$\phi = -2\times10^3\times75\times10^{-4} = -150\times10^{-1}$$

$$= -15\,\mathrm{Nm^2\,C^{-1}}$$



(ii) Electric flux through slanted surface

$$\phi = \int E\, ds \cos\theta$$

$$\phi = \int E\, ds \cos 60^\circ$$

$$= \frac{1}{2}\int E\, ds$$

$$\int ds = s = l \times x$$

$$\Delta ABC, \quad \sin 30^\circ = \frac{5\times10^{-2}}{x}$$

$$x = \frac{5\times10^{-2}}{\frac{1}{2}} = 10\times10^{-2}\,\text{m}$$

$$s = 10\times10^{-2}\times15\times10^{-2} = 150\times10^{-4}\,\text{m}^2$$

$$\phi = \frac{1}{2}\times2\times10^3\times150\times10^{-4}$$

$$= 15\,\text{Nm}^2\,\text{C}^{-1}.$$

(iii) Electric flux through entire surface of the box

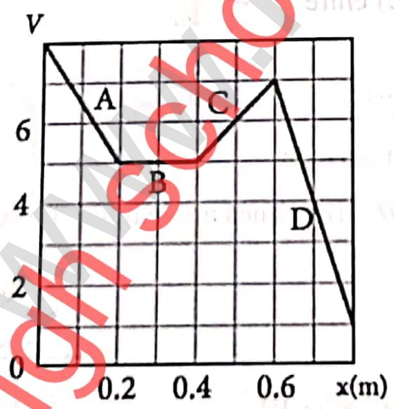
$$\phi = \phi_{\substack{\text{vertical}\\ \text{surface}}} + \phi_{\substack{\text{slanted}\\ \text{surface}}} + \phi_{\text{ends}}$$

At the ends $\quad \phi_{\text{ends}} = 0,$

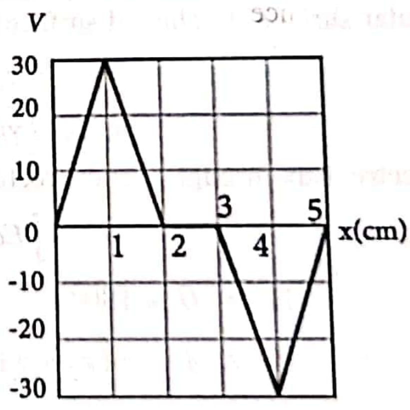
$$\phi = -15+15+0=0$$

Through vertical surface, field lines are entering, hence negative $\phi = -15$. Through the slanted surface, same amount of lines are going outward from the surface $\phi = +15$.

8. The electrostatic potential is given as a function of x in figure (a) and (b). Calculate the corresponding electric fields in regions A, B, C and D. Plot the electric field as a function of x for the figure (b).



(a)



(b)

Solution

$$\text{Electric field} \quad E = -\left(\frac{dV}{dx}\right)$$

Fig. (a) (a) Region A

$$\therefore E = 0$$

$$E = -\frac{dV}{dx} = \frac{-(5-8)}{(0.2-0)} = \frac{3}{0.2} = 15\,\text{Vm}^{-1}$$

(b) Region B

There is no change in potential with respect to x

$$\therefore dV = 0 \; ; \; \therefore E = 0$$

(c) Region C

$$E = \frac{-dV}{dx} = \frac{-(7-5)}{0.6-0.4} = \frac{-2}{0.2} = -10\,\text{Vm}^{-1}$$

(d) Region D

$$E = \frac{-dV}{dx} = -\left(\frac{1-7}{0.8-0.6}\right) = \frac{6}{0.2} = 30\,\text{Vm}^{-1}$$

Fig. (b)

$$E = \frac{-dV}{dx}$$

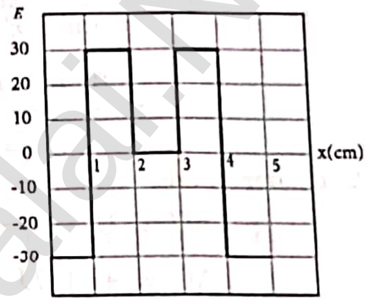
(i) $E = \frac{-(30-0)}{(1-0)} = -30\,\text{Vm}^{-1}$

(ii) $E = \frac{-(0-30)}{(2-1)} = +30\,\text{Vm}^{-1}$

(iii) $E = \frac{-(0-0)}{(3-2)} = 0\,\text{Vm}^{-1}$

(iv) $E = \frac{-(-30-0)}{(4-3)} = 30\,\text{Vm}^{-1}$

(v) $E = -\frac{[0-(-30)]}{(5-4)} = -30\,\text{Vm}^{-1}$



9. A spark plug in a bike or a car is used to ignite the air-fuel mixture in the engine. It consists of two electrodes separated by a gap of around 0.6 mm gap as shown in the figure. To create the spark, an electric field of magnitude $3\times10^6 Vm^{-1}$ is required. (a) What potential difference must be applied to produce the spark? (b) If the gap is increased, does the potential difference increase, decrease or remains the same? (c) find the potential difference if the gap is 1 mm.

Given

$$d = 0.6mm = 0.6\times10^{-3}\,\text{m}$$
$$E = 3\times10^6\,\text{Vm}^{-1}$$

Solution

(a)

$$E = \frac{V}{d}$$
$$V = E.d = 3\times10^6\times0.6\times10^{-3}$$
$$= 18\times10^2 = 1800\,\text{V}$$

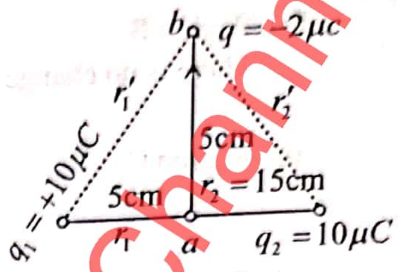
(b) If the gap d increases, potential difference increases ($\because V \propto d$).

(c)

$$d = 1\text{mm}$$
$$V = E\times d = 3\times10^6\times1\times10^{-3}$$
$$= 3\times10^3 = 3000\,\text{V}$$

10. A point charge of $+10\mu C$ is placed at a distance of 20cm from another identical point charge of $+10\mu C$. A point charge of $-2\mu C$ is moved from point a to b as shown in the figure. Calculalte the change in potential energy of the system? Interpret your result.

Solution



$$r_1' = \sqrt{5^2+5^2} = 5\sqrt{2}\text{ cm}$$

$$r_2' = \sqrt{15^2+5^2} = 5\sqrt{10}\text{ cm}$$

Initial P.E. when $-2\mu C$ is at 'a',

$$U_i = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q}{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{q_2 q}{r_2}$$

$$= \frac{9\times10^9\times(-2\times10^{-6})(10\times10^{-6})}{10^{-2}}\left[\frac{1}{5}+\frac{1}{15}\right] \Rightarrow U_i = -4.8\text{J}$$

Final P.E. when $-2\mu C$ is at 'b',

$$U_f = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q}{r_1'} + \frac{1}{4\pi\varepsilon_0}\frac{q_2 q}{r_2'}$$

$$= \frac{9\times10^9\times(-2\times10^{-6})(10\times10^{-6})}{10^{-2}}\left[\frac{1}{5\sqrt{2}}+\frac{1}{5\sqrt{10}}\right] \Rightarrow U_f = -3.683\text{J}$$

$\therefore$ change in P.E.

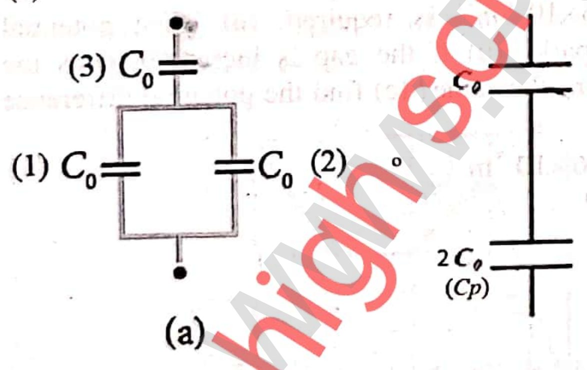
$$\Delta U = U_f - U_i = -3.683-(-4.8)$$

$$\therefore \Delta U = +1.117\text{J} \;\square +1.12\text{ J}$$

Since ΔU is positive, work has to be done to move 'q' from 'a' to 'b'.

11. Calculate the resultant capacitances for each of the following combinations of capacitors.

(a)



Capacitor 1 & 2 are parallel.

$$\therefore C_P = C_o + C_o = 2C_o$$

Capacitors C_P and C_3 are in series

$$\frac{1}{C_s} = \frac{1}{C_P}+\frac{1}{C_3}$$

$$= \frac{1}{2C_o}+\frac{1}{C_o} = \frac{3}{2C_o}$$

$$C_s = \frac{2C_0}{3}$$

(b)

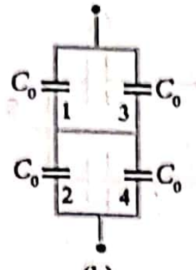
Capacitors 1&3 are in parallel.

$$C_P = C_o + C_o$$

$$= 2C_o$$

Capacitors 2 & 4 are in parallel

$$C_P' = C_o + C_o$$



(b)

≡

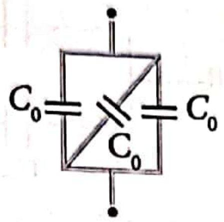
$= 2C_o$

C_p and C_p' are in series.

$$\frac{1}{C_s} = \frac{1}{C_p} + \frac{1}{C_p'}$$

$$= \frac{1}{2C_o} + \frac{1}{2C_o} \Rightarrow C_s = C_o$$

(c)



(c)

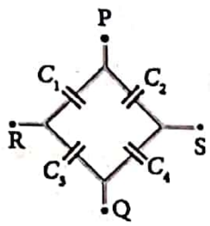
All the capacitances are in parallel

Resultant capacitance C_p

$$C_p = C_o + C_o + C_o$$

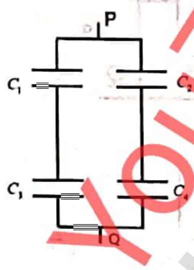
$$C_p = 3C_o$$

(d)



(d)

≡



(1) For across PQ

C_1 and C_3 are in series

$$\frac{1}{C_{S_1}} = \frac{1}{C_1} + \frac{1}{C_3}$$

$$C_{S_1} = \frac{C_1 C_3}{C_1 + C_3}$$

C_2 and C_4 are in series

$$\frac{1}{C_{S_2}} = \frac{1}{C_2} + \frac{1}{C_4}$$

$$C_{S_2} = \frac{C_2 C_4}{C_2 + C_4}$$

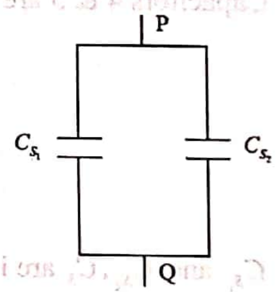
C_{S_1} and C_{S_2} are parallel

$$C_{PQ} = C_{S_1} + C_{S_2}$$

$$C_{PQ} = \frac{C_1 C_3}{C_1 + C_3} + \frac{C_2 C_4}{C_2 + C_4}$$

$$C_{PQ} = \frac{C_1 C_3 (C_2 + C_4) + C_2 C_4 (C_1 + C_3)}{(C_1 + C_3)(C_2 + C_4)}$$

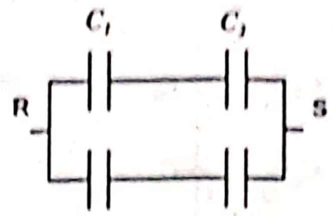
$$= \frac{C_1 C_2 C_3 + C_1 C_3 C_4 + C_1 C_2 C_4 + C_2 C_3 C_4}{(C_1 + C_3)\ (C_2 + C_4)}$$



(2) Equivalent capacitance between R and S

C_1 & C_2 are in series, $\frac{1}{C_{S_1}} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C_{S_1} = \frac{C_1 C_2}{C_1 + C_2}$

C_3 & C_4 are in series, $\frac{1}{C_{S_2}} = \frac{1}{C_3} + \frac{1}{C_4} \Rightarrow C_{S_2} = \frac{C_3 C_4}{C_3 + C_4}$

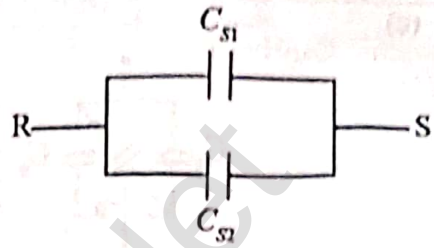


Now C_{S_1} and C_{S_2} are in parallel

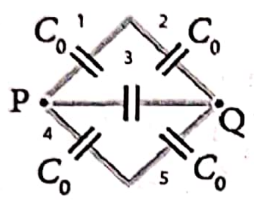
$$C_{RS} = C_{S_1} + C_{S_2} = \frac{C_1 C_2}{C_1 + C_2} + \frac{C_3 C_4}{C_3 + C_4}$$

$$\therefore C_{RS} = \frac{C_1C_2C_3 + C_1C_2C_4 + C_1C_3C_4 + C_2C_3C_4}{(C_1 + C_2)(C_3 + C_4)}$$

$$\Rightarrow \quad C_{PQ} = C_{RS}$$



(e)



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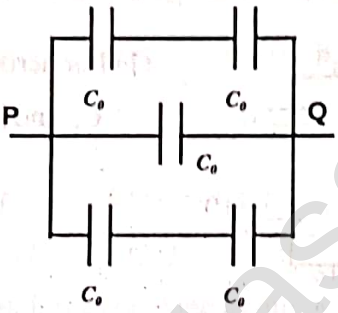


Capacitors 1 & 2 are in series

$$\frac{1}{C_{S_1}} = \frac{1}{C_o} + \frac{1}{C_o} = \frac{2}{C_o}$$

$$C_{S_1} = \frac{C_o}{2}$$

Capacitors 4 & 5 are in series

$$\frac{1}{C_{S_2}} = \frac{1}{C_o} + \frac{1}{C_o} = \frac{2}{C_o}$$

$$C_{S_2} = \frac{C_o}{2}$$

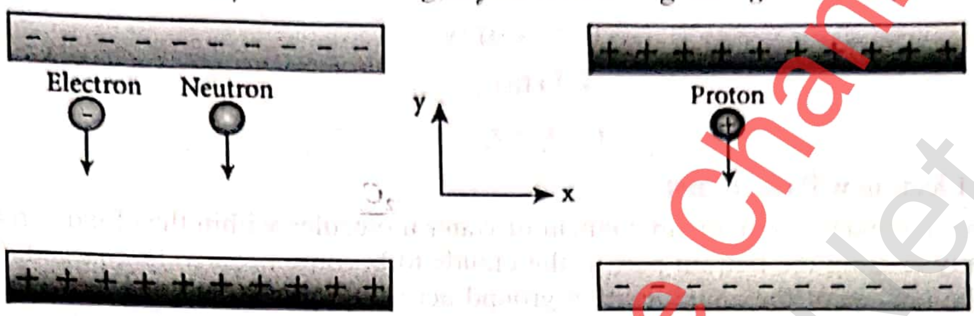
C_{S_1} and C_{S_2}, C_3 are in parallel.

$$C_p = C_{S_1} + C_{S_2} + C_3$$

$$= \frac{C_o}{2} + \frac{C_o}{2} + C_o$$

$$= C_o + C_o = 2C_o$$

12. An electron and a proton are allowed to fall through the separation between the plates of a parallel plate capacitor of voltage 5V and separation distance $h = 1$ mm as shown in the figure. (a) Calculate the time of flight for both electron and proton (b) Suppose if a neutron is allowed to fall, what is the time of flight? (c) Among the three, which one will reach the bottom first? (Take $m_p = 1.5\times10^{-27}$ kg, $m_e = 9.1\times10^{-31}$ kg and $g = 10\,\text{ms}^{-2}$)



Solution

$$h = 1mm = 1\times10^{-3}\text{m} \;;\; V = 5\text{V} \;;\; m_p = 1.6\times10^{-27}\text{ kg}$$

$$m_e = 9.1\times10^{-31}\text{ kg} \;;\; e = 1.6\times10^{-19}\text{C}$$

(a) For electron, $E = \frac{V}{d} = \frac{5}{1\times10^{-3}} = 5\times10^{3}\ \text{NC}^{-1}$

(Gravitational force is ignored) $s = ut + \frac{1}{2}at^2$

$$u = 0 \quad a = \frac{F}{m_e} = \frac{eE}{m_e}$$

$$h = 0 + \frac{1}{2}\frac{Ee}{m_e}t_e^2 \qquad (\because s = h)$$

$$t_e^2 = \frac{2hm_e}{eE}$$

$$t_e^2 = \frac{2\times9.11\times10^{-31}\times1\times10^{-3}}{5\times10^{3}\times1.6\times10^{-19}}$$

$$= \frac{2\times9.11\times10^{-18}}{8} = 2.278\times10^{-18}$$

$$t_e = \sqrt{2.278\times10^{-18}}$$

$$t_e = 1.509\times10^{-9}\text{s} \approx 1.5\text{ ns}.$$

for proton, $t_p^2 = \frac{2hm_p}{eE} = \frac{2\times1\times10^{-3}\times1.6\times10^{-27}}{1.6\times10^{-19}\times5\times10^{3}}$

(Gravitational force is ignored) $= 0.4\times10^{-14}$

$$\therefore t_p \approx 0.63\times10^{-7}\text{s}$$

$$t_p = 63\times10^{-9}\text{s} = 63\text{ns}$$

(b) for neutron $h = 0 + \frac{1}{2} g t_n^2 \quad (\because a = g)$

(free fall under gravity) $t_n = \sqrt{\frac{2h}{g}} \quad \left(\because h = \frac{1}{2} g t_n^2\right)$

$$t_n = \sqrt{\frac{2 \times 1 \times 10^{-3}}{10}}$$

$$= \sqrt{2 \times 10^{-4}}$$

$$= 1.414 \times 10^{-2} \text{s}$$

$$= 14.14 \text{ms}$$

(c) $t_e < t_p < t_n$

Electron will reach first.

13. During a thunder storm, the movement of water molecules within the clouds creates friction, partially causing the bottom part of the clouds to become negatively charged. This implies that the bottom of the cloud and the ground act as a parallel plate capacitor. If the electric field between the cloud and ground exceeds the dielectric breakdown of the air $(3 \times 10^6 Vm^{-1})$, lightning will occur.

(a) If the bottom part of the cloud is 1000 m above the ground, determine the electric potential difference that exists between the cloud and ground.

(b) In a typical lightning phenomenon, around 25C of electrons are transferred from cloud to ground. How much electrostatic potential energy is transferred to the ground?

Given : $E = 3 \times 10^6 \text{ Vm}^{-1} \quad q = 25\text{C}$

$d = 1000 \text{m}$

(a) Electric potential difference between the cloud and the ground $= V$

$$V = E.d \quad \left[\because E = \frac{V}{d}\right]$$

$$= 3 \times 10^6 \times 1000$$

$$V = 3 \times 10^9 \text{ V}.$$

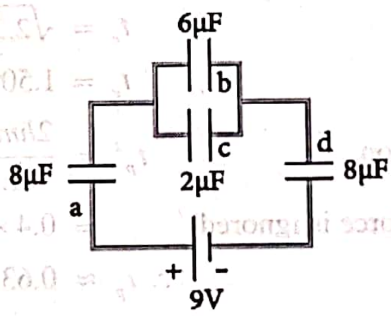
(b) Potential energy transferred to the ground $= W$

$$W = Vq$$

$$= 3 \times 10^9 \times 25$$

$$= 75 \times 10^9 \text{J}.$$

14. For the given capacitor configuration (a) Find the charges on each capacitor (b) potential difference across them (c) energy stored in each capacitor



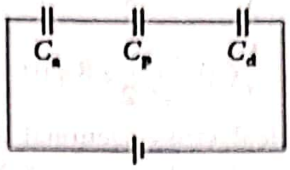
Given : $C_a = 8\mu\text{F}, \quad C_b = 6\mu\text{F}, \quad C_c = 2\mu\text{F}, \quad C_d = 8\mu\text{F}, \quad V = 9\text{V}$

Solution

Capacitors b and c are connected in parallel.

$$C_p = C_b + C_c$$
$$= 6 + 2 = 8\mu F$$

C_a, C_p, C_d are in series.



$$\frac{1}{C_s} = \frac{1}{C_a} + \frac{1}{C_p} + \frac{1}{C_d} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$$

$$C_s = \frac{8}{3}\ \mu F.$$

(a) Total charge flowing, $q = C_s V = \frac{8}{3} \times 10^{-6} \times 9 = 24\,\mu C$

Same charge $(24\mu C)$ will flow through C_a, C_p and C_d. ($\because$ they are in series)

$$V_{C_a} + V_{C_p} + V_{C_d} = 9$$

$$V = \frac{Q}{C}\ ;\ Q \text{ is constant and } C_a = C_p = C_d$$

$$\Rightarrow V_{C_a} = V_{C_p} = V_{C_d} = V$$

$$3V = 9$$

$$V = \frac{9}{3} = 3V$$

$$Q_a = C_a V = 8 \times 10^{-6} \times 3 = 24 \times 10^{-6} C = 24\mu C$$
$$Q_b = C_b V = 6 \times 10^{-6} \times 3 = 18 \times 10^{-6} C = 18\mu C$$
$$Q_c = C_c V = 2 \times 10^{-6} \times 3 = 6 \times 10^{-6} C = 6\mu C$$
$$Q_d = C_d V = 8 \times 10^{-6} \times 3 = 24 \times 10^{-6} C = 24\mu C.$$

(b) Potential difference across each capacitor

$$V_a = \frac{Q_a}{C_a} = \frac{24 \times 10^{-6}}{8 \times 10^{-6}} = 3V$$

$$V_b = \frac{Q_b}{C_b} = \frac{18 \times 10^{-6}}{6 \times 10^{-6}} = 3V$$

$$V_c = \frac{Q_c}{C_c} = \frac{6 \times 10^{-6}}{2 \times 10^{-6}} = 3V$$

$$V_d = \frac{Q_c}{C_d} = \frac{24 \times 10^{-6}}{8 \times 10^{-6}} = 3V$$

(c) Energy stored in the cacitors are

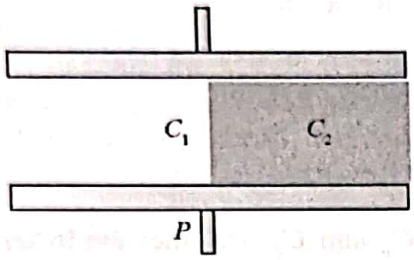
$$U_a = \frac{1}{2}C_aV^2 = \frac{1}{2}\times 8\times 10^{-6}\times 3\times 3 = 36\mu J$$

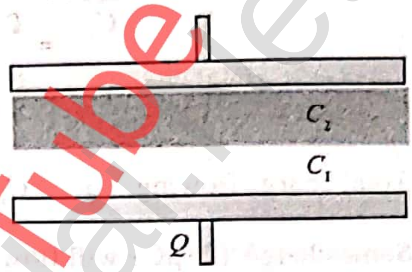
$$U_b = \frac{1}{2}C_bV^2 = \frac{1}{2}\times 2\times 10^{-6}\times 3\times 3 = 27\mu J$$

$$U_c = \frac{1}{2}C_cV^2 = \frac{1}{2}2\times 10^{-6}\times 3\times 3 = 9\mu J$$

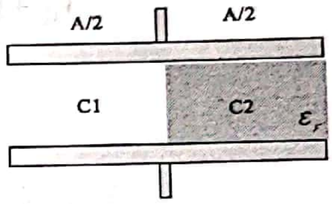
$$U_d = \frac{1}{2}C_dV^2 = \frac{1}{2}\times 8\times 10^{-6}\times 3\times 3 = 36\mu J.$$

15. Capacitors P and Q have identical cross sectional areas A and separation d. The space between the capacitors is filled with a dielectric of dielectric constant ε_r as shown in the figure. Calculate the capacitance of capacitors P and Q.





(a) In Capacitor P, as C_1 and C_2 are connected to the same potential difference, $C_1 \& C_2$ are in parallel

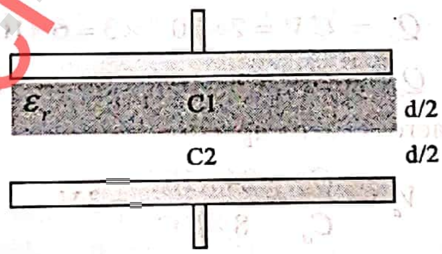


$\Rightarrow C_p = C_1 + C_2$

$$C_1 = \frac{\varepsilon_0(A/2)}{d} \quad ; \quad C_2 = \frac{\varepsilon_r\varepsilon_0 A/2}{d}$$

$$\therefore \; C_p = \frac{\varepsilon_0 A}{2d} + \frac{\varepsilon_r\varepsilon_0 A}{2d} \Rightarrow C_p = \frac{\varepsilon_0 A}{2d}(1+\varepsilon_r)$$

(b) In Capacitor Q, as C_1 and C_2 have the same charge, they are in series



$$\text{Capacitance of } C_1 = \frac{\varepsilon_0 A}{\frac{d}{2}} = \frac{2\varepsilon_0 A}{d}$$

$$\text{Capacitance of } C_2 = \frac{\varepsilon_0\varepsilon_r A}{\frac{d}{2}} = \frac{2\varepsilon_0\varepsilon_r A}{d}$$

If C_Q is the effective capacitance

Total capacitance $\frac{1}{C_Q} = \frac{1}{C_1} + \frac{1}{C_2}$

$$\frac{1}{C_Q} = \frac{d}{2\varepsilon_0 A} + \frac{d}{2\varepsilon_0 \varepsilon_r A}$$

$$= \frac{d}{2\varepsilon_0 \varepsilon_r A} + \frac{d}{2\varepsilon_0 A}$$

$$\frac{1}{C_Q} = \frac{d}{2\varepsilon_0 A}\left[\frac{1}{\varepsilon_r} + 1\right]$$

$$\frac{1}{C_Q} = \frac{d}{2\varepsilon_0 A}\left(\frac{1+\varepsilon_r}{\varepsilon_r}\right)$$

$$C_Q = \frac{2\varepsilon_0 A}{d}\left(\frac{\varepsilon_r}{1+\varepsilon_r}\right).$$

Chapter 2

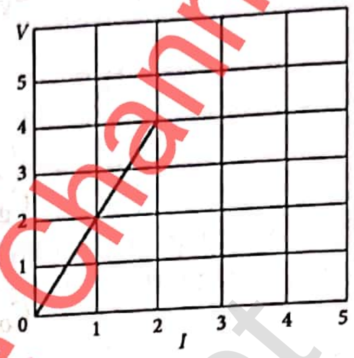
Current Electricity

Objective type Questions

1. The following graph shows current versus voltage values of some unknown conductor. What is the resistance of this conductor?

(a) 2 ohm (b) 4 ohm

(c) 8 ohm (d) 1 ohm



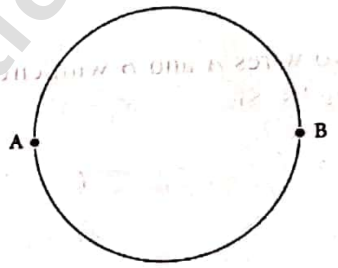
Solution

According to Ohm's law, $V = IR \qquad R = \frac{V}{I}$

$$\text{Slope } R = \frac{\Delta V}{\Delta I} = \frac{4-0}{2-0} = \frac{4}{2} = 2 \text{ ohm}$$

[Option : (a)]

2. A wire of resistance 2 ohms per metre is bent to form a circle of radius $1m$. The equivalent resistance between its two diametrically opposite points A and B as shown in figure is



(a) $\pi\Omega$ (b) $\frac{\pi}{2}\Omega$

(c) $2\pi\Omega$ (d) $\frac{\pi}{4}\Omega$

Solution

Total resistance of the wire $R = 2\pi r \times 2 = 4\pi r = 4\pi \times 1 = 4\pi \quad (\because r = 1)$

Two diametrically opposite points correspond to two semi circular lengths connected in parallel. Since they have half the length of the original wire, each one will have resistance $= \frac{R}{2} = \frac{4\pi}{2} = 2\pi$.

The resistances are in parallel.

$$\therefore \frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{2\pi} + \frac{1}{2\pi} = \frac{1}{\pi}$$

$$R_p = \pi\Omega$$

[Option : (a)]

3. A toaster operating at $240V$ has a resistance of 120Ω. Its power is

(a) 400 W (b) 2 W (c) 480 W (d) 240 W

Solution

$$P = VI = \frac{V^2}{R} = \frac{240 \times 240}{120} = 240 \times 2 = 480\,\text{W}$$

[Option : (c)]

4. A carbon resistor of $(47 \pm 4.7)k\Omega$ to be marked with rings of different colours for its identification. The colour code sequence will be

(a) yellow–green–violet–gold (b) yellow–violet–orange–silver

(c) violet–yellow–orange–silver (d) green–orange–violet–gold

Solution

B	B	R	O	Y	G	B	V	G	W
0	1	2	3	4	5	6	7	8	9

4–yellow, 7–violet, $1k\Omega = 10^3\Omega$ –orange ; ; $4.7k\Omega = 10\%$ of $47k\Omega$

$\Rightarrow$ tolerance $= 10\%$ $\therefore$ silver

[Option : (b)]

5. What is the value of the resistance of the following resistor?

(a) $100k\Omega$ (b) $10k\Omega$

(c) $1k\Omega$ (d) $1000k\Omega$

Solution

Brown – 1 Black – 0 Yellow – 10^4

$$10 \times 10^4 = 100 \times 10^3 = 100k\Omega$$

[Option : (a)]

6. Two wires A and B with circular cross section are made up of the same material with equal lengths. Suppose $R_A = 3R_B$ then what is the ratio of radius of wire A to that of B?

