10th STD

Reg. No.

PUBLIC EXAMINATION - APRIL 2024

PART - III

Mathematics (With Answers)

Time Allowed : 3.00 Hours] [Maximum Marks : 100

Instructions : (1) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.

(2) Use **Blue** or **Black** ink to write and underline and pencil to draw diagrams.

Note : This question paper contains **four** parts.

Part - I

Note : (i) Answer **all** the questions. **14 × 1 = 14**

(ii) Choose the most appropriate answer from the given **four** alternatives and write the option code and the corresponding answer.

1. If $n(A \times B) = 6$ and $A = \{1, 3\}$, then $n(B)$ is :
(a) 1 (b) 2 (c) 3 (d) 6

2. If $f : A \to B$ is a bijective function and if $n(B) = 7$, then $n(A)$ is equal to :
(a) 7 (b) 49 (c) 1 (d) 14

3. The least number that is divisible by all the numbers from 1 to 10 (both inclusive) is :
(a) 2025 (b) 5220 (c) 5025 (d) 2520

4. An A.P. consists of 31 terms. If its 16^{th} term is m, then the sum of all the terms of this A.P. is :
(a) $16\,m$ (b) $62\,m$ (c) $31\,m$ (d) $\frac{31}{2}m$

5. Which of the following should be added to make $x^4 + 64$ a perfect square?
(a) $4x^2$ (b) $16x^2$ (c) $8x^2$ (d) $-8x^2$

6. Graph of a linear equation is a ________ .
(a) straight line (b) circle
(c) parabola (d) hyperbola

7. If in ΔABC, $DE \parallel BC$, AB = 3.6 cm, AC = 2.4 cm and AD = 2.1 cm then the length of AE is :
(a) 1.4 cm (b) 1.8 cm
(c) 1.2 cm (d) 1.05 cm

8. How many tangents can be drawn to the circle from an exterior point?
(a) One (b) Two
(c) Infinite (d) Zero

9. The area of triangle formed by the points (–5, 0), (0, –5) and (5, 0) is :
(a) 0 sq. units (b) 25 sq. units
(c) 5 sq. units (d) 10 sq. units

10. If $x = a\tan\theta$ and $y = b\sec\theta$, then :
(a) $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$ (b) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
(c) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (d) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$

11. The curved surface area of a right circular cylinder of height 4 cm and base diameter 10 cm is :
(a) 40 π sq. units (b) 20 π sq. units
(c) 14 π sq. units (d) 80 π sq. units

12. The ratio of the volumes of a cylinder, a cone and a sphere, if each has the same diameter and same height is :
(a) 1 : 2 : 3 (b) 2 : 1 : 3
(c) 1 : 3 : 2 (d) 3 : 1 : 2

13. Which of the following values cannot be a probability of an event?
(a) 0 (b) 0.5 (c) 1.05 (d) 1

14. The probability of getting a job for a person is $\frac{x}{3}$. If the probability of not getting the job is $\frac{2}{3}$, then the value of x is :
(a) 2 (b) 1 (c) 3 (d) 1.5

Part - II

Note : Answer **any 10** questions. Question No.**28** is **compulsory.** **10 × 2 = 20**

15. If $A \times B = \{(3, 2), (3, 4), (5, 2), (5, 4)\}$ then find A and B.

16. If $f(x) = 3x - 2$, $g(x) = 2x + k$ and $fog = gof$, then find the value of k.

17. 'a' and 'b' are two positive integers such that $a^b \times b^a = 800$. Find 'a' and 'b'.

