

MATHEMATICAL PHYSICS – UNIT 1 – SPECIAL FUNCTIONS

Each question carries four options namely A, B, C and D. Choose one correct option and mark in appropriate place

- 1. Legendre Polynomial is given $P_n(x) = \frac{1}{2^n} \frac{d^n}{dx^n} (x^2 - 1)^n$ when $n = 2$, the polynomial is given by**

- a) 1
- b) x
- c) $\frac{1}{2}(3x^2 - 1)$
- d) $\frac{1}{2}(5x^2 - 1)$

$$\text{For } n=2 \rightarrow P_n(x) = P_2(x) = \frac{1}{2}(3x^2 - 1)$$

- 2. The value of $\Gamma(-n) = ?$**

- a) $(n - 1)!$
- b) $(n + 1)!$
- c) ∞
- d) $-\infty$

- 3. The value of $\Gamma(0) = ?$**

- a) $(n)!$
- b) $(n + 1)!$
- c) ∞
- d) $-\infty$

- 4. The value of $\Gamma\left(\frac{-3}{2}\right) = ?$**

- a) $-2\sqrt{\pi}$
- b) $\frac{4}{3}\sqrt{\pi}$
- c) $\sqrt{\pi}$
- d) $\frac{\sqrt{\pi}}{2}$

- 5. The value of $\Gamma(n)\Gamma(1 - n) = ?$**

- a) $\frac{\pi}{\sin m\pi}$
- b) $\frac{\pi}{\sin n\pi}$

c) $\frac{m\pi}{\sin n\pi}$

d) $\frac{n\pi}{\sin m\pi}$

6. Express $7x^4 - 9x^2 + 2$ in terms of Legendre polynomials for $P_2(x) = \frac{1}{2}(3x^2 - 1)$, $P_3(x) = \frac{1}{2}(5x^3 - 3x)$, and $P_4(x) = \frac{1}{8}(35x^4 - 30x^2 + 3)$

a) $\frac{2}{5}(4P_4 - 5P_2 + P_0)$

b) $8P_4 - P_2 + P_0$

c) $\frac{8}{5}(P_4 - 4P_3 + 2P_2 - P_0)$

d) $\frac{1}{7}(P_4 - 2P_3 + 2P_2 - P_0)$

Given expression the coefficient of $x^4 = 7$

In P_4 the coefficient of $x^4 = \frac{35}{8}$

Comparing the coefficients of x^4 in given expression and P_4

$$\frac{x^4}{7} = \frac{8}{35}P_4 \rightarrow x^4 = \frac{8}{5}P_4$$

Also in given expression there is no x^3 and x term. Therefore no P_3 exist in answer.

Thus option A is only suitable

7. Given $H_4'(x) = 64x^3 - 96x$ Find the Hermite polynomials of $H_3(x) = ?$

a) $192x^2 - 96$

b) $16x^4 - 48x^3$

c) $8x^3 - 12x$

d) $4x^3 - 6x$

Recurrence relation: $H'_n(x) = 2n H_{n-1}(x)$

$$H_{n-1}(x) = \frac{1}{2n} H'_n(x)$$

$$H_3(x) = \frac{1}{2 \times 4} H'_4(x) = \frac{1}{8} (64x^3 - 96x) = 8x^3 - 12x$$

8. The weight function which makes the Laguerre polynomials an orthogonal

- a) e^{-x}
- b) e^x
- c) e^{x^2}
- d) e^{-x^2}

9. If m is an integer, less than n $\int_{-1}^1 x^m P_n(x) dx$ is

- a) 0
- b) 1
- c) $2^{n+1} n!^2$
- d) $\frac{2^{n+1} n!^2}{(2n+1)!}$

10. If n is an integer, $H_n(0)$ is..

- a) 1
- b) 0
- c) $(-1)^n \frac{n!}{(\frac{n}{2})!}$
- d) $\frac{n!}{(\frac{n}{2})!}$