(a) 3 (b) $\sqrt{3}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{3}$

Solution

$$R_A = 3R_B \text{ (given)} \quad R = \frac{\rho l}{\pi r^2}$$

$$R \;\alpha\; \frac{1}{r^2} \quad (\because \rho \text{ and } l \text{ are same})$$

$$R_A = \frac{1}{r_A^2} \quad R_B = \frac{1}{r_B^2} \quad \frac{R_A}{R_B} = \frac{r_B^2}{r_A^2}$$

$$\frac{r_A^2}{r_B^2} = \frac{1}{3}$$

$$\frac{r_A}{r_B} = \frac{1}{\sqrt{3}}$$

[Option : (c)]

7. A wire connected to a power supply of $230V$ has power dissipitation P_1. Suppose the wire is cut into two equal pieces and connected parallel to the same power supply. In this case power dissipation is P_2. The ratio of $\frac{P_2}{P_1}$ is

(a) 1 (b) 2 (c) 3 (d) 4

Solution

$$P_1 = \frac{V^2}{R_1} \quad P_2 = \frac{V^2}{R_2} \quad P \alpha \frac{1}{R}$$

$$\frac{1}{R_2} = \frac{1}{\frac{R_1}{2}} + \frac{1}{\frac{R_1}{2}} = \frac{2}{R_1} + \frac{2}{R_1} = \frac{4}{R_1}$$

$$R_2 = \frac{R_1}{4}$$

$$R_1 = 4R_2$$

$$\frac{P_1}{P_2} = \frac{R_2}{R_1} = \frac{R_2}{4R_1} = \frac{1}{4}$$

$$\frac{P_2}{P_1} = 4.$$

[Option : (d)]

8. In India electricity is supplied for domestic use at 220 V. It is supplied at 110 V in USA. If the resistance of a 60 W bulb for use in India is R, the resistance of the bulb of the same wattage for use in USA will be

(a) R (b) $2R$ (c) $\frac{R}{4}$ (d) $\frac{R}{2}$

Solution

$$V_I = 220\,\text{V} \quad P_I = 60\,\text{W}$$

$$V_U = 110\,\text{V} \quad P_U = 60\,\text{W}$$

$$P_I = \frac{V_I^2}{R_I}; \quad P_U = \frac{V_U^2}{R_U}$$

$$R_I = \frac{V_I^2}{P_I} = \frac{220\times220}{60} = \frac{48400}{60}$$

$$R_U = \frac{V_U^2}{P_U} = \frac{110\times110}{60} = \frac{12100}{60}$$

$$\frac{R_U}{R_I} = \frac{12100}{60}\times\frac{60}{48400} = \frac{1}{4}$$

$$R_U = \frac{R_I}{4} = \frac{R}{4}$$

[Option : (c)]

9. In a large building, there are 15 bulbs of 40 W, 5 bulbs of 100W, 5 fans of 80W and 1 heater of 1 kW are connected. The voltage of electric main is 220V. The minimum capacity of the main fuse of the building will be

(a) 14 A (b) 8 A (c) 10 A (d) 12A

Solution

$$\text{Total power} = 15\times40+5\times100+5\times80+1\times1000$$

$$= 600+500+400+1000$$

$$= 2500\,\text{W}$$

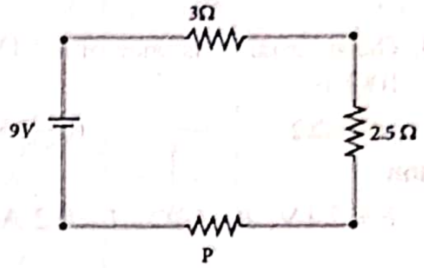
$$V = 220\ \text{V}$$

$$I = \frac{P}{V} = \frac{2500}{220} = 11.363\,\text{A} \approx 12\,\text{A}.$$

[Option : (d)]

10. There is a current of 1.0A in the circuit shown below. What is the resistance of P?

(a) 1.5Ω (b) 2.5Ω
(c) 3.5Ω (d) 4.5Ω



Solution

$$I = 1\text{A} \quad R_s = 3+2.5+P = (5.5+P)\Omega$$

$$R_s = \frac{V}{I} = \frac{9}{1} = 9\Omega$$

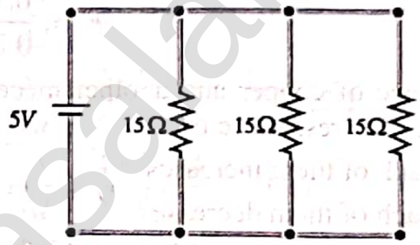
$$5.5+P = 9$$

$$\therefore P = 9-5.5 = 3.5\Omega.$$

[Option : (c)]

11. What is the current drawn out from the battery?

(a) 1 A (b) 2 A
(c) 3 A (d) 4 A



Solution

$$\frac{1}{R_p} = \frac{1}{15}+\frac{1}{15}+\frac{1}{15} = \frac{3}{15}$$

$$R_p = \frac{15}{3} = 5\Omega$$

$$I = \frac{V}{R_p} = \frac{5}{5} = 1\,\text{A}.$$

[Option : (a)]

12. The temperature coefficient of resistance of a wire is $0.00125\,\text{per}^\circ C$. At 300$K$, its resistance is 1Ω. The resistance of the wire will be 2Ω at

(a) 1154 K (b) 1100 K (c) 1400 K (d) 1127 K

Solution

$$T_1 = 300\,\text{K} = 300-273 = 27^\circ\text{C} \quad R_2 = 2\Omega \quad R_1 = 1\Omega$$

$$\alpha = 0.00125/^\circ\text{C}$$

$$\alpha = \frac{R_2 - R_1}{R_1 T_2 - R_2 T_1}$$

$$125\times10^{-5} = \frac{2-1}{(1\times T_2 - 2\times 27)} \Rightarrow (T_2 - 54) = \frac{1}{125\times10^{-5}} = 800$$

$$T_2 = 800+54 = 854°C$$

$$T_2 = 854+273 = 1127\,K$$

[Option : (d)]

13. The internal resistance of a 2.1V cell which gives a current of $0.2A$ through a resistance of 10Ω is

(a) 0.2Ω (b) 0.5Ω (c) 0.8Ω (d) 1.0Ω

Solution

$E = 2.1$V, $R = 10\Omega$, $I = 0.2$ A, $r =$ internal resistance

$$I = \frac{E}{R+r}$$

$$0.2 = \frac{2.1}{10+r}$$

$$0.2(10+r) = 2.1$$

$$2+0.2r = 2.1$$

$$0.2r = 2.1-2$$

$$r = \frac{0.1}{0.2} = \frac{1}{2} = 0.5\Omega.$$

[Option : (b)]

14. A piece of copper and another piece of germanium are cooled from room temperature to 80 K. The resistance of

(a) each of them increases

(b) each of them decreases

(c) copper increases and of germanium decreases

(d) copper decreseas and of germanium increases

Solution

For copper α is positive. $\therefore$ as temperature decreases, its resistivity also decreases.

For germanium α is negative. $\therefore$ as temperature decreases, its resistivity increases.

[Option : (d)]

15. In Joule's heating law, when R and t are kept constant, if the H is taken along the y axis and I^2 along the x-axis, the graph is

(a) straight lin (b) parabola (c) circle (d) ellipse

Solution

$$H = I^2Rt$$

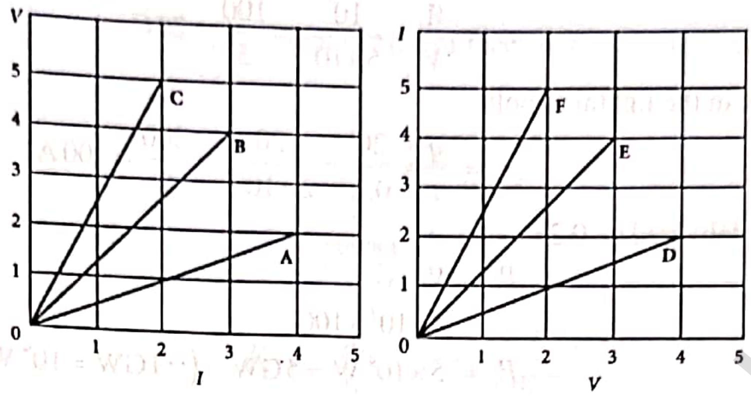
$$H \;\alpha\; I^2 \;(\because R,t \text{ constant})$$

$\Rightarrow (I^2 - H)$ graph will be linear.

[Option : (a)]

Exercise Problems

1. The following graphs represent the current versus voltage and voltage versus current for the six conductors A, B, C, D, E and F. Which conductor has least resistance and which has maximum resistance?



Solution

According to Ohms law, the resistance of the conductor, $R = \frac{V}{I}$

$$\text{For graph } I, \text{ slope } = \frac{\Delta V}{\Delta I} = R$$

$$\text{For conductor } A, R_A = \frac{\Delta V}{\Delta I} = \frac{2-0}{4-0} = \frac{2}{4} = \frac{1}{2} = 0.5\Omega$$

$$\text{For conductor } B, R_B = \frac{\Delta V}{\Delta I} = \frac{4-0}{3-0} = \frac{4}{3} = 1.33\Omega$$

$$\text{For conductor } C, R_C = \frac{\Delta V}{\Delta I} = \frac{5-0}{2-0} = \frac{5}{2} = 2.5\Omega$$

$$\text{For graph } II, R = \frac{1}{\text{slope}} = \frac{\Delta V}{\Delta I}$$

$$\text{For conductor } D, R_D = \frac{\Delta V}{\Delta I} = \frac{4-0}{2-0} = \frac{4}{2} = 2\Omega$$

$$\text{For conductor } E, R_E = \frac{\Delta V}{\Delta I} = \frac{3-0}{4-0} = \frac{3}{4} = 0.75\Omega$$

$$\text{For conductor } F, R_F = \frac{\Delta V}{\Delta I} = \frac{2-0}{5-0} = \frac{2}{5} = 0.4\Omega$$

$$R_F \text{ has minimum resistance } = 0.4\Omega$$

$$R_C \text{ has maximum resistance } = 2.5\Omega$$

2. Lightning is very good example of natural current. In typical lightning, there is $10^9 J$ energy transfer across the potential difference of 5×10^7 V during a time interval of 0.2 s. Using this information, estimate the following quantities (a) total amount of charge transferred between cloud and ground (b) the current in the lightning bolt (c) the

power delivered in 0.2 s.

Solution

(a) Total amount of energy transferred between cloud and ground = W

$$W = Vq$$

$$q = \frac{W}{V} = \frac{10^9}{5\times10^7} = \frac{100}{5} = 20C$$

(b) Current in the lightning bolt

$$I = \frac{q}{t} = \frac{20}{0.2} = \frac{20}{2\times10^{-1}} = \frac{200}{2} = 100A$$

(c) Power delivered in $0.2s$

$$P = VI$$

$$= 5\times10^7 \times 100$$

$$P = 5\times10^9 \text{ W} = 5\text{GW} \quad (\because 1\text{GW} = 10^9 \text{ W})$$

3. A copper wire of $10^{-6} m^2$ area of cross section carries a current of $2A$. If the number of electrons per cubic meter is 8×10^{28} calculate the current density and average drift velocity.

Given :

$A = 10^{-6} m^2$, $I = 2A$ $n = 8\times10^{28}$

$J = ?$ $vd = ?$

Solution

(i) Current density $J = \frac{I}{A} = \frac{2}{10^{-6}} = 2\times10^6 \text{ Am}^{-2}$

(ii) Drift velocity $v_d = \frac{J}{ne} = \frac{2\times10^6}{8\times10^{28}\times1.6\times10^{-19}}$

$$v_d = 0.156\times10^{-3} \text{ms}^{-1}$$

$$= 15.6\times10^{-5} \text{ms}^{-1}.$$

4. The resistance of a nichrome wire at $0°C$ is 10Ω. If it's temperature coefficient of resistance is $0.004/°C$, find its resistance at boiling point of water. Comment on the result.

Given :

$R_0 = 10\Omega$, $\alpha = 0.004/°C$, $t = 100°C$

$t_o = 0°C$ $R_t = ?$

Solution

(i)

$$R_t = R_o(1+\alpha\Delta T) \quad ; \quad \Delta T = 100°\text{C} - 0°\text{C}$$

$$R_{100} = 10(1+0.004\times100) = 10(1+0.4)$$

$$R_{100} = 14\Omega$$

Resistance of nichrome (an alloy) increases with rise in temperature.

5. The rod given in the figure is made up of two different materials. Both have square cross sections of 3mm side. The resistivity of the first material is $4\times10^{-3}\Omega\text{m}$ and it is 25cm long while second material has resistivity of $5\times10^{-3}\Omega\text{m}$ and is of 70cm long. What is the

resistance of rod between its ends?

Given :

$\rho_1 = 4\times10^{-3}\Omega\,m$ $\rho_2 = 5\times10^{-3}\Omega\,m$

$l_1 = 25cm$ $l_2 = 70cm$

$A_1 = A_2 = 9\times10^{-6}\,m^2$ ($\because$ length of side of the square $= 3\times10^{-3}m$)

Solution

(i)

$$R_1 = \frac{\rho_1 l_1}{A_1} = \frac{4\times10^{-3}\times25\times10^{-2}}{9\times10^{-6}}$$

$$= \frac{100\times10^{-5}}{9\times10^{-6}} = \frac{1000}{9}\Omega$$

$$R_2 = \frac{\rho_2 l_2}{A_2} = \frac{5\times10^{-3}\times70\times10^{-2}}{9\times10^{-6}}$$

$$= \frac{3500}{9}\Omega$$

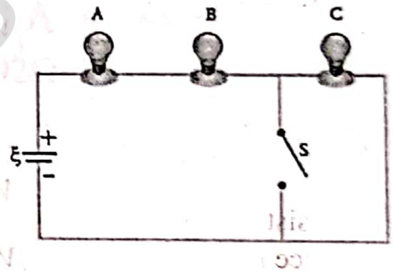
Resistance of the rod between its ends

$$R = R_1 + R_2$$

$$= \frac{1000}{9} + \frac{3500}{9} = \frac{4500}{9}$$

$$R = 500\Omega$$

6. Three identical lamps each having a resistance R are connected to the battery of emf as shown in the figure



Suddenly the switch S is closed. (a) Calculate the current in the circuit when S is open and closed (b) What happens to the intensities of the bulbs A, B and C. (c) Calculate the voltage across the three bulbs when S is open and closed (d) Calculate the power delivered to the circuit when S is opened and closed (e) Does the power delivered to the circuit decreases, increases or remain same?

Given

(a) <u>Current</u> : Let the current in the circuit when S is open be I

$$V = IR \qquad V = \xi$$

The bulbs are connected in series

$$R_s = R_1 + R_2 + R_3$$

$$\text{but } R_1 = R_2 = R_3 = R$$

$$R_s = 3R$$

When switch is open $I = \frac{V}{R_S} = \frac{\xi}{3R}$

When switch is closed (no current flows through bulb C)

$$I = \frac{V}{R_s} = \frac{\xi}{R+R} = \frac{\xi}{2R}$$

(b) Intensity :

When switch is open

Three bulbs are connected in series, the same current flows through them and so there is no change in intensity.

∴ All the bulbs will glow with equal intensity.

When the switch is closed, two bulbs A and B are in series combination. So the intensities across the two bulbs increases and both A and B will have equal intensity.

As S is closed, all the current flows through it and no current flows through bulb C ; so the bulb C will not glow.

(c) Voltage across the bulbs :

When switch is open ;

In series connection, the current is same

$$V_A = \frac{\xi}{3R} \times R = \frac{\xi}{3}$$

$$V_B = \frac{\xi}{3R} \times R = \frac{\xi}{3}$$

$$V_C = \frac{\xi}{3R} \times R = \frac{\xi}{3}$$

When switch is closed, bulbs A and B are in series combination and bulb C is in parallel combination. So no current flows through C. Hence $I = 0$.

$$V_A = IR$$

$$= \frac{\xi}{2R} \times R = \frac{\xi}{2}$$

$$V_B = \frac{\xi}{2R} \times R = \frac{\xi}{2}$$

$$V_C = IR = 0$$

(d) Power delivered

when switch is open

$$P_A = V_A I_A = \left(\frac{\xi}{3}\right) \times \left(\frac{\xi}{3R}\right)$$

$$P_A = \frac{\xi^2}{9R}$$

$$P_B = V_B I_B = \left(\frac{\xi}{3}\right) \times \left(\frac{\xi}{3R}\right) = \frac{\xi^2}{9R}$$

$$P_C = V_C I_C = \left(\frac{\xi}{3}\right) \times \left(\frac{\xi}{3R}\right) = \frac{\xi^2}{9R}$$

$$\text{Total power} = \frac{\varepsilon^2}{9R} + \frac{\varepsilon^2}{9R} + \frac{\varepsilon^2}{9R} = \frac{\varepsilon^2}{3R}$$

When switch is closed

$$P_A = V_A I_A = \frac{\xi}{2} \times \frac{\xi}{2R} = \frac{\xi^2}{4R}$$

$$P_B = V_B I_B = \frac{\xi}{2} \times \frac{\xi}{2R} = \frac{\xi^2}{4R}$$

$$P_C = V_C I_C = 0.$$

$$\text{and total power} = \frac{\varepsilon^2}{4R} + \frac{\varepsilon^2}{4R} + 0 = \frac{\varepsilon^2}{2R} \quad \left(\because \frac{\varepsilon^2}{2R} > \frac{\varepsilon^2}{3R}\right)$$

(e) The total power delivered to the circuit increases.

7. An electronics hobbyist is building a radio which requires 150Ω in her circuit but she has only $220\Omega, 79\Omega$ and 92Ω resistors available. How can she connect the available resistors to get desired value of resistance?

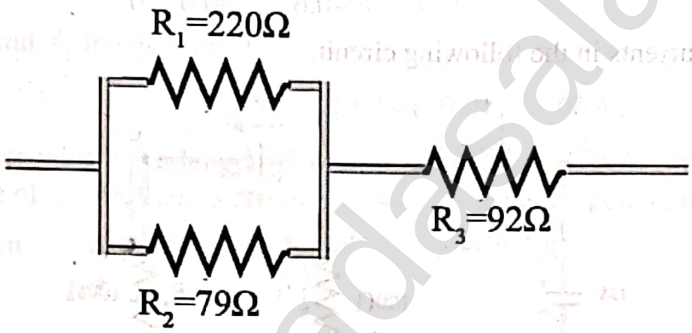
Given

Required resistance $= 150\Omega$

Available resistances $R_1 = 220\Omega \quad R_2 = 79\Omega$ and $R_3 = 92\Omega$

Solution

For getting 150Ω resistance as resultant, two of the resistances R_1 and R_2 should be connected in parallel and third one should be connected in series with it.



When R_1 and R_2 are in parallel,

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R_p = \frac{R_1 R_2}{R_1 + R_2}$$

$$= \frac{220 \times 79}{220 + 79} = \frac{17380}{299} = 58.12 \approx 58\Omega$$

When $R_p + R_3$ are in series

$$R_s = R_p + R_3$$

$$= 58 + 92 = 150\Omega$$

8. A cell supplies a current of $0.9A$ through a 2Ω resistor and a current of $0.3A$ through a 7Ω resistor. Calculate internal resistance of the cell.

Given :

$I_1 = 0.9A \qquad R_1 = 2\Omega$

$I_2 = 0.3A \qquad R_2 = 7\Omega$

Let the internal resistance of the cell be 'r'

Solution

$$I_1 = \frac{E}{R_1 + r}$$

$$E = I_1(R_1 + r) \quad \text{... (1)}$$

$$\text{Similarly, } E = I_2(R_2 + r) \quad \text{... (2)}$$

$$I_1(R_1 + r) = I_2(R_2 + r)$$

$$I_1R_1 + I_1r = I_2R_2 + I_2r$$

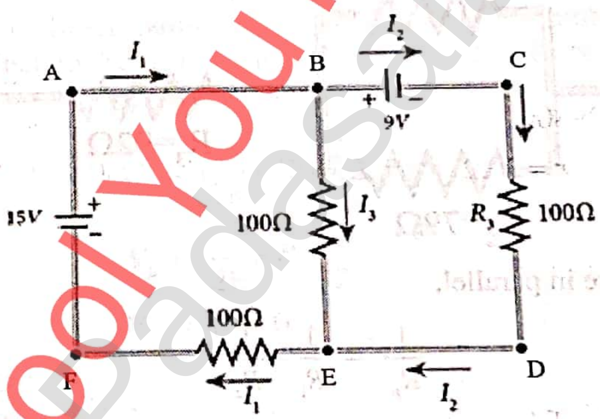
$$I_1R_1 - I_2R_2 = I_2r - I_1r$$

$$r = \frac{I_1R_1 - I_2R_2}{I_2 - I_1}$$

$$r = \frac{(0.9 \times 2) - (0.3 \times 7)}{0.3 - 0.9}$$

$$= \frac{1.8 - 2.1}{-0.6} = \frac{-0.3}{-0.6} = \frac{3}{6} = 0.5\Omega$$

9. Calculate the currents in the following circuit.



Solution

Applying Kirchoff's 1st law at Junction B

$$I_1 - I_2 - I_3 = 0$$

$$I_3 = I_1 - I_2 \quad \text{... (1)}$$

Applying Kirchoff's 2nd law to the closed path $ABEFA$

$$100I_3 + 100I_1 = 15\text{V}$$

$$100(I_3 + I_1) = 15\text{V}$$

Substituting for I_3

$$100(I_1 - I_2) + 100I_1 = 15\text{V} \quad ; \quad 100I_1 - 100I_2 + 100I_1 = 15\text{V}$$

$$200I_1 - 100I_2 = 15\text{V} \quad \text{... (2)}$$

Applying Kirchoff's 2[nd] law to the closed path $BEDCB$

$$100I_3 - 100I_2 = 9\,\text{V}$$

$$100(I_1 - I_2) - 100I_2 = 9\,\text{V}$$

$$100I_1 - 100I_2 - 100I_2 = 9\,\text{V} \quad ; \quad 100I_1 - 200I_2 = 9\,\text{V} \qquad \dots (3)$$

$(3) \times 2 \Rightarrow$

$$200I_1 - 400I_2 = 18\,\text{V}$$

$(2) \Rightarrow$

$$200I_1 - 100I_2 = 15\,\text{V}$$

Subtracting

$$-300I_2 = 3\,\text{V}$$

$I_2 = \frac{-3}{300} = -0.01\,\text{A}$ [$\Rightarrow$ assumed direction is opposite to actual direction of current]

Substitute I_2 in equation (2)

$$200I_1 - 100(-0.01) = 15$$

$$200I_1 + 1 = 15$$

$$200I_1 = 14$$

$$I_1 = \frac{14}{200} = \frac{7}{100} = 0.07\,\text{A}$$

Substitute I_1 and I_2 in equation (1)

$$I_3 = 0.07 - (-0.01) = 0.08\,\text{A}.$$

10. A potentiometer wire has a length of $4m$ and resistance of 20Ω. It is connected in series with resistance of 2980Ω and a cell of emf $4V$ calculate the potential along the wire.

Given : $l = 4\,\text{m}$ $\quad r = 20\Omega$ $\quad E = 4V$ $\quad r' = 2980\Omega$

Solution

Effective resistance of two resistors in series combination,

$$R = r + r'$$

$$= 20 + 2980 = 3000\Omega$$

Current in the potentiometer wire,

$$I = \frac{E}{R} = \frac{4}{3000}$$

Potential across the potentiometer wire $V = I \times r$

$$= \frac{4}{3000} \times 20$$

$$V = \frac{80}{3000} = \frac{8}{300}\,\text{V} \approx 0.026\,\text{V}$$

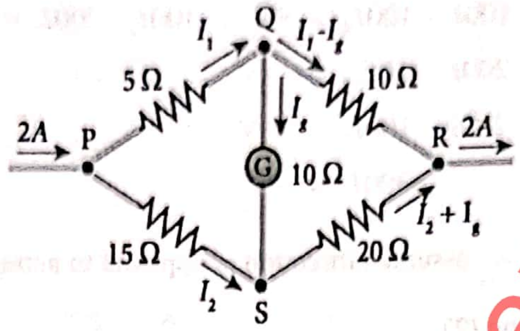
Potential gradient $\quad \varepsilon = \frac{V}{l}$

$$\varepsilon = \frac{0.026}{4}$$

$$= 0.0065$$

$$\varepsilon = 0.66 \times 10^{-2}\ \mathrm{Vm}^{-1}.$$

11. Determine the current flowing through the galvanometer (G) as shown in figure.



$$I = 2\,\mathrm{A}$$

Applying Kirchoff's 1st law at P, $I_2 = I - I_1$

Applying Kirchoff's 2nd law to the closed path $PQSP$

$$5I_1 + 10I_g - 15I_2 = 0$$

$$5I_1 + 10I_g - 15(I - I_1) = 0$$

$$10I_g - 15I + 20I_1 = 0$$

$$10I_g - 30 + 20I_1 = 0 \qquad (\because I = 2\,\mathrm{A})$$

$$20I_1 + 10I_g = 30$$

$$2I_1 + I_g = 3 \qquad \text{... (1)}$$

Appling Kirchoff's II law to the closed path $QRSQ$

$$10(I_1 - I_g) - 20(I_2 + I_g) - 10I_g = 0$$

$$10I_1 - 10I_g - 20I_2 - 20I_g - 10I_g = 0$$

$$10I_1 - 40I_g - 20I_2 = 0$$

$$10I_1 - 40I_g - 20(I - I_1) = 0$$

$$10I_1 - 40I_g - 20I - 20I_1 = 0$$

$$30I_1 - 40I_g - 40 = 0 \qquad (\because I = 2\,\mathrm{A})$$

$$30I_1 - 40I_g = 40$$

$$3I_1 - 4I_g = 4 \qquad \text{... (2)}$$

$(1) \times 4 \Rightarrow$

$$8I_1 + 4I_g = 12$$

Adding

$$11I_1 = 16\ ;\ I_1 = \frac{16}{11}\mathrm{A}.$$

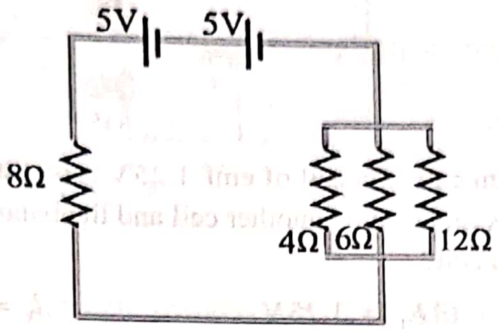
Substituting I_1 in (1)

$$\frac{2 \times 16}{11} + I_g = 3$$

$$32 + 11I_g = 33$$

$$11I_g = 33-32 \;;\; I_g = \frac{1}{11}\text{A}.$$

12. Two cells each of $5V$ are connected in series across a 8Ω resistor and three parallel resistor $4\Omega, 6\Omega$ and 12Ω. Draw a circuit diagram for the above arrangement. Calculate (i) the current drawn from the cell (ii) Current through each resistor.



Resistors $4\Omega, 6\Omega$ and 12Ω are connected in parallel

$$\frac{1}{R_p} = \frac{1}{4}+\frac{1}{6}+\frac{1}{12}$$

$$\frac{1}{R_p} = \frac{6+4+2}{24} = \frac{12}{24} = \frac{1}{2}$$

$$R_p = 2\Omega.$$

Resistors 8Ω is connected in series with 2Ω (R_p)

$$R_s = 8+2 = 10\Omega.$$

$$\text{Total emf} \quad E = E_1 + E_2 = 5+5 = 10\text{V}$$

(a) Current drawn from the cells

$$I = \frac{E}{R_S} = \frac{10}{10} = 1\text{A}$$

(b) P.d across 8Ω resistor $\quad V = IR = 1\times 8 = 8\text{V}$

Potential across $R_p = 1\times 2 = 2V$ ($4\Omega, 6\Omega$ and 12Ω in parallel combination)

Current through 4Ω $\quad I = \frac{V}{R} = \frac{2}{4} = 0.5\text{A}$

through 6Ω $\quad I = \frac{V}{R} = \frac{2}{6} = 0.33\text{A}$

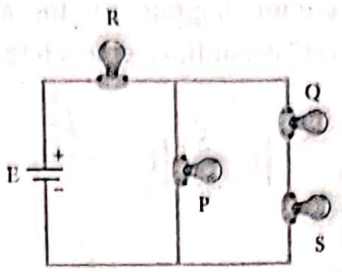
through 12Ω $\quad I = \frac{V}{R} = \frac{2}{12} = 0.17\text{A}.$

13. Four bulbs *P, Q, R, S* are connected in a circuit of unknown arrangement. When each bulb is removed one at a time and replaced the following behaviour is observed.

	P	*Q*	*R*	*S*
P removed	–	on	on	on
Q removed	on	–	on	off

R removed	off	off	–	off
S removed	on	off	on	–

Solution



14. In a potentiometer arrangement, a cell of emf $1.25\,\text{V}$ gives a balance point at $35\,\text{cm}$ length of the wire. If the cell is replaced by another cell and the balance point shifts to $63\,\text{cm}$. What is the emf of the second cell?

