3. ALGEBRA

Two marks

1. The father's age is six times his son's age. Six years hence the age of father will be four times his son's age. Find the present ages (in years) of the son and father.

2. Solve $2x-3y = 6, x+y = 1$

3. Find the LCM of the following
i) x^4-1, x^2-2x+1 ii) $x-27, (x-3)^2, x^2-9$ iii)p^2-3p+2, p^2-4 iv)$2x^2-5x-3, 4x^2-36$

4. Reduce the rational expressions to its lowest form
i) $\frac{x^2-16}{x^2+8x+16}$ ii) $\frac{9x^2+81x}{x^3+8x^2-9x}$ iii) $\frac{x^2-11x+18}{x^2-4x+4}$

5. Find the excluded values of the following expressions

i) $\frac{7p}{8p2+13p+5}$ ii) $\frac{t}{t^2-5t+6}$ iii) $\frac{x^3-27}{x^3+x^2-6x}$

6. Simplify
i) $\frac{x^3}{x-y} + \frac{y^3}{y-x}$ ii) $\frac{x(x+1)}{x-2} + \frac{x(1-x)}{x-2}$

7. Find the square root of the following expressions
i) $256(x-1)^8(x-b)^4(x-c)^{16}(x-d)^{20}$ ii) $16x^2+9y^2-24xy+24x—18y+9$

8. Find the zeroes of the quadratic expression $x^2+8x+12$

9. Write down the quadratic equation in general form for which sum and product of the roots are given below
i) 9, 14 ii) 5/3, 4 iii) -3/2, -1 iv) -7/2, 5/2

10. Find the sum and product of the roots for each of the following quadratic equations
i) $x^2+8x-65=0$ ii) $2x^2+5x+7$ iii) $x^2+3x-28=0$ iv) $x^2+3x=0$

11. Solve the following quadratic equations by factorization method
i)$4x^2-7x-2=0$ ii)$2x^2-x+\frac{1}{8}$ iii) $x^4-13x^2+42=0$

12. Solve the following quadratic equations by completing the square method

i) $x^2-3x-2=0$ ii) $2x^2-x-1=0$ iii) $9x^2-12x+4$

13. Solve the following quadratic equations by formula method

i) $x^2-2x-2=0$ ii) $2x^2-3x-3=0$ iii) $2x^2-5x+2$

14 If the difference between a number and its reciprocal is 5/4. Find the number.

15. Determine the nature of roots for the following quadratic equations

i)$x^2-x-20=0$ ii) $9x^2-24x=16=0$ iii)$2x^2-2x+9=0$

16. If the difference between the roots of the equation x2-13x+k=0is 17 find k

17. Define diagonal Matrix

18. Define transpose of a matrix.

19 .If a matrix has 16 elements, what are the possible orders it can have?

20. If a matrix has 18 elements, what are the possible orders it can have? What if it has 6 elements?

21. Construct a 3x3 matrix whose elements are

i) $a_{ij} = i^2j^2$ ii) $a_{ij} = |i-2j|$ iii) $\frac{(i+j)^3}{3}$

22. If A= $\begin{pmatrix} 5 & 4 & 3 \\ 1 & -7 & 9 \\ 3 & 8 & 2 \end{pmatrix}$ then find the transpose of A

23.. If A= $\begin{pmatrix} 5 & 2 & 2 \\ 17 & 0.7 & 5/2 \\ 8 & 3 & 1 \end{pmatrix}$ then verify $(A^T)^T=A$

24. Find the values of *x, y* and *z* from the following equations

i) $\begin{pmatrix} x+y & 2 \\ 5+z & xy \end{pmatrix} = \begin{pmatrix} 6 & 2 \\ 5 & 8 \end{pmatrix}$ ii) $\begin{pmatrix} 12 & 3 \\ x & 3/2 \end{pmatrix} = \begin{pmatrix} y & z \\ 3 & 5 \end{pmatrix}$

25. If A = $\begin{pmatrix} 1 & 4 & 5 \\ 7 & 5 & 2 \\ 1 & 3 & 5 \end{pmatrix}$ and B = $\begin{pmatrix} 8 & 6 \\ 5 & 4 \\ 2 & 9 \end{pmatrix}$ find A +B

26. If $A = \begin{bmatrix} 7 & 8 & 6 \\ 1 & 3 & 9 \\ -4 & 3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{bmatrix}$ find 2A +B