18. Simplify : $\frac{4x^2y}{2z^2} \times \frac{6xz^3}{20y^4}$

19. Find the sum and product of the roots for following quadratic equation. $x^2 + 8x - 65 = 0$

20. A man goes 18 m due East and then 24 m due North. Find the distance of his current position from the starting point.

21. If the points A (–3, 9), B(a, b) and C(4, –5) are collinear and if $a + b = 1$, then find a and b.

22. Find the equation of a straight line which has slope $\frac{-5}{4}$ and passing through the point (–1, 2).

23. Prove that $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \operatorname{cosec}\theta + \cot\theta$.

24. If the base area of a hemispherical solid is 1386 sq. metres, then find its total surface area.

25. Find the volume of cylinder whose height is 2 m and base area is 250 sq. m.

26. Find the range and coefficient of range of the following data :
25, 67, 48, 53, 18, 39, 44

27. What is the probability that a leap year selected at random will contain 53 Saturdays?

28. Find the HCF of 23 and 12.

Part - III

Note : Answer **any 10** questions. Question No.**42** is **compulsory**. **10 × 5 = 50**

29. Let A = $\{x \in \mathbb{N} \mid 1 < x < 4\}$, B = $\{x \in W \mid 0 \le x < 2\}$ and C = $\{x \in \mathbb{N} \mid x < 3\}$. Then verify that $A \times (B \cup C) = (A \times B) \cup (A \times C)$

30. Let A = {0, 1, 2, 3} and B = {1, 3, 5, 7, 9} be two sets. Let f : A → B be a function given by $f(x) = 2x + 1$. Represent this function
(i) by arrow diagram (ii) in a table form
(iii) as a set of ordered pairs
(iv) in a graphical form

31. Find the sum of $9^3 + 10^3 + \ldots\ldots\ldots\ldots\ldots 21^3$

32. Find the square root of $64x^4 - 16x^3 + 17x^2 - 2x + 1$.

33. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ show that $A^2 - 5A + 7I_2 = 0$.

34. State and prove Thales Theorem.

35. Find the area of quadrilateral whose vertices are at (–9, –2), (–8, –4), (2, 2) and (1, –3).

36. Find the equation of the perpendicular bisector of the line joining the points A(–4, 2) and B(6, –4).

37. Two ships are sailing in the sea on either sides of a lighthouse. The angle of elevation of the top of the lighthouse as observed from the ships are 30° and 45° respectively. If the lighthouse is 200m high, find the distance between the two ships. $(\sqrt{3} = 1.732)$

38. If the radii of the circular ends of a frustum which is 45 cm high are 28 cm and 7 cm, find the volume of the frustum.

39. A right circular cylindrical container of base radius 6 cm and height 15 cm is full of ice-cream. The ice-cream is to be filled in cones of height 9 cm and base radius 3 cm, having a hemispherical cap. Find the number of cones needed to empty the container.

40. Find the coefficient of variation of 24, 26, 33, 37, 29, 31.

41. Two dice are rollled once. Find the probability of getting an even number on the first die or the total of face sum 8.

42. Find the sum to n terms of the series 7 + 77 + 777 + ...

Part - IV

Note : Answer **all** the questions. **2 × 8 = 16**

43. (a) Construct a ΔPQR which the base PQ = 4.5 cm, $\angle R = 35°$ and the median RG from R to PQ is 6 cm. **(OR)**

(b) Draw a circle of diameter 6 cm. from a point P, which is 8 cm. away from its centre. Draw the two tangents. PA and PB to the circle and measure their lengths.

44. (a) Draw the graph of $y = 2x^2 - 3x - 5$ and hence solve $2x^2 - 4x - 6 = 0$. **(OR)**

(b) Draw the graph of $xy = 24$, $x, y > 0$. Using the graph find,
(i) y when $x = 3$ and (ii) x when $y = 6$.

❁❁❁❁

ANSWERS

Part - I

1.	(c)	3	**2.**	(a)	7
3.	(d)	2520	**4.**	(c)	31 m
5.	(b)	$16x^2$	**6.**	(a)	straight line
7.	(a)	1.4 cm	**8.**	(b)	Two
9.	(b)	25 sq. units	**10.**	(a)	$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$
11.	(a)	40 π sq. units	**12.**	(d)	3 : 1 : 2
13.	(c)	1.05	**14.**	(b)	1

Part - II

15. $A \times B$ = {(3,2), (3,4), (5,2), (5,4)}
We have A = {set of all first coordinates of elements of $A \times B$}. ∴ A = {3, 5}
B = {set of all second coordinates of elements of $A \times B$}. ∴ B = {2,4}.
Thus A = {3,5} and B = {2,4}.

