Register number

PART-III

MATHEMATICS

MODEL QUESTION PAPER-2021

Time allowed: 3.00 Hours] [Maximum marks: 90

Instructions:

- Check the question paper for fairness of printing. If there is any lack of fairness, inform the hall supervisor immediately.
- Use **Blue** or **Black** ink to write and underline and pencil to draw diagrams.

PART-I

Note: Choose the most appropriate answer from the given **four** alternatives and write the option code and the corresponding answer. **20 x 1=20**

1. If the volume of the parallelepiped with $\vec{a}+\vec{b}, \vec{b}+\vec{c}$ and $\vec{c}+\vec{a}$ is 8 cubic units then the volume of parallelepiped with $\vec{a}X\vec{b},\ \vec{b}X\vec{c}$ and $\vec{c}X\vec{a}$ as coterminous edges is:

(a) 8 cubic units (b) 16 cubic units (c) 64 cubic units (d) 27 cubic units

2. The number of solution of the equation $sin^{-1}\frac{3x}{5}+sin^{-1}\frac{4x}{5}=sin^{-1}x, x\,\epsilon\,[-1,1]$ is:

(a) 0 (b) 1 (c) 2 (d) 3

3. Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{9}+\frac{y^2}{25}=1$ is:

(a) 30 (b) 15 (c) $\sqrt{15}$ (d) $\frac{1}{5}$

4. If A is a square matrix of order 3 such that |adj A| = 64 then the value of |A| is:

(a) 4 (b) 8 (c) 10 (d) 16

5. The volume of cube is increasing at the constant rate of 3 cm^3/s then the rate of change of the cube when its edge is 5cm is:

(a) 25 cm^3/sec (b) 25 cm/sec (c) $\frac{1}{25}$ cm^2/sec (d) $\frac{1}{25}$ cm/sec

6. Let P be the point (1,0) and Q a point on a locus $y^2 = 8x$, then locus of PQ is:

(a) $y^2 - 4x + 2 = 0$ (b) $y^2 + 4x + 2 = 0$ (c) $y^2 - 4x - 2 = 0$ (d) $y^2 + 4x - 2 = 0$

7. If the equations $x^2 + 2x + 3 = 0$ and $x^2 + 4x + 2 = 0$ have a common root, that it must be equal to:

(a) – 8 (b) $\frac{1}{2}$ (c) – 4 (d) option (a) or (b)

8. If $\hat{\imath}, \hat{\jmath}, \hat{k}$ are the three unit vectors and mutually perpendicular, then $[\hat{\imath}, \hat{\jmath}, \hat{k}]$ is equal to:

(a) 0 (b) – 1 (c) 1 (d) None of these

9. The area of the region bounded by the parabolas $y^2 = 6x$ and $x^2 = 6y$ is:

(a) 2 sq. units (b) 4 sq. units (c) 8 sq. units (d) 12 sq. units

10. If $y = e^{x^2}$, $x = 1$ and $dx = 1$ then the value of dy is:

(a) 2 (b) 2e (c) $2e^2$ (d) 0.2e

11. The order and degree of the differential equation $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = \frac{d^2y}{dx^2}$ is:

(a) 1, 1 (b) 2, 2 (c) 2, 3 (d) 2, 1

12. The general solution of $\frac{dy}{dx} = 2xe^{x^2-y}$ is:

(a) $e^{x^2-y} = c$ (b) $e^{-y} + e^{x^2} = c$ (c) $e^y = e^{x^2} + c$ (d) $e^{x^2+y} = c$

13. The value of $\int_0^\infty \frac{x^2}{2^x}\,dx$ is:

(a) $\frac{4}{\log 2}$ (b) $\frac{8}{\log 16}$ (c) $\frac{2}{\log 8}$ (d) $\frac{2}{\log 4}$

14. The rectangular form of the complex number $\begin{vmatrix} i^2 & i \\ -i & i \end{vmatrix}$ is:

(a) $1 - i$ (b) $-1 - i$ (c) $1 + i$ (d) $-1 + i$

15. For a continuous random variable X, the cumulative distribution function satisfies:

(a) $0 < F(X) < 1$ (b) $F(-\infty) = 1$ (c) $F(+\infty) = 1$ (d) $F(X)$ is discontinuous

16. The value of $\lim_{x \to 1^-} \left(\frac{\log(1-x)}{\cot(\pi x)}\right)$ is:

(a) 0 (b) 1 (c) ∞ (d) – 2

17. The Cartesian equation of the plane passing through the points A(2, 2, - 1), B(3, 4, 2) and C(7, 0, 6) is:

(a) $5x - 2y + 3z = 17$ (b) $5x + 2y + 3z = 17$ (c) $5x - 2y - 3z = 17$ (d) $5x + 2y - 3z = 1$