27. If $A = \begin{bmatrix} 0 & 4 & 9 \\ 8 & 3 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{bmatrix}$

Find the value of i) B-5A ii)3A-9B

28. If $A = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$ find AB and BA.

29. Solve $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

30. If $A = \begin{bmatrix} \cos\theta & 0 \\ 0 & \cos\theta \end{bmatrix}$ and $B = \begin{bmatrix} \sin\theta & 0 \\ 0 & \sin\theta \end{bmatrix}$ then show that $A^2+B^2=I$

31. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ Prove that $AA^T=I$

32. Verify that $A^2=I$ when $A = \begin{bmatrix} 5 & -4 \\ 6 & -5 \end{bmatrix}$

Five marks

1. Solve the following system of linear equations in three variables
i) 3x-2y+z =2, 2x+3y-z=5, x + y + z=6
ii) x+ 2y –z =5, x- y + z =-2, -5x-4y+z=-11
iii)x + y+ z=5, 2x-y+z=59, x-2y+3z = 16

2. Discuss the nature of solutions of the following system of equations
i) 3x-y+z = 1, 2x-y+2z=1, -x-y +z = 2
ii)x+2y-z= 6, -3x-2y+5z= -12, x-2z=3
iii) 2y+z=3(-x+1), -x+3y-z=-4, 3x+2y+z=-1/2

3. Vani, her father and her grand father have an average age of 53. One-half of her grand father's age plus one-third of her father's age plus one fourth of Vani's age is 65. Four years ago if Vani's grandfather was four times as old as Vani then how old are they all now

4. There are 12 pieces of five, ten and twenty rupee currencies whose total value is ₹105. When first 2 sorts are interchanged in their numbers its value will be increased by ₹20. Find the number of currencies in each sort

5. Find the GCD of the polynomials

i) x^3+x^2-x+2 and $2x^3-5x+5x-3$

ii) $6x^3-30x^2+60x-48$ and $3x^3-12x^2+21x-18$

iii) $3x^4+6x^3-12x^2-24x$ and $4x^4+14x^3+8x^2-8x$

iv) $3x^3+3x^2+3x+3$ and $6x^3+12x^2+6x+12$

6. Simplify $\dfrac{2a^2+5a+3}{3a^2+7a+6} \div \dfrac{a^2+6a+5}{-5a+-35a-50}$

7If $x = \dfrac{a^2+3a-4}{3a^2-3}$ and $y = \dfrac{a^2+2a-8}{2a^2-2a-4}$ find the value of x^2y^{-2}

8. Simplify $\dfrac{1}{x^2-5x+6} + \dfrac{1}{x^2-3x+2} - \dfrac{1}{x^2-8x+15}$

9. If $A = \dfrac{2x+1}{2x-1}$ and $B = \dfrac{2x-1}{2x+1}$ find $\dfrac{1}{A-B} - \dfrac{2B}{A^2-B^2}$

10. If $A = \dfrac{x}{X+1}$ and $B = \dfrac{1}{x+1}$ find $\dfrac{(A+B)^2+(A-B)^2}{A \div B} = \dfrac{2(x^2+1)}{x(x+1)^2}$

11. Find the square root of

i) $(6x^2+x-1)(3x^2+2x-1)(2x^2+3x+1)$ ii) $(4x^2-9x+2)(7x^2-13x-2)(28x^2-3x-1)$

12. Find the square root of the following polynomials by division method

i) $\dfrac{x^2}{y^2} - \dfrac{10x}{y} + 27 - \dfrac{10y}{x} + \dfrac{y^2}{x^2}$

ii) $\dfrac{4x^2}{y^2} + \dfrac{20x}{y} + 13 - \dfrac{30y}{x} + \dfrac{9y^2}{x^2}$

iii) $64x^4-16x^3+17x^2-2x+1$

iv) $x^4-12x^3+42x^2-36x+9$

v) $121x^4-198x^3+183x^2+216x+144$

13. Find the values of *a* and *b* if the following polynomials are perfect squares

i) $9x^4+12x^3+28x^2+ax+b$ ii) $4x^4-12x^3+37x^2+bx+a$

iii) $ax^4+bx^3+361x^2+220x+100$

14. Find the values of *m* and *n*if the following polynomials are perfect squares

i) $x^4-8x^3+mx^2+nx+16$ ii) $\frac{1}{x^4} - \frac{6}{x^3} + \frac{13}{x^2} + \frac{m}{x} + n$

15. A passenger train takes 1 hr more than an express train to travel a distance of 240 km from Chennai to Virudhachalam. The speed of passenger train is less than that of an express train by 20 km per hour. Find the average speed of both the trains.