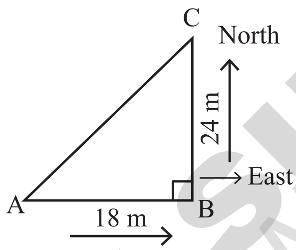
16.
$f(x) = 3x - 2,\; g(x) = 2x + k$
$fog(x) = f(g(x)) = f(2x + k)$
$= 3(2x + k) - 2 = 6x + 3k - 2$
Thus, $fog(x) = 6x + 3k - 2$
$gof(x) = g(3x - 2) = 2(3x - 2) + k$
Thus, $gof(x) = 6x - 4 + k$
Given that $fog = gof$
$\therefore\; 6x + 3k - 2 = 6x - 4 + k$
$6x - 6x + 3k - k = -4 + 2 \Rightarrow k = -1$

17. The number 800 can be factorized as
$800 = 2\times 2 \times 2 \times 2 \times 2 \times 5 \times 5 = 2^5 \times 5^2$
Hence, $a^b \times b^a = 2^5 \times 5^2$
This implies that $a = 2$ and $b = 5$ (or) $a = 5$ and $b = 2$

18. $\frac{4x^2y}{2z^2} \times \frac{6xz^3}{20y^4} = \frac{3x^3yz^3}{5y^4z^2} = \frac{3x^3z}{5y^3}$

19. Let α and β be the roots of the given quadratic equation
$x^2 + 8x - 65 = 0$
$a = 1, b = 8, c = -65$
$\alpha + \beta = -\frac{b}{a} = -8$ and $\alpha\beta = \frac{c}{a} = -65$
$\alpha + \beta = -8;\; \alpha\beta = -65$

20. Let A be the position of the man.



Using Pythagoras theorem
$AC^2 = AB^2 + BC^2$ [∵ AB = 18 and BC = 24]
$= (18)^2 + (24)^2 = 324 + 576 = 900$
$AC = \sqrt{900} = 30$ m
∴ The distance from the starting point is 30 m.

21. $A(-3, 9),\; B(a, b),\; C(4, -5)$
$\downarrow \qquad \downarrow \qquad \downarrow$
$(x_1, y_1) \quad (x_2, y_2) \quad (x_3, y_3)$
are collinear points, $a + b = 1$ (given)

$\therefore$ Area of the $\Delta = \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix}$ sq. units

$= \frac{1}{2}\begin{vmatrix} -3 & a & 4 & -3 \\ 9 & b & -5 & 9 \end{vmatrix} = 0$ (∵ points are collinear)

$(-3b - 5a + 36) - (9a + 4b + 15) = 0$
$(-3b - 4b) + (-5a - 9a) + (36 - 15) = 0$
$-7b - 14a = -21$
$-7(b + 2a) = -21$
$b + 2a = 3$
$(b + a) + a = 3$
$1 + a = 3$
$= a = 2 \Rightarrow b = 1 - 2 = -1$
$a = 2$
$b = -1$

22. $m = \frac{-5}{4}$, point $= (-1, 2) = (x_1, y_1)$
$\Rightarrow\; y - 2 = \frac{-5}{4}(x - (-1))$
[∵ $y - y_1 = m(x - x_1)$]
$\Rightarrow\; y - 2 = \frac{-5}{4}(x + 1)$
$\Rightarrow\; 4(y - 2) = -5(x + 1)$
$\Rightarrow\; 4y - 8 = -5x - 5$
$\Rightarrow\; 5x + 4y = 3 \Rightarrow 5x + 4y - 3 = 0$

23. $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \sqrt{\frac{1+\cos\theta}{1-\cos\theta}\times\frac{1+\cos\theta}{1+\cos\theta}}$
[multiply numerator and denominator by the conjugate of $1 - \cos\theta$]
$\sqrt{\frac{(1+\cos\theta)^2}{1-\cos^2\theta}} = \frac{1+\cos\theta}{\sqrt{\sin^2\theta}}$ [∵ $\sin^2\theta + \cos^2\theta = 1$]
$= \frac{1+\cos\theta}{\sin\theta} = \text{cosec}\,\theta + \cot\theta$

24. Let r be the radius of the hemisphere.
Given that, base area $= \pi r^2 = 1386$ sq. m
T.S.A. $= 3\pi r^2$ sq.m $= 3 \times 1386 = 4158$
Therefore, T.S.A. of the hemispherical solid is 4158 m^2.