11. The orthonormal property of Hermite polynomial

- a) $2^n n! \pi \delta_{mn}$
- b) $2^n n! \sqrt{\pi} \delta_{mn}$
- c) $2^n \sqrt{\pi} \delta_{mn}$
- d) $2n! \sqrt{\pi} \delta_{mn}$

Orthogonality of Hermite Polynomial

$$\int_{-\infty}^{+\infty} e^{-x^2} H_n(x) H_m(x) dx = 2^n n! \sqrt{\pi} \delta_{mn}$$

12. The value of $H_{2n}(0)$ is

- a) $\frac{2n!}{n!}$
- b) $(-1)^n \frac{2n!}{n!}$
- c) $(-1)^n \frac{2n!}{n!}$
- d) $(-1)^n \frac{2n}{n}$

13. If P If $P_n(x)$ is the legendre polynomial of order n then $3x^2 + 3x + 1$ can be expressed as

- a) $P_2 + 3P$
- b) $4P_2 + 2P_1 + P_0$
- c) $3P_2 + 3P_1 + P_0$
- d) $2P_2 + 3P_1 + 2P_0$

using the following relations

- (i) $P_0(x) = 1 \Rightarrow 1 = P_0$
- (ii) $P_1(x) = x \Rightarrow x = P_1$
- (iii) $P_2(x) = \frac{1}{2}(3x^2 - 1) \Rightarrow x^2 = \frac{1}{3}P_0 + \frac{2}{3}P_2$

Substituting $x^2, x, 1$ values in the expression $3x^2 + 3x + 1$

$$\begin{aligned}
 3x^2 + 3x + 1 &= 3\left(\frac{1}{3}P_0 + \frac{2}{3}P_2\right) + 3P_1 + P_0 = P_0 + 2P_2 + 3P_1 + P_0 \\
 &= 2P_2 + 3P_1 + 2P_0
 \end{aligned}$$

14. The polynomial $2 - 3x + 4x^2$ in terms of Legendre polynomials is..

- a) $\frac{10}{3}P_0(x) - 3P_1(x) + \frac{8}{3}P_2(x)$
- b) $\frac{5}{2}P_0(x) - \frac{3}{2}P_1(x) + \frac{8}{2}P_2(x)$
- c) $P_0(x) - 8P_1(x) + 16P_2(x)$
- d) $\frac{10}{3}P_0(x) + 3P_1(x) - \frac{8}{3}P_2(x)$

using the following relations

$$\begin{aligned}
 \text{(i)} \quad P_0(x) &= 1 & \Rightarrow 1 = P_0 \\
 \text{(ii)} \quad P_1(x) &= x & \Rightarrow x = P_1 \\
 \text{(iii)} \quad P_2(x) &= \frac{1}{2}(3x^2 - 1) \Rightarrow x^2 = \frac{1}{3}P_0 + \frac{2}{3}P_2
 \end{aligned}$$

Substituting $x^2, x, 1$ values in the expression $2 - 3x + 4x^2$

$$\begin{aligned}
 2 - 3x + 4x^2 &= 2P_0 - 3P_1 + 4\left(\frac{1}{3}P_0 + \frac{2}{3}P_2\right) \\
 &= \frac{10}{3}P_0(x) - 3P_1(x) + \frac{8}{3}P_2(x)
 \end{aligned}$$

15. Let $P_n(x)$ be the Legendre polynomial then $P_n(-x)$ is equal to..

- a) $(-1)^{n+1}P'_n(x)$
- b) $(-1)^n P'_n(x)$
- c) $P'_n(x)$
- d) $(-1)^n P_n(x)$

16. If $\int_{-1}^1 P_n(x)dx = 2$ then n is,

- a) 1
- b) 0
- c) -1
- d) 2

$$\int_{-1}^1 P_n(x) dx = 2$$

When $n = 0 \rightarrow \int_{-1}^1 P_0(x) dx = \int_{-1}^1 dx = [x]_{-1}^1 = 2$

17. The Legendre polynomial $P_n(x)$ has..

- (a) **n real zeros between 0 and 1**
- (b) **n zeros of which only one is between 1 and -1**
- (c) **$2n - 1$ real zero between 1 and -1**
- (d) **none of these**