Given $E_1 = 1.25\,\text{V}$ $\quad l_1 = 35\,\text{cm} = 35 \times 10^{-2}\,\text{m}$

$E_2 = ?$ $\quad l_2 = 63\,\text{cm} = 63 \times 10^{-2}\,\text{m}$

Solution $E \,\alpha\, l$

$$\frac{E_1}{l_1} = \frac{E_2}{l_2}$$

$$E_2 = E_1 \times \frac{l_2}{l_1}$$

$$= 1.25 \times \frac{63 \times 10^{-2}}{35 \times 10^{-2}} = 125 \times 10^{-2} \times \frac{63}{35}$$

$$= 125 \times 10^{-2} \times \frac{9}{5} = 25 \times 9 \times 10^{-2} = 225 \times 10^{-2}$$

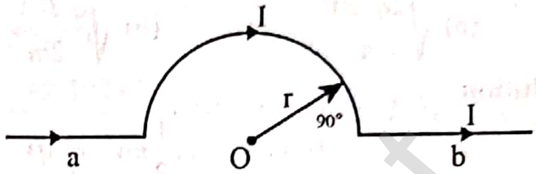
$$E = 2.25\,\text{V}$$

Chapter 3

Magnetism and Magnetic Effects of Electric Current

Objective type Questions

1. The magnetic field at the centre O of the following current loop is



(a) $\frac{\mu_o I}{4r}\otimes$ (b) $\frac{\mu_o I}{4r}$ e

(c) $\frac{\mu_o I}{2r}\otimes$ (d) $\frac{\mu_o I}{2r}$ e

Solution

Magnetic field at O due to straight portions is zero.
The magnetic field at O due to semi-circular part is

$$B = \int \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2} \;;\; \sin\theta = 1$$

$$B = \frac{\mu_0 I}{4\pi r}\int dl \;;\; \int dl = \pi r$$

$$\therefore B = \frac{\mu_0 I}{4r}.$$

Aliter :

For a circular coil,

$$B = \frac{\mu_0 I}{2r}$$

For a semi-circular coil,

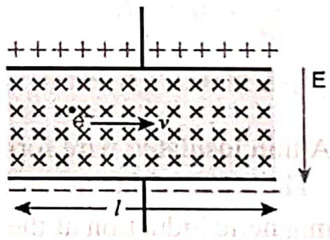
$$B = \frac{\mu_0 I}{2r}\times\frac{1}{2}$$

$$B = \frac{\mu_0 I}{4r}$$

Using 'Right Hand Rule', direction of $\vec{B}$ is inward $\otimes$

[Option : (a)]

2. An electron moves straight inside a charged parallel plate of uniform charge density σ. The time taken by the electron to cross the parallel plate capacitor in a straight line when the plates of the capacitor are kept under constant magnetic field of induction $\overline{B}$ is



(a) $\varepsilon_o \frac{elB}{\sigma}$ (b) $\varepsilon_o \frac{lB}{\sigma l}$ (c) $\varepsilon_o \frac{lB}{e\sigma}$ (d) $\varepsilon_o \frac{lB}{\sigma}$

Solution

Since the electron moves in a straight line, without any deviation, $eE = e\upsilon B$

$$\upsilon = \frac{E}{B}$$

But $\upsilon = \frac{l}{t}$ ($t =$ time of transit)

Also, $E = \frac{\sigma}{\varepsilon_0}$ [Refer section 1.6.4 (iii) – P. 41]

$$\Rightarrow \upsilon = \frac{\sigma}{B\varepsilon_o}$$

$$\therefore \quad \frac{\sigma}{B\varepsilon_o} = \frac{l}{t}$$

$$t = \frac{B\varepsilon_o l}{\sigma}$$

[Option : (d)]

3. A particle having mass m and charge q is accelerated through a potential difference V. The force experienced by the particle when it is kept under perpendicular magnetic field $\vec{B}$ is

(a) $\sqrt{\frac{2q^3BV}{m}}$ (b) $\sqrt{\frac{q^3B^2V}{2m}}$ (c) $\sqrt{\frac{2q^3B^2V}{m}}$ (d) $\sqrt{\frac{2q^3BV}{m^3}}$

Solution

$$\frac{1}{2}mv^2 = qV$$

$$v = \sqrt{\frac{2qV}{m}}$$

$$F = Bqv = Bq\sqrt{\frac{2qV}{m}} = \sqrt{\frac{2q^3B^2V}{m}}$$

[Option : (c)]

4. A circular coil of radius 5cm has 50 turns carries a current of $3A$. The magnetic dipole moment of the coil is

(a) $1.0\,\text{Am}^2$ (b) $1.2\,\text{Am}^2$ (3) $0.5\,\text{Am}^2$ (d) $0.8\,\text{Am}^2$

Solution

Magnetic dipole moment associated with the coil

$$p_m = NIA = NI\pi R^2$$

$$= 50\times3\times3.14\times\left(5\times10^{-2}\right)^2 = 1.17$$

$$\approx 1.2\,\text{Am}^2$$

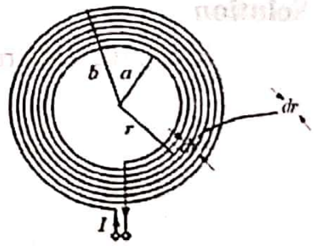
[Option : (b)]

5. A thin insulated wire forms a plane spiral of $N=100$ tight turns carrying a current $I=8mA$. The radii of inside and outside turns are $a=50\,\text{mm}$ and $b=100\,\text{mm}$ respectively. The magnetic induction at the centre of the spiral is

(a) $5\mu\text{T}$ (b) $7\mu\text{T}$ (3) $8\mu\text{T}$ (d) $10\mu\text{T}$

Solution

Let I be the current, r be the radius. Then for a circular coil of N turns, magnetic field at its centre $B=\frac{\mu_o I}{2r}N$.



In the case of this spiral coil, let N be the total number of turns, then no. of turns per unit length $=\frac{N}{b-a}$.

The no. of turns in a distance of dr, $dn=\frac{N}{b-a}dr$ .... (1)

$\therefore$ The magnetic field due to dn turns,

$$dB = \frac{\mu_o I}{2r}dn = \frac{\mu_o I}{2r}\left(\frac{N}{b-a}\right)dr$$

Total magnetic field due to spiral,

$$B = \int_a^b \frac{\mu_o I}{2r}\left(\frac{N}{b-a}\right)dr$$

$$= \frac{\mu_o IN}{2(b-a)}\int_a^b \frac{dr}{r} = \frac{\mu_o NI}{2(b-a)}\ln\left(\frac{b}{a}\right)$$

$$B = \frac{\mu_o NI}{2(b-a)}\ln\frac{b}{a} = \frac{4\pi\times10^{-7}\times100\times8\times10^{-3}\ln\frac{100}{50}}{2(100-50)\times10^{-3}}$$

$$= \frac{4\times3.14\times10^{-7}\times100\times8\times10^{-3}\times(2.303\times\log_{10}2)}{2\times50\times10^{-3}} \quad [\because \ln2 = 2.303\times\log_{10}2]$$

$$= 6.96\times10^{-6} \simeq 7\times10^{-6}\text{T}$$

$$B = 7\mu\text{T}.$$

[Option : (b)]

6. Three wires of equal lengths are bent in the form of loops, one of the loops is circle another is a semi circle and the third one is a square. They are placed in a uniform magnetic field and same electric current is passed through them. Which of the following loop configuration will experience greater torque?

(a) circle (b) semi-circle (c) square (d) all of them

Solution

For circular loop

$$l = 2\pi r$$

$$r = \frac{l}{2\pi}$$

$$A_c = \pi r^2 = \frac{\pi l^2}{4\pi^2} = \frac{l^2}{4\pi}$$

$$A_c \simeq 0.08l^2$$

For semi-circular loop

$$l = \pi r + 2r$$

$$r = \frac{l}{2+\pi}$$

$$A_{sc} = \frac{\pi r^2}{2} = \frac{\pi}{2}\left(\frac{l}{2+\pi}\right)^2 = \frac{\pi l^2}{2(\pi+2)^2}$$

$$A_{sc} = 0.059l^2$$

For square loop

$$l = 4a \Rightarrow a = \frac{l}{4}$$

$$A_{sq} = a^2 = \frac{l^2}{16}$$

$$\Rightarrow A_{sq} \simeq 0.0625l^2$$

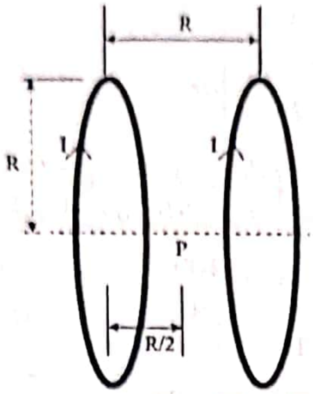
$$\tau = NBIA \Rightarrow \tau \propto A$$

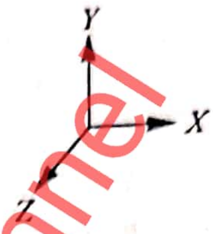
$$A_c > A_{sq} > A_{sc} \Rightarrow \tau_c > \tau_{sq} > \tau_{sc}.$$

[Option : (a)]

7. Two identical coils each with N turns and radius R are placed coaxially at a distance R as shown in figure. If I is the current passing through the loops in the same direction, then the magnetic field at a point P at a distance $\frac{R}{2}$ from the centre y each coil is

(a) $\frac{8N\mu_o I}{\sqrt{5}R}$ (b) $\frac{8N\mu_o I}{5^{\frac{3}{2}}R}$ (c) $\frac{8N\mu_o I}{5R}$ (d) $\frac{4N\mu_o I}{\sqrt{5}R}$





Solution

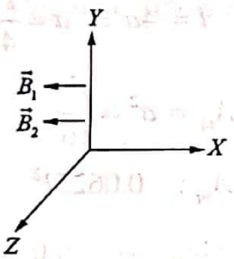
Both coils are in YZ plane.

Magnetic field at any point on the axis of a circular loop carrying current is

$$B = \frac{\mu_o NIR^2}{2\left(x^2+R^2\right)^{\frac{3}{2}}}$$

Point P is at a distance of $x = \frac{R}{2}$ from both the coils.

$$B_1 = B_2 = \frac{\mu_o NIR^2}{2\left(\frac{R^2}{4}+R^2\right)^{\frac{3}{2}}} = \frac{\mu_o NIR^2}{2\left(\frac{5R^2}{4}\right)^{\frac{3}{2}}} = \frac{\mu_o NIR^2}{2\left(\frac{5}{4}\right)^{\frac{3}{2}}\cdot R^3}$$



$$= \frac{\mu_o NI}{\frac{1}{4}\cdot 5^{3/2}\cdot R} \qquad \left(\because 4^{3/2} = 4\sqrt{4} = 8\right)$$

$$= \frac{4\mu_o NI}{5^{3/2}R}$$

$$\vec{B}_1 = B(-\hat{i}) = \vec{B}_2$$

$$\Rightarrow \quad B = B_1 + B_2 = 2B_1 = \frac{8\mu_o NI}{5^{3/2}R}.$$

[Option : (b)]

8. A wire of length l carrying a current along the y direction is kept in a magnetic field give by $\vec{B} = \frac{\beta}{\sqrt{3}}\left(\hat{i}+\hat{j}+\hat{k}\right)T$. The magnitude of Lorentz force acting on the wire is

(a) $\sqrt{\frac{2}{\sqrt{3}}}\beta Il$ (b) $\sqrt{\frac{1}{3}}\beta Il$ (c) $\sqrt{2}\beta Il$ (d) $\sqrt{\frac{1}{2}}\beta Il$

Solution

$$\vec{Il} = Il\hat{j}\ ; \quad \vec{B} = \frac{\beta}{\sqrt{3}}\left(\hat{i}+\hat{j}+\hat{k}\right)$$

$$\vec{F} = \left(\vec{Il}\times\vec{B}\right) = Il\hat{j}\times\frac{\beta}{\sqrt{3}}\left(\hat{i}+\hat{j}+\hat{k}\right)$$

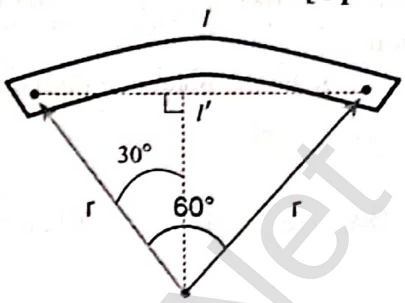
$$\vec{F} = \frac{Il\beta}{\sqrt{3}}\left[-\hat{k}+\vec{0}+\hat{i}\right]$$

$$\left|\vec{F}\right| = \frac{Il\beta}{\sqrt{3}}\sqrt{(-1)^2+(+1)^2} = \frac{Il\beta}{\sqrt{3}}\times\sqrt{2}$$

$$= \sqrt{\frac{2}{3}}Il\beta.$$

[Option : (a)]

9. A bar magnet of length l and magnetic moment p_m is bent in the form at an arc as shown in figure. The new magnetic dipole moment will be



(a) p_m (b) $\frac{3}{\pi}p_m$

(c) $\frac{2}{\pi}p_m$ (d) $\frac{1}{2}p_m$

Solution

For straight magnet $p_m = q_m \times l$

For bent magnet $p'_m = q_m \times l'$

$$l' = 2r\sin 30$$

$$= 2r\times\frac{1}{2} = r$$

Let l be the arc length.

$$\therefore r = \frac{3l}{\pi}$$

$$l' = r = \frac{3l}{\pi}$$

$$p'_m = q_m \times l' = q_m \times \frac{3l}{\pi} = \frac{3}{\pi}p_m.$$

[Option : (b)]

10. A non-conducting charged ring of charge q mass m and radius r is rotated about its axis with constant angular speed ω. Find the ratio of its magnetic moment with angular momentum.

(a) $\frac{q}{m}$ (b) $\frac{2q}{m}$ (c) $\frac{q}{2m}$ (d) $\frac{q}{4m}$

Solution

Magnetic moment, $\mu_L = I.A$

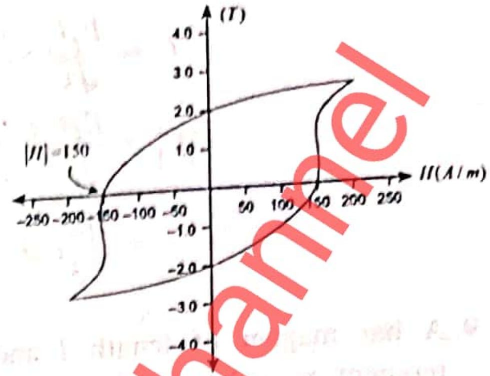
$$= \frac{q}{T}\cdot\pi r^2 = \frac{q\omega}{2\pi}\cdot\pi r^2 = \frac{q\omega r^2}{2} \quad \left(\because T = \frac{2\pi}{\omega}\right)$$

Angular momentum, $L = mr^2\omega$

$$\frac{\mu_L}{L} = \frac{q\omega r^2/2}{mr^2\omega} = \frac{q}{2m}$$

[Option : (c)]

11. The BH curve of a Ferro magnetic material is shown in the figure. The material is placed inside a long solenoid which contains 1000 turns / cm. The current that should be passed in the solenoid to damagnatize the ferromagnet completely is



(a) 1 mA (b) 1.25 mA

(c) 1.50 mA (d) 1.75 mA

Solution

Let B be the magnetic field due to long solenoid and H be the magnetising field.

$$B = \mu_0 nI \quad ; \quad H = \frac{B}{\mu_0} = nI$$

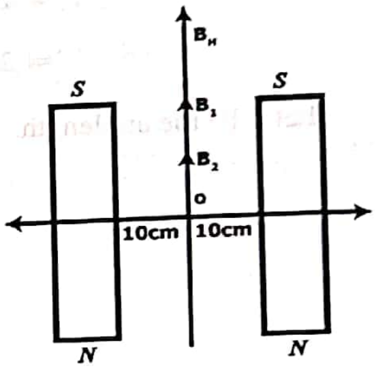
$$I = \frac{H}{n} \qquad n = 1000/\text{cm} = \frac{1000}{10^{-2}}\ \text{m}^{-1}$$

$$I = \frac{150 \times 10^{-2}}{1000}$$

$$= 1.5 \times 10^{-3}\ \text{A} = 1.5\ \text{mA}.$$

[Option : (c)]

12. Two short bar magnets have magnetic moments $1.20\ \text{Am}^2$ and $1.00\ \text{Am}^2$ respectively. They are kept on a horizontal table parallel to each other with their north poles pointing towards South. They have a common magnetic equator and are seperated by a distance of 20.0cm. The value of the resultant horizontal magnetic induction at the mid point O of the line joining their centres is (horizontal components of Earth's magnetic induction is $3.6 \times 10^{-5}\ \text{Wbm}^{-2}$).



(a) $3.60 \times 10^{-5}\ \text{Wbm}^{-2}$ (b) $3.5 \times 10^{-5}\ \text{Wbm}^{-2}$

(c) $2.56 \times 10^{-4}\ \text{Wbm}^{-2}$ (d) $2.2 \times 10^{-4}\ \text{Wbm}^{-2}$

Solution

$$B = B_1 + B_2 + B_H$$

$$= \frac{\mu_o}{4\pi}\frac{Pm_1}{r^3} + \frac{\mu_o}{4\pi}\frac{Pm_2}{r^3} + B_H \quad [\because \text{ Point } O \text{ is on the equatorial line of both magnets}]$$

$$= \frac{\mu_o}{4\pi r^3}[Pm_1 + Pm_2] + B_H$$

$$= \frac{10^{-7}}{(0.1)^3}[1.2 + 1] + 3.6 \times 10^{-5}$$

$$= 2.2 \times 10^{-4} + 3.6 \times 10^{-5} = 22 \times 10^{-5} + 3.6 \times 10^{-5}$$

$$= 25.6 \times 10^{-5}$$

$$= 2.56 \times 10^{-4}\ \text{Wbm}^{-2}.$$

[Option : (c)]

13. The vertical component of Earth's magnetic field at a place is equal to the horizontal component. What is the value of angle of dip at this place?

(a) 30° (b) 45° (c) 60° (d) 90°

Solution

$$B_V = B_H \text{ (given)}$$

$$\tan\delta = \frac{B_V}{B_H} = 1, \text{ where } \delta \text{ is the dip at the place.}$$

$$\delta = 45°.$$

[Option : (b)]

14. A flat dielectric disc of radius R carries an excess charge on its surface. The surface charge density is σ. The disc rotates about an axis perpendicular to its plane passing through the centre with angular velocity ω. Find the magnitude of the torque on the disc if it is placed in a uniform magnetic whose strength is B which is directed perpendicular to the axis of rotation.

(a) $\frac{1}{4}\sigma\omega\pi BR$ (b) $\frac{1}{2}\sigma\omega\pi BR^2$ (c) $\frac{1}{4}\sigma\omega\pi BR^3$ (d) $\frac{1}{4}\sigma\omega\pi BR^4$

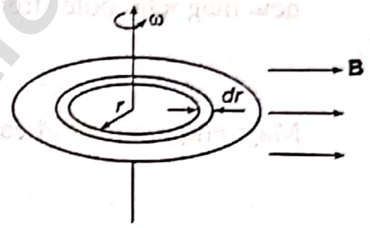
Solution

$$dI = \frac{dq}{T} = \frac{\omega dq}{2\pi} = \frac{\omega}{2\pi}\sigma.2\pi r dr = \omega\sigma r dr$$

[$\because$ $dq = \sigma dA$; for ring-shaped portion, $dA = 2\pi r dr$]

$$dp_m = (dI)A = \omega\sigma r dr \times \pi r^2 = \sigma\omega\pi r^3 dr$$

$$p_m = \int dp_m = \int_0^R \sigma\omega\pi r^3 dr = \frac{1}{4}\sigma\omega\pi R^4$$



$$\tau = p_m B\sin\theta \ ; \ \theta = 90° \ \therefore \ \tau = \frac{1}{4}\sigma\omega\pi R^4 B$$

[Option : (d)]

15. The potential energy of magnetic dipole whose dipole moment is $\vec{p}_m = (-0.5\hat{i} + 0.4\hat{j})Am^2$ kept in uniform magnetic field $\vec{B} = 0.2\hat{i}T$

(a) −0.1J (b) −0.8J (c) 0.1J (d) 0.8J

Solution

According to (2) $\overline{F}$ acts downward during bob's leftward motion $\Rightarrow$

Given :

$$p_m = (-0.5\hat{i} + 0.4\hat{j})Am^2 \ ; \ B = 0.2\hat{i}T$$

Solution

$$U = -\vec{p}_m \cdot \vec{B} = -\left[\left(-0.5\hat{i} + 0.4\hat{j}\right)\cdot\left(0.2\hat{i}\right)\right] \quad (\because \ \hat{i}\cdot\hat{i} = 1 \ ; \ \hat{j}\cdot\hat{i} = 0)$$

$$= -(-0.5\times0.2) + (0.4\times0)$$

$$= +(0.5\times0.2)$$

$$= 0.1\text{J}.$$

[Option : (c)]

Exercise Problems

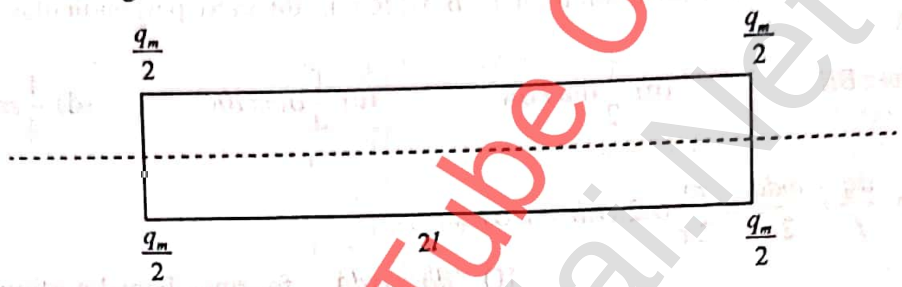
1. A bar magnet having a magnetic moment $\vec{p}_m$ is cut into four pieces (i.e.) first cut into two pieces along the axis of the magnet and each piece is further cut into two pieces. Compute the magnetic moment of each pieces.

Given

Let the pole strength and magnetic moment of original magnet be q_m and $\vec{p}_m$ respectively.

Solution

When the bar magnet is cut into two pieces along the axis of the magnet,



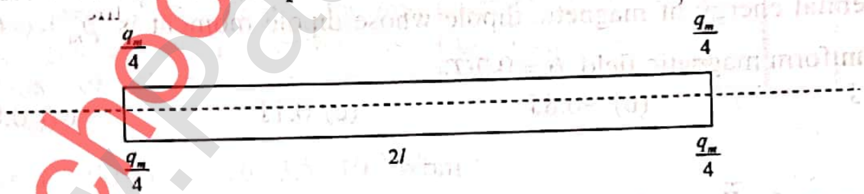
new magnetic pole strength, $q'_m = \frac{q_m}{2}$

$$\text{Magnetic length} = 2l$$

Magnetic moment of each piece, $p'_m = q'_m \times 2l$

$$= \frac{q_m}{2} \times 2l = \frac{p_m}{2} \quad (\because q_m \times 2l = p_m)$$

If each piece is further cut into pieces along the axis of the magnet,



new magnetic pole strength

$$q''_m = \frac{q_m}{4}$$

$$\text{Magnetic length} = 2l$$

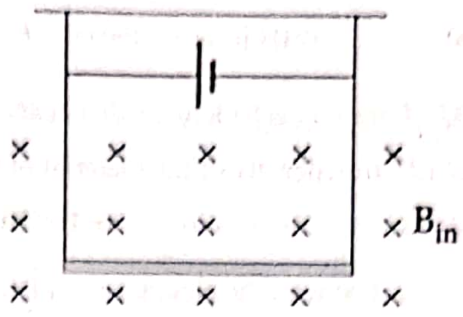
Magnetic moment of each piece $p''_m = q''_m \times 2l$

$$= \frac{q_m}{4} \times 2l$$

$$= \frac{p_m}{4} \quad [\because p_m = 2lq_m]$$

$$\vec{p}_{new} = \frac{\vec{p}_m}{4}$$

2. A conductor of linear mass density $0.2\,gm^{-1}$ suspended by two flexible wire as shown in figure. Suppose the tension in the supporting wires is zero when it is kept inside the magnetic field of 1T whose direction is into the page compute the current inside the current and also the direction for the current. Assume $g = 10\,ms^{-2}$.



Solution

To have zero tension in the wires, the vertical upward magnetic force should be balanced by the downward force due to the weight of the conductor.

$$\text{Linear mass density} = \frac{m}{l} = 0.2\times10^{-3}\,\text{kgm}^{-1} \quad (\because 1\,\text{g} = 10^{-3}\,\text{kg})$$

$$B = 1T \quad g = 10\,\text{ms}^{-2}$$

Upward magnetic force on wire $F_u = BIl$

Downward force (weight) $F_d = mg$

$$BIl = mg \qquad \therefore I = \left(\frac{m}{l}\right)\times\frac{g}{B}$$

$$I = 0.2\times10^{-3}\times10$$

$$I = 0.2\times10^{-2}$$

$$= 2\times10^{-3} = 2\,\text{mA}$$

3. A circular coil with cross sectional area 0.1cm^2 is kept in a uniform magnetic field of strength 0.2T. If the current passing in the coil is 3A and the plane of the loop is perpendicular to the direction of magnetic field. Calculate

(a) total torque on the coil

(b) total force on the coil

(c) average force on each electron in the coil due to the magnetic field of the free electron density for the material of the wire is $10^{28}\,m^{-3}$

Given

$$A = 0.1\text{cm}^2 = 0.1\times10^{-4}\,\text{m}^2$$

$$B = 0.2\text{T}$$

$$I = 3\text{A}.$$

Solution

Since the plane of the loop is held perpendicular to the direction of magnetic field $[\hat{n}\perp\bar{B}]$, the angle between the normal to the plane of the coil and the direction of magnetic field is zero.

(a) Total torque on the coil

$$\tau = IBA\sin\theta \quad (N=1)$$

$$= 3\times0.2\times0.1\times10^{-4}\times0 = 0 \quad (\because \sin0^\circ = 0)$$

(b) Total force on the coil $F = 0$ $\left(\text{using the formula } \overrightarrow{dF} = I\overrightarrow{dl} \times \overrightarrow{B}\right)$

[$\because$ force on each element $\overrightarrow{dl}$ is cancelled by the force on a diametrically opposite element)

(c) Electron density of the material of the wire

$$= 10^{28}\ \text{m}^{-3}$$

Force on the electron $= -e\left(\overrightarrow{v_d} \times \overrightarrow{B}\right)$ (v_d = drift velocity of electron)

$$F = Bev_d \sin\theta \quad \text{(here the angle } \theta \text{ is between } \overrightarrow{dl} \text{ and } \overrightarrow{B}\text{)}$$

$$F = Bev_d \quad (\because \theta = 90°)$$

$$\text{Also, } I = nAev_d \Rightarrow v_d = \frac{I}{neA}$$

$$F = Be \times \frac{I}{neA} = \frac{BI}{nA}$$

$$= \frac{0.2 \times 3}{10^{28} \times 0.1 \times 10^{-4}}$$

$$F = 6 \times 10^{-24}\ \text{N} = 0.6 \times 10^{-23}\ \text{N}.$$

4. A bar magnet is placed in a uniform magnetic field whose strength is 0.8T. If the bar magnet oriented at an angle $30°$ with the external field, experiences a torque of $0.2\,\text{Nm}$, calculate (i) the magnetic moment of the magnet, (ii) the work done by an applied force in moving it from most stable configuration to the most unstable configuration and also compute the work done by the applied magnetic field in this case.

Given :

Magnetic field strength $B = 0.8\text{T}$; Angle of orientation $\theta = 30°$; Torque $\tau = 0.2\,\text{Nm}$

(i) Magnetic moment of the magnet

$$\tau = p_m B \sin\theta$$

$$p_m = \frac{\tau}{B\sin\theta} = \frac{0.2}{0.8\sin 30°}$$

$$= \frac{0.2}{0.8 \times \frac{1}{2}} = \frac{0.2}{0.4} = 0.5\,\text{Am}^2$$

P.E. of the bar magnet $= -\vec{p}_m \cdot \vec{B} = -p_m B\cos\theta$

(ii) P.E. of the bar magnet in the most stable configuration,

$$U_i = -p_m B \quad (\because \theta = 0°)$$

P.E. of the bar magnet in the most unstable configuration

$$U_f = p_m B \quad (\theta = 180°)$$

Work done by the external force $= U_f - U_i$

$$= p_m B - (-p_m B)$$

$$= 2p_m B$$

$$W = 2\times0.5\times0.8$$

$$W = 0.8\,J$$; this work is done against the magnetic field

$\Rightarrow$ work done by the magnetic field, $W_{mag} = -0.8\,J$

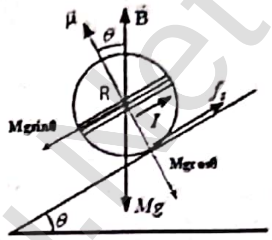
5. A non conducting sphere has mass of 100g and radius 20cm. A flat compact coil of wire with turns 5 is wrapped tightly around it with each turns concentric with the sphere. This sphere is placed on an inclined plane such that the plane of the coil is parallel to the inclined plane. A uniform magnetic field of 0.5T exists in the region in vertically upward to rest the sphere in equilibrium.

Given:

$$m = 100g = 100\times10^{-3}\,kg \qquad B = 0.5T$$

$$R = 20\times10^{-2}\,m \qquad N = 5$$

Solution



For the sphere to be in translational equilibrium, without any movement,

$$f_s - Mg\sin\theta = 0 \qquad \text{... (1)}$$

Sphere is in rotational equilibrium also. About the centre of the sphere, $\overline{B}$ produces a clockwise moment $(= -\mu B\sin\theta)$; friction produces an anti-clockwise moment $(= f_s R)$. Net moment $= 0$

$$\therefore f_s R - \mu B\sin\theta = 0 \qquad \text{... (2)}$$

sub. (1) in (2)

$$(Mg\sin\theta)R = \mu B\sin\theta$$

$$\Rightarrow \mu B = MgR$$

$$\mu = NIA = NI(\pi R^2)$$

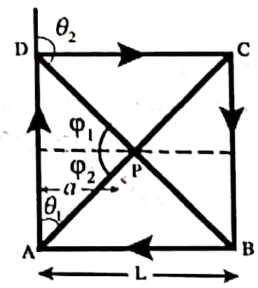
$$\therefore NI(\pi R^2)B = MgR$$

$$I = \frac{Mg}{N\pi BR}$$

$$I = \frac{100\times10^{-3}\times10}{5\times\pi\times0.5\times20\times10^{-2}} = \frac{1}{0.5\pi} = \frac{2}{\pi}$$

$$I = \frac{2}{\pi}\,A.$$

6. Calculate the magnetic field at the centre of a square loop which carries a current of 1.5A, length of each side being 50cm.



Given : $I = 1.5A \qquad L = 50cm = 50\times10^{-2}\,m$

Solution

Magnetic field due to current carrying straight conductor, $B = \dfrac{\mu_o I}{4\pi a}(\sin\phi_1 + \sin\phi_2)$.

$$a = \frac{L}{2} \qquad \phi_1 = 45^\circ \qquad \phi_2 = 45^\circ$$

For a square, there are 4 sides. So the magnetic field at the centre of the square due to four sides is

$$B = 4 \times \frac{\mu_0 I}{4\pi \times \frac{L}{2}} \left[\sin 45^\circ + \sin 45^\circ\right]$$

(since the fields due to current in all the four sides act in the same direction i.e., $\otimes$)

$$= \frac{4 \times 4\pi \times 10^{-7} \times 1.5}{4\pi \times \frac{50 \times 10^{-2}}{2}} \left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right]$$

$$= \frac{4 \times 1.5 \times 10^{-7}}{25 \times 10^{-2}} \times \frac{2}{\sqrt{2}} = \frac{4 \times 1.5 \times 2}{25\sqrt{2}} \times 10^{-5} = 0.33936 \times 10^{-5}$$

$$\therefore\ B = 3.4 \times 10^{-6}\,\mathrm{T}.$$

Chapter 4

Electro Magnetic Induction and Alternating Current

Objective type Questions

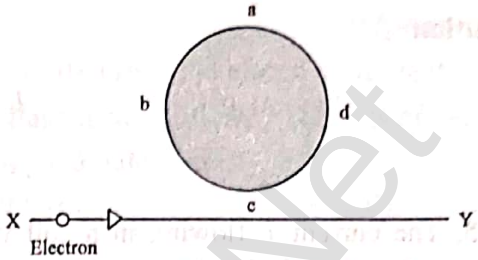
1. An electron moves on a straight line path XY as shown in the figure. The coil $abcd$ is adjacent to the path of the electron. What will be the direction of current, if any, induced in the coil.