25. Let r and h be the radius and height of the cylinder respectively
Given that, height $h = 2$ m, base area $= 250$ m^2
Now, volume of a cylinder $= \pi r^2 h$ cu. units
$= \textit{base area} \times h$
$= 250 \times 2 = 500$ m^3
Therefore, volume of the cylinder = 500 m^3

26. Largest value $L = 67$; Smallest value $S = 18$
Range $R = L - S = 67 - 18 = 49$
Coefficient of range $= \frac{L - S}{L + S}$
Coefficient of range $= \frac{67 - 18}{67 + 18} = \frac{49}{85} = 0.576$

27. A leap year has 366 days. So it has 52 full weeks and 2 days. 52 Saturdays must be in 52 full weeks.

The possible chances for the remaining two days will be the sample space.

S = {Sun - Mon, Mon - Tue, Tue - Wed, Wed-Thu, Thu-Fri, Fri-Sat, Sat-Sun}

$n(S) = 7$

Let A be the event of getting 53^{rd} Saturday.

Then A = {Fri - Sat, Sat - Sun}; $n(A) = 2$

Probability of getting 53 Saturdays in a leap year is

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{7}$$

28. HCF of 23 and 12

$23 = 12 \times 1 + 11$

$12 = 11 \times 1 + 1$

$11 = 1 \times 11 + 0 \qquad \therefore$ HCF = 1

Part - III

29. $A = \{x \Sigma N \mid 1 < x < 4\} = \{2, 3\}$

$B = \{x \Sigma W\ 0 \le x < 2\} = \{0, 1\}$

$C = \{x \Sigma N \mid x < 3\} = \{1, 2\}$

$A \times (B \cup C) = (A \times B) \cup (A \times C)$

$(B \cup C) = \{0,1\} \cup \{1,2\} = \{0,1,2\}$

$A \times (B \cup C) = \{2,3\} \times \{0,1,2\} = \{(2,0,), (2,1), (2,2), (3,0), (3,1), (3,2)\}$ .... (1)

$A \times B = \{2,3\} \times \{0,1\} = \{(2,0,), (2,1), (3,0), (3,1)\}$

$A \times C = \{2,3\} \times \{1,2\} = \{(2,1), (2,2), (3,1), (3,2)\}$

$(A \times B) \cup (A \times C) = \{(2,0,), (2,1), (3,0), (3,1)\} \cup \{(2,1), (2,2), (3,1), (3,2)\}$

$= \{(2, 0), (2, 1)\ (2, 2)\ (3, 0), (3, 1), (3, 2)\}$ ..... (2)

From (1) and (2), $A \times (B \cup C) = (A \times B) \cup (A \times C)$ is verified.

30. $A = \{0, 1, 2, 3\}$, $B = \{1, 3, 5, 7, 9\}$

$f(x) = 2x + 1$

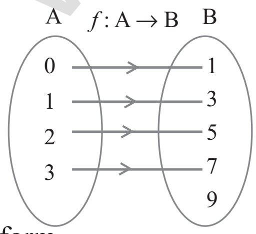
$f(0) = 2(0) + 1 = 1$

$f(1) = 2(1) + 1 = 3$

$f(2) = 2(2) + 1 = 5$

$f(3) = 2(3) + 1 = 7$

(i) An arrow diagram



(ii) A table form

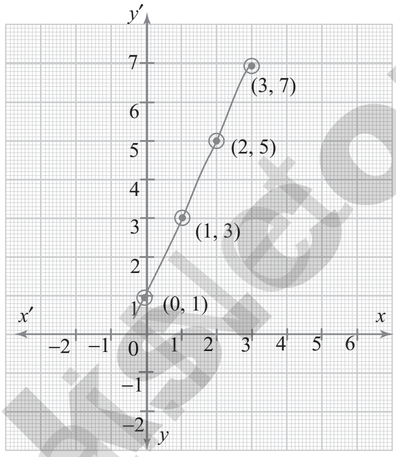
x	0	1	2	3
$f(x)$	1	3	5	7

(iii) A set of ordered pairs.