18. The incorrect equation among the following....

- (a) $P_0(x) = 1$
- (b) $P_1(x) = x$
- (c) $P_n(-x) = (-1)^{n+1} P_n(x)$
- (d) $P'_n(-x) = (-1)^{n+1} P'_n(x)$

The correct relation is

$$P_n(-x) = (-1)^n P_n(x)$$

19. The value of $\Gamma\left(\frac{1}{2}\right) = ?$

- (a) 0
- (b) $\frac{\pi}{2}$
- (c) π
- (d) $\sqrt{\pi}$

20. The value of $\beta(2, z)$ is ...

- (a) $\frac{1}{z}$
- (b) $\frac{1}{z+1}$
- (c) $\frac{1}{z(z+1)}$

(d) $\frac{z-1}{z+1}$

$$\beta(m, n) = \frac{(m-1)! (n-1)!}{(n+m-1)!}$$

$$\beta(2, z) = \frac{(1)! (z-1)!}{(2+z-1)!}$$

$$\beta(2, z) = \frac{(z-1)!}{(z+1)!} = \frac{(z-1)!}{(z+1) z (z-1)!} \quad (\text{since } (z+1)! = (z+1)z(z-1)!)$$

$$\beta(2, z) = \frac{1}{(z+1) z}$$

21. The value of $\Gamma(1 + m) \Gamma(1 - m)$ is

- (a) $\frac{m\pi}{\sin m\pi}$
- (b) $\frac{\pi}{\sin m\pi}$
- (c) $\frac{\pi}{m \sin m\pi}$
- (d) $\frac{\sin(1-m)\pi}{\sin(1+m)\pi}$

22. The Legendre duplication formula is..

- (a) $\left(m + \frac{1}{2}\right)! = \frac{\pi(2m)!!}{2^m}$
- (b) $\left(m + \frac{1}{2}\right)! = \frac{\pi^{\frac{1}{2}}(2m+1)!!}{2^{m+1}}$
- (c) $\left(m + \frac{1}{2}\right)! = \frac{\pi^{\frac{1}{2}}(2m-1)!!}{2^{m-1}}$
- (d) $\left(m - \frac{1}{2}\right)! = \frac{\pi^{\frac{1}{2}}(2m-1)!!}{2^{m-1}}$

23. $1, 3, 5, \dots, (2n - 1) = \frac{2^n}{\sqrt{\pi}} \Gamma(p)$, then the value of p is...

- (a) $\left(n + \frac{1}{2}\right)$
- (b) $\left(n - \frac{1}{2}\right)$
- (c) $\left(n + \frac{1}{3}\right)$
- (d) $\left(n + \frac{2}{3}\right)$

$$1 \cdot 3 \cdot 5 \cdots (2n - 1) \sqrt{\pi} = 2^n \Gamma\left(n + \frac{1}{2}\right)$$

$$\rightarrow 1 \cdot 3 \cdot 5 \cdots (2n - 1) = \frac{2^n}{\sqrt{\pi}} \Gamma\left(n + \frac{1}{2}\right)$$

Given expression: $1, 3, 5, \dots, (2n - 1) = \frac{2^n}{\sqrt{\pi}} \Gamma(p)$

Comparing the above two equations: $\Gamma(p) = \Gamma\left(n + \frac{1}{2}\right)$

i.e., $(p) = \left(n + \frac{1}{2}\right)$

24. The value of integral $\int_0^{\infty} \sqrt{\frac{k}{y}} e^{-ky} dy$ is

- (a) $\Gamma\left(\frac{1}{2}\right)$
- (b) $\Gamma\left(\frac{3}{2}\right)$
- (c) $\Gamma\left(\frac{\pi}{2}\right)$
- (d) $\Gamma\left(\frac{k}{2}\right)$