(a) The current will reverse its direction as the electron goes past the coil

(b) No current will be induced

(c) $abcd$

(d) $adcb$



Solution

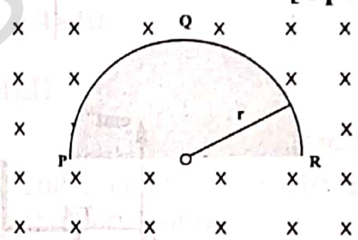
When electron moves towards the loop, flux linked with the coil (which is into the coil ⊗) increases and induced current will be in anti-clockwise direction $(abcd)$ according to Lens law.

When electron moves away from the loop, flux decreases (into the coil ⊗) and induced current will be in the clockwise direction $(adcb)$, to oppose it.

[**Note :** *One should remember that the direction of conventional current in the wire is opposite to the direction of electron flow*].

[Option : (a)]

2. A thin semi-circular conducting ring (PQR) of radius r is falling with its plane vertical in a horizontal magnetic field B, as shown in the figure. The potential difference developed across the ring when its speed υ, is



(a) zero

(b) $\dfrac{B\upsilon\pi r^2}{2}$ and P is at higher potential

(c) $\pi rB\upsilon$ and R is at higher potential

(d) $2rB\upsilon$ and R is at higher potential

Solution

The emf induced in the semi circular conductor $\varepsilon = Bvl = Bv(2r)$. When the conductor falls, the magnetic flux passing into it increases; according to Lens law, the induced current will flow in the anti-clockwise direction. (i.e. along RQP). This implies that R is at a higher potential and P is at a lower potential.

[Option : (d)]

3. The flux linked with a coil at any instant t is given by $\Phi_B = 10t^2 - 50t + 250$. The induced emf at $t = 3\text{s}$ is

(a) -190V (b) -10V (c) 10V (d) 190V

Solution

$$\varepsilon = -\frac{d\phi}{dt} = \frac{-d}{dt}(10t^2 - 50t + 250)$$

$$= -20t + 50$$

When $t = 3s$ $\quad \varepsilon = -20(3) + 50$

$$\varepsilon = -10\,V.$$

[Option : (b)]

4. When the current charges from $+2A$ to $-2A$ in $0.05s$, an emf of $8V$ is induced in a coil. The co-efficient of self-induction of the coil is

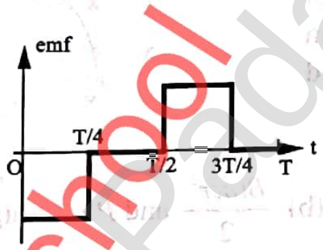
(a) $0.2\,H$ (b) $0.4\,H$ (c) $0.8\,H$ (d) $0.1\,H$

Solution

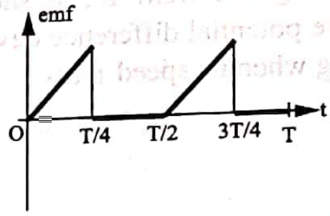
$$L = \frac{\varepsilon}{dI/dt} = \frac{8}{4/0.05} = \frac{0.4}{4} = 0.1\,H$$

[Option : (d)]

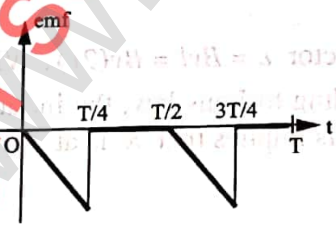
5. The current i flowing in a coil varies with time is shown in the figure. The variation of induced emf with time would be



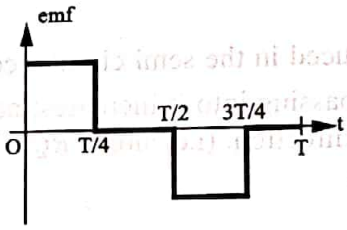
(a)



(b)



(c)



(d)

Solution

$$\text{Induced emf } \varepsilon = -L\frac{di}{dt}$$

For $0 \le t \le \frac{T}{4}$, $(i\text{-}t)$ graph is a straight line with positive constant slope.

$\therefore \frac{di}{dt} =$ constant, emf $\varepsilon = -ve$ constant

For $\frac{T}{4} \leq t \leq \frac{T}{2}$, i is constant. $\therefore \frac{di}{dt} = 0 \Rightarrow \varepsilon = 0$

$\frac{T}{2} \leq t \leq \frac{3T}{4}$, $(i\text{-}t)$ graph is a straight line with negative constant slope.

$\therefore \frac{di}{dt}$ = constant, $\varepsilon = +$ve constant

For $\frac{3T}{4} \leq t \leq T$, i is zero. $\because \frac{di}{dt} = 0 \Rightarrow \varepsilon = 0$

[Option : (a)]

6. A circular coil with a cross sectional area of 4cm^2 has 10 turns. It is placed at the centre of a long solenoid that has 15 turns / cm and a cross sectional area of 10cm^2.. The axis of the coil coincides with the axis of the solenoid what is their mutual inductance?

(a) $7.54\,\mu\text{H}$ (b) $8.54\,\mu\text{H}$ (c) $9.54\,\mu\text{H}$ (d) $10.54\,\mu\text{H}$

Solution

$$n_1 = \frac{N_1}{l}\ ;\ n_2 = \frac{N_2}{l}$$

$$M = \mu_0 n_1 n_2 A_2 l = \frac{\mu_0 N_1 N_2 A_2}{l} = \mu_0 n_1 N_2 A_2$$

$$n_1 = \frac{N_1}{l} = \frac{15}{10^{-2}} \text{ turns}/m$$

$$M = 4\pi \times 10^{-7} \times \frac{15}{10^{-2}} \times 10 \times 4 \times 10^{-4}$$

$$= 7.54 \times 10^{-6}\,\text{H} = 7.54\,\mu\text{H}.$$

[Option : (a)]

7. In a transformer the number of turns in the primary and secondary are 410 and 1230 respectively. If the current in primary is $6\,\text{A}$ then that is secondary coil is

(a) 2 A (b) 18 A (c) 12 A (d) 1 A

Solution

For an ideal transformer,

$$\frac{I_s}{I_p} = \frac{N_p}{N_s}\ \therefore I_s = \frac{N_p}{N_s} \times I_p$$

$$I_s = \frac{410}{1230} \times 6 = 2\text{A}.$$

[Option : (a)]

8. A step down transformer reduces the supply voltage from 220V to 11V and increase the current from 6A to 100A. Then its efficiency is

(a) 1.2 (b) 0.83 (c) 0.12 (d) 0.9

Solution

$$\eta = \frac{E_s I_s}{E_p I_p} = \frac{11 \times 100}{220 \times 6} = 0.83$$

[Option : (b)]

9. In an electrical circuit R, L, C and AC voltage source are all connected in series. When L is removed from the circuit, the phase difference between the voltage and current in the circuit is $\frac{\pi}{3}$. Instead if C is removed from the circuit the phase difference is again $\frac{\pi}{3}$. The power factor of the circuit is

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) 1 (d) $\frac{\sqrt{3}}{2}$

Solution

When L is removed the phase difference between the voltage and current is

$$\tan\phi_1 = \frac{X_C}{R}$$

$$\tan\frac{\pi}{3} = \frac{X_C}{R} \quad \text{or} \quad X_C = R\tan 60°$$

$$X_C = \sqrt{3}R$$

When C is removed the phase difference between the voltage and current is

$$\tan\phi_2 = \frac{X_L}{R}$$

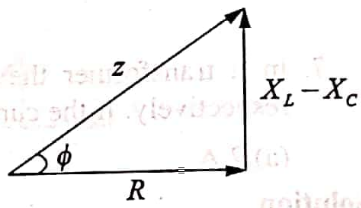
$$\tan\frac{\pi}{3} = \frac{X_L}{R} \quad \text{or} \quad X_L = R\tan 60°$$

$$X_L = \sqrt{3}R$$

As $X_L = X_C$, the series LCR circuit is in resonance.

Impedance of the curcuit $Z = \sqrt{R^2+(X_L - X_C)^2}$

$Z = R$. $(\because X_L = X_C)$



Power factor $\cos\phi = \frac{R}{Z} = \frac{R}{R} = 1$.

[Option : (c)]

10. In a series RL circuit, the resistance and inductive reactance are the same. Then the phase difference between the voltage and current in the circuit is

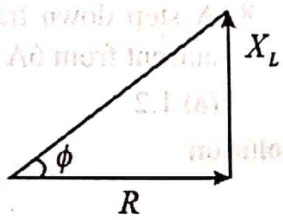
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{6}$ (d) zero

Solution

In RL circuit, $\tan\phi = \frac{X_L}{R}$

If $X_L = R$ then $\tan\phi = 1$

$$\phi = \frac{\pi}{d}.$$



[Option : (a)]

11. In a series resonant RLC circuit the voltage across 100Ω resistor is $40\,V$. The resonant frequency ω is 250 rad/s. If the value of C is $4\mu F$, then the voltage across L is

(a) $600\,V$ (b) $4000\,V$ (c) $400\,V$ (d) $1\,V$

Solution

$$X_C = \frac{1}{C\omega} = \frac{1}{4\times10^{-6}\times250} = 10^3\,\Omega$$

At resonance $X_C = X_L$

$$I = \frac{V}{R} = \frac{40}{100} = 0.4\,A$$

$$V_L = I X_L = I X_C = 0.4\times1000 = 400\,V.$$

[Option : (c)]

12. An inductor $20\,mH$, a capacitor $50\mu F$ and a resistor 40Ω are connected in series across a source of emf $V = 10\sin 340t$. The power loss in AC circuit is

(a) 0.76 W (b) 0.89 W (c) 0.46 W (d) 0.67 W

Solution

Comparing with $V = V_m \sin\omega t$, $\omega = 340$; $V_m = 10$

$$X_L = L\omega = 20\times10^{-3}\times340 = 6.8\Omega$$

$$X_C = \frac{1}{C\omega} = \frac{1}{50\times10^{-6}\times340} \approx 58.8\Omega$$

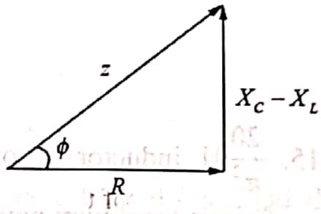
$X_C - X_L = 52\Omega$

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

$$= \sqrt{40^2 + 52^2}$$

$$= \sqrt{1600 + 2704}$$

$$Z = 65.6\Omega.$$



$$\cos\phi = \frac{R}{Z} = \frac{40}{65.6} \; ; \; I_m = \frac{V_m}{Z} = \frac{10}{65.6}$$

$$\text{Power loss } = V_{rms} I_{rms} \cos\phi = \frac{10}{\sqrt{2}} \times \frac{10}{\sqrt{2}\times65.6} \times \frac{40}{65.6} = 0.46\,W$$

$$\left(\because \; V_{rms} = \frac{V_m}{\sqrt{2}} \; ; \; I_{rms} = \frac{I_m}{\sqrt{2}}\right)$$

[Option : (c)]

13. The instantaneous values of alternating current and voltage in a circuit are $i = \frac{1}{\sqrt{2}}\sin(100\pi t)A$ and $V = \frac{1}{\sqrt{2}}\sin\left(100\pi t + \frac{\pi}{3}\right)V$. The average power in watts consumed in the circuit is

(a) $\frac{1}{4}$ (b) $\frac{\sqrt{3}}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{8}$

Solution

Compared with $V_m(\sin\omega t + \phi)$ and $I_m \sin\omega t$, $V_m = I_m = \frac{1}{\sqrt{2}}$; $\phi = \frac{\pi}{3}$

$$P_{av} = \frac{E_m I_m}{2}\cos\phi = \frac{1}{2}\times\frac{1}{\sqrt{2}}\times\frac{1}{\sqrt{2}}\cos\frac{\pi}{3} \quad \left(\because \phi = \frac{\pi}{3}\right)$$

$$= \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = \frac{1}{8}$$

[Option : (d)]

14. In an oscillating LC circuit, the maximum charge on the capacitor is Q. The charge on the capacitor when the energy is stored equally between the electric and magnetic field is

(a) $\frac{Q}{2}$ (b) $\frac{Q}{\sqrt{3}}$ (c) $\frac{Q}{\sqrt{2}}$ (d) Q

Solution

When maximum charge is stored, $U_E = \frac{Q^2}{2C}$

At the given instant, $U'_E = \frac{Q'^2}{2C}$

For given instant $U'_E = \frac{1}{2}U_E$

$$\frac{Q'^2}{2C} = \frac{1}{2}\frac{Q^2}{2C}$$

$$Q' = \frac{Q}{\sqrt{2}}.$$

[Option : (c)]

15. $\frac{20}{\pi^2}$H inductor is connected to a capacitor of capacitance C. The value of C in order to impart maximum power at 50 Hz is

(a) 50μF (b) 0.5μF (c) 500μF (d) 5μF

Solution

For maximum power, resonance prevails

$$\therefore X_L = X_C$$

$$\Rightarrow f = \frac{1}{2\pi\sqrt{LC}} \; ; \; f^2 = \frac{1}{4\pi^2 LC}$$

$$C = \frac{1}{4\pi^2 L f^2}$$

$$= \frac{1}{4\pi^2 \times \frac{20}{\pi^2}\times 50\times 50}$$

$$= \frac{1}{200000} = \frac{1}{2\times 10^5} = 0.5\times 10^{-5}$$

$$= 5\mu\text{F}.$$

[Option : (d)]

Exercise Problems

1. A square coil of side 30 cm with 500 turns is kept in a uniform magnetic field of 0.4 T. The plane of the coil is inclined at an angle of 30° to the field. Calculate the magnetic flux through the coil.

Given :

$$\text{Side of square coil} = 30\text{cm}$$
$$= 30\times10^{-2}\ \text{m}$$
$$A = \text{Area of the coil} = \text{side}\times\text{side}$$
$$= (30\times10^{-2})\times(30\times10^{-2})\ \text{m}^2$$
$$A = 900\times10^{-4}\ \text{m}^2$$
$$\text{Magnetic field } B = 0.4\text{T}$$
$$\text{Angle of orientation} = \theta = 90^\circ - 30^\circ = 60^\circ$$

(θ is the angle between normal to the area of the coil and the direction of magnetic field).

$$\text{No. of turn } N = 500$$

Solution

Magnetic flux through the coil,

$$\phi = NBA\cos\theta$$
$$\phi = 500\times0.4\times900\times10^{-4}\times\cos 60^\circ$$
$$= 45\times10^{4}\times10^{-4}\times0.4\times\frac{1}{2}$$
$$= 45\times0.2$$
$$\phi = 9\ \text{Wb.}$$

2. A straight metal wire crosses a magnetic field of flux 4 mWb in a time 0.4 s. Find the magnitude of the emf induced in the wire.

Given : Change in magnetic flux $d\phi = 4\times10^{-3}$ Wb, time (t) = 0.4 s

Magnitude of emf = ?

Solution :

$$\text{Magnitude of emf } \varepsilon = \frac{d\phi}{dt}$$
$$\varepsilon = \frac{4\times10^{-3}}{0.4} = 10\times10^{-3}$$
$$\varepsilon = 10\,\text{mV}.$$

3. The magnetic flux passing through a coil perpendicular to its plane is a function of time and is given by $\phi_B = (2t^3 + 4t^2 + 8t + 8)$ wb. If the resistance of the coil is 5 Ω, determine the induced current through the coil at a time t = 3 second.

Given : Magnetic flux linked with the coil, $\phi_B = 2t^3 + 4t^2 + 8t + 8$

Resistance of coil $R = 5\Omega$

Time $= 3s$

Induced current = ?

Solution

$$\text{Magnitude of induced emf} = \varepsilon = \frac{d\phi}{dt}.$$

$$= \frac{d}{dt}(2t^3 + 4t^2 + 8t + 8)$$

$$\varepsilon = 6t^2 + 8t + 8$$

$$\text{when } t = 3\text{s}$$

$$\varepsilon = 6(3)^2 + 8(3) + 8$$

$$= 54 + 24 + 8$$

$$\varepsilon = 86V$$

$$\text{Induced current } I = \frac{\varepsilon}{R}$$

$$I = \frac{86}{5}.$$

$$I = 17.2A$$

4. A closely wound circular coil of radius 0.02 m is placed perpendicular to the magnetic field. When the magnetic field is changed from 8000 T to 2000 T in 6 s, an emf of 44 V is induced in it. Calculate the number of turns in the coil.

Given : Radius of the coil = $0.02\,\text{m} = 2\times10^{-2}$ m

Area of the coil $A = \pi r^2 = 3.14\times(2\times10^{-2})^2 = 3.14\times4\times10^{-4}$

$\therefore A = 12.56\times10^{-4}\text{m}^2$.

Change in magnetic field $dB = 8000\text{T} - 2000\text{T} = 6000\text{T}$

Time, $dt = 6\text{s}$

Induced emf $\varepsilon = 44$ V

N = Number of turn of coil = ?

Solution

$$\phi = NBA\cos\theta$$

$\theta = 0°$ ($\because$ plane of the coil is $\perp r$ to magnetic field)

$\therefore \phi = NBA$

$$\text{Magnitude of induced emf } \varepsilon = \frac{d\phi}{dt}$$

$$\varepsilon = \frac{d}{dt}(NBA) = NA\frac{dB}{dt}$$

$$N = \frac{\varepsilon}{A\frac{dB}{dt}}$$

$$N = \frac{44}{12.56\times10^{-4}\times\frac{6000}{6}}$$

$$= \frac{44\times10^4\times10^{-3}}{12.56} = 3.5031\times10$$
$$= 35.031$$
$$N = 35 \text{ turns.}$$

5. A rectangular coil of area 6 cm^2 having 3500 turns is kept in a uniform magnetic field of 0.4 T. Initially, the plane of the coil is perpendicular to the field and is then rotated through an angle of 180° in 1 second. If the resistance of the coil is 35 Ω, find the amount of charge flowing through the coil.

Given :

Area of rectangular coil $= 6cm^2 = 6\times10^{-4} m^2$

No. of turns, $N = 3500$

Magnetic Field, $B = 0.4T$

Resistance of the coil, $R = 35\Omega$

Initially plane of coil is $\perp r$ to the field $\therefore \theta_1 = 0°$

Then rotated through angle $\theta_2 = 180°$.

Solution

$$\phi = NBA\cos\theta$$
$$\phi_1 = NBA\cos\theta_1 = NBA\cos 0° = NBA$$
$$\phi_2 = NBA\cos\theta_2 = NBA\cos 180° = -NBA$$
$$d\phi = \phi_2 - \phi_1 = -NBA - NBA = -2NBA$$

Induced emf
$$\varepsilon = -\frac{d\phi}{dt} = -\frac{(-2NBA)}{dt}$$
$$\varepsilon = \frac{2NBA}{dt} = \frac{2\times3500\times0.4\times6\times10^{-4}}{1}$$
$$\varepsilon = 16.8\times10^3\times10^{-4}$$
$$= 168\times10^{-2} V$$

Current flowing through coil

$$I = \frac{q}{t} = \frac{\varepsilon}{R} \qquad q = \frac{\varepsilon t}{R}$$

Charge flowing, $q = \frac{168\times10^{-2}\times1}{35} = 4.8\times10^{-2}$

$$q = 48\times10^{-3} C.$$

6. An induced current of 2.5 mA flows through a single conductor of resistance 100 Ω. Find out the rate at which the magnetic flux is cut by the conductor.

Given :

Induced current $I = 2.5$ mA

Resistance $= 100\Omega$

Rate of change of magnetic flux $\frac{d\phi}{dt} = ?$

Solution

Rate at which the magnetic flux is cut by the conductor,

$$\varepsilon = \frac{d\phi}{dt} = i \times R$$
$$= 2.5 \times 10^{-3} \times 100$$
$$= 250 \times 10^{-3}$$
$$\varepsilon = 250\,\text{mWbs}^{-1}$$

7. A fan of metal blades of length 0.4 m rotates normal to a magnetic field of 4×10^{-3}T. If the induced emf between the centre and edge of the blade is 0.02 V, determine the rate of rotation of the blade.

Given : Length of the blade $l = 0.4$m

Magnetic field $B = 4 \times 10^{-3}$T

Induced emf $\varepsilon = 0.02$ V

Area swept $A = \pi r^2$

[when the fan rotates, the length of the blade will be equal to the radius, $\therefore l = r$]

Area $A = \pi \times (0.4)^2$

Rate of rotation of blade = ?

Solution

$$\phi = NBA\cos\theta \qquad \theta = 0°$$
$$\phi = NBA$$

Magnitude of induced emf $\varepsilon = \frac{d\phi}{dt}$

$$\varepsilon = \frac{d}{dt}(NBA)$$
$$\varepsilon = B\frac{dA}{dt} = \frac{BdA}{dt} \quad (\because N = 1)$$

$\frac{1}{dt} = n$ is the rate of rotation of the blade, $\therefore \; n = \frac{\varepsilon}{BdA} = \frac{0.02}{4 \times 10^{-3} \times \pi \times (0.4)^2}$

$$n = \frac{0.02 \times 10^3}{2.0096} = 0.009952 \times 10^3$$
$$= 9.95 \text{ revolutions / second.}$$

8. A bicycle wheel with metal spokes of 1 m long rotates in Earth's magnetic field. The plane of the wheel is perpendicular to the horizontal component of Earth's field of 4×10^{-5}T. If the emf induced across the spokes is 31.4 mV, calculate the rate of revolution of the wheel.

Given : Length of the spoke $l = 1$m

Induced emf, $\varepsilon = \frac{1}{2}B\omega l^2$ (refer e.g. 4.9)

Horizontal component of earth's magnetic field

$B = 4 \times 10^{-5}$T

Induced emf $\varepsilon = 31.4\,\text{mV} = 31.4 \times 10^{-3}$ V

Rate of rotation of wheel, $\omega = ?$

$$\text{Area swept, } A = \pi r^2$$

$$\varepsilon = \frac{1}{2}B(2\pi n)l^2$$

Rate of revolution,

$$n = \frac{\varepsilon}{\pi B l^2} = \frac{31.4\times10^{-3}}{3.14\times4\times10^{-5}\times1^2}$$

$$= \frac{10\times10^2}{4}$$

$$n = 250 \text{ rev/s}$$

9. Determine the self-inductance of 4000 turn air-core solenoid of length 2m and diameter 0.04 m.

Given :

$$\text{No. of turns, } N = 4000$$

$$\text{Length of solenoid, } l = 2m$$

$$\text{Diameter of solenoid, } D = 0.04m$$

$$\text{Radius of solenoid, } r = 0.02m$$

$$\text{Area} = \pi r^2 = \pi\times(0.02)^2$$

$$\text{Self inductance } L = ?$$

Solution

$$\text{Self inductance } L = \mu_o n^2 A l \; ; \; n = \frac{N}{l}$$

$$L = \frac{\mu_o N^2 A}{l}$$

$$L = \frac{4\pi\times10^{-7}\times4000\times4000\times\pi\times(0.02)^2}{2}$$

$$= 16\times8\times9.8596\times10^{-5}$$

$$= 1262.028\times10^{-5} \approx 1262\times10^{-5}\text{ H}$$

$$L = 12.62\text{ mH}$$

10. A coil of 200 turns carries a current of 4 A. If the magnetic flux through each turn of the coil is 6×10^{-5} Wb, find the magnetic energy stored in the medium surrounding the coil.

Given : No. of turns in the coil, $N = 200$

Magnetic flux linked with each turn of the coil, $\phi = 6\times10^{-5}$ wb

Current in the coil $I = 4A$

Magnetic energy stored in coil $U_B = ?$

Solution

$$U_B = \frac{1}{2}LI^2$$

$$\text{Self inductance } L = \frac{N\phi_B}{I}$$

$$U_B = \frac{1}{2}\frac{N\phi_B}{I}I^2 = \frac{1}{2}N\phi_B I$$

$$= \frac{1}{2}\times 200\times 6\times 10^{-5}\times 4$$

$$= 24\times 10^{-3}$$

$$U_B = 0.024\,\text{J}$$

11. A 50 cm long solenoid has 400 turns per cm. The diameter of the solenoid is 0.04 m. Find the magnetic flux of a turn when it carries a current of 1 A.

Given : Length of the solenoid $l = 50\text{cm} = 50\times 10^{-2}\,\text{m}$

No. of turns $n = 400/\text{cm} = 400/10^{-2}\,\text{m}$

$$= 400\times\frac{10^2}{m} = 4\times\frac{10^4}{m}$$

$\therefore$ $n =$ No. of turns per unit length of the solenoid $= 4\times 10^4$

Diameter of the solenoid $= 0.04\,\text{m}$

Radius of the solenoid $r = 0.02\,\text{m}$

Current passing through solenoid, $I = 1\,\text{A}$

Area of solenoid $A = \pi r^2 = 3.14\times(0.02)^2\,\text{m}^2$

Magnetic flux of a turn = ?

Solution

Magnetic flux per turn, $\phi_B = \mu_0 niA$ (for solenoid)

$$= 4\times 3.14\times 10^{-7}\times 4\times 10^4\times 1\times 3.14\times(0.02)^2$$

$$= 0.63\times 10^{-4}\,\text{Wb}$$

12. A coil of 200 turns carries a current of 0.4 A. If the magnetic flux of 4 mwb is linked with each turn of the coil, find the inductance of the coil.

Given No. of turns $N = 200$

Current through coil $I = 0.4A$

Magnetic flux with each turn, $\phi = 4\,\text{mWb} = 4\times 10^{-3}\,\text{Wb}$

Inductance of the coil $L = ?$

Solution

$$L = \frac{N\phi}{I}$$

$$= \frac{200\times 4\times 10^{-3}}{0.4} = \frac{800\times 10^{-3}}{0.4} = 2\,\text{H}$$

$L = 2\,\text{H}$.

13. Two air core solenoids have the same length of 80 cm and same cross-sectional area 5 cm^2. Find the mutual inductance between them if the number of turns in the first solenoid is 1200 turns and that in the second solenoid is 400 turns.

Given :

Length of each solenoid $l = 80\text{cm} = 80\times10^{-2}\text{m}$

Area cross-section of each solenoid $A = 5\text{cm}^2 = 5\times10^{-4}\text{m}^2$

No. of turns in first solenoid $N_1 = 1200$

No. of turns in second solenoid $N_2 = 400$

Mutual inductance = ?

Solution

Mutual inductance M between the solenoids is given by $M = \mu_0 n_1 n_2 Al$

$$M = \mu_0 \frac{N_1}{l}\frac{N_2}{l}Al = \frac{\mu_0 N_1 N_2 A}{l}$$

$$= \frac{4\pi\times10^{-7}\times1200\times400\times5\times10^{-4}}{80\times10^{-2}}$$

$$= 4\pi\times10^{-7}\times1200\times25\times10^{-4}\times10^{2} = 376800\times10^{-9}$$

$$= 0.3768\times10^{-3}\text{H}$$

$$M = 0.38\,\text{mH}.$$

14. A long solenoid having 400 turns per cm carries a current 2A. A 100 turn coil of cross-sectional area 4cm^2 is placed co-axially inside the solenoid so that the coil is in the field produced by the solenoid. Find the emf induced in the coil if the current through the solenoid reverses its direction in 0.04 sec.

Given : n_1 = No. of turns per unit length of the solenoid,

$$\frac{N_1}{l} = 400 \text{ turns/per cm} = 400\times10^{2}\text{ / m}$$

$$i = 2A \ ; \ N_2 = 100 \text{ turns} \ ; \ A_2 = 4\times10^{-4}\text{m}^2 \ ; \ t = 0.04 \text{ s}$$

Solution

Magnetic field produced by the solenoid,

$$B_1 = \mu_o n_1 i_1$$

$$B_1 = 4\times3.14\times10^{-7}\times400\times10^{2}\times2 = 10.04\times10^{-2}\text{ Wbm}^{-2}$$

Magnetic flux through each turn of the coil is

$$\phi_{21} = B_1 A_2 \cos\theta \ ; \ \theta = 0^\circ$$

$$= 10.04\times10^{-2}\times4\times10^{-4}\times1 = 4.02\times10^{-5}\text{Wb}$$

When the direction of current is revered, magnetic flux through each turn is,

$$\phi_{21} = B_1 A_2 \cos\theta \quad (\theta = 180^\circ)$$

$$= -B_1 A_2 = -4.02\times10^{-5}\text{ Wb}$$

$$\text{Induced emf, } \varepsilon_2 = -N_2 \frac{d\phi_{21}}{dt} = \frac{100 \times \left[-4.02\times10^{-5} - 4.02\times10^{-5}\right]}{0.04}$$

$$\varepsilon_2 = \frac{100 \times 8.04 \times 10^{-5}}{0.04} = 0.2\,\text{V}.$$

15. A 200 turn coil of radius 2 cm is placed co-axially within a long solenoid of 3 cm radius. If the turn density of the solenoid is 90 turns per cm, then calculate mutual inductance of the coil.

Given : No. of turns in the coil, $N_2 = 200$

Turn density of long solenoid, $\frac{N_1}{l} = 90 \times 10^2\,\text{m}^{-1}$

Radius of the coil, $r_2 = 2\text{cm} = 2\times10^{-2}\text{m}$

Area of cross-section of the coil, $A_2 = \pi r_2^2$

$= \pi \times \left(2\times10^{-2}\right)^2 = \pi \times 4\times10^{-4}\,\text{m}^2$

Radius of long solenoid $r_1 = 3\text{cm} = 3\times10^{-2}\text{m}$

Mutual inductance = ?

Solution

Mutual inductance M between the solenoids is given by $M = \mu_0 n_1 n_2 A l$

$$M = \mu_0 \frac{N_1}{l}\frac{N_2}{l} A l = \frac{\mu_0 N_1 N_2 A_2}{l}$$

$$M = \mu_0 \left(\frac{N_1}{l}\right)(N_2 A_2)$$

$$= 4\pi\times10^{-7}\times200\times90\times10^2\times\pi\times4\times10^{-4}$$

$$= 288\times9.8596\times10^{-6} = 2839.56\times10^{-6}\,\text{H}$$

$$M \sqcup 2.84\,\text{mH}.$$

16. The solenoids S_1 and S_2 are wound on an iron-core of relative permeability 900. The area of their cross-section and their length are the same and are 4cm^2 and 0.04 m respectively. If the number of turns in S_1 is 200 and that in S_2 is 800, calculate the mutual inductance between the coils. If the current in solenoid 1 is increased form 2A to 8A in 0.04 second. Calculate the induced emf in solenoid 2.

Given : Relative permeability $\mu_r = 900$

No. of turns in solenoid S_1 is $N_1 = 200$

No. of turns in solenoid S_2 is $N_2 = 800$

Area of cross section of solenoid S_2 is $A_2 = 4\text{cm}^2 = 4\times10^{-4}\text{m}^2$

Length of solenoid S_1 is $l_1 = 0.04\,\text{m}$

Change of current in the solenoid 1, $dI = I_2 - I_1 = 8 - 2 = 6\text{A}$

Time taken $dt = 0.04\,\text{s}$

Induced emf = ?