$f = \{(0, 1), (1, 3), (2, 5), (3, 7)\}$

(iv) A Graph $f = \{(x, f(x)) / x \in A\}$

$= \{(0, 1), (1, 3), (2, 5), (3, 7)\}$



31. $9^3 + 10^3 + ... + 21^3$

$= (1^3 + 2^3 + 3^3 + ... + 21^3) - (1^3 + 2^3 + 3^3 + ... + 8^3)$

$$= \left[\frac{21 \times (21+1)}{2}\right]^2 - \left[\frac{8 \times (8+1)}{2}\right]^2$$

$= (231)^2 - (36)^2 =$ **52065**

32. $64x^4 - 16x^3 + 17x^2 - 2x + 1$.

	$8x^2 - x + 1$	
$8x^2$	$64x^4 - 16x^3 + 17x^2 - 2x + 1$	
	$64x^4$	(–)
$16x^2 - x$	$-16x^3 + 17x^2$	
	$-16x^3 + x^2$	(–)
$16x^2 - 2x + 1$	$16x^2 - 2x + 1$	
	$16x^2 - 2x + 1$	(–)
	0	

Therefore, $\sqrt{64x^4 - 16x^3 + 17x^2 - 2x + 1}$

$= |8x^2 - x + 1|$

33. L.H.S $= A^2 - 5A + 7I_2$

$$A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} (9-1) & (3+2) \\ (-3-2) & (-1+4) \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$5A = 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}$$

$$7I_2 = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$A^2 - 5A + 7I_2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

Hence verified.

34. **Statement** : A straight line drawn parallel to a side of triangle intersecting the other two sides, divides the sides in the same ratio.

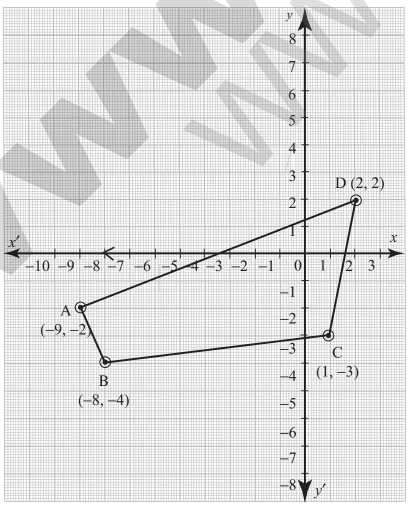
Proof : *Given :* In ΔABC, D is a point on AB and E is a point on AC.

To prove : $\frac{AD}{DB} = \frac{AE}{EC}$

Construction : Draw a line DE ∥ BC

No.	Statement	Reason
1.	∠ABC = ∠ADE = ∠1	Corresponding angles are equal because DE ∥ BC
2.	∠ACB = ∠AED = ∠2	Corresponding angles are equal because DE ∥ BC
3.	∠DAE = ∠BAC = ∠3	Both triangles have a common angle
	Δ ABC ~ ΔADE	By AAA similarity
	$\frac{AB}{AD} = \frac{AC}{AE}$	Corresponding sides are proportional
	$\frac{AD+DB}{AD} = \frac{AE+EC}{AE}$	Split AB and AC using the points D and E.
4.	$1+\frac{DB}{AD} = 1+\frac{EC}{AE}$	On simplification
	$\frac{DB}{AD} = \frac{EC}{AE}$	Cancelling 1 on both sides
	$\frac{AD}{DB} = \frac{AE}{EC}$	Taking reciprocals
Hence proved.		

35.