25. The value of integral $\beta(a, b) \beta(a + b, c)$ is

- (a) $\Gamma(a) \Gamma(b) \Gamma(c)$
- (b) $\frac{\Gamma(a) \Gamma(b) \Gamma(c)}{\Gamma(a+b+c)}$
- (c) $\Gamma(b) \Gamma(a+b)$
- (d) $\Gamma(c) \Gamma(a+b) \Gamma(a)$

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Relation between **Beta** and **Gamma** functions

$$\beta(m, n) = \frac{\Gamma_m \Gamma_n}{\Gamma_{(m+n)}}$$

$$\beta(a, b) = \frac{\Gamma_a \Gamma_b}{\Gamma_{(a+b)}}$$

$$\beta(a + b, c) = \frac{\Gamma_{(a+b)} \Gamma_c}{\Gamma_{(a+b+c)}}$$

$$\beta(a, b) \beta(a + b, c) = \frac{\Gamma_a \Gamma_b}{\Gamma_{(a+b)}} \times \frac{\Gamma_{(a+b)} \Gamma_c}{\Gamma_{(a+b+c)}}$$

$$\beta(a, b) \beta(a + b, c) = \frac{\Gamma_a \Gamma_b}{\Gamma_{(a+b+c)}} \times \frac{\Gamma_c}{\Gamma_{(a+b+c)}} = \frac{\Gamma_a \Gamma_b \Gamma_c}{\Gamma_{(a+b+c)}}$$

26. If **m** and **n** are positive integers then the value of $\beta(m, n)$ is

- (a) $\frac{n!m!}{(m+n)!}$
- b)** $\frac{(n-1)! (m-1)!}{(m+n-1)!}$
- c) $\frac{(n+1)! (m+1)!}{(m+n+2)!}$
- d) $\frac{(m-1)! (n-1)!}{(m+n-2)!}$

27. The value of $\Gamma(m) \Gamma(1 - m)$ is

- (a) $\sin m \sin(1 - m)$
- b)** $\frac{\pi}{\sin m\pi}$
- c) $\frac{\pi}{\sin(m-1)\pi}$
- d) $\frac{\sin(m-1)\pi}{\sin(m+1)\pi}$

28. The value of $\int_0^{\frac{\pi}{2}} \sin^p \theta \cos^q \theta d\theta$ is...

- a)** $\frac{\Gamma(p) \Gamma(q)}{\Gamma(p+q)}$
- b)** $\frac{\Gamma(p+1) \Gamma(q+1)}{\Gamma(p+q+2)}$
- c) $\frac{\Gamma(p+1) \Gamma(q+1)}{2 \Gamma(p+q+2)}$
- d) $\frac{\Gamma(\frac{p+1}{2}) \Gamma(\frac{q+1}{2})}{2 \Gamma(p+q+2)}$

29. What is the ratio of $\frac{\Gamma(-\frac{3}{2})}{\Gamma(\frac{3}{2})}$

- (a) 1 (b) $\frac{3}{8}$ (c) $\frac{8}{3}$ (d) $\frac{2}{3}$

$$\Gamma\left(-\frac{3}{2}\right) = \frac{4}{3}\sqrt{\pi}$$

$$\Gamma\left(\frac{3}{2}\right) = \frac{1}{2}\sqrt{\pi}$$

$$\frac{\Gamma(-\frac{3}{2})}{\Gamma(\frac{3}{2})} = \frac{\frac{4}{3}\sqrt{\pi}}{\frac{1}{2}\sqrt{\pi}} = \frac{8}{3}$$

30. The digamma function is defined as

- (a) *digamma representation of gamma function*
- (b) *derivative of logarithmic Gamma function*
- (c) **logarithmic gamma function**
- (d) *derivative of Gamma function*

31. The relation between beta and gamma function is

- (a) $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{2\Gamma(m+n)}$
- (b) $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$
- (c) $\beta(m, n) = \frac{2\Gamma(m-1)\Gamma(n-1)}{\Gamma(m+n-2)}$
- (d) $\frac{\Gamma(\frac{m+1}{2})\Gamma(\frac{n+1}{2})}{\Gamma(\frac{m+n+2}{2})}$