Solution

$$\mu = \mu_0\mu_r \; ; \; M = \mu_0 n_1 n_2 A_2 l$$

$$M = \frac{\mu_0\mu_r N_1 N_2 A_2}{l} = \mu_0\mu_r\left(\frac{N_1}{l}\right)N_2 A_2$$

$$M = \frac{4\pi\times10^{-7}\times900\times200\times800\times4\times10^{-4}}{0.04}$$

$$= 4\pi\times90\times4\times4\times10^{-4} = 18086\times10^{-4}\,\text{H}$$

$$M \sqcup 1.81\,\text{H}.$$

Induced emf solenoid 2, $\varepsilon = -M\dfrac{dI}{dt}$

$$= -1.81\times\frac{6}{0.04} = -2.715\times10^{+2}$$

$$\varepsilon = -271.5\,\text{V}.$$

17. A step-down transformer connected to main supply of 220 V is used to operate 11V, 88W lamp. Calculate (i) Transformation ratio and (ii) Current in the primary.

Given :

$$V_P = 220\,\text{V}, V_s = 11\,\text{V};$$

$$\text{Output power} = 88\,\text{W}$$

$$\text{Transformer ratio} = ? \quad \text{Primary current} = ?$$

Solution

Ideal transformer equation is $\dfrac{V_s}{V_P} = \dfrac{N_s}{N_P} = \dfrac{I_P}{I_s} = k$

(i) Voltage transformation ratio $= k$

$$k = \frac{V_S}{V_P} = \frac{11}{220} = \frac{1}{20}$$

(ii) Primary current

$$\text{Output power} = \text{Input power} = V_P \times I_P \quad \text{(for an ideal transformer)}$$

$$I_P = \frac{\text{output power}}{V_P} = \frac{88}{220} = \frac{4}{10} = 0.4\,\text{A}$$

Current in the primary, $I_P = 0.4\,\text{A}$.

18. A 200V/120V step-down transformer of 90% efficiency is connected to an induction stove of resistance 40 Ω. Find the current drawn by the primary of the transformer.

Given : $V_P = 200\,\text{V}$ $V_S = 120\,\text{V}$ $\eta = 90\% = \dfrac{90}{100} = 0.9$ $R_S = 40\Omega$ $I_P = ?$

Solution

$$\eta = \frac{\text{output power}}{\text{input power}} = \frac{V_S I_S}{V_P I_P}$$

$$I_S = \frac{V_S}{R_S} = \frac{120}{40} = 3\,\text{A}$$

$$\eta = \frac{120 \times 3}{200 \times I_P} = \frac{90}{100} = 0.9$$

$$I_P = \frac{120 \times 3}{200 \times 0.9}$$

$$I_P = 2\,A.$$

19. The 300 turn primary of a transformer has resistance 0.82 Ω and the resistance of its secondary of 1200 turns is 6.2 Ω. Find the voltage across the primary if the power output from the secondary at 1600V is 32 kW. Calculate the power losses in both coils when the transformer efficiency is 80%.

Given : $\eta = 80\%$ $N_P = 300$ $R_P = 0.82\Omega$ $R_s = 6.2\Omega$ $N_s = 1200$ $V_s = 1600\,V$

Output power $= 32\,kW$

Power loss in the primary $= ?$

Power loss in the secondary $= ?$

Solution

$$\eta = \frac{\text{Output power}}{\text{Input power}} = \frac{V_S I_S}{V_P I_P}$$

$$\frac{80}{100} = \frac{32 \times 10^3}{V_P I_P}$$

$$V_P I_P = \frac{32 \times 10^3 \times 100}{80}$$

$$= 40\,kW$$

$$V_S I_S = 32 \times 10^3\,W$$

$$I_S = \frac{32 \times 10^3}{1600} = 20\,A$$

$$\frac{N_S}{N_P} = \frac{V_S}{V_P}$$

$$V_P = \frac{V_S \times N_P}{N_S} = \frac{1600 \times 300}{1200} = 400$$

$$V_P = 400\,V$$

$$\text{Input power} = V_P I_P$$

$$= 400 \times I_P = 40 \times 10^3$$

$$I_P = \frac{40 \times 10^3}{400} = 100\,A$$

$$\text{Power loss in primary} = I_P^2 \times R_P$$

$$= (100)^2 \times 0.82$$

$$= 8200 = 8.2\,kW$$

$$\text{Power loss in the secondary} = I_S^2 \times R_S$$

$$= (20)^2 \times 6.2$$
$$= 2480$$
$$= 2.48\,\text{kW}.$$

20. Calculate the instantaneous value at 60°, average value and RMS value of an alternating current whose peak value is $20A$.

Given : Peak value of current $I_m = 20\,\text{A}$

Phase angle, $\phi = 60°$

Solution

(i) Instantaneous current

$$i = I_m \sin \omega t = I_m \sin \phi$$
$$i = 20 \times \sin 60$$
$$= 20 \times \frac{\sqrt{3}}{2} = \frac{20 \times 1.732}{2} = 17.32\,\text{A}$$

(ii) Average value of current

$$I_{av} = 0.637\, I_m$$
$$= 0.637 \times 20$$
$$= 12.74\ \text{A}$$

(iii) RMS value of current

$$I_{RMS} = 0.707 I_m$$
$$= 0.707 \times 20$$
$$= 14.14\,\text{A}.$$

Chapter 5

Electromagnetic waves

Objective type Questions

1. The dimension of $\frac{1}{\mu_o \varepsilon_o}$ is

(a) $\left[LT^{-1}\right]$ (b) $\left[L^2T^{-2}\right]$ (c) $\left[L^{-1}T\right]$ (d) $\left[L^{-2}T^2\right]$

Solution

Speed of light in vacuum, $c = \frac{1}{\sqrt{\mu_o \varepsilon_o}}$; $c^2 = \frac{1}{\mu_o \varepsilon_o}$

Dimension of $\frac{1}{\mu_o \varepsilon_o}$ is the dimension of c^2

$$c^2 = \left[LT^{-1}\right]^2 = \left[L^2T^{-2}\right].$$

[Option : (b)]

2. If the amplitude of the magnetic field is $3\times10^{-6}\,\mathrm{T}$, then amplitude of the electric field for an electromagnetic wave is

(a) $100\,\mathrm{Vm^{-1}}$ (b) $400\,\mathrm{Vm^{-1}}$ (c) $600\,\mathrm{Vm^{-1}}$ (d) $900\,\mathrm{Vm^{-1}}$

Solution

Amplitude of electric field $E_o = B_o c \quad \left[\because\ c = \frac{E_o}{B_o}\right]$

$$= 3\times10^{-6}\times3\times10^{8}$$

$$= 9\times10^{2} = 900\,\mathrm{Vm^{-1}}.$$

[Option : (d)]

3. Which of the following electromagnetic radiations is used for viewing objects through fog?

(a) Microwave (b) Gamma rays (c) X-rays (d) Infrared

Solution

Infrared.

[Option : (d)]

4. Which of the following is false for electromagnetic waves?

(a) Transverse (b) Non-mechanical waves

(c) Longitudinal (d) Produced by accelerating charges

Solution

Longitudinal.

[Option : (c)]

5. Consider an oscillator, which has a charged particle, oscillating about its mean position with a frequency of $300\,\mathrm{MHz}$. The wavelength of electromagnetic waves produced by this oscillator is

(a) 1 m (b) 10 m (c) 100 m (d) 1000 m

Solution

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{300\times10^6} = 1\,\text{m}$$

[Option : (a)]

6. The electric and the magnetic field associated with an electromagnetic wave, propagating along negative x-axis can be represented by

(a) $\vec{E} = E_o\hat{j}$ and $\vec{B} = B_o\hat{k}$ (b) $\vec{E} = E_o\hat{k}$ and $B_o\hat{j}$

(c) $\vec{E} = E_o\hat{i}$ and $\vec{B} = B_o\hat{j}$ (d) $\vec{E} = E_o\hat{j}$ and $B_o\hat{i}$

Solution

Direction of propagation of e.m. wave will be along the direction of $(\vec{E}\times\vec{B}) \Rightarrow (\hat{k}\times\hat{j}) = -\hat{i}$

$\vec{E} = E_o\hat{k}$ and $B_o\hat{j}$

[Option : (b)]

7. In an electromagnetic wave travelling in free space, the *rms* value of the electric field is $3\,\text{Vm}^{-1}$. The peak value of the magnetic field is

(a) $1.414\times10^{-8}\,\text{T}$ (b) $1.0\times10^{-8}\,\text{T}$ (c) $2.828\times10^{-8}\,\text{T}$ (d) $2.0\times10^{-8}\,\text{T}$

Solution

$$E_{rms} = \frac{E_o}{\sqrt{2}} \qquad E_o = 3\times\sqrt{2}\ \text{Vm}^{-1}$$

$$c = \frac{E_o}{B_o} \Rightarrow \qquad B_o = \frac{E_o}{c} = \frac{3\sqrt{2}}{3\times10^8} = 1.414\times10^{-8}\,\text{T}$$

[Option : (a)]

8. An e.m. wave is propagating in a medium with velocity $\vec{v} = v\hat{i}$. The instantaneous oscillating electric field of this e.m. wave is along $+y$-axis, then the direction of oscillating magnetic field of the e.m. wave will be along :

(a) $-y$ direction (b) $-x$ direction (c) $+z$ direction (d) $-z$ direction

Solution

The direction of propagation of e.m. wave will be along the direction of $(\vec{E}\times\vec{B})$;

$\because\ v = v\hat{i}$, velocity is along $+x$ direction and $\vec{E}$ is along $+y$ direction, $\Rightarrow$ $\vec{B}$ will be along $+z$ direction. i.e., $\hat{i}\times\hat{j} = \hat{k}$

[Option : (c)]

9. If the magnetic monopole exists, then which of the Maxwell's equation to be modified?

(a) $\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_o}$ (b) $\oint \vec{B}\cdot d\vec{A} = 0$

(c) $\oint \vec{B}\cdot d\vec{l} = \mu_o i_c + \mu_o\varepsilon_o \frac{d}{dt}\oint \vec{E}\cdot d\vec{A}$ (d) $\oint \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\Phi_B$

Solution

$\oint \vec{B}\cdot d\vec{A} = 0$

[Option : (b)]

10. Fraunhofer lines are an example of _____ spectrum.
(a) line emission (b) line absorption (c) band emission (d) band absorption

Solution

Dark lines observed in the solar spectrum are Fraunhofer lines. (line absorption spectrum)

[Option : (b)]

11. Which of the following is an electromagnetic wave?
(a) α-rays (b) β-rays (c) γ-rays (d) all of them

Solution

γ-rays

[Option : (c)]

12. Which one of them is used to produce a propagating electromagnetic wave?
(a) an accelerating charge (b) a charge moving with constant velocity
(c) a stationary charge (d) an uncharged particle

Solution

an accelerating charge

[Option : (a)]

13. If $E = E_o \sin\left[10^6 x - \omega t\right]$ be the electric field of plane electromagnetic wave, the value of ω is
(a) 0.3×10^{-14} rad s^{-1} (b) 3×10^{-14} rad s^{-1}
(c) 0.3×10^{14} rad s^{-1} (d) 3×10^{14} rad s^{-1}

Solution

$$E = E_o \sin\left[10^6 x - \omega t\right] \text{ ; compared with } E = E_o \sin\left[kx - \omega t\right]$$

$$k = 10^6$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{(c/f)} = \frac{2\pi f}{c} = \frac{\omega}{c} \quad \left(\because \lambda = \frac{c}{f} \; ; \; \omega = 2\pi f\right)$$

$$\Rightarrow \quad \omega = kc = 10^6 \times 3\times10^8 = 3\times10^{14} \text{ rad } s^{-1}$$

[Option : (d)]

14. Which of the following is not true for electromagnetic waves?
(a) it transports energy
(b) it transports momentum
(c) it transports angular momentum
(d) in vacuum, it travels with different speeds which depend on their frequency

Solution

in vacuum, it travels with different speeds which depend on their frequency

[Option : (d)]

15. The electric and magnetic fields of an electromagnetic wave are
(a) in phase and perpendicular to each other
(b) out of phase and not perpendicular to each other
(c) in phase and not perpendicular to each other
(d) out of phase and perpendicular to each other

Solution

in phase and perpendicular to each other.

[Option : (a)]

Exercise Problems

1. Consider a parallel plate capacitor whose plates are closely spaced. Let R be the radius of the plates and the current in the wire connected to the plates is 5A, calculate the displacement current through the surface passing between the plates by directly calculating the rate of change of flux of electric field through the surface.

Given : Current in the wire connected to plates $= I_C = 5\,\text{A}$

Radius of the plates $= R$

Area of the plates $= A$

Displacement current $I_d = ?$

Solution

$$= I_c = 5\,\text{A}$$

Electric flux, through the space between the plates,

$$\phi_E = \frac{q}{\varepsilon_0}$$

$$\text{Displacement current} = I_d = \varepsilon_0 \frac{d\phi_E}{dt} = \varepsilon_0 \frac{d}{dt}\left(\frac{q}{\varepsilon_0}\right)$$

$$= \frac{dq}{dt} = I_c$$

$$I_d = I_c = 5\,\text{A}.$$

2. A transmitter consists of LC circuit with an inductance of $1\,\mu\text{H}$ and a capacitance of $1\,\mu\text{F}$. What is the wavelength of the electromagnetic waves it emits?

Given : Inductance $L = 1\,\mu\text{H} = 1\times10^{-6}\,\text{H}$

Capacitance $C = 1\,\mu\text{F} = 1\times10^{-6}\,\text{F}$

λ, wavelength of the electromagnetic wave emitted $= ?$

Solution

$$\text{Frequency of the wave emitted, } f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{10^{-6}\times10^{-6}}}$$

$$= \frac{1}{2\pi\times10^{-6}}\,\text{Hz}. \quad ; \quad f = 1.592\times10^{5}\,\text{Hz}$$

$$\text{Wavelength } \lambda = \frac{c}{f} \quad ; \quad c = 3\times10^{8}\,\text{m/s} \quad (c = \text{velocity of light})$$

$$\lambda = \frac{3\times10^{8}}{\frac{1}{2\pi\times10^{-6}}} = 3\times2\pi\times10^{2}$$

$$\lambda = 18.84\times10^{2}\,\text{m}.$$

3. A pulse of light of duration $10^{-6}\,\text{s}$ is absorbed completely by a small object initially at rest. If the power of the pulse is $60\times10^{-3}\,\text{W}$, calculate the final momentum of the object.

Given : Duration of absorption of light $t = 10^{-6}\,\text{s}$

Power of the pulse $= 60\times10^{-3}\,\text{N}$

Final momentum $= ?$

Solution

$$\text{Energy } U = \text{power} \times \text{time} = P \times t$$

$$\text{Momentum, } p = \frac{U}{c} = \frac{P \times t}{c} \quad (c = \text{velocity of light})$$

$$p = \frac{60 \times 10^{-3} \times 10^{-6}}{3 \times 10^{8}}$$

$$= 20 \times 10^{-17}\ \text{kgms}^{-1}.$$

4. Let an electromagnetic wave propagates along the x direction, the magnetic field oscillates at a frequency of 10^{10} Hz and has an amplitude of 10^{-5} T, acting along the y-direction. Then, compute the wavelength of the wave. Also write down the expression for electric field in this case.

Given :

$$\text{Frequency of e.m wave } f = 10^{10}\ \text{Hz}$$

$$\text{Amplitude of magnetic field } B_0 = 10^{-5}\ \text{T}$$

$$\text{Amplitude of oscillating electric field } E_0 = ?$$

Solution

$$\text{Velocity of light } c = \frac{E_o}{B_o}$$

$$E_o = c \times B_o$$

$$= 3 \times 10^{8} \times 10^{-5}$$

$$E_0 = 3 \times 10^{3}\ \text{NC}^{-1}$$

$$\text{Wavelength } \lambda = \frac{c}{f} = \frac{3 \times 10^{8}}{10^{10}}$$

$$\lambda = 3 \times 10^{-2}\ \text{m}$$

$$\omega = 2\pi f = 2 \times 3.14 \times 10^{10}$$

$$= 6.28 \times 10^{10}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi f}{c} = \frac{6.28 \times 10^{10}}{3 \times 10^{8}}$$

$$= 2.09 \times 10^{2}\ \text{m}^{-1}$$

Expression for electric field $E(x,t) = E_o \sin(kx - \omega t)$

$$E(x,t) = 3 \times 10^{3} \sin\left[2.09 \times 10^{2} x - 6.28 \times 10^{10} t\right] NC^{-1}.$$

Direction of propagation (along $+x$ direction) will be along direction of $(\bar{E} \times \bar{B})$; as $\vec{B}$ is along the $+y$ direction, $\vec{E}$ will be along $-z$ direction (i.e., $-\hat{k}$)

$$\Rightarrow \bar{E}(x,t) = 3 \times 10^{3} \sin(2.9 \times 10^{2} x - 6.28 \times 10^{10} t)(-\hat{k})\text{NC}^{-1}$$

5. If the relative permeability and relative permittivity of the medium are 1.0 and 2.25, respectively, then find the speed of the electromagnetic wave in this medium.

Given :

$$\text{Relative permeability } \mu_r = 1.0$$

Relative permittivity ε_r = 2.25

Speed of the electromagnetic wave = ?

Solution

Speed of electromagnetic wave υ in the medium is given by

$$\upsilon = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}} = \frac{1}{\sqrt{\varepsilon_0\mu_0}} \times \frac{1}{\sqrt{\varepsilon_r\mu_r}} = c \times \frac{1}{\varepsilon_r\mu_r}$$

$$= 3\times10^8 \times \frac{1}{\sqrt{2.25\times1}} = \frac{3\times10^8}{1.5} = 2\times10^8\,\text{ms}^{-1}$$

$$\upsilon = 2\times10^8\,\text{ms}^{-1}.$$

Chapter 6

Ray Optics

Objective type Questions

1. The speed of light in an isotropic medium depends on,
   (a) its intensity (b) its wavelength
   (c) the nature of propagation (d) the motion of the source w.r.t medium

[Option : (b)]

2. A rod of length 10 cm lies along the principal axis of a concave mirror of focal length 10 cm in such a way that its end closer to the pole is 20 cm away from the mirror. The length of the image is
   (a) 2.5 cm (b) 5cm (c) 10 cm (d) 15cm

Solution

$u_A = -20\text{cm}, \ u_B = -30\text{cm}$ and $f = -10\text{cm}$

Length of image AB is $v_B - v_A$

From miror formula

$$\frac{1}{v_A} = \frac{1}{f} - \frac{1}{u_A} \text{ and } \frac{1}{v_B} = \frac{1}{f} - \frac{1}{u_B}$$

$$\frac{1}{v_A} = \frac{1}{-10} - \frac{1}{-20} = \frac{1}{-20} \Rightarrow v_A = -20\text{ cm}$$

$$\frac{1}{v_B} = \frac{1}{-10} - \frac{1}{-30} = \frac{1}{-15} \Rightarrow v_B = -15\text{ cm}$$

$$\therefore v_B - v_A = -15 - (-20) = 5 \text{ cm}$$

[Option : (b)]

3. An object is placed in front of a convex mirror of focal length of f. Then the maximum and minimum distance of an object from the mirror such that the image formed is real and magnified, are
   (a) $2f$ and c (b) c and ∞ (c) f and O (d) None of these

Solution

In a convex mirror, the image is always virtual, erect and diminished, whatever be the object distance.

[Option : (d)]

4. For light incident from air on a slab of refractive index 2, the maximum possible angle of refraction is,
   (a) 30° (b) 45° (c) 60° (d) 90°

Solution

$$n_1 = 1 \quad \text{(first medium is air medium, } \therefore \text{ refractive index } = 1)$$

$$n_2 = 2$$

From Snell's law $\quad n_1 \sin i = n_2 \sin r$

$$\therefore \sin r = \frac{1}{2}\sin i$$

For maximum r, $\sin i = 1$

$$\Rightarrow \sin r = \frac{1}{2}$$

$$\Rightarrow r = 30°.$$

[Option : (a)]

5. If the velocity and wavelength of light in air are υ_a and λ_a and that in water is υ_w and λ_w, then the refractive index of water are,

(a) $\frac{\upsilon_w}{\upsilon_a}$ (b) $\frac{\upsilon_a}{\upsilon_w}$ (c) $\frac{\lambda_w}{\lambda_a}$ (d) $\frac{\upsilon_a \lambda_a}{\upsilon_w \lambda_w}$

Solution

$$n = \frac{\lambda_a}{\lambda_w}$$

Since, $\upsilon \propto \lambda$

$$\therefore n = \frac{\upsilon_a}{\upsilon_w}$$

[Option : (b)]

6. Stars twinkle due to,

(a) reflection (b) total internal reflection

(c) refraction (d) polarisation

[Option : (c)]

7. When a biconvex lens of glass having refractive index 1.47 is dipped in a liquid, it acts as a plane sheet of glass. This implies that the liquid must have refractive index

(a) less than one (b) less than that of glass

(c) greater than that of glass (d) equal to that of glass

Solution

When a biconvex lens acts as a plane sheet of glass, the light ray is undeviated. This implies that the liquid must have refractive index equal to that of glass.

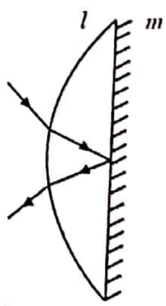
[Option : (d)]

8. The radius of curvature of a curved surface at a thin planoconvex lens is 10cm and the refractive index is 1.5. If the plane surface is silvered, then the focal length will be:

(a) 5 cm (b) 10 cm (c) 15 cm (d) 20 cm

Solution

Power of the silvered lens, $P = 2P_l + P_m$ silvered side (mirror) is plane. (refer section 6.6.7 – eq. 6.78)



$$\therefore f_m = \infty \Rightarrow P_m = \frac{1}{f_m} = 0$$

$$\therefore P = 2P_l \text{ (or) } \frac{1}{f} = \frac{2}{f_l} \quad \dots (1)$$

Lens-maker's formula

$$\frac{1}{f_l} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.5-1)\left(\frac{1}{10} - \frac{1}{\infty}\right) = \frac{0.5}{10}$$

Substitute in (1),

$$\frac{1}{f} = 2\times\frac{0.5}{10} \Rightarrow f = 10\text{cm}$$

[Option : (b)]

9. An air bubble in glass slab of refractive index 1.5 (near normal incidence) is 5 cm deep when viewed from one surface and 3 cm deep when viewed from the opposite face. The thickness of the slab is,

(a) 8 cm (b) 10 cm (c) 12 cm (d) 16cm

Solution

$$n = 1.5$$

$$d_1' = 5 \text{ cm and } d_2' = 3\text{cm}$$

$$\text{Thickness of slab} = d_1 + d_2 = nd_1' + nd_2'$$

$$= (1.5\times5)+(1.5\times3)$$

$$= 4.5+7.5 = 12$$

[Option : (c)]

10. A ray of light travelling in a transparent medium of refractive indedx n falls, on a surface separating the medium from air at an angle of incidence of $45°$. The ray can undergo total internal reflection for the following n,

(a) $n=1.25$ (b) $n=1.33$ (c) $n=1.4$ (d) $n=1.5$

Solution

For total internal reflection, $n\sin i_c = 1 \Rightarrow n = \frac{1}{\sin i_c}$

for this medium, $45° > i_c$ i.e., $i_c < 45°$

$$\Rightarrow \sin i_c < \sin 45° \text{ (or) } \sin i_c < \frac{1}{1.414}$$

$$\therefore \frac{1}{\sin i_c} > 1.414 \text{ (or) } n > 1.414$$

$$\therefore n = 1.5$$

[Option : (d)]

Exercise Problems

1. An object of 4 cm height is placed at 6 cm in front of a concave mirror of radius of curvature 24 cm. Find the position, height, magnification and nature of the image.

Solution

$$R = 2f \Rightarrow f = \frac{R}{2} = \frac{-24}{2} = -12 \text{ cm}$$

(i) Position : $\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \Rightarrow \frac{1}{(-12)} = \frac{1}{v} + \frac{1}{(-6)}$ $\therefore v = 12$ cm

(ii) Magnification : $m = \frac{-v}{u} = \frac{-12}{-6} = 2$

(iii) $m = \frac{h'}{h} \Rightarrow 2 = \frac{h'}{4} \Rightarrow h' = 8$ cm

(iv) Image is erect, virtual, twice the height of the object and is formed behind the mirror.

2. An object is placed in front of a concave mirror of focal length 20 cm. The image formed is three times the size of the object. Calculate two possible distances of the object from the mirror.

Data

Focal length of concave mirror, according to sign convention, $f = -20$ cm

Solution

$m = 3$ for virtual and $m = -3$ for real images

$$m = \frac{f}{(f-u)} \; ; \Rightarrow \frac{1}{m} = \frac{(f-u)}{f} = 1 - \frac{u}{f}$$

$$\frac{u}{f} = 1 - \frac{1}{m} \; ; \; \therefore u = f\left(1 - \frac{1}{m}\right)$$

For virtual image m is positive, $m = +3$

$$\therefore u = -20\left(1 - \frac{1}{3}\right) = \frac{-40}{3}$$

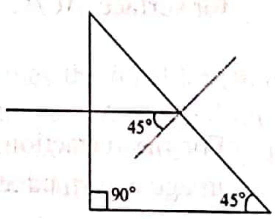
$$u = \frac{-40}{3} \text{ cm}$$

For real image m is negative, $m = -3$

$$\therefore u = f\left(1 - \frac{1}{m}\right) = -20\left(1 - \frac{1}{(-3)}\right) = -\frac{80}{3}$$

$$u = \frac{-80}{3} \text{ cm}$$

3. A beam of light consisting or red, green and blue is incident on a right-angled prism as shown in figure. The refractive index of the matrial of the prism for the above red, green and blue colours are 1.39,1.44 and 1.47 respectively. What are the colours suffer total internal reflection?



Solution

Let c be the critical angle

$$\sin c = \frac{1}{n} \Rightarrow c = \sin^{-1}\left(\frac{1}{n}\right)$$

$$c_{red} = \sin^{-1}\left(\frac{1}{1.39}\right) \approx 46^\circ \; ; \; c_{green} = \sin^{-1}\left(\frac{1}{1.44}\right) \approx 44^\circ \; ; \; c_{blue} = \sin^{-1}\left(\frac{1}{1.47}\right) \approx 43^\circ$$

For TIR to take place, i must be greater than c.

Angle of incidence $i = 45^\circ$ (Fig.). As $i > c_{green}$ and $i > c_{blue}$, green and blue undergo total internal reflection. $i < c_{red}$, red does not undergo.

4. An object is placed at a certain distance from a convex lens of focal length 20cm. Find the distance of the object if the image obtained is magnified 4 times.

Data

$$f = 20\,\text{cm} \; ; \; m = 4 \; ; \; u = ?$$

Solution

According to sign convention, $f = +20$; $|m| = 4$

Depending upon object distance, a real image $(m = -4)$ or an virtual image $(m = +4)$ can be formed.

Here, we consider that $m = +4$.

magnification of a convex lens, $m = \frac{f}{f+u}$; $\frac{1}{m} = \frac{f+u}{f} = \left(1 + \frac{u}{f}\right)$

$$\therefore \frac{u}{f} = \left(\frac{1}{m} - 1\right) \Rightarrow u = \left(\frac{1}{m} - 1\right) f$$

$$f = +20 \; ; \; m = +4$$

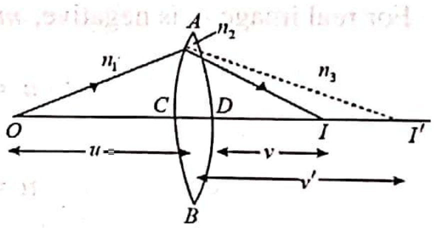
$$u = \left(\frac{1}{4} - 1\right) 20 = -15$$

$$\therefore u = -15 \text{ cm}$$

5. Obtain the lens maker's formula for a lens of refractive index n_2 which is separating two media of refractive indices n_1 and n_3 on the left and right respectively.

Solution

For the refraction on surface ACB, object distance is u in medium n_1 whereas refraction occurs entirely on medium n_2 with the (virtual) image at I'. Thus, image distance is v'. Applying refraction equation for surface ACB,



$$\frac{n_2}{v'} - \frac{n_1}{u} = \frac{n_2 - n_1}{R_1} \quad \ldots (1)$$

For the refraction on surface ADB, the virtual image at I' serves as the object and final image is formed at I in medium n_3. Therefore,

$$\frac{n_3}{v} - \frac{n_2}{v'} = \frac{n_3 - n_2}{R_2} \quad \ldots (2)$$

Adding equations (1) and (2), we get

$$\frac{n_3}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R_1} + \frac{n_3 - n_2}{R_2} \quad \ldots (3)$$

When $u = \infty$, $v = f$ substitute in equation (3),

$$\frac{n_3}{f} = \frac{n_2 - n_1}{R_1} + \frac{n_3 - n_2}{R_2} \quad \ldots (4)$$

Dividing equation (4) by n_3,

$$\frac{1}{f} = \frac{1}{n_3}\left[\frac{n_2 - n_1}{R_1} + \frac{n_3 - n_2}{R_2}\right].$$

6. A thin converging lens made of glass with refractive index 1.5 has a power of $+5.0D$. When this lens is immersed in a liquid of refractive index n, it acts as a divergent lens of focal length 100 cm. What must be the value of n?

Data

Focal length of diverging lens $f = -100$ cm

Power of convex lens $P_1 = 5.0D$

Refractive index $n = ?$

Solution

Len's maker's formula, $\frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Focal length of convex lens, $f_1 = \frac{1}{P_1} = \frac{1}{5} = 0.2\text{ m} = 20\text{ cm}$

For converging lens, $\frac{1}{20} = (1.5 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ ($\because$ for air medium $n_1 = 1$) ... (1)

for diverging lens, f is negative

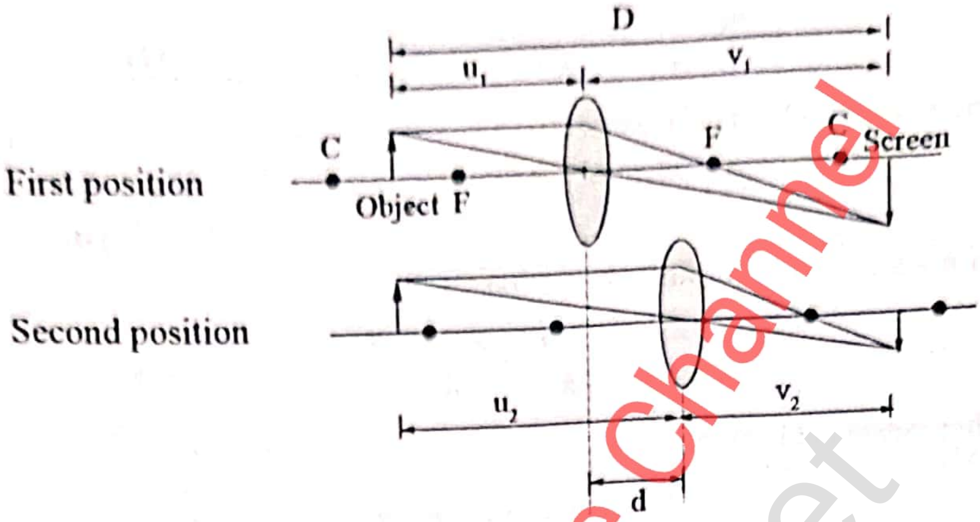
$$\Rightarrow \frac{1}{-100} = \left(\frac{1.5}{n} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \ldots (2)$$

Dividing equation (1) by equation (2) and simplifying,

$$n = \frac{5}{3}$$

$$n = 1.66.$$

7. If the distance D between an object and screen is greater than 4 times the focal length of a convex lens, then there are two positions of the lens for which images are formed on the screen. This method is called conjugate foci method. If d is the distance between the two positions of the lens, obtain the equation for focal length of the convex lens.