18. Which one of the following is tautology?

(a) $(p \wedge q) \rightarrow (p \rightarrow q)$
(b) $(p \wedge q) \vee (p \rightarrow q)$
(c) $(p \wedge q) \wedge (p \rightarrow q)$
(d) $(p \wedge q) \rightarrow (p \leftrightarrow q)$

19. If $A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$ and $A^8 \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$, then the equation has:

(a) One solution (b) two solutions (c) Three solutions (d) No solutions

20. How many rows are needed for the statement $((p \wedge q) \vee (\neg r \rightarrow \neg s)) \rightarrow (\neg t \leftrightarrow v)$?

(a) 8 (b) 32 (c) 64 (d) 128

PART-II

Note: Answer **any 7** questions. Q.no:30 is **compulsory.** **7x2=14**

21. Evaluate: $\left[i^{18} + \left(\frac{1}{i}\right)^{25}\right]^3$

22. Find the domain of the function $sin^{-1}\left(\frac{|x|-2}{3}\right) + cos^{-1}\left(\frac{1-|x|}{4}\right)$.

23. Prove that the acute angle between the lines $\vec{r} = (\hat{\imath} + 2\hat{\jmath} + 4\hat{k}) + t(2\hat{\imath} + 2\hat{\jmath} + \hat{k})$ and the straight line passing through the points (5, 1, 4) and (9, 2, 12) is $cos^{-1}\left(\frac{2}{3}\right)$.

24. Using linear approximation, estimate the value of $\sqrt{49.5}$.

25. If $z = x + iy, w = \frac{1-iz}{z-i}$ and $|w| = 1$, then show that z is purely real.

26. Find the equation of the circle passing through the point (2, 4) and having its centre at the intersection of the lines $x - y = 4$ and $2x + 3y = -7$.

27. Prove that $\int_0^{\pi} \frac{x}{1+\sin x}\, dx = \pi \int_0^{\frac{\pi}{2}} \frac{1}{1+\cos x}\, dx$.

28. Find the order and degree of the differential equation $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{4/8} = y\frac{d^3y}{dx^3}$.

29. Find the constant C such that the function $f(x) = \begin{cases} cx^2 & 1 \le x < 4 \\ 0 & otherwise \end{cases}$ is a density function and compute P(3<x).

30. Write down the (i) Conditional statement (ii) Converse statement (iii) Inverse statement (iv) Contrapositive statement for the two statements p and q given below:

p: Today is Monday **q:** 4 + 4 = 8

PART-III

Note: Answer **any 7** questions. Q.no: 40 is **compulsory.** **7x3=21**

31. A chemist has one solution which is 50% acid and another solution which is 25% acid. How much each should be mixed to make 10 litres of 40% acid solution? (Use Cramer's rule)

32. Solve the equation $x^3 - 9x^2 + 14x + 24 = 0$ if it is given that two of its roots are in the ratio 3:2.

33. An equation of the elliptical part of an optical lens system is $\frac{x^2}{16} + \frac{y^2}{9} = 1$. The parabolic part of the system has a focus in common with the right focus of the ellipse. The vertex of the parabola is at the origin and the parabola opens to the right. Determine the equation of parabola.

34. Find the direction cosines of the straight line passing through the points (5, 6, 7) and (7, 9, 13). Also find the parametric form of vector equation and the Cartesian equation of the straight line passing through two given points.

35. Salt is poured from a conveyer belt at a rate of 30 cubic metre per minute forming a conical pile with a circular base whose height and diameter of base are always equal. How fast is height of pile increasing when the pile is 10 metre high?

36. Find a linear approximation for $f(x) = x^3 - 5x + 12$ at the indicated point $x_0 = 2$.

37. Show that $\Gamma(n) = 2\int_0^\infty e^{-x^2} x^{2n-1}\, dx$.

38. Solve: $\left(1 + 2e^{\frac{x}{y}}\right) dx + 2e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right) dy = 0$.

39. A random variable X has the following probability mass function.

x	1	2	3	4	5	6	7
f(x)	k	2k	6k	5k	6k	10k	5k

Find (i) $P(2 < X < 6)$ (ii) $P(2 \le X < 5)$ (iii) $P(X \le 4)$

40. Find the length of transverse and conjugate axes, eccentricity and coordinate of foci, vertices, length of latus rectum and the equation of directrix of Hyperbola $25x^2 - 36y^2 = 225$.