32. If n is a positive integer then the value of $\Gamma(n)$ is...

- (a) $n!$
- (b) $(n-1)!$
- (c) $(n-2)!$
- (d) $(n+1)!$

33. The value of $\beta(z, 1)$ is

- (a) $\frac{1}{z}$
- (b) $\frac{1}{z+1}$
- (c) $\frac{1}{z(z+1)}$
- (d) $\frac{z}{(z+1)}$

$$\beta(m, n) = \frac{(n-1)! (m-1)!}{(m+n-1)!}$$

$$\beta(z, 1) = \frac{(z-1)! (1-1)!}{(z+1-1)!} = \frac{(z-1)! 0!}{(z)!}$$

$$= \frac{(z-1)! 0!}{(z) (z-1)!} = \frac{1}{z}$$

34. The value of integral $\int_0^{\frac{\pi}{2}} (\tan \theta)^{\frac{1}{2}} d\theta$ is...

- (a) $\frac{\Gamma(\frac{3}{2})}{2}$
- (b) $\frac{\Gamma(\frac{3}{4}) \Gamma(\frac{1}{4})}{2}$
- (c) $\frac{\Gamma(\frac{3}{4}) \Gamma(\frac{3}{2})}{2}$
- (d) $\frac{\Gamma(\frac{3}{2}) \Gamma(\frac{1}{2})}{\sqrt{\pi}}$

35. Find the value of $\beta(1, 2)$ is

- (a) 0.25
- (b) 0.75
- (c) 0.05
- (d) 0.5

$$\beta(m, n) = \frac{(n-1)! (m-1)!}{(m+n-1)!}$$

$$\beta(1, 2) = \frac{(1-1)! (2-1)!}{(1+2-1)!} = \frac{0! 1!}{(2)!}$$

$$= \frac{1}{2} = 0.5$$

36. Find the value of $\beta(2, 4)$ is

- (a) 0.25
- (b) 0.75
- (c) 0.05
- (d) 0.5

$$\beta(m, n) = \frac{(n-1)! (m-1)!}{(m+n-1)!}$$

$$\beta(2, 4) = \frac{(2-1)! (4-1)!}{(2+4-1)!} = \frac{1! 3!}{(5)!}$$

$$= \frac{3 \times 2}{5 \times 4 \times 3 \times 2} = \frac{1}{20} = 0.05$$

37. Find the value of $\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)$ is

- (a) $\pi 2^{\frac{1}{2}}$
- (b) $\sqrt{\pi} 2$
- (c) $\sqrt{\pi} 2^{\frac{1}{2}}$
- (d) none of these

$$\Gamma m \Gamma(1 - m) = \frac{\pi}{\sin m\pi}$$

Put $m = 1/4$

$$\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right) = \frac{\pi}{\sin\frac{\pi}{4}} = \frac{\pi}{\frac{1}{\sqrt{2}}} = \pi\sqrt{2}$$

38. Find the value of $\Gamma(m)\Gamma(n) = ?$ If $m > 0$ & $n > 0$

- (a) $\Gamma(m + n)$
- (b) $\Gamma(m + n) \beta(m, n)$
- (c) $\Gamma(m - n) \beta(m, n)$
- (d) $\Gamma(m - n)$

$$\beta(m, n) = \frac{(m-1)! (n-1)!}{(m+n-1)!}$$

$$\Gamma(m)\Gamma(n) = (m-1)! (n-1)!$$

$$\Gamma(m + n) = (m + n - 1)!$$

$$\Rightarrow \beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$$

$$\Rightarrow \Gamma(m)\Gamma(n) = \Gamma(m + n) \beta(m, n)$$

39. $\beta(m, n) =$

- (a) $\beta(m + 1) + \beta(n + 1)$
- (b) $\beta(m + 1) \cdot \beta(n + 1)$
- (c) $\beta(m + 1, n) + \beta(m, n + 1)$
- (d) $\beta(m + 1, n) \cdot \beta(m, n + 1)$