Solution

$$D = u + v > 4f$$

Conjuugate points As all the light rays from the object to the image are **reversible,** this implies that if the object were placed where the image is, an image would be formed at the original object position.

$$\text{i.e., } |u_2| = |v_1| \ \& \ |v_2| = |u_1|$$

From the figure

$$D = u_1 + v_1 \quad \text{.... (1)}$$

$$d = v_1 - v_2 = v_1 - u_1 \quad \text{... (2)}$$

$(1)-(2)$ gives

$$D - d = 2u_1 \Rightarrow u_1 = \frac{D-d}{2} \quad \text{... (3)}$$

$(1)+(2)$ gives

$$D + d = 2v_1 \Rightarrow v_1 = \frac{D+d}{2} \quad \text{... (4)}$$

Lens equation is $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

For convex lens forming a **real** image,

$$v = +v_1$$

$$u = -u_1$$

$$\therefore \frac{1}{f} = \frac{1}{v_1} - \frac{1}{-u_1} = \frac{1}{v_1} + \frac{1}{u_1} \quad \text{... (5)}$$

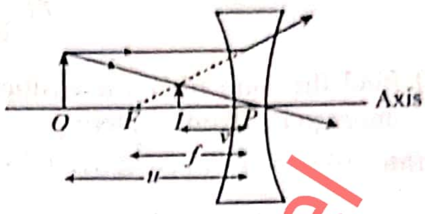
Substituting (3) & (4) in (5),

$$\frac{1}{f} = \frac{2}{D+d} + \frac{2}{D-d} = \frac{4D}{D^2 - d^2}$$

$$\therefore f = \frac{D^2 - d^2}{4D}$$

8. Prove that a concave lens can only form a virtual, erect and diminished image.

Solution



The image formation in a concave lens is shown above. The image is formed by the diverging rays on the left side of the lens; therefore, for a diverging lens, f is negative. Clearly u is also negative.

From lens equation, $\frac{1}{v} = \frac{1}{f} + \frac{1}{u}$ (or) $v = \frac{fu}{f+u}$

with sign convention, $v = \frac{(-f)(-u)}{(-f)+(-u)} \Rightarrow \frac{+ve}{-ve} \Rightarrow \begin{cases} v \text{ is negative} \\ \text{image is virtual} \end{cases}$

Similarly, $m = \frac{v}{u} \Rightarrow \frac{-v}{-u} \Rightarrow$ positive

As m is positive, the image is erect.

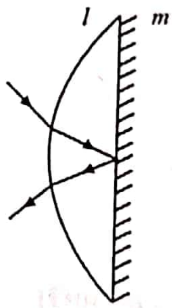
From diagram, $|v| < |u| \Rightarrow m < 1$

$\therefore$ image is diminished.

9. A point object is placed at 20 cm from a thin, plano-convex lens of focal length 15 cm whose plane surface is silvered. Locate the position and nature of the final image.

Solution

Data



Object distance $|u| = 20$ cm

focal length of convex lenz $f_l = 15$ cm

(refer section 6.6.7 – eq. 6.78)

Power of the silvered lens $P = 2P_l + P_m$

For the silvered surface (i.e., mirror, m) as it is a plane surface

$$f_m = \infty \Rightarrow P_m = \frac{1}{f_m} = 0$$

$$P = \frac{1}{-f}$$

$$\frac{1}{-f} = 2P_l = \frac{2}{f_l} = \frac{2}{15} \Rightarrow f = \frac{-15}{2} \text{ cm}$$

Silvered lens is a modified mirror.

Using mirror formula and sign convention, (i.e., $u = -20$)

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{-2}{15} - \frac{1}{(-20)} = \frac{-5}{60} = \frac{-1}{12}$$

$$v = -12 \text{ cm}$$

∴ image will be formed 12 cm from the lens on the same side of the object.

$$m = \frac{-v}{u} = \frac{-(-12)}{-20} = -\frac{3}{5} \Rightarrow \begin{cases} \text{image is inverted and real,} \\ \text{moreover, image is diminished} \end{cases}$$

10. Find the ratio of the intensities of lights with wavelengths 500 nm and 300 nm which undergo Rayleigh scattering.

Solution

$$I \alpha \frac{1}{\lambda^4} \Rightarrow \frac{I_1}{I_2} = \left(\frac{\lambda_2}{\lambda_1}\right)^4 = \left(\frac{300 \times 10^{-9}}{500 \times 10^{-9}}\right)^4 = \frac{81}{625}.$$

Chapter 7

Wave Optics

Objective type Questions

1. A plane glass is placed over a various coloured letters (violet, green, yellow, red). The letter which appears to be raised more is,

(a) red (b) yellow (c) green (d) violet

Solution

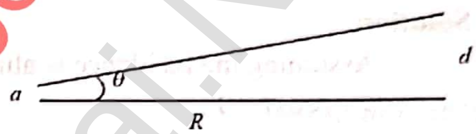
[Option : (d)]

2. Two point white dots are 1 mm apart on a black paper. They are viewed by eye of pupil diameter 3 mm approximately. The maximum distance at which these dots can be resolved by the eye is, [take wavelength of light, $\lambda = 500\,\text{nm}$].

(a) 1m (b) 5m (c) 3m (d) 6m

Solution

$$a = 1\,\text{mm} \;;\; d = 3\,\text{mm} \;;\; \lambda = 500\,\text{nm}$$



$$a\sin\theta = 1.22\lambda$$

$$a\left(\frac{d}{R}\right) = 1.22\lambda \quad \text{(nearly)}$$

$$\frac{a}{R} = \frac{1.22\lambda}{d}$$

$$\frac{1\times10^{-3}}{R} = \frac{1.22\times500\times10^{-9}}{3\times10^{-3}}$$

$$R = \frac{3\times10^{-3}\times1\times10^{-3}}{1.22\times500\times10^{-9}}$$

$$R = \frac{3000}{1.22\times500} = \frac{30}{6.1} \approx 5\,\text{m}$$

[Option : (b)]

3. In a Young's double-slit experiment, the slit separation is doubled. To maintain the same fringe spacing on the screen, the screen-to-slit distance D must be changed to,

(a) $2D$ (b) $\frac{D}{2}$ (c) $\sqrt{2}D$ (d) $\frac{D}{\sqrt{2}}$

Solution

$$\beta = \frac{\lambda D}{d}$$

Here $d' \rightarrow 2d$ but β remains same,

$$\beta' = \frac{\lambda D'}{d'} = \beta \Rightarrow \frac{\lambda D'}{2d} = \frac{\lambda D}{d}$$

$$\therefore D' = 2D.$$

[Option : (a)]

4. Two coherent monochromatic light beams of intensities I and 4I are superposed. The maximum and minimum possible intensities in the resulting beams are

(a) 5I and I (b) 5I and 3I (c) 9I and I (d) 9I and 3I

Solution

$$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$$
$$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$$
$$I_{max} = \left(\sqrt{I} + \sqrt{4I}\right)^2 = 9I$$
$$I_{min} = \left(\sqrt{I} - \sqrt{4I}\right)^2 = I$$

[Option : (c)

5. When light is incident on a soap film of thickness 5×10^{-5} cm, the wavelength of light reflected maximum in the visible region is $5320\,Å$. Refractive index of the film will be,

(a) 1.22 (b) 1.33 (c) 1.51 (d) 1.83

Solution

Assuming the incidence is almost normal for minimum value of n i.e., $r=0$ and $n=1$,
$\Rightarrow \cos r = 1$

$$\therefore 2\mu t = (2n+1)\frac{\lambda}{2} \Rightarrow \mu = \frac{2n+1}{2t}\cdot\frac{\lambda}{2}$$
$$t = 5\times10^{-5}\times10^{-2}\text{ m}$$
$$\mu = \frac{(2+1)}{2\times5\times10^{-7}}\;\frac{5320\times10^{-10}}{2} = 1.33$$

[Option : (b)

6. First diffraction minimum due to a single slit of width 1.0×10^{-5} cm is at $30°$. Then wavelength of light used is,

(a) 400 Å (b) 500 Å (c) 600 Å (d) 700 Å

Solution

$$a\sin\theta = n\lambda$$
$$1.0\times10^{-7}\sin(30°) = 1\times\lambda \quad \text{(for first minimum, } n=1)$$
$$\lambda = \frac{10^{-7}\times\sin 30°}{1} = 0.5\times10^{-7}\text{ m}$$
$$\therefore \lambda = 500\text{ Å}$$

[Option : (b)

7. A ray of light strikes a glass plate at an angle $60°$. If the reflected and refracted rays are perpendicular to each other, the refractive index of the glass is,

(a) $\sqrt{3}$ (b) $\frac{3}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) 2

Solution

If the reflected and refracted rays are perpendicular to each other, angle of incidence is the polarising angle i_p.

$$\text{i.e.,}\quad i = i_p.$$

By Brewster's law

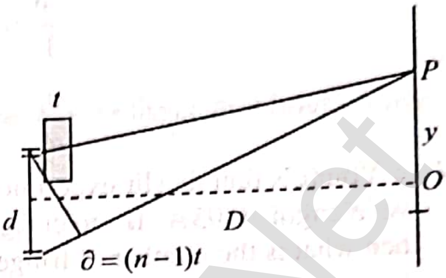
$$\mu = \tan(i_p)$$

$$\mu = \tan 60^o = \sqrt{3}.$$

[Option : (a)]

8. One of the Young's double slits is covered with a glass plate as shown in figure. The position of central maximum will,
(a) get shifted downwards (b) get shifted upwards
(c) will remain the same (d) data insufficient to conclude

Solution



When both the slits are open, the path difference at O will be zero and hence the central maximum will be at O. As the upper slit is covered with the glass plate, there will be a path difference at O (due to optical path in glass plate).

Therefore, light from lower slit has to travel additional path in air to equal the path length of light from upper slit.

Thus central maximum will now be at P i.e., shifted upwards.

[Option : (b)]

9. Light transmitted by Nicol prism is,
(a) partially polarised (b) unpolarised
(c) plane polarised (d) elliptically polarised

Solution

[Option : (c)]

10. The transverse nature of light is shown in,
(a) interference (b) diffraction
(c) scattering (d) polarisation

Solution

Both longitudinal and transverse waves exhibit diffraction, interference and scattering; but only transverse waves exhibit polarisation.

[Option : (d)]

Exercise Problems

1. The ratio of maximum and minimum intensities in an interference pattern is 36:1. What is the ratio of the amplitudes of the two inferfering waves?

Solution

$$\frac{I_{max}}{I_{min}} = \frac{(A_{max})^2}{(A_{min})^2} = \frac{(a_1+a_2)^2}{(a_1-a_2)^2}$$

$$\frac{36}{1} = \left(\frac{a_1+a_2}{a_1-a_2}\right)^2 \Rightarrow a_1+a_2 = 6(a_1-a_2) \Rightarrow 5a_1 = 7a_2$$

$$\therefore \frac{a_1}{a_2} = \frac{7}{5}.$$

2. In Young's double slit experiment, 62 fringes are seen in visible region for sodium light of wavelength $5893\,\text{Å}$. If violet light of wavelength $4359\,\text{Å}$ is used in place of sodium light, then what is the number of fringes seen?

Data

$$n_1 = 62 \;;\; \lambda_1 = 5893\,\text{Å} \;;\; \lambda_2 = 4359\,\text{Å}$$

$$n_2 = ?$$

Solution

In an interference pattern, the distance of a bright band from the centre is $y = \frac{Dn\lambda}{d}$; D,d are constants. For different wavelengths, $n\lambda =$ const.

$$n_1\lambda_1 = n_2\lambda_2$$

$$n_2 = \frac{n_1\lambda_1}{\lambda_2}$$

$$= \frac{62\times 5893\text{Å}}{4359\text{Å}} = 83.819$$

$$\Rightarrow n_2 = 84.$$

3. Light of wavelength 600 nm that falls on a pair of slits producing interference pattern on a screen in which the bright fringes are separated by 7.2 mm. What must be the wavelength of another light which produces bright fringes separated y 8.1 mm with the same apparatus?

Solution

$$\beta = \frac{D\lambda}{d} \Rightarrow \frac{\beta_1}{\beta_2} = \frac{\lambda_1}{\lambda_2}$$

$$\therefore \lambda_2 = \frac{\beta_2}{\beta_1}\lambda_1 = \frac{8.1\times10^{-3}}{7.2\times10^{-3}}\times 600 = 675\,\text{nm}$$

4. A beam of light of wavelength 600 nm from a distant source falls on a single slit 1mm wide and resulting diffraction pattern is observed on a screen 2 m away. What is the distance between the first dark fringe on either side of the central bright fringe?

Data

Wavelength of light $\lambda = 600\,\text{nm}$

Distance of screen $D = 2\,\text{m}$

slit width $a = 1\,\text{mm}$

Solution

For a single slit, for first minimum, $a\sin\theta = \lambda$; $(n = 1)$

$$\sin\theta = \frac{y}{D} \quad \therefore a\frac{y}{D} = \lambda$$

$$\therefore \quad y = \lambda\frac{D}{a}$$

$$= 600\times10^{-9}\,\frac{2}{1\times10^{-3}} = 1200\times10^{-6}\,\text{m}$$

$$\therefore \quad y = 1.2\,\text{mm}.$$

Distance between two dark fringes which are on either side of the central bright fringe

$$= 2\times1.2\,\text{mm} = 2.4\,\text{mm}.$$

5. Light of wavelength 5000 Å produces diffraction pattern of a single slit of width 2.5 mm. What is the maximum order of diffraction possible?

Solution

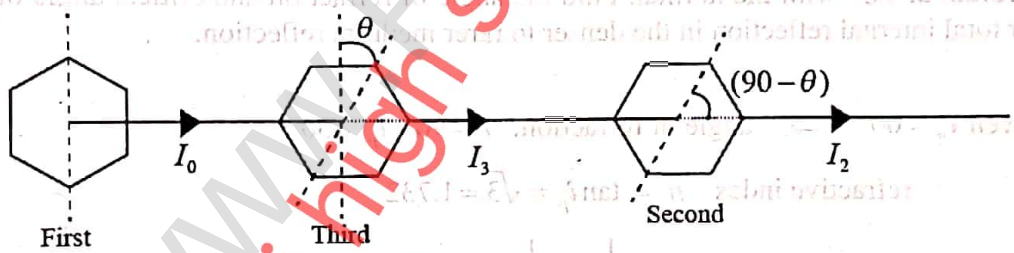
$$a\sin\theta = n\lambda \;;\; \theta = 90 \text{ (for max. order)}$$

$$\therefore \quad a = n\lambda \;\Rightarrow\; n = \frac{a}{\lambda} = \frac{2.5\times10^{-6}}{5\times10^{-7}} = 5.$$

6. I_0 is the intensity of light existing between two Polaroids kept with their axes perpendicular to each other. A third polaroid is introduced between them. What must be the angle between the axes of first and the newly introduced polaroid to get the maximum light from the whole arrangement?

Solution

I_0 is the intensity of light entering the 3rd (introduced) polaroid; let it make an angle θ with first polaroid. As first and second polaroids are perpendicular to each other, the angle between the axes of third and second polaroids is $(90° - \theta)$ (fig.)



$\therefore$ intensity of light emerging from 3rd polaroid, $I_3 = I_0\cos^2\theta$

Intensity of light emerging from 2nd polaroid

$$I_2 = I_3\cos^2(90-\theta)$$

$$= I_3\sin^2\theta = (I_0\cos^2\theta)\sin^2\theta$$

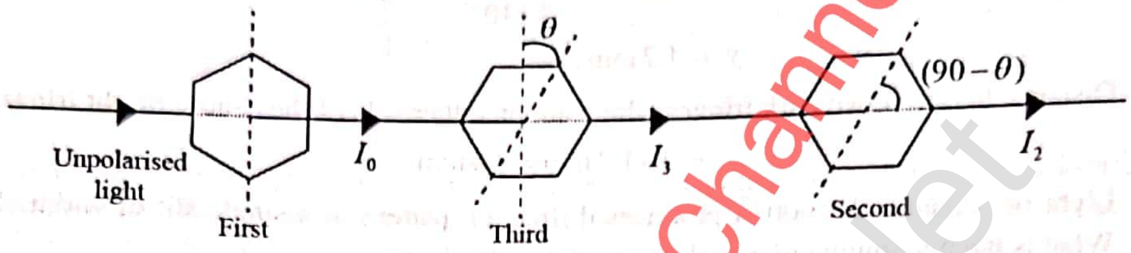
$$= \frac{I_0}{4}(2\sin\theta\cos\theta)^2 = \frac{I_0}{4}(\sin 2\theta)^2$$

I_2 will be maximum when $2\theta = 90°$ (or) $\theta = 45°$.

7. An unpolarised light of intensity $32\,Wm^{-2}$ passes through three polaroids such that the axes of the first and the last polaroids are at 90°. What is the angle between the axes of the first and the middle polaroids so that the emerging light has an intensity of only $3\,WM^{-2}$?

Solution

Intensity of light emerging from the first polaroid will be half of the incident unpolarised light, i.e., $\frac{32}{2} = 16\,Wm^{-2}$. Let it be $I_0 = 16\,Wm^{-2}$. Let θ be the angle between the axes of the 1^{st} and the middle polaroid.



The light emerging from the middle polaroid, $I_3 = I_0 \cos^2\theta$

light emerging from the last polaroid, $I_2 = I_3 \cos^2(90-\theta) = I_3 \sin^2\theta$

$$= (I_0 \cos^2\theta)\sin^2\theta$$

$$= \frac{I_0}{4}(\sin 2\theta)^2$$

From the given data, $I_2 = 3\ Wm^{-2}$, $\Rightarrow \frac{16}{4}(\sin 2\theta)^2 = 3$

$$\Rightarrow \sin 2\theta = \frac{\sqrt{3}}{2}$$

$$\therefore \theta = 30^\circ$$

8. The reflected light is found to be plane polarised when an unpolarised light falls on a denser medium at 60° with the normal. Find the angle of refraction and critical angle of incidence for total internal reflection in the denser to rarer medium reflection.

Solution

Given $i_p = 60^\circ \Rightarrow$ angle of refraction, $r = 90 - i_p = 30^\circ$

refractive index $n = \tan i_p = \sqrt{3} = 1.732$

$$\sin i_c = \frac{1}{n} = \frac{1}{1.732} = 0.5774$$

$$\therefore i_c = \sin^{-1}(0.5774) = 35^\circ 16'.$$

9. The near point and the far point for a person are 50 cm and 500 cm respectively. Calculate the power of the len's the person should wear to read a book held in hand at 25 cm. What maximum distance is clearly visible for the person with this lens on the eye?

Solution

(i) $u = -25$ cm ; $v = -50$ cm

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}=\frac{1}{-50}-\frac{1}{(-25)}=\frac{1}{50}$$

$$P=\frac{1}{f\,(\text{in m})}=\frac{1}{0.5}=2\text{D (convex lens)}$$

(ii) $f=50$ cm ; $v=-500$ cm ; $u=?$

$$\frac{1}{u}=\frac{1}{v}-\frac{1}{f}=\frac{1}{-500}-\frac{1}{50}=\frac{-11}{500}\quad\therefore\ u=-\frac{500}{11}=-45.45$$

(or) $|u|=45.45\text{cm}.$

10. A compound microscope has a magnifying power of 100 when the image is formed at infinity. The objective has a focal length of 0.5cm and the tube length is 6.5cm. What is the focal length of the eyepiece?

Solution

As the image is formed at infinity,

$$m=\frac{L}{f_0}\times\frac{D}{f_e}\Rightarrow f_e=\frac{L}{f_0}\times\frac{D}{m}$$

$$\therefore\quad f_e=\frac{6.5\times25}{0.5\times100}=3.25\text{ cm}$$

Chapter 8

Dual Nature of Radiation and Matter

Objective type Questions

1. The wavelength λ_e of an electron and λ_p of a photon of same energy E are related by

(a) $\lambda_p \propto \lambda_e$ (b) $\lambda_p \propto \sqrt{\lambda_e}$ (c) $\lambda_p \propto 1/\sqrt{\lambda_e}$ (d) $\lambda_p \propto \lambda_e^2$

Solution

De broglie wavelength for photon

$$\lambda_p = \frac{hc}{E} \quad \left(\because \; E = h\upsilon_p = \frac{hc}{\lambda_p}\right)$$

For electron $$\lambda_e = \frac{h}{\sqrt{2mE}} \; ; \; \lambda_e^2 = \frac{h^2}{2mE}$$

$$\Rightarrow \lambda_e^2 = \frac{h^2\lambda_p}{2mhc}$$

$$\therefore \lambda_p \propto \lambda_e^2$$

[Option : (d)

2. In an electron microscope, the electrons are accelerated by a voltage of 14 kV. If the voltag is changed to 224 kV, then the de Broglie wavelength associated with the electrons would

(a) increase by 2 times (b) decrease by 2 times

(c) decrease by 4 times (d) increase by 4 times

Solution

$$\lambda_e = \frac{12.27\text{Å}}{\sqrt{V}} \; ; \; \lambda_e \propto \frac{1}{\sqrt{V}}$$

$$\frac{\lambda_e'}{\lambda_e} = \sqrt{\frac{V}{V'}} = \sqrt{\frac{14\times10^3}{224\times10^3}} = \sqrt{\frac{1}{16}} = \frac{1}{4}$$

$$\lambda_e' = \frac{\lambda_e}{4}.$$

[Option : (c

3. The wave associated with a moving particle of mass 3×10^{-6} g has the same wavelength a an electron moving with a velocity $6\times10^6\,\text{ms}^{-1}$. The velocity of particle is

(a) $1.82\times10^{-18}\,\text{ms}^{-1}$ (b) $9\times10^{-2}\,\text{ms}^{-1}$

(c) $3\times10^{-31}\,\text{ms}^{-1}$ (d) $1.82\times10^{-15}\,\text{ms}^{-1}$

Solution

$$m = 3\times10^{-6}\text{ g} = 3\times10^{-6}\times10^{-3}\text{ kg} = 3\times10^{-9}\text{ kg}$$

$$\lambda = \lambda_e$$

$$\lambda = \frac{h}{mv} \; ; \; \lambda_e = \frac{h}{m_e v_e}$$

$$\Rightarrow \quad mv = m_e v_e$$

$$v = \frac{m_e v_e}{m}$$

$$v = \frac{9\times10^{-31}\times6\times10^{6}}{3\times10^{-9}} = 18\times10^{-16}$$

$$\therefore \quad v = 1.8\times10^{-15}\,\text{ms}^{-1}.$$

[Option : (d)]

4. When a metallic surface is illuminated with radiation of wavelength λ, the stopping potential is V. If the same surface is illuminated with radiation of wavelength 2λ, the stopping potential is $V/4$. The threshold wavelength for the metallic surface is

(a) 4λ (b) 5λ (c) $\frac{5}{2}\lambda$ (d) 3λ

Solution

By Einstein photo electric equation

$$h(v - v_0) = eV_o$$

For wavelength λ, $hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) = eV$ ... (1)

For wavelength 2λ, $hc\left(\frac{1}{2\lambda} - \frac{1}{\lambda_o}\right) = \frac{eV}{4}$ ... (2)

Solving equations (1) and (2)

$$\lambda_0 = 3\lambda$$

[Option : (d)]

5. If a light of wavelength 330nm is incident on a metal with work function 3.55eV, the electrons are emitted. Then the wavelength of the emitted electron is

(Take $h = 6.6\times10^{-34}$ Js)

(a) $< 2.75\times10^{-9}$ m (b) $\leq 2.75\times10^{-9}$ m (c) $\leq 2.75\times10^{-12}$ m (d) $< 2.75\times10^{-10}$ m

Solution

By Einstein photo electric equation

$$hv - \phi = \frac{1}{2}mv^2_{\max} = K_{\max}$$

$$K_{\max} = \frac{hc}{\lambda} - \phi$$

$$= \frac{6.6\times10^{-34}\times3\times10^{8}}{3.3\times10^{-7}} - 3.55\times1.6\times10^{-19} \quad (\because 1\text{eV} = 1.6\times10^{-19}\text{J})$$

$$K_{\max} = 6\times10^{-19} - 5.68\times10^{-19} = 0.32\times10^{-19}\,\text{J}$$

De Broglie wavelength of electron

$$\lambda_e = \frac{h}{\sqrt{2mE}}$$

Here $E = K_{max}$ and λ_e is λ_{min}

$$\lambda_{min} = \frac{6.6\times10^{-34}}{\sqrt{2\times9\times10^{-31}\times0.32\times10^{-19}}} = 2.75\times10^{-9}\text{ m}$$

the emitted electron will have wavelength $\geq 2.75\times10^{-9}$ m.

[Option : (b)]

6. A photoelectric surface is illuminated successively by monochromatic light of wavelength λ and $\frac{\lambda}{2}$. If the maximum kinetic energy of the emitted photoelectrons in the second case is 3 times that in the first case, the work function at the surface of material is

(a) $\frac{hc}{\lambda}$ (b) $\frac{2hc}{\lambda}$ (c) $\frac{hc}{3\lambda}$ (d) $\frac{hc}{2\lambda}$

Solution

Einstein photo electric equation for first case

$$\frac{hc}{\lambda} - \phi = \frac{1}{2}mv_{max}^2 \quad \text{... (1)}$$

for second case

$$\frac{hc}{\frac{\lambda}{2}} - \phi = 3\left(\frac{1}{2}mv_{max}^2\right) \quad \text{... (2)}$$

Sub. (1) in (2),

$$\Rightarrow \frac{2hc}{\lambda} - \phi = 3\left(\frac{hc}{\lambda} - \phi\right)$$

$$3\phi - \phi = 3\frac{hc}{\lambda} - 2\frac{hc}{\lambda}$$

$$\phi = \frac{hc}{2\lambda}.$$

[Option : (d)]

7. In photoelectric emission, a radiation whose frequency is 4 times threshold frequency of a certain metal is incident on the metal. Then the maximum possible velocity of the emitted electron will be

(a) $\sqrt{\frac{hv_o}{m}}$ (b) $\sqrt{\frac{6hv_o}{m}}$ (c) $2\sqrt{\frac{hv_o}{m}}$ (d) $\sqrt{\frac{hv_o}{2m}}$

Solution

$$hv - hv_o = \frac{1}{2}mv_{max}^2$$

$$v = 4v_0 \Rightarrow h4v_0 - hv_o = \frac{1}{2}mv_{max}^2$$

$$3h\nu_o = \frac{1}{2}m\upsilon_{max}^2$$

$$\upsilon_{max}^2 = \frac{6h\nu_o}{m}$$

$$\upsilon = \sqrt{\frac{6h\nu_o}{m}}$$

[Option : (b)]

8. Two radiations with photon energies 0.9eV and 3.3eV respectively are falling on a metallic surface successively. If the work function of the metal is 0.6eV, then the ratio of maximum speeds of emitted electrons will be

(a) 1:4 (b) 1:3 (c) 1:1 (d) 1:9

Solution

$$h\nu - h\nu_o = \frac{1}{2}m\upsilon_{max}^2$$

For 0.9eV photon, $(0.9-0.6) = \frac{1}{2}m\upsilon_1^2$

For 3.3eV photon, $(3.3-0.6) = \frac{1}{2}m\upsilon_2^2$

by dividing $\frac{0.3}{2.7} = \frac{\upsilon_1^2}{\upsilon_2^2}$

$$\upsilon_1 : \upsilon_2 = 1:3.$$

[Option : (c)]

9. A light source of wavelength 520 nm emits 1.04×10^{15} photons per second while the second source of 460 nm produces 1.38×10^{15} photons per second. Then the ratio of power of second source to that of first source is

(a) 1.00 (b) 1.02 (c) 1.5 (d) 0.98

Solution

Power of source = no. of photons emitted per second × energy of photon

$\left(\because\ P = \frac{E_{Total}}{t} = \frac{n}{t}\times E\right)$

$$E = \frac{hc}{\lambda}\ ;\ t = 1\text{s} \Rightarrow P = nE = n\frac{hc}{\lambda}$$

$$\therefore\ P_1 = n_1\frac{hc}{\lambda_1}\ ;\ P_2 = n_2\frac{hc}{\lambda_2}$$

$$\frac{P_2}{P_1} = \frac{n_2\lambda_1}{n_1\lambda_2} = \frac{1.38\times10^{15}\times520\times10^{-9}}{1.04\times10^{15}\times480\times10^{-9}} = 1.5$$

[Option : (c)]

10. The mean wavelength of light from sun is taken to be 550 nm and its mean power is 3.8×10^{26} W. The number of photons received by the human eye per second on the average from sunlight is of the order of

(a) 10^{45} (b) 10^{42} (c) 10^{54} (d) 10^{51}

Solution

$$E = \frac{hc}{\lambda} = \frac{6.6\times10^{-34}\times3\times10^{8}}{5.5\times10^{-7}} = 3.6\times10^{-19}\,\text{J}$$

number of photons emitted per second

$$n = \frac{P}{E} = \frac{3.8\times10^{26}}{3.6\times10^{-19}} ;\ 10^{45}$$

[Option : (a)]

11. The threshold wavelength for a metal surface whose photoelectric work function is 3.313eV is

(a) 4125Å (b) 3750Å (c) 6000Å (d) 2062.5Å

Solution

$$\phi = h\nu_o = \frac{hc}{\lambda_o}$$

$$\lambda_o = \frac{hc}{\phi} = \frac{6.626\times10^{-34}\times3\times10^{8}}{3.313\times1.6\times10^{-19}} = 3.75\times10^{-7} = 3750\times10^{-10}\,\text{m}$$

$$= 3750\times10^{-10}m$$

[Option : (b)]

12. A light of wavlength 500nm is incident on a photosensitive plate of photoelectric work function 1.235eV. The kinetic energy of the photo electrons emitted is (Take $h = 6.6\times10^{-34}$ Js)

(a) 0.58eV (b) 2.48eV (c) 1.24eV (d) 1.16eV

Solution

$$\frac{hc}{\lambda} - \phi = \frac{1}{2}mv^2 = K$$

$$K = \frac{6.6\times10^{-34}\times3\times10^{8}}{500\times10^{-9}\times1.6\times10^{-19}} - 1.235 = 2.475 - 1.235 = 1.24\,\text{eV}$$

[Option : (c)]

13. Photons of wavelength λ are incident on a metal. The most energetic electrons ejected from the metal are bent into a circular are of radius R by a perpendicular magnetic field having magnitude B. The work function of the metal is

(a) $\frac{hc}{\lambda} - m_e + \frac{e^2B^2R^2}{2m_e}$ (b) $\frac{hc}{\lambda} + 2m_e + \left[\frac{eBR}{2m_e}\right]^2$

(c) $\frac{hc}{\lambda} - m_ec^2 - \frac{e^2B^2R^2}{2m_e}$ (d) $\frac{hc}{\lambda} - 2m_e + \left[\frac{eBR}{2m_e}\right]^2$

Solution

$$\frac{hc}{\lambda} - \phi = \frac{1}{2}mv_{max}^2$$

$$\text{centripetal force} = \frac{m_e v_{max}^2}{R} = Bev_{max}$$

$$v_{max} = \frac{BeR}{m_e}$$

$$\frac{hc}{\lambda} - \phi = \frac{1}{2}m_e\left(\frac{BeR}{m_e}\right)^2 \Rightarrow \phi = \frac{hc}{\lambda} - \left(\frac{1}{2}m_e\left(\frac{eBR}{m_e}\right)^2 \times \frac{4}{4}\right)$$

$$\therefore \phi = \frac{hc}{\lambda} - 2m_e\left(\frac{eBR}{2m_e}\right)^2$$

[Option : (d)]

14. The work functions for metals A, B and C are 1.92eV, 2.0eV and 5.0eV respectively. The metals which will emit photoelectrons for a radiation of wavelength 4100Å is/are

(a) A only (b) both A and B (c) all these metals (d) none

Solution

$$\text{For metal } A,\ \lambda_0 = \frac{hc}{\phi_A} = \frac{6.626\times10^{-34}\times3\times10^8}{1.92\times1.6\times10^{-19}} = 6.471\times10^{-7}m = 6471\,\text{Å}$$

$$\text{For metal } B,\ \lambda_0 = \frac{hc}{\phi_B} = \frac{6.626\times10^{-34}\times3\times10^8}{2.0\times1.6\times10^{-19}} = 6.212\times10^{-7}m = 6212\,\text{Å}$$

$$\text{For metal } C,\ \lambda_0 = \frac{hc}{\phi_C} = \frac{6.626\times10^{-3}\times3\times10^8}{5.0\times1.6\times10^{-19}} = 2.485\times10^{-7}m = 2485\,\text{Å}$$

$\lambda = 4100$ Å

For photo electric effect $\lambda < \lambda_0$

$\therefore$ for both A and B photo electric effect possible.