$A(-9, -2), B(-8, -4), C(1, -3), D(2, 2)$ with $(x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4)$

Area of the quadrilateral

$$= \frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{vmatrix} \text{ sq. units}$$

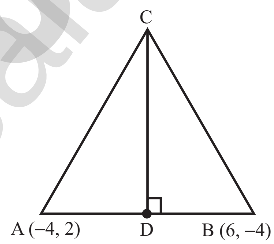
$$= \frac{1}{2}\begin{vmatrix} -9 & -8 & 1 & 2 & -9 \\ -2 & -4 & -3 & 2 & -2 \end{vmatrix} \text{ sq. units.}$$

$$= \frac{1}{2}[(36+24+2-4)-(16-4-6-18)]$$

$$= \frac{1}{2}[58-(-12)] = \frac{1}{2}(70) = 35 \text{ sq. units}$$

36. Mid Point AB is DC

$\Rightarrow$ D is $\left(\frac{-4+6}{2}, \frac{2+(-4)}{2}\right) = \left(\frac{2}{2}, \frac{-2}{2}\right) = (1, -1)$



Slope of AB $= \frac{-4-2}{6-(-4)} = \frac{-6}{10} = \frac{-3}{5}$

∴Slope of CD $= \frac{-1}{-3/5} = \frac{5}{3}$ [∵ CD ⊥ AB]

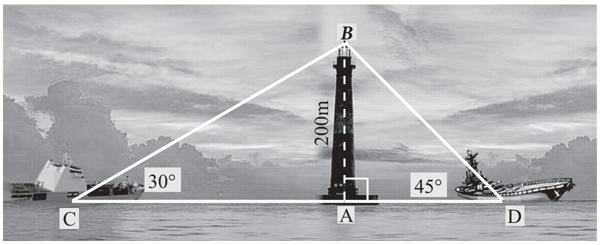
∴Equation of CD is

$y-(-1) = \frac{5}{3}(x-1)$

$3(y+1) = 5x-5 \Rightarrow 3y+3 = 5x-5$

$5x - 3y - 8 = 0$ is the required equation of the line.

37. Let AB be the lighthouse. Let C and D be the positions of the two ships.



Then, AB = 200 m. ∠ACB = 30°, ∠ADB = 45°

In the right angled ΔBAC, $\tan 30° = \frac{AB}{AC}$

$$\frac{1}{\sqrt{3}} = \frac{200}{AC} \Rightarrow AC = 200\sqrt{3} \quad ...(1)$$

In the right angled ΔBAD, $\tan 45° = \frac{AB}{AD}$

$$1 = \frac{200}{AD} \Rightarrow AD = 200 \quad ...(2)$$

Now, $CD = AC + AD = 200\sqrt{3} + 200$ [by (1) and (2)]

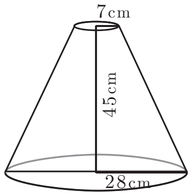
$CD = 200(\sqrt{3} + 1) = 200 \times 2.732 = 546.4$

Distance between two ships is 546.4 m.

38. Let h, r and R be the height, top and bottom radii of the frustum.

Given that, h = 45 cm,

R = 28 cm, r = 7 cm



Volume $= \frac{1}{3}\pi[R^2 + Rr + r^2]h$ cubic units.

$$= \frac{1}{3} \times \frac{22}{7} \times [28^2 + (28 \times 7) + 7^2] \times 45$$

$$= \frac{1}{3} \times \frac{22}{7} \times 1029 \times 45 = 48510$$

Therefore, volume of the frustum is 48510 cm^3

39. Let h and r be the height and radius of the cylinder respectively.

Given that, h = 15 cm, r = 6 cm

Volume of the container $V = \pi r^2 h$ cubic units.

$$= \frac{22}{7} \times 6 \times 6 \times 15$$

Let, $r_1 = 3$ cm, $h_1 = 9$ cm be the radius and height of the cone.

Also, $r_1 = 3$ cm is the radius of the hemispherical cap.

Volume of one ice cream cone = (Volume of the cone + Volume of the hemispherical cap)

$$= \frac{1}{3}\pi r_1^2 h_1 + \frac{2}{3}\pi r_1^3 = \frac{1}{3} \times \frac{22}{7} \times 3 \times 3 \times 9 + \frac{2}{3} \times \frac{22}{7} \times 3 \times 3 \times 3$$

$$= \frac{22}{7} \times 9\,(3 + 2) = \frac{22}{7} \times 45$$

Number of cones $= \dfrac{\text{volume of the cylinder}}{\text{volume of one ice cream cone}}$

Number of ice cream cones needed

$$= \frac{\frac{27}{7} \times 6 \times 6 \times 15}{\frac{22}{7} \times 45} = 12$$

Thus 12 ice cream cones are required to empty the cylindrical container.