16. A bus covers a distance of 90 km at a uniform speed. Had the speed been 15 km/hour more it would have taken 30 minutes less for the journey. Find the original speed of the bus.

17. A pole has to be erected at a point on the boundary of a circular ground of diameter 20 m in such a way that the difference of its distances from two diametrically opposite fixed gates P and Q on the boundary is 4 m. Is it possible to do so? If answer is yes at what distance from the two gates should the pole be erected?

18.In a town of 8000 people, 1300 are over 50 years and 3000 are females. It is known that 30% of the females are over 50 years. What is the probability that a chosen individual from the town is either a female or over 50 years?

19. There is a square field whose side is 10 m. A square flower bed is prepared in its centre leaving a gravel path all round the flower bed. The total cost of laying the flower bed and gravelling the path at ₹3 and ₹4 per square metre respectively is ₹364. Find the width of the gravel path.

20. Prove that the equation $x^2(p^2+q^2)+2x(pr+qs)+r^2+s^2=0$ has no real roots.
If $ps=qr$, then show that the roots are real and equal.

21. If α, β *are* the roots of the equation$x^2+7x+10=0$ find the values of

i)$\alpha-\beta$ ii)$\alpha^2+\beta^2$ iii)$\alpha^3-\beta^3$ iv) $\alpha^4+\beta^4$ v) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ vi) $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

22. If α, β *are* the roots of the equation $3x^2+7x-2=0$ find the values of

i) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ ii) $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

23. If α, β *are* the roots of the equation $2x^2-x-1=0$ then form the equation whose roots are

i) $\frac{1}{\alpha}, \frac{1}{\beta}$ ii) $\alpha^2\beta + \beta^2\alpha$ iii) $2\alpha+2\beta$

24. If α, β *are* the roots of the equation $x^2+6x-4=0$ then form the equation whose roots are

i) α^2 and β^2 ii) $\frac{2}{\alpha}$ and $\frac{2}{\beta}$ iii) $\alpha^2\beta$ and $\beta^2\alpha$

25. If α, β *are* the roots of the equation $7x^2+ax+2=0$ and if $\beta - \alpha = -13/7$ find the values of a.

26. Find x and Y if $X+Y = \begin{bmatrix} 7 & 0 \\ 3 & 5 \end{bmatrix}$ and $X-Y = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$

27. solve for x, y $\begin{bmatrix} x2 \\ y2 \end{bmatrix} + 2\begin{bmatrix} -2x \\ -y \end{bmatrix} = \begin{bmatrix} -5 \\ 8 \end{bmatrix}$

28. If A = (1 -1 2) $B = \begin{bmatrix} 1 & -1 \\ 2 & 1 \\ 1 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

then show that (AB) C=A (BC)

29. If $A = \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 \\ -4 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} -7 & 6 \\ 3 & 2 \end{bmatrix}$

Verify that A(B + C) = AB + AC

30. If $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 2 & -1 \\ -1 & 4 \\ 0 & 2 \end{bmatrix}$ show that $(AB)^T=B^TA^T$

31. If $A = \begin{pmatrix} 5 & 2 & 9 \\ 1 & 2 & 8 \end{pmatrix}$ $B = \begin{pmatrix} 1 & 7 \\ 1 & 2 \\ 5 & -1 \end{pmatrix}$ show that $(AB)^T = B^T A^T$

32. If $A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$ show that $A^2 - 5A + 7I_2 = 0$

33. If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ then show that $A^2 - (a+d)A = (bc-ad)I_2$

34. If $A = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 4 & 0 \\ 1 & 5 \end{pmatrix}$, $C = \begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix}$

Show that i) $A(BC) = (AB)C$ ii) $(A - B)C = AC - BC$ iii) $(A-B)^T = A^T - B^T$

Prepared by
S.MURUGAVEL M.Sc.,B.Ed.,
TGT in Mathematics
Puducherry state
Email id: murugavel213@gmail.com

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