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$$\beta(m, n) = \frac{(m-1)! (n-1)!}{(m+n-1)!}$$

$$\beta(m+1, n) = \frac{m!(n-1)!}{(m+n)!}$$

$$\beta(m, n+1) = \frac{(m-1)!(n)!}{(m+n)!}$$

$$\beta(m+1, n) + \beta(m, n+1) = \frac{m!(n-1)!}{(m+n)!} + \frac{(m-1)!(n)!}{(m+n)!}$$

$$= \frac{m(m-1)!(n-1)!}{(m+n)!} + \frac{(m-1)!(n)(n-1)!}{(m+n)!}$$

$$= \frac{(m-1)!(n-1)!}{(m+n)!} (m+n) = \frac{(m-1)!(n-1)!}{(m+n)(m+n-1)!} (m+n)$$

$$= \frac{(m-1)!(n-1)!}{(m+n-1)!} = \beta(m, n)$$

40. $\beta(m, n) =$

(a) $(m+n)\beta(m, n)$

(b) $\frac{(m+n)}{n} \beta(m, n)$

(c) $\frac{(m+n)}{n} \beta(m, n+1)$

(d) $\frac{(m-n)}{n} \beta(m, n+1)$

$$\beta(m, n) = \frac{(m-1)! (n-1)!}{(m+n-1)!}$$

$$\beta(m, n+1) = \frac{(m-1)!(n)!}{(m+n)!} = \frac{(m-1)! n (n-1)!}{(m+n) (m+n-1)!}$$

$$= \frac{n}{(m+n)} \frac{(m-1)! (n-1)!}{(m+n-1)!} = \frac{n}{(m+n)} \beta(m, n)$$

$$\beta(m, n) = \frac{(m+n)}{n} \beta(m, n+1)$$

41. The value of $\beta(x, y + 1) = ?$

(a) $\left(\frac{x}{x+y}\right) \beta(x, y)$

(b) $\left(\frac{x}{x-y}\right) \beta(x, y)$

(c) $\left(\frac{y}{x+y}\right) \beta(x, y)$

(d) $\left(\frac{y}{x-y}\right) \beta(x, y)$

From above problem

$$\beta(m, n + 1) = \frac{n}{(m+n)} \beta(m, n)$$

put $m = x$ & $n = y$

$$\beta(x, y + 1) = \frac{y}{(x+y)} \beta(x, y)$$

42. The value of $\beta(x + 1, y) = ?$

(a) $\left(\frac{x}{x-y}\right) \beta(x, y)$

(b) $\left(\frac{x}{x+y}\right) \beta(x, y)$

(c) $\left(\frac{y}{x+y}\right) \beta(x, y)$

(d) $\left(\frac{y}{x-y}\right) \beta(x, y)$

43. The value of $\frac{2}{3}P_2(x) + \frac{1}{3}P_0(x) = ?$

(a) x

(b) x^2

(c) x^3

(d) $x^2 + \frac{2}{3}$

$$\frac{2}{3}P_2(x) + \frac{1}{3}P_0(x) = \frac{2}{3} \times \frac{1}{2} (3x^2 - 1) + \frac{1}{3}(1) = \frac{1}{3} \times 3x^2 - \frac{1}{3} + \frac{1}{3} = x^2$$

44. If the solution of Legendre differential equation as infinite series is reduced to finite series, then the solution is called.

- (a) *polynomial*
- (b) *binomial*
- (c) *Trivial*
- (d) *green function*

45. In terms of Legendre polynomials $x^2 + x$ is equal to..

- (a) $P_0 + P_1 + 2P_2$
- (b) $\frac{1}{3}P_0 + P_1 + \frac{1}{3}P_2$
- (c) $\frac{1}{3}P_0 + P_1 + \frac{2}{3}P_2$
- (d) $\frac{1}{3}P_0 + \frac{2}{3}P_2$

$$\begin{aligned} x &= P_1 \\ x^2 &= \frac{1}{3}P_0 + \frac{2}{3}P_2 \\ x^2 + x &= \frac{1}{3}P_0 + \frac{2}{3}P_2 + P_1 \end{aligned}$$