[Option : (b)]

15. Emission of electrons by the absorption of heat energy is called _______ emission.

(a) photoelectric (b) field (c) thermionic (d) secondary

Solution

[Option : (c)]

Exercise Problems

1. How many photons per second emanate from a 50 mW laser of 640 nm ?

Solution

Data :

$$P = 50\text{mW} = 50\times10^{-3}\text{W} = 50\times10^{-3}\text{Js}^{-1}$$

$$\lambda = 640\text{nm} = 640\times10^{-9}\text{m}$$

Energy emitted per second $= 50\times10^{-3}$J

If 'n' is the number of photons emitted per second, then $P = nE$

$$P = nh\nu = \frac{nhc}{\lambda}$$

$$50\times10^{-3} = \frac{n\times6.626\times10^{-34}\times3\times10^{8}}{640\times10^{-9}}$$

$$n = \frac{50\times10^{-3}\times640\times10^{-9}}{6.626\times10^{-34}\times3\times10^{8}} = 1610\times10^{14} = 16.10\times10^{16}$$

$$\approx 1.6\times10^{17}\ \text{s}^{-1}$$

2. Calculate the maximum kinetic energy and maximum velocity of the photoelectrons emitted when the stopping potential is 81 V for the photoelectric emission experiment.

Solution :

Data : $V_0 = 81\text{V}$

$$K_{max} = eV_0 = 1.6\times10^{-19}\times81 = 129.6\times10^{-19} = 1.296\times10^{-17}\text{J}$$

$$K_{max} = \frac{1}{2}mv_{max}^2$$

$$v_{max} = \sqrt{\frac{2eV_0}{m}} = \sqrt{\frac{2\times1.296\times10^{-17}}{9.1\times10^{-31}}} = 0.5337\times10^{7}$$

$$\approx 0.53\times10^{7} = 5.3\times10^{6}\ \text{ms}^{-1}$$

3. Calculate the energies of the photons associated with the following radiation : (i) violet light of 413 nm (ii) X -rays of 0.1 nm (iii) radio waves of 10 m .

(i) $\lambda = 413\text{nm} = 413\times10^{-9}\text{m}$

$$E = \frac{hc}{\lambda}$$

$$E = \frac{6.626\times10^{-34}\times3\times10^{8}}{413\times10^{-9}} = 0.048\times10^{-17}\text{J}$$

$$= 4.8\times10^{-19}\text{J}.$$

$$= \frac{4.8\times10^{-19}}{1.6\times10^{-19}}\text{eV}\quad\left[\because\ 1\text{J} = \frac{1}{1.6\times10^{-19}}\text{eV}\right]$$

$$E = 3\text{eV}$$

(ii) $\lambda = 0.1\text{ nm} = 0.1\times10^{-9}\text{ m} \quad ; \quad E = \dfrac{hc}{\lambda}$

$$E = \frac{6.626\times10^{-34}\times3\times10^{8}}{0.1\times10^{-9}}$$

$$= 1.9878\times10^{-16}\text{ J}$$

$$= \frac{1.9878\times10^{-15}}{1.6\times10^{-19}}\text{eV} \ \square\ 1.242\times10^{3}$$

$$E = 1242\text{ eV}$$

(iii) $\lambda = 10\text{ m} \quad ; \quad E = \dfrac{hc}{\lambda}$

$$E = \frac{6.626\times10^{-34}\times3\times10^{8}}{10} = 19.878\times10^{-27}\text{ J}$$

$$= \frac{19.878\times10^{-27}}{1.6\times10^{-19}}\text{eV}$$

$$E = 1.242\times10^{-7}\text{eV}.$$

4. A 150 W lamp emits light of mean wavelength of 5500 Å. If the efficiency is 12%, find out the number of photons emitted by the lamp in one second.

Solution

Data : $P = 150\text{ W} = 150\text{ Js}^{-1}$

$$\lambda = 5500\text{ Å} = 5500\times10^{-10}\text{ m}$$

$$\text{Efficiency} = 12\%$$

$$\therefore E = 12\% \text{ of } P$$

$$= \frac{12\times150}{100} = 18\text{ Js}^{-1}$$

$$E = nh\nu = \frac{nhc}{\lambda}$$ (n = number of photons emitted per second)

$$18 = \frac{n\times6.626\times10^{-34}\times3\times10^{8}}{5500\times10^{-10}}$$

$$n = \frac{18\times5500\times10^{-10}}{6.626\times10^{-34}\times3\times10^{8}}$$

$$n = 4.98\times10^{19}.$$

5. How many photons of frequency 10^{14} Hz will make up 19.86 J of energy?

Solution

Data $\nu = 10^{14}\text{ Hz} \quad ; \quad E = 19.86\text{ J}$

$E = nh\nu$ (where n is number of photons)

$$n = \frac{E}{h\nu} = \frac{19.86}{6.626\times10^{-34}\times10^{14}} = 2.997\times10^{20} \ \square\ 3\times10^{20}$$

6. What should be the velocity of the electron so that its momentum equals that of 4000 Å wavelength photon?

Solution

Data : $\lambda = 4000\,\text{Å} = 4000 \times 10^{-10} m = 4 \times 10^{-7}\,\text{m}$

Momentum of the photon $P_P = \frac{h}{\lambda}$

Momentum of the electron $P_e = m\upsilon$

$$P_e = P_P \Rightarrow m\upsilon = \frac{h}{\lambda}$$

$$\upsilon = \frac{h}{m\lambda} = \frac{6.626 \times 10^{-34}}{9.1 \times 10^{-31} \times 4 \times 10^{-7}}$$

$$= 0.182 \times 10^{4}$$

$$= 1820\,\text{ms}^{-1}.$$

7. When a light of frequency 9×10^{14} Hz is incident on a metal surface, photoelectrons are emitted with a maximum speed of $8 \times 10^{5}\,\text{ms}^{-1}$. Determine the threshold frequency of the surface.

Solution

Data :

$$\nu = 9 \times 10^{14}\,\text{Hz} \;;\; \upsilon_{max} = 8 \times 10^{5}\,\text{ms}^{-1}$$

$$K_{max} = \frac{1}{2} m\upsilon^2_{max} = h(\nu - \nu_0)$$

$$\frac{1}{2} \times 9.1 \times 10^{-3} \times (8 \times 10^{5})^2 = 6.626 \times 10^{-34}(9 \times 10^{14} - \upsilon_0)$$

$$(9 \times 10^{14} - \upsilon_0) = \frac{9.1 \times 10^{-31} \times (8 \times 10^{5})^2}{2 \times 6.626 \times 10^{-34}}$$

$$= \frac{9.1 \times 64 \times 10^{-21}}{2 \times 6.626 \times 10^{-34}} = 43.95 \times 10^{13} = 4.395 \times 10^{14}$$

$$\upsilon_0 = 9 \times 10^{14} - 4.395 \times 10^{14}$$

$$= 4.605 \times 10^{14}\,\text{Hz}.$$

8. When a 6000 Å light falls on the cathode of a photo cell, photoemission takes place. If a potential of 0.8 V is required to stop emission of electron, then determine the (i) frequency of the light (ii) energy of the incident photon (iii) work function of the cathode material (iv) threshold frequency and (v) net energy of the electron after it leaves the surface.

Solution

Data : $\lambda = 6000\,\text{Å} = 6000 \times 10^{-10}\,\text{m}$

$V_0 = 0.8\,\text{V}$

(i) Frequency of the incident light,

$$\nu = \frac{c}{\lambda}$$

$$= \frac{3\times10^{8}}{6000\times10^{-10}}$$

$$\therefore \nu = 5\times10^{14}\text{ Hz}.$$

(ii) Energy of photon incident on the cathode

$$E = h\nu = 6.626\times10^{-34}\times5\times10^{14}$$

$$E = 3.313\times10^{-19}\text{J}$$

$$= \frac{3.313\times10^{-19}}{1.6\times10^{-19}}\text{eV} = 2.07\text{eV}.$$

(iii)
$$h\nu = h\nu_0 + \frac{1}{2}m\upsilon^2$$

$$= \phi_0 + eV_o \quad \left(\because \frac{1}{2}m\upsilon^2 = eV_o\right)$$

$$\Rightarrow \phi_0 = h\nu - eV_0 = \frac{2.07\times1.6\times10^{-19} - 0.8\times1.6\times10^{-19}}{1.6\times10^{-19}}\text{eV}$$

$$= 2.07 - 0.8$$

$$= 1.27\text{eV}$$

(iv) Threshold frequency $= \nu_0$

$$h\nu_0 = \phi_0$$

$$\nu_0 = \frac{\phi_0}{h} = \frac{1.27\times1.6\times10^{-19}\text{J}}{6.626\times10^{-34}} = 3.07\times10^{14}\text{ Hz}$$

(v) Net energy of the electron after it leaves the surface = its kinetic energy k

$$k = eV_0$$

$$= 1.6\times10^{-19}\times0.8$$

$$= 1.28\times10^{-19}\text{J} = \frac{1.28\times10^{-19}}{1.6\times10^{-19}} = 0.8\text{eV}$$

9. A 3310 Å photon liberates an electron from a material with energy 3×10^{-19}J while another 5000 Å photon ejects an electron with energy 0.972×10^{-19}J from the same material. Determine the value of Planck's constant and the threshold wavelength of the material.

Solution

Data :
$\lambda_1 = 3310\text{Å} = 3310\times10^{-10}\text{m}$; $K_1 = 3\times10^{-19}\text{J}$

$\lambda_2 = 5000\text{Å} = 5000\times10^{-10}\text{m}$; $K_2 = 0.972\times10^{-19}\text{J}$; $h = ?$

$$K = \frac{1}{2}m\upsilon^2 = h(\nu - \nu_0) = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$$

$$\therefore K_1 = hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_0}\right) \quad \text{... (1)}$$

$$K_2 = hc\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_0}\right) \quad \text{... (2)}$$

Subtracting equation (2) from equation (1)

$$K_1 - K_2 = hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$$

$$(3-0.972)\times10^{-19} = h\times3\times10^8\times10^{10}\left(\frac{1}{3310} - \frac{1}{5000}\right)$$

$$h = \frac{2.028\times10^{-19}\times5000\times3310}{3\times10^{18}\times1690}$$

$$h = 6.62\times10^{-34}\,\text{Js}$$

equation (2) $\Rightarrow \dfrac{K_2}{hc} = \dfrac{1}{\lambda_2} - \dfrac{1}{\lambda_0}$

$$\frac{1}{\lambda_0} = \frac{1}{\lambda_2} - \frac{K_2}{hc}$$

$$\frac{1}{\lambda_0} = \frac{1}{5000\times10^{-10}} - \frac{0.972\times10^{-19}}{3\times10^8\times6.626\times10^{-34}}$$

$$= 2\times10^6 - 0.4890\times10^6 = 1.511\times10^6$$

$$\lambda_0 \; ; \; 6.618\times10^{-7}\,\text{m}.$$

$$\lambda_0 = 6618\,\text{Å}$$

10. At the given point of time, the earth receives energy from sun at $4\,\text{calcm}^{-2}\,\text{min}^{-1}$. Determine the number of photons received on the surface of the Earth per cm^2 per minute. (Given : Mean wavelength of sun light = 5500 Å ; 1 cal = 4.2 J)

Data :

Energy received from the sun $= 4\,\text{calcm}^{-2}\,\text{min}^{-1}$

Energy of a single photon of wavelength 5500Å $= E = \dfrac{hc}{\lambda}$

$$E = \frac{6.626\times10^{-34}\times3\times10^8}{5500\times10^{-10}} = \frac{19.878\times10^{-19}}{5.5}$$

$$E = 3.614\times10^{-19}\,\text{J}$$

Number of photons received by 1cm^2 area of earth's surface in one minute $= n = \dfrac{P}{E}$

$$P = 4\times4.2\,\text{J cm}^{-2}\,\text{min}^{-1}$$

$$P = nE \; ; \; n = \frac{P}{E} = \frac{4\times4.2}{3.614\times10^{-19}} = 4.65\times10^{19}.$$

11. UV light of wavelength 1800 Å is incident on a lithium surface whose threshold wavelength 4965 Å. Determine the maximum energy of the electron emitted.

Solution

Data

$$\lambda = 1800\,\text{Å} = 1800\times10^{-10}\,\text{m} \; ; \; \lambda_0 = 4965\text{Å} = 4965\times10^{-10}\,\text{m}$$

$$K_{max} = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$$

$$K_{max} = 6.626\times10^{-34}\times3\times10^{8}\times10^{10}\left(\frac{1}{1800}-\frac{1}{4965}\right)$$

$$= 19.878\times10^{-16}\left(\frac{3165}{1800\times4965}\right)$$

$$K_{max} = 7.040\times10^{-19}\,J$$

$$= \frac{7.040\times10^{-19}}{1.6\times10^{-19}}\,eV = 4.4\,eV$$

$$K_{max} = 4.4\,eV.$$

12. Calculate the de Broglie wavelength of a proton whose kinetic energy is equal to $81.9\times10^{-15}\,J$. (Given : mass of proton is 1836 times that of electron).

Solution

Data

$$E = 81.9\times10^{-15}\,J \;;\; m = 1836m_e = 1836\times9.11\times10^{-31}\,kg$$

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$\lambda = \frac{6.626\times10^{-34}}{\sqrt{2\times9.11\times10^{-31}\times1836\times81.9\times10^{-15}}}$$

$$\lambda = \frac{6.626\times10^{-34}}{1655\times10^{-23}}$$

$$\lambda \;;\; 4\times10^{-14}\,m.$$

13. A deuteron and an alpha particle are accelerated with the same potential. Which one of the two has (i) greater value of de Broglie wavelength associated with it and (ii) less kinetic energy? Explain.

Solution

$$2m_d = m_\alpha \;;\; q_\alpha = 2e \;;\; q_d = e$$

(i) de Broglie wavelength

$$\lambda = \frac{h}{\sqrt{2mqV}}$$, where q is the charge on the particle.

For deutron, $$\lambda_d = \frac{h}{\sqrt{2m_d q_d V}} = \frac{h}{\sqrt{2m_d eV}}$$

For α particle, $$\lambda_\alpha = \frac{h}{\sqrt{2m_\alpha q_\alpha V}} = \frac{h}{\sqrt{2(2m_d)\times2e\times V}} = \frac{h}{2\sqrt{2m_d eV}} = \frac{\lambda_d}{2}$$

$$\therefore \lambda_d = 2\lambda_\alpha$$

Deutron has greater wavelength (twice the wavelength of α particle).

(ii) Kinetic energy $K_d = eV$ $\quad (\because K = qV)$

$$K_\alpha = 2eV$$

$$K_\alpha = 2K_d$$

$$K_d = \frac{K_\alpha}{2}$$

$\therefore$ Deutron has less kinetic energy (half of kinetic energy of α).

14. An electron is accelerated through a potential difference of 81V. What is the de Broglie wavelength associated with it? To which part of electromagnetic spectrum does this wavelength correspond?

Solution

$$V_0 = 81V$$

$$\lambda_e = \frac{12.27\text{Å}}{\sqrt{V}}$$

$$\lambda_e = \frac{12.27\text{Å}}{9};\ 1.36\text{Å}$$

This wavelength falls in X-ray spectrum.

15. The ratio between the de Broglie wavelengths associated with protons, accelerated through a potential of 512 V and that of alpha particle accelerated through a potential of X volts is found to be one. Find the value of X.

Solution

$$V_p = 512\,\text{V} \ ;\ V_\alpha = X$$

$$m_\alpha \sqcup 4m_p \ ;\ q_\alpha = 2e \ ;\ q_p = e$$

$$\lambda_p = \frac{h}{\sqrt{2m_p eV_p}} \ ;\ \lambda_\alpha = \frac{h}{\sqrt{2m_\alpha(2e)V_\alpha}}$$

$$\frac{\lambda_\alpha}{\lambda_p} = \frac{\sqrt{2m_p eV_p}}{\sqrt{2\times 4m_p(2e)X}} = 1 \quad \text{(given)}$$

Squaring,

$$1 = \frac{2m_p eV_p}{2\times 4m_p \times 2e \times X}$$

$$\Rightarrow X = \frac{V_p}{8} = \frac{512}{8} = 64\,\text{V}$$

Chapter 9

Atomic and Nuclear Physics

Objective type Questions

1. Suppose an alpha particle accelerated by a potential of V volt is allowed to collide with a nucleus whose atomic number is Z, then the distance of closest approach of alpha particle to the nucleus is

(a) $14.4\frac{Z}{V}$ Å (b) $14.4\frac{V}{Z}$ Å (c) $1.44\frac{Z}{V}$ Å (d) $1.44\frac{V}{Z}$ Å

Solution

Kinetic energy obtained through potential $= qV = 2eV$ ($\because$ charge of α-particle, $q = 2e$)

According to law of conservation of energy,

Kinetic energy = Electric potential energy

$$\therefore 2eV = \frac{1}{4\pi\varepsilon_0}\frac{2e \times Ze}{r_0}$$

$$r_0 = \frac{1}{4\pi\varepsilon_0}\frac{Ze}{V} = 9\times10^9 \times 1.6\times10^{-19} \times \frac{Z}{V} \Rightarrow r_0 = 14.4\frac{Z}{V} \text{ Å}$$

[Option : (a)]

2. In a hydrogen atom, the electron revolving in the fourth orbit, has angular momentum equal to

(a) h (b) $\frac{h}{\pi}$ (c) $\frac{4h}{\pi}$ (d) $\frac{2h}{\pi}$

Solution

$$l_n = \frac{nh}{2\pi}$$

$$l_4 = \frac{4h}{2\pi} = \frac{2h}{\pi}$$

[Option : (d)]

3. Atomic number of H-like atom with ionization potential $122.4\,\text{V}$ for $n = 1$ is

(a) 1 (b) 2 (c) 3 (d) 4

Solution

$$V_{\text{ionization}} = \frac{13.6}{n^2}Z^2 \Rightarrow Z^2 = \frac{122.4 \times 1^2}{13.6} = 9 \Rightarrow Z = 3.$$

[Option : (c)]

4. The ratio between the radii of the first three orbits of hydrogen atom is

(a) 1:2:3 (b) 2:4:6 (c) 1:4:9 (d) 1:3:5

Solution

$$r_n \propto n^2 \Rightarrow r_1 : r_2 : r_3 = 1:4:9$$

[Option : (c)]

5. The charge of cathode rays is

(a) positive (b) negative (c) neutral (d) not defined

Solution

[Option : (b)]

6. In J.J. Thomson e/m experiment, electrons accelerated through 2.6 kV enter the region of crossed electric and magnetic fields of strength $3.0\times10^4\ Vm^{-1}$ and $1.0\times10^{-3}\ T$, respectively and pass through it undeflected. Then the specific charge is :

(a) $1.6\times10^{10}\ Ckg^{-1}$ (b) $1.7\times10^{11}\ Ckg^{-1}$ (c) $1.5\times10^{11}\ Ckg^{-1}$ (d) $1.8\times10^{11}\ Ckg^{-1}$

Solution

Electrons are undeflected,

$$\therefore eE = BeV$$

$$\therefore \upsilon = \frac{E}{B} = \frac{3\times10^4}{1\times10^{-3}}$$

$$= 3\times10^7\ ms^{-1}$$

kinetic energy = work done by the $p.d$

$$\frac{1}{2}m\upsilon^2 = eV \quad \therefore \frac{e}{m} = \frac{\upsilon^2}{2V} = \frac{(3\times10^7)^2}{2\times2.6\times10^3} = 1.7\times10^{11}\ Ckg^{-1}$$

[Option : (b)]

7. The ratio of the wavelengths for the transition from $n=2$ to $n=1$ in Li^{++}, He^{+} and H is

(a) 1:2:3 (b) 1:4:9 (c) 3:2:1 (d) 4:9:36

Solution

According to Bohr's theory,

$$\text{wave number, } \frac{1}{\lambda} = Z^2R\left[\frac{1}{n^2}-\frac{1}{m^2}\right] \Rightarrow \lambda \propto \frac{1}{Z^2}$$

For Li^{++} atom, $Z=3$; for He^{+} atom, $Z=2$; for H atom, $Z=1$;

Therefore $\lambda_{Li^{++}} : \lambda_{He^{+}} : \lambda_{H} = \frac{1}{9} : \frac{1}{4} : 1 = 4:9:36$.

[Option : (d)]

8. The electric potential of an electron is given by $V = V_0 \ln\left(\frac{r}{r_0}\right)$, where r_0 is a constant. Assume that Bohr atom model is valid, then variation of radius of n^{th} orbit r_n with the principal quantum number n is

(a) $r_n \propto \frac{1}{n}$ (b) $r_n \propto n$ (c) $r_n \propto \frac{1}{n^2}$ (d) $r_n \propto n^2$

Solution

Electric potential $V = V_0 \ln\left(\frac{r}{r_0}\right)$; electrical potential energy, $U = eV = eV_0 \ln\left(\frac{r}{r_0}\right)$;

$$\text{Electric force, } F = -\frac{dU}{dr} = -eV_0\frac{d}{dr}\left[\ln\left(\frac{r}{r_0}\right)\right] = -eV_0\left(\frac{1}{r}\right)$$

$$\left[\because \frac{d}{dx}[\ln kx] = \frac{1}{kx}\cdot k = \frac{1}{x}\right]$$

This force acts as the centripetal force, therefore,

$$|F| = \frac{eV_0}{r} = \frac{mv^2}{r} \Rightarrow v^2 = \frac{eV_o}{m}, \quad v = \sqrt{\frac{eV_0}{m}}$$

$$L = mvr = \frac{nh}{2\pi} \Rightarrow r = \frac{nh}{2\pi mv}$$

$$\Rightarrow r = \frac{nh}{2\pi m}\sqrt{\frac{m}{eV_0}} = \frac{nh}{2\pi\sqrt{meV_0}} \Rightarrow r \propto n$$

[Option : (b)]

9. If the nuclear radius of ^{27}Al is 3.6 fermi, the approximate nuclear radius of ^{64}Cu is

(a) 2.4 (b) 1.2 (c) 4.8 (d) 3.6

Solution

$$r = R_0 A^{\frac{1}{3}} \Rightarrow \frac{r_2}{r_1} = \left(\frac{A_2}{A_1}\right)^{\frac{1}{3}} = \left(\frac{64}{27}\right)^{\frac{1}{3}} = \frac{4}{3} \quad \therefore r_2 = \frac{4}{3} r_1;$$

$$r_1 = 3.6\,\text{F} \qquad \therefore r_2 = \frac{4}{3} \times 3.6 = 4.8\,\text{F}$$

[Option : (c)]

10. The nucleus is approximately spherical in shape. Then the surface area of nucleus having mass number A varies as

(a) $A^{\frac{2}{3}}$ (b) $A^{\frac{4}{3}}$ (c) $A^{\frac{1}{3}}$ (d) $A^{\frac{5}{3}}$

Solution

$$\text{Surface energy} = 4\pi r^2 = 4\pi\left(R_0 A^{\frac{1}{3}}\right)^2 \Rightarrow S.E. \propto A^{\frac{2}{3}}$$

[Option : (a)]

11. The mass of a ^7_3Li nucleus is $0.042u$ less than the sum of the masses of all its nucleons. The binding energy per nucleon of ^7_3Li nucleus is nearly

(a) 46 MeV (b) 5.6 MeV (c) 3.9 MeV (d) 23 MeV

Solution

$$\text{Binding energy, } B.E. = \Delta m \times 931\text{ MeV} = 0.042 \times 931\text{ MeV} \quad [\because 1u \equiv 931\text{ MeV}]$$

$$\therefore B.E / A = \frac{0.042 \times 931}{7} = 5.586\text{ MeV}; \quad 5.6\text{ MeV}$$

[Option : (b)]

12. M_p denotes the mass of the proton and M_n denotes mass of a neutron. A given nucleus of binding energy B, contains Z protons and N neutrons. The mass $M(N,Z)$ of the nucleus is given by (where c is the speed of light)

(a) $M(N,Z) = NM_n + ZM_p - Bc^2$ (b) $M(N,Z) = NM_n + ZM_p + Bc^2$

(c) $M(N,Z) = NM_n + ZM_p - B/c^2$ (d) $M(N,Z) = NM_n + ZM_p + B/c^2$

Solution

$$B = \Delta mc^2 \; ; \quad \Delta m = \frac{B}{c^2}$$

$$\Delta m = ZM_p + NM_n - M(N,Z)$$

$$M(N,Z) = ZM_p + NM_n - \Delta m$$
$$= ZM_p + NM_n - \frac{B}{c^2}$$

[Option : (c)]

13. A radioactive nucleus (initial mass number A and atomic number Z) emits 2α and 2 positrons. The ratio of number of neutrons to that of protons in the final nucleus will be

(a) $\frac{A-Z-4}{Z-2}$ (b) $\frac{A-Z-2}{Z-6}$ (c) $\frac{A-Z-4}{Z-6}$ (d) $\frac{A-Z-12}{Z-4}$

Solution

The radioactive decay can be represented as,

$${}^{A}_{Z}X \rightarrow {}^{A-8}_{Z-6}Y + 2\,{}^{4}_{2}He + 2\,{}^{0}_{1}e$$

No. of neutrons in the end $= (A-8)-(Z-6) = A-Z-2$

No. of protons in the end $= Z-6$.

[Option : (b)]

14. The half-life period of a radioactive element A is same as the mean life time of another radioactive element B. Initially both have the same number of atoms. Then

(a) A and B have the same decay rate initially

(b) A and B decay at the same rate always

(c) B will decay at faster rate than A

(d) A will decay at faster rate than B

Solution

$$T_A = \tau_B = \frac{T_B}{0.6931} = 1.44T_B \Rightarrow T_A > T_B \left(\because T = \frac{0.6931}{\lambda} = 0.6931\tau \;\&\; \frac{1}{0.6931};\; 1.44\right)$$

Activity, $R_A = \lambda_A N = \frac{0.6931}{T_A}N,\; R_B = \lambda_B N = \frac{0.6931}{T_B}N$

$$\Rightarrow \frac{R_B}{R_A} = \frac{T_A}{T_B} > 1 \Rightarrow R_B > R_A$$

[Option : (c)]

15. A radioactive element N_0 has number of nuclei at $t=0$. The number of nuclei remaining after half of a half-life $\left(\text{that is, at time } t = \frac{1}{2}T_{\frac{1}{2}}\right)$

(a) $\frac{N_0}{2}$ (b) $\frac{N_0}{\sqrt{2}}$ (c) $\frac{N_0}{4}$ (d) $\frac{N_0}{8}$

Solution

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n, \text{ also } n = \frac{t}{T_{\frac{1}{2}}}$$

$$\text{Here, } t = \frac{T_{\frac{1}{2}}}{2}, \quad \therefore n = \frac{T_{\frac{1}{2}}/2}{T_{\frac{1}{2}}} = \frac{1}{2} \Rightarrow \frac{N}{N_0} = \left(\frac{1}{2}\right)^{\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

[Option : (b)]

1. Consider two hydrogen atoms H_A and H_B in ground state. Assume that hydrogen atom H_A is at rest and hydrogen atom H_B is moving with a speed and make head-on collide on the stationary hydrogen atom H_A. After the strike, both of them move together. What is minimum value of the kinetic energy of the moving hydrogen atom H_B, such that any one of the hydrogen atoms reaches first excitation state.