PART-IV

Note: Answer **all** the questions: **7x5=35**

41. In a murder investigation, a corpse was found by a detective at exactly 8 pm. Being alert, the detective also measured the body temperature and found it to be 70°F. Two hours later, the detective measured the body temperature again and found it to be 60°F. If the room temperature is 50°F, and assuming that body temperature of a person before death was 98.6°F at what time did the murder occur?

$$[\log(2.43) = 0.88789, \quad \log(0.5) = -0.69315]$$

(or)

Using the properties of integral, evaluate $\int_0^{\sin^2 x} \sin^{-1}\sqrt{t}\, dt + \int_0^{\cos^2 x} \cos^{-1}\sqrt{t}\, dt$.

42. Prove that the curves $x^2 = 4y$ and $y^2 = 4x$ divide the area of the square bounded by $x = 0, x = 4$ and $y = 0, y = 4$ into three equal parts.

(or)

A steel plant is capable of producing x tonnes per day of a low grade steel and y tonnes per day of high grade steel, where $y = \frac{40-5x}{10-x}$. if the fixed market price of low-grade steel is half that of high-grade steel, then what should be the optimal productions in low grade steel and high grade steel in order to have maximum receipts.

43. Solve the first order linear differential equation $dy = \cos x(2 - y\ cosec\ x)dx$ given that $y\left(\frac{\pi}{2}\right) = 2$.

(or)

Two coast guard stations are located 600 km apart at points A(0, 0) and B(0, 600). A distress signal from the ship at P is received at slight different times by two stations. It is determined that the ship is 200 km farther from the station A than it is from station B. determine the equation of hyperbola that passes through the location of the ship.

44. An automobile company uses three types of steels A, B and C for producing three types of cars P, Q and R. steel requirement for each type of cars are given below. Find the number of cars of each type which can be produced using 29, 13 and 16 tonnes of steel for three types. (Use Gaussian elimination method).

Steel \ Cars	P	Q	R
A	2	3	4
B	1	1	2
C	3	2	1

(or)

Solve: $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$

45. If G is the centroid of ΔABC prove that (area of ΔGAB) = (area of ΔGBC) = (area of ΔGCA) = $\frac{1}{3}$(area of ΔABC).

(or)

The probability density function of x is given by $f(x) = \quad f(x) = \begin{cases} 0, & x < 0 \\ ke^{-\frac{x}{3}}, & x > 0 \end{cases}$

Find (i) Value of k (ii) Distribution function (iii) P(X < 3) (iv) P(5 ≤ X) (v) P(X ≤ 4)

46. Prove that $p \to (q \to r) \equiv (p \wedge q) \to r$ without using truth table.

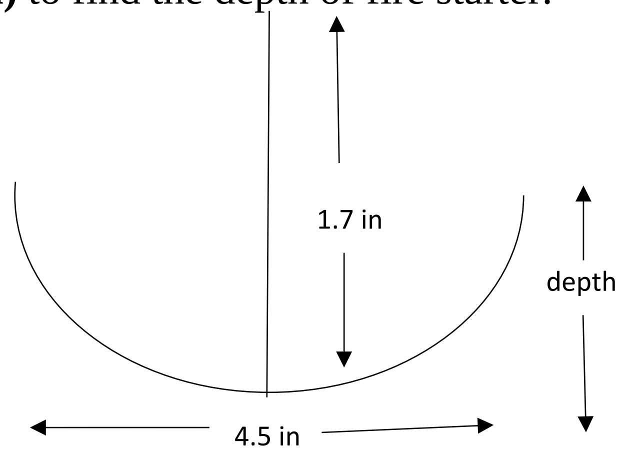
(or)

Find the parametric vector, non-parametric vector and Cartesian form of the equations of the plane passing through non-collinear points (3, 6, -2), (-1, -2, 6) and (6, -4, -2).

47. A cross section of a design for a travel sized solar fire starter is shown in below diagram. The sun's rays reflect off the parabolic mirror towards an object attached to the igniter. Because the igniter is located at the focus of parabola, the reflected rays cause object to burn in just seconds.

(i) Find the equation of parabola that modes the fire starter. Assume that vertex of parabolic mirror is the origin of coordinate plane.

(ii) Use the equation found in **(i)** to find the depth of fire starter.



(or)

(i) Find the complex number satisfying the equation $z + \sqrt{2}|z+1| + i = 0$.

(ii) Verify the following properties:

(a) $|z_1 + z_2| \le |z_1| + |z_2|$

(b) $|z_1 - z_2| \ge ||z_1| - |z_2||$

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