40.

x	$d = x - \bar{x}$	d^2
24	–6	36
26	–4	16
33	3	9
37	7	49
29	–1	1
31	1	1
180	$\Sigma d = 0$	112

$$\bar{x} = \frac{\Sigma x}{n} = \frac{180}{6} = 30$$

$$\sigma = \sqrt{\frac{\Sigma d^2}{n}} = \sqrt{\frac{112}{6}} = \sqrt{18.66} = 4.32$$

$\therefore$ Co-efficient of variation C.V $= \frac{\sigma}{\bar{x}} \times 100$

$$C.V = \frac{4.32}{30} \times 100\% = 14.4\%$$

41. Two dice rolled once.

$$S = \begin{Bmatrix} (1,1), & (1,2), & (1,3), & (1,4), & (1,5), & (1,6) \\ (2,1), & (2,2), & (2,3), & (2,4), & (2,5), & (2,6) \\ (3,1), & (3,2), & (3,3), & (3,4), & (3,5), & (3,6) \\ (4,1), & (4,2), & (4,3), & (4,4), & (4,5), & (4,6) \\ (5,1), & (5,2), & (5,3), & (5,4), & (5,5), & (5,6) \\ (6,1), & (6,2), & (6,3), & (6,4), & (6,5), & (6,6) \end{Bmatrix}$$

$n(S) = 36$

Happening of an even number in the first die is A.

$$A = \begin{Bmatrix} (2,1), & (2,2), & (2,3), & (2,4), & (2,5), & (2,6) \\ (4,1), & (4,2), & (4,3), & (4,4), & (4,5), & (4,6) \\ (6,1), & (6,2), & (6,3), & (6,4), & (6,5), & (6,6) \end{Bmatrix}$$

$n(A) = 18$

$$P(A) = \frac{n(A)}{n(S)} = \frac{18}{36}$$

Happening of a total of face sum is 8 is B.

$B = \{(2, 6), (3,5), (4, 4), (5, 3), (6, 2)\}$

$n(B) = 5$

$$P(B) = \frac{n(B)}{n(S)} = \frac{5}{36}$$

$(A \cap B) = \{(2, 6), (4, 4), (6, 2)\}$

$$n(A \cap B) = 3 \Rightarrow P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{3}{36}$$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{18}{36} + \frac{5}{36} - \frac{3}{36} = \frac{18 + 5 - 3}{36}$$

$$= \frac{20^{5}}{36_{9}} = \frac{5}{9}$$

42. We have,

$7 + 77 + 777 + ...$

$S_n = 7[1 + 11 + 111 + ... n \text{ terms}]$

$= \frac{7}{9}[9 + 99 + 999 + ...n \text{ terms}]$

$= \frac{7}{9}\{(10-1) + (10^2-1) + (10^3-1) + ... (10^n-1)\}$

$= \frac{7}{9}\{(10 + 10^2 + 10^3 + ... + 10^n) - (1+1+1+1)\ n \text{ terms}]$

$= \frac{7}{9}\left\{10 \times \frac{(10^n-1)}{10-1} - n\right\} = \frac{7}{9}\left\{\frac{10}{9}(10^n-1) - n\right\}$

$= \frac{70}{81}(10^n - 1) - n\ - \frac{7n}{9}$

Part - IV

43. (a) Construction:

Step 1 : Draw a line segment PQ = 4.5 cm

Step 2 : At P, draw PE such that ∠QPE = 35°.

Step 3 : At P, draw PF such that ∠EPF = 90°.

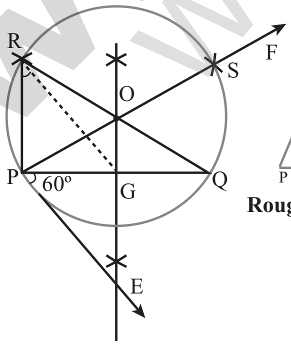
Step 4 : Draw $\perp^r$ bisector to PQ which intersects PF at O.