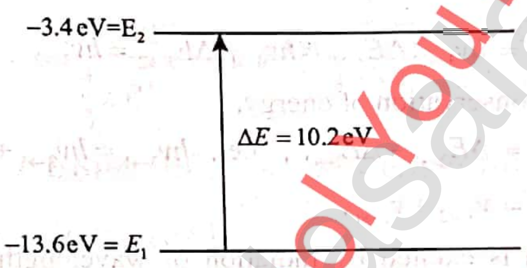
Solution

The given problem can be represented as follows :

According to law of conservation of momentum,

Total momentum before collision = Total momentum after collision

$$\sqrt{2K_i m} = \sqrt{2K_f 2m} \Rightarrow K_i = 2K_f \quad \text{or} \quad K_f = \frac{K_i}{2} \qquad \left(\because K = \frac{p^2}{2m};\ p = \sqrt{K.2m}\right)$$



According to law of conservation of energy,

$$K_i = K_f + \Delta E \Rightarrow K_i = \frac{K_i}{2} + \Delta E \Rightarrow \Delta E = \frac{K_i}{2}$$

$$\therefore K_i = 2\Delta E = 2 \times 10.2 = 20.4\,\text{eV}$$

$$(\because E_1 = -13.6\text{eV},\ E_2 = -3.4\text{eV} \ \text{and} \ E_2 - E_1 = \Delta E = 10.2\text{eV})$$

2. In the Bohr atom model, the frequency of transitions is given by the following expression $\nu = Rc\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$, where $n < m$, consider the following transitions :

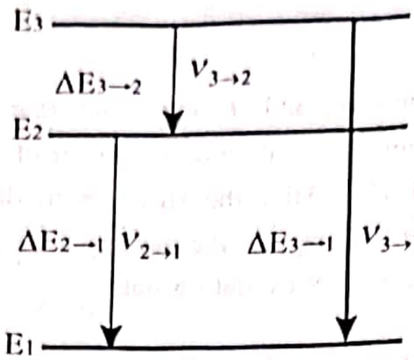
Transitions	$m \rightarrow n$
1	$3 \rightarrow 2$
2	$2 \rightarrow 1$
3	$3 \rightarrow 1$

Show that the frequency of these transitions obey sum rule (which is known as Ritz combination principle).

Solution

Method 1

The transitions of electron can be given as follows :



Frequency of photon $\nu_{m\to n} = Rc\left(\frac{1}{n^2}-\frac{1}{m^2}\right)$

$$\nu_{2\to1} = Rc\left[\frac{1}{1^2}-\frac{1}{2^2}\right] = Rc\left[\frac{3}{4}\right];\ \nu_{3\to2} = Rc\left[\frac{1}{2^2}-\frac{1}{3^2}\right] = Rc\left[\frac{5}{36}\right]$$

$$\nu_{3\to2}+\nu_{2\to1} = Rc\left[\frac{8}{9}\right]$$

$$\nu_{3\to1} = Rc\left[\frac{1}{1^2}-\frac{1}{3^2}\right] = Rc\left[\frac{8}{9}\right] = \nu_{3\to2}+\nu_{2\to1}$$

Method 2 :

$$\Delta E_{3\to1} = h\nu_{3\to1};\Delta E_{2\to1} = h\nu_{2\to1};\Delta E_{3\to2} = h\nu_{3\to2}$$

According to law of conservation of energy,

$$\Delta E_{3\to1} = \Delta E_{2\to1}+\Delta E_{3\to2}\ ;\ \text{i.e., } h\nu_{3\to1} = h\nu_{2\to1}+h\nu_{3\to2}\ ;$$

$$\therefore\ \nu_{3\to1} = \nu_{3\to2}+\nu_{2\to1}.$$

3. (a) A hydrogen atom is excited by radiation of wavelength 97.5 nm. Find the principal quantum number of the excited state.

(b) Show that the total number of lines in emission spectrum is $\frac{n(n-1)}{2}$ and compute the total number of possible lines in emission spectrum.

Solution

(a) $\lambda = 97.5\,\text{nm}$

Energy corresponding to radiation of wavelength λ is given by

$$\Delta E = \frac{hc}{\lambda} = \frac{6.626\times10^{-34}\times3\times10^{8}}{97.5\times10^{-9}} = 0.2039\times10^{-17}\,\text{J} = \frac{0.204\times10^{-17}}{1.6\times10^{-19}} = 12.75\,\text{eV}$$

$$E_n = E_1+\Delta E = -13.6+12.75 = -0.85\,\text{eV}$$

$$E_n = -\frac{13.6}{n^2} \Rightarrow n^2 = \frac{-13.6}{-0.85} = 16$$

$$n = 4.$$

$\therefore$ Principal quantum number of excited state is 4.

(b) (i) Let n be the quantum number of the excited state to which the electron transition takes place ;

from $n=2$, the number of downward transitions possible for electron $=1$ (i.e., $2\to1$) ;

from $n=3$, the number is $1+2$; $(2\to 1, 3\to 2 \ \& \ 3\to 1)$

from $n=4$, the number is $1+2+3$.

$(2\to 1 \ ; \ 3\to 2, \ 3\to 1 \ ; \ 4\to 3, 4\to 2, 4\to 1)$

....

from n, the number is $1+2+3+\cdots(n-1)$

$\Rightarrow$ from n, the total number of downward transitions of electron

$$\frac{(n-1)(n-1+1)}{2} = \frac{n(n-1)}{2}$$

(ii) the excited state in (a) is 4

$$\therefore \text{ the number of possible lines } = \frac{n(n-1)}{2} = \frac{4\times 3}{2} = 6.$$

4. Calculate the radius of the earth if the density of the earth is equal to the density of the nucleus. [mass of earth $5.97\times 10^{24}\,kg$].

Solution

$$\text{Density } \rho = \frac{m}{V} \ ; \ \rho_{\text{nucleus}} = 2.3\times 10^{17}\ \text{kg.m}^{-3}$$ [Ref. : Textbook Example 9.8]

Given : $\rho_{\text{nucleus}} = \rho_{\text{earth}}$

$$\therefore 2.3\times 10^{17} = \frac{5.97\times 10^{24}}{\frac{4}{3}\pi R^3}, \Rightarrow R^3 = \frac{5.97\times 10^{24}\times 3}{4\times 3.14\times 2.3\times 10^{17}} = \frac{17.91}{28.89}\times 10^7$$

$$= 0.6199\times 10^7, \quad \text{or} \quad R^3 = 6.199\times 10^6.$$

$$\therefore R = \left(6.199\times 10^6\right)^{\frac{1}{3}}$$

Calculation	$\frac{1}{3}\log(6.199) = 0.7923/3 = 0.2641$
	$AL(0.2641) = 1.837$

$\therefore$ Radius of earth, $R = 1.837\times 10^2\ \text{m} \approx 180\,\text{m}$

5. Calculate the mass defect and the binding energy per nucleon of the $^{108}_{47}Ag$ nucleus, [atomic mass of $Ag = 107.905949$].

Solution

Mass defect, $\Delta m = \left[Zm_H + (A-Z)m_n\right] - M_{\text{atom}}$

$$\therefore \Delta m = \{47\times 1.007825 + 61\times 1.008665 - 107.905949\}u$$ [Ref. : Section 9.4.3]

$$= 0.990391\,u$$

$$BE = \Delta m\times 931\,\text{MeV} = 0.990391\times 931 = 922.054021\,\text{MeV}$$

$$\overline{BE} = \frac{BE}{A} = \frac{922.0}{108} = 8.537 \approx 8.5\,\text{MeV}$$

6. Half lives of two radioactive elements A and B are 20 minutes and 40 minutes respectively. Initially, the samples have equal number of nuclei. Calculate the ratio of decayed numbers of A and B nuclei after 80 minutes.

Solution

Half-life period of $A, T_A = 20\,\text{min}$; half-life period of $B, T_B = 40$ min.

The number of atoms in the beginning, $N_{0A} = N_{0B}$.

Time $t = 80$ min $\Rightarrow t = 4T_A = 2T_B$

i.e., for $t = 80$ min, the number of half-lives, $n_A = 4$; $n_B = 2$.

After this time, the fraction of atoms remaining undecayed,

$$\frac{N_A}{N_{0A}} = \left(\frac{1}{2}\right)^4 = \frac{1}{16} \;;\; \text{Similarly, } \frac{N_B}{N_{0B}} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

Then, the fraction of atoms of A which have decayed $= 1 - \frac{1}{16} = \frac{15}{16}$

the fraction of atoms of B which have decayed $= 1 - \frac{1}{4} = \frac{3}{4}$.

Ratio of these fractions $= \frac{15/16}{3/4} = \frac{5}{4}$ or $5:4$.

7. On your birthday, you measure the activity of the sample ^{210}Bi which has a half-life of 5.01 days. The initial activity that you measure is $1\mu\text{Ci}$. (a) What is the approximate activity of the sample on your next birthday? Calculate (b) the decay constant (c) the mean life (d) initial number of atoms. $[e^{-50.51} \approx 10^{-22}]$

Solution

Data : $T_{\frac{1}{2}} = 5.01\text{d}$, $R_0 = 1\times10^{-6}\,\text{Ci}$, $t = 1\text{a} = 365.25\text{d}$

[Symbols used here : day = d, year = a]

$$\text{Decay constant } \lambda = \frac{0.6931}{T_{\frac{1}{2}}} = \frac{0.6931}{5.01\text{d}} = 0.1383\text{d}^{-1}$$

(a) Activity after one year $R = R_0 e^{-\lambda t} = 1\times10^{-6}\times e^{-0.1383\times365.25} = 10^{-6}\times e^{-50.51}$

$\therefore R = 10^{-22}\,\mu\text{Ci}$ $\quad [\because e^{-50.51} \approx 10^{-22}]$

(b) $\lambda = 0.1383\text{d}^{-1}$. Also

$$\lambda = \frac{0.6931}{5.01\times24\times60\times60\text{s}} \approx 1.6\times10^{-6}\,\text{s}^{-1}$$

(c) Mean life $\tau = 1/\lambda = \frac{1}{0.1383\text{d}^{-1}} = 7.23\text{d}$

(d) Number of atoms present initially

$$N_0 = \frac{R_0}{\lambda} = \frac{1\times10^{-6}\times3.7\times10^{10}\,\text{s}^{-1}}{1.6\times10^{-6}\,\text{s}^{-1}} \quad [\because 1\text{Ci} = 3.7\times10^{10}\,\text{s}^{-1}]$$

$$= 2.31\times10^{10}.$$

8. Calculate the time required for 60% of a sample of radon to undergo decay, Given $T_{1/2}$ of radon = 3.8 days.

Solution

$T_{1/2} = 3.8\text{d}$. Amount decayed = 60%. $\therefore$ amount remaining undecayed = 40%.

According to radioactive law of disintegration, $N = N_0 e^{-\lambda t}$.

$$\frac{40}{100} N_0 = N_0 e^{-\lambda t} \Rightarrow \frac{2}{5} = e^{-\lambda t}, \Rightarrow 2.5 = e^{\lambda t}$$

Taking log on both sides, in $\ln(2.5) = \lambda t$ and $\lambda = \frac{0.6931}{T_{1/2}} = \frac{0.6931}{3.8}$.

$$t = \frac{\ln 2.5}{\lambda} = \frac{0.9163}{0.6931/3.8} = \frac{0.9163 \times 3.8}{0.6931} = 5.023\text{d} \quad [\because \ln 2.5 = 2.303 \times \log_{10} 2.5 = 0.9163]$$

9. Assuming that energy released by the fission of a single $^{235}_{92}U$ nucleus is $200\,\text{MeV}$, calculate the number of fissions per second required to produce 1 watt power.

Solution

$$\text{Amount of energy released per fission} = 200\,\text{MeV}$$

$$= 200 \times 10^6 \times 1.6 \times 10^{-19}\,\text{J} = 320 \times 10^{-13}\,\text{J}$$

Let N be the number of fissions needed to produce an electric power of $1\,\text{W}$.

$$\text{Total electric power generated} = \text{Energy per fission} \times N.$$

$$\Rightarrow 1\,\text{W} = 1\,\text{J/s} = (320 \times 10^{-13}\,\text{J}) \times N$$

$$\therefore N = \frac{1\,\text{J/s}}{320 \times 10^{-13}\,\text{J}} = 3.125 \times 10^{10}\,\text{s}^{-1}$$

10. Show that the mass of radium $\left(^{226}_{88}Ra\right)$ with an activity of 1 curie is almost a gram. Given $T_{1/2} = 1600$ years.

Solution

$$\text{Activity } R = \lambda N, \quad N = \frac{R}{\lambda} = \frac{R}{0.6931/T_{1/2}} = \frac{RT_{1/2}}{0.6931}$$

$$R = 1\text{Ci} = 3.7 \times 10^{10}\,\text{s}^{-1}$$

$$T_{1/2} = 1600\,\text{a} = 1600 \times 365 \times 24 \times 60 \times 60\,\text{s} \qquad (1a = 1 \text{ year})$$

$$N = \frac{(3.7 \times 10^{10}\,\text{s}^{-1}) \times (1600 \times 365 \times 24 \times 60 \times 60\,\text{s})}{0.6931} = 2.694 \times 10^{21} \text{ atoms}$$

$$\text{Mass of } (6.022 \times 10^{23}) \text{ atoms of } ^{226}_{88}Ra = 226\text{g}$$

$$\Rightarrow \text{mass of } 2.694 \times 10^{21} \text{ atoms of } ^{226}_{88}Ra = \frac{226}{6.022 \times 10^{23}} \times 2.694 \times 10^{21} = 1.011g \approx 1g$$

11. Charcoal pieces of a tree is found from an archeological site. The carbon-14 content of this charcoal is only 17.5% that of equivalent sample of carbon from a living tree. What is the age of tree?

Solution

$$N = \frac{17.5}{100} N_0.$$

According to radioactive law of disintegration, $N = N_0 e^{-\lambda t}$

$$\frac{17.5}{100} N_0 = N_0 e^{-\lambda t} \Rightarrow e^{\lambda t} = \frac{100}{17.5}$$

Taking log on both sides,

$$\lambda t = \ln\left(\frac{100}{17.5}\right) = 2.303\times\log_{10}\left(\frac{100}{17.5}\right) \Rightarrow t = \frac{2.303\times\log_{10}\left(\frac{100}{17.5}\right)}{\lambda}$$

$$\lambda = \frac{0.6931}{T_{\frac{1}{2}}} = \frac{0.6931}{5730}\,yr^{-1} \quad (\because \text{ half-life of radiactive } {}^{14}_{6}C \text{ is 5730 years})$$

$$\Rightarrow t = \frac{2.303\times(\log_{10}100-\log_{10}17.5)}{\left(\frac{0.6931}{5730}\right)} = \frac{2.303\times5730\times(2-1.2430)}{0.6931}$$

$$\Rightarrow t = 14410 \text{ yr} \approx 1.44\times10^{4} \text{ yrs}$$

Chapter 10

Electronics and Communication

Objective type Questions

1. The barrier potential of a silicon diode is approximately
   (a) 0.7 V (b) 0.3 V (c) 2.0 V (d) 2.2 V

Solution

[Option : (a)]

2. If a small amount of antimony (Sb) is added to germanium crystal,
   (a) it becomes a p-type semiconductor
   (b) the antimony becomes an acceptor atom
   (c) there will be more free electrons than hole in the semiconductor
   (d) its resistance is increased

Solution

[Option : (c)]

3. In an unbiased p-n junction, the majority charge carriers (that is, holes) in the p-region diffuse into n-region because of
   (a) the potential difference across the p-n junction
   (b) the higher hole concentration in p-region than that in n-region
   (c) the attraction of free electrons of n-region
   (d) All of the above

Solution

[Option : (b)]

4. If a positive half-wave rectified voltage is fed to a load resistor, for which part of a cycle there will be current flow through the load?
   (a) $0°-90°$ (b) $90°-180°$ (c) $0°-180°$ (d) $0°-360°$

Solution

[Refer : Textbook Fig. 10.17]

[Option : (c)]

5. The zener diode is primarily used as
   (a) Rectifier (b) Amplifier (c) Oscillator (d) Voltage regulator

Solution

[Option : (d)]

6. The principle based on which a solar cell operates is
   (a) Diffusion (b) Recombination
   (c) Photovoltaic action (d) Carrier flow

Solution

[Option : (c)]

7. The light emitted in an LED is due to
(a) Recombination of charge carriers
(b) Reflection of light due to lens action
(c) Amplification of light falling at the junction
(d) Large current capacity

Solution

[Option : (a)]

8. The barrier potential of a p-n junction depends on
(i) type of semiconductor material (ii) amount of doping (iii) temperature.
Which one of the following is correct?
(a) (i) and (ii) only
(b) (ii) only
(c) (ii) and (iii) only
(d) (i) (ii) and (iii)

Solution

[Option : (d)]

9. To obtain sustained oscillation in an oscillator,
(a) Feedback should be positive
(b) Feedback factor must be unity
(c) Phase shift must be 0 or 2π
(d) All the above

Solution

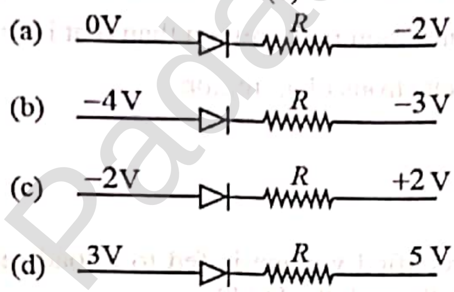
[Option : (d)]

10. If the input to the NOT gate is $A = 1011$, its output is
(a) 0100 (b) 1000 (c) 1100 (d) 0011

Solution

[Option : (a)]

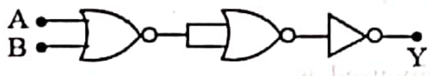
11. Which one of the following represents forward bias diode?
(a) (b) (c) (d)



Solution

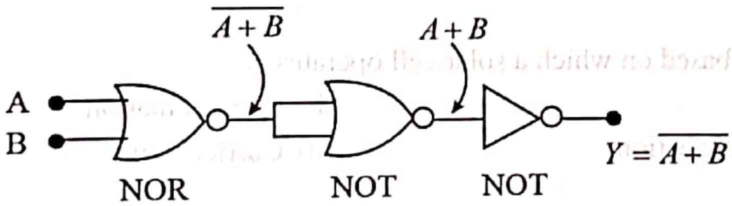
(a) $0\,V > -2\,V$ [Option : (a)]

12. The given electrical network is equivalent to



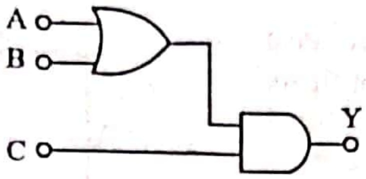
(a) AND gate (b) OR gate (c) NOR gate (d) NOT gate

Solution



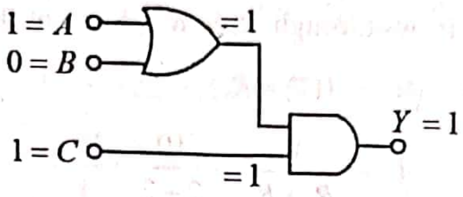
[Option : (C)]

13. The output of the following circuit is 1 when the input ABC is



(a) 101 (b) 100 (c) 110 (d) 010

Solution



[Option : (a)]

14. The variation of frequency of carrier wave with respect to the amplitude of the modulating signal is called

(a) Amplitude modulation (b) Frequency modulation
(c) Phase modulation (d) Pulse width modulation

Solution

[Option : (b)]

15. The frequency range of 3 MHz to 30 MHz is used for

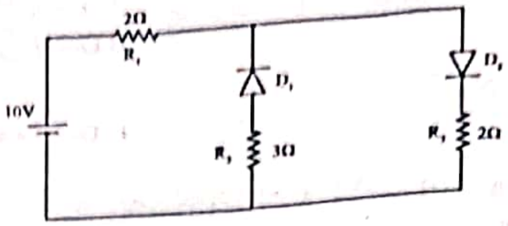
(a) Ground wave propagation (b) Space wave propagation
(c) Sky wave propogation (d) Satellite communication

Solution

[Option : (c)]

Exercise Problems

1. The given circuit has two ideal diodes connected as shown in figure below. Calculate the current flowing through the resistance R_1.



Solution

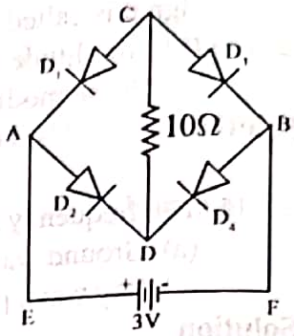
Diode D_1 is in reverse bias and D_2 is in forward bias hence, current flows through R_1 & R_3 (does not flow through R_2)

$$\therefore \quad V = I(R_1 + R_3)$$

$$I = \frac{V}{R_1 + R_3} = \frac{10}{2+2} = \frac{10}{4}$$

$$I = 2.5\,A.$$

2. Four silicon diodes and a 10Ω resistor are connected as shown in figure below. Each diode has a resistance of 1Ω. Find the current that flows through the 10Ω resistor.



Solution

Diodes D_1 and D_4 are reverse biased,

Diodes D_2 and D_3 are forward biased.

The diodes shown in the figure are real diodes, therefore, diodes D_2 and D_3 both have a potential barrier of $V_B = 0.7$ V each.

$$\therefore \quad V = 3-(0.7+0.7) = 1.6\,V$$

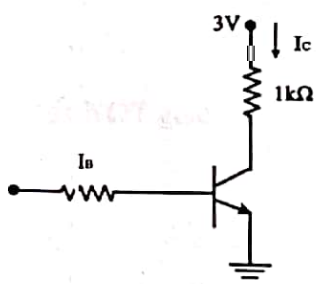
$$V = I(R_2 + R + R_3)$$

$$\Rightarrow I = \frac{V}{R_2 + R + R_3}$$

$$= \frac{1.6}{1+10+1} = \frac{1.6}{12}$$

$$I \approx 0.13\,A.$$

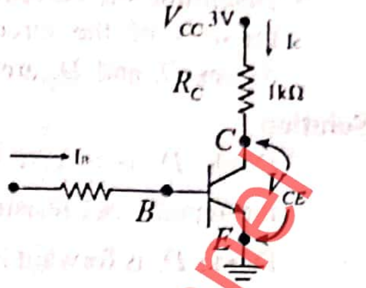
3. Assuming $V_{CEsat} = 0.2\,V$ and $\beta = 50$, find the minimum base current (I_B) required to drive the transistor given in the figure to saturation.



Solution

$$V_{CE} = 0.2\,V$$
$$\beta = 50$$

$$V_{CC} = 3\,V$$
$$R_C = 10^3\Omega$$



Appling Kirchoff's voltage law, $V_{CC} - I_C R_C - V_{CE} = 0$

$$I_C = \frac{V_{CC} - V_{CE}}{R_C}$$

$$I_C = \frac{3-0.2}{10^3}$$

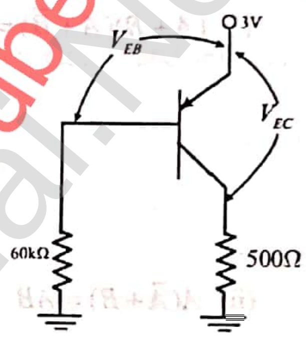
$$I_C = 2.8 \times 10^{-3}\,\text{A}$$

$$\beta = \frac{I_C}{I_B} \Rightarrow I_B = \frac{I_C}{\beta} = (2.8 \times 10^{-3})/\beta$$

$$I_B = (2.8 \times 10^{-3})/50 = 0.056 \times 10^{-3}\,\text{A}$$

$$I_B = 56\,\mu\text{A}.$$

4. In the circuit shown in the figure, the BJT has a current gain (β) of 50. For an emitter-base voltage $V_{EB} = 600\,\text{mV}$, calculate the emitter-collector voltage V_{EC} (in volts).



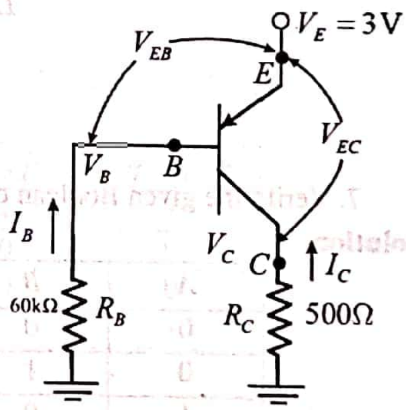
Solution

$$V_E = 3\,\text{V} \; ; \; \beta = 50$$

$$V_{EB} = 600\,\text{mV} = 0.6\,\text{V}$$

$$R_B = 60k\Omega = 60 \times 10^3\,\Omega$$

$$V_{CE} = ?$$



$$V_B = V_E - V_{EB} = 3 - 0.6 = 2.4\,\text{V}$$

$$V_B = I_B R_B$$

$$\therefore I_B = \frac{V_B}{R_B}$$

$$= \frac{2.4}{60 \times 10^3}$$

$$I_B = 0.04 \times 10^{-3}\,\text{A}$$

$$\beta = \frac{I_C}{I_B} \quad \therefore I_C = \beta I_B$$

$$= 50 \times 0.04 \times 10^{-3}$$

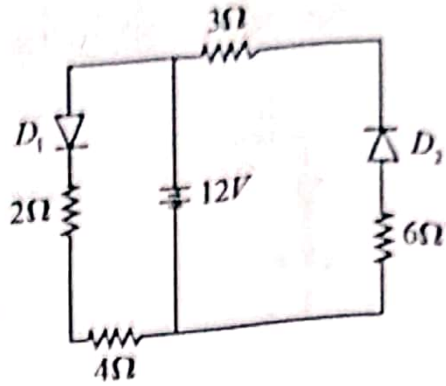
$$I_C = 2 \times 10^{-3}\,\text{A}$$

$$V_C = I_C R_C = 2 \times 10^{-3} \times 500 = 1\,\text{V}$$

$$V_{EC} = V_E - V_C = 3 - 1 = 2\,\text{V}$$

$$\therefore V_{EC} = 2\,\text{V}.$$

5. Determine the current flowing through 3Ω and 4Ω resistors of the circuit given below. Assume that diodes D_1 and D_2 are ideal diodes.



Solution

Diode D_2 is reverse biased. Hence no current flows through the 3Ω resistors. i.e., $I_{3\Omega} = 0$.

Diode D_1 is forward biased. As it is an ideal diode,

$$I = \frac{V}{R_s}$$

$$I_{4\Omega} = \frac{12}{2+4} = 2\,\text{A}.$$

6. Prove the following Boolean expressions using the laws and theorems of Boolean algebra.

Solution

(i) $(A+B)(A+\bar{B}) = A$

$$\begin{aligned} LHS &= AA + A\bar{B} + BA \quad (\because B\bar{B} = 0) \\ &= A + A\bar{B} + AB \quad (\because AA = A) \\ &= A(1 + \bar{B} + B) \\ &= A \quad (\because 1 + \bar{B} + B = 1) \end{aligned}$$

(ii) $A(\bar{A} + B) = AB$

$$\begin{aligned} LHS &= A\bar{A} + AB \\ &= AB \quad (\because A\bar{A} = 0) \end{aligned}$$

(iii) $(A+B)(A+C) = A + BC$

$$\begin{aligned} LHS &= AA + AC + BA + BC \\ &= A + AC + AB + BC \quad (AA = A) \\ &= A(1 + C + B) + BC \\ &= A + BC \quad (\because 1 + C + B = 1) \end{aligned}$$

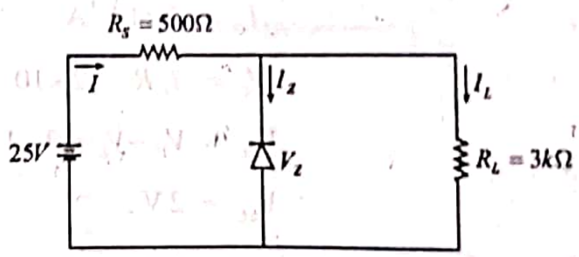
7. Verify the given Boolean equation $A + \bar{A}B = A + B$ using truth table.

Solution

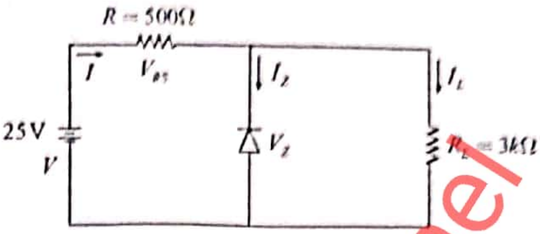
A	B	$\bar{A}$	$\bar{A}B$	$A+\bar{A}B$	$A+B$
0	0	1	0	0	0
0	1	1	1	1	1
1	0	0	0	1	1
1	1	0	0	1	1

$A + \bar{A}B = A + B$. Thus proved.

8. In the given figure of a voltage regulator, a zener diode of breakdown voltage 15 V is employed. Determine the current through the load resistance, the total current and the current through the diode. Use diode approximation.



Solution



$V = 25\text{V}$; $V_Z = 15\text{V}$

$\therefore$ potential across $R_L = 15\text{V}$

$\Rightarrow \quad I_L = \dfrac{V_L}{R_L} = \dfrac{15}{3\times10^3} = 5\times10^{-3}\text{A} = 5\text{mA}$

According to Kirchhoff's second law,

$V = V_{RS} + V_Z$

\ Potential across R_S

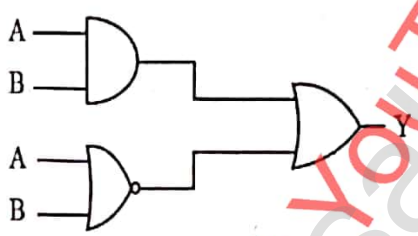
$V_{RS} = V - V_Z = 25 - 15 = 10\text{V}$

$\Rightarrow \quad I = \dfrac{V_{RS}}{R_S} = \dfrac{10}{500} = 0.02\text{A} = 20\text{mA}$

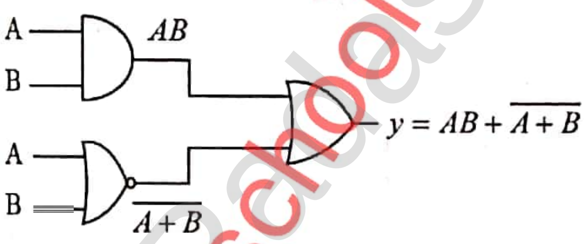
Applying Kirchhoff's current law, $I = I_Z + I_L$

$\therefore \quad I_Z = I - I_L = 20 - 5 = 15\text{mA}$.

9. Write down the Boolean equation for the output y of the given circuit and give its truth table.



Solution



A	B	AB	$A+B$	$\overline{A+B}$	$AB+\overline{A+B}$
0	0	0	0	1	1
0	1	0	1	0	0
1	0	0	1	0	0
1	1	1	1	0	1

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