46. If $P_n(x)$ is Legendre Polynomial then $\int_{-1}^1 [P_n(x)]^2 dx$ is equal to

- (a) 1
- (b) $\frac{1}{2}(2n - 1)$
- (c) $\frac{2}{2n+1}$
- (d) $\frac{n(n+1)}{2}$

47. If $P_n(x)$ is Legendre Polynomial then the value of $\int_{-1}^1 P_n(x) P_m(x) dx$ ($m \neq n$) is equal to...

- (a) 0
- (b) 1
- (c) $\frac{2}{(2m+1)(2n+1)}$
- (d) $\frac{m!n!}{2(m+n)}$

48. The Rodrigues formula for $P_n(x)$ the Legendre polynomial of degree n is expressed as $P_n(x) = K \frac{d^n}{dx^n} |(x_2 - 1)^n|$ then the value of K is..

- (a) $\frac{n!}{2^n}$
- (b) $\frac{2^n}{n!}$
- (c) $\frac{1}{2^n n!}$
- (d) $\frac{1}{2^n (n!)^2}$

49. The value of integral $\int_{-1}^1 (P_0 + P_1 + 2P_2) P_2 dx$ is equal to

- (a) $\frac{2}{5}$
- (b) $\frac{4}{5}$
- (c) 2
- (d) 0

$$\begin{aligned}
 \int_{-1}^1 (P_0 + P_1 + 2P_2) P_2 dx &= \int_{-1}^1 \left\{ \left(1 + x + 2 \times \frac{1}{2} (3x^2 - 1) \right) \right\} \frac{1}{2} (3x^2 - 1) dx \\
 &= \int_{-1}^1 \{(1 + x + 3x^2 - 1)\} \frac{1}{2} (3x^2 - 1) dx \\
 &= \int_{-1}^1 \{(x + 3x^2)\} \frac{1}{2} (3x^2 - 1) dx \\
 &= \frac{1}{2} \int_{-1}^1 (3x^3 - x + 9x^4 - 3x^2) dx \\
 &= \frac{1}{2} \left\{ \frac{3x^4}{4} - \frac{x^2}{2} + \frac{9x^5}{5} - \frac{3x^3}{3} \right\}_{-1}^1 = \frac{1}{2} \left\{ \frac{3}{4} - \frac{1}{2} + \frac{9}{5} - \frac{3}{3} - \frac{3}{4} + \frac{1}{2} + \frac{9}{5} - \frac{3}{3} \right\} \\
 &= \frac{1}{2} \left\{ +\frac{9}{5} - 1 + \frac{9}{5} - 1 \right\} = \frac{1}{2} \left\{ \frac{18}{5} - 2 \right\} = \frac{1}{2} \left\{ \frac{8}{5} \right\} = \frac{4}{5}
 \end{aligned}$$

50. Which of the following is not true?

- (a) $H_{2n}(0) = (-1)^n \frac{2n!}{n!}$
- (b) $H_{2n+1}(0) = 0$
- (c) $H'_{2n}(0) = 0$
- (d) $H'_{2n+1}(0) = 0$

51. The value of $H_2(x)$ is

- (a) $x^2 - 1$
- (b) $2x^2 - 1$
- (c) $4x^2 - 2$
- (d) $\frac{1}{2}(3x^2 - 1)$

52. The value of $\int_0^\infty e^{-x} [L_n(x)]^2 dx$ is

- (a) 0
- (b) 1
- (c) $n!$
- (d) $\frac{1}{2^n n!}$

53. Which of the following is not equal to 1?

- (a) $P_0(x)$
- (b) $J_0(x)$
- (c) $H_0(x)$
- (d) $L_0(x)$

54. The value of $P_3(x)$

- (a) $\frac{1}{2}(3x^2 - 1)$
- (b) $\frac{1}{2}(5x^3 - 3x)$
- (c) $\frac{1}{2}(5x^3 - 1)$
- (d) $3x$