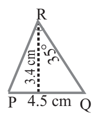
Step 5 : With O centre OP as radius draw a circle.

Step 6 : From G mark arcs of 6 cm on the circle. Mark them as R and S.

Step 7 : Join PR and RQ.

Step 8 : PQR is the required triangle.



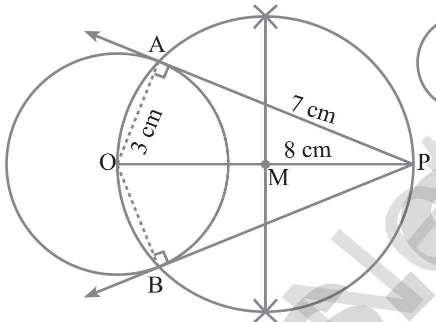


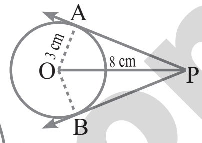
Rough diagram

(OR)

(b) Given, diameter $(d) = 6$ cm,

We find radius $(r) = (d) = 6 = \frac{6}{2} = 3$ cm





Rough diagram

Construction

Step 1 : With centre at O, draw a circle of radius 3 cm.

Step 2 : Draw a line OP of length 8 cm.

Step 3 : Draw a perpendicular bisector of OP, which cuts OP at M.

Step 4 : With M as centre and MO as radius, draw a circle which cuts previous circle at A and B.

Step 5 : Join AP and BP. AP and BP are the required tangents. Thus length of the tangents are PA = PB = 7.4 cm.

Verification :

In the right angle triangle OAP,

$PA^2 = OP^2 - OA^2 = 8^2 - 3^2 = 64 - 9 = 55$

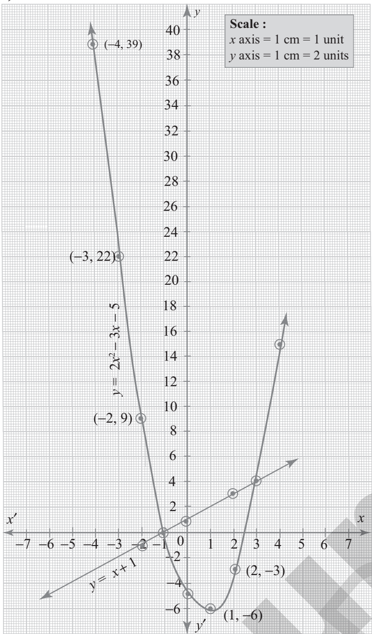
$PA = \sqrt{55} = 7.4$ cm

(approximately)

44. (a)

x	–4	–3	–2	–1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$2x^2$	32	18	8	2	0	2	8	18	32
$-3x$	12	9	6	3	0	–3	–6	–9	–12
-5	–5	–5	–5	–5	–5	–5	–5	–5	–5
$y = 2x^2 - 3x - 5$	39	22	9	0	–5	–6	–3	4	15

Draw the parabola using the points (–4, 39), (–3, 22), (–2, 9), (–1, 0), (0, –5), (1, –6), (2, –3), (3, 4), (4, 15).



To solve $2x^2 - 4x - 6 = 0$, subtract it from $y = 2x^2 - 3x - 5$

$$y = 2x^2 - 3x - 5$$
$$0 = 2x^2 - 4x - 6$$
$$(-)\quad(+)\quad(+)$$
$$y = x + 1 \text{ is a straight line}$$

x	–2	0	2
1	1	1	1
$y = x + 1$	–1	1	3

Draw a straight line using the points (–2, –1), (0, 1), (2, 3). The points of intersection of the parabola and the straight line forms the roots of the equation.

The x-coordinates of the points of intersection forms the solution set.

∴ Solution {–1, 3}

(OR)

(b)

x	1	2	3	4	6
y	24	12	8	6	4

From the table we observe that as x increases y decreases. This type of variation is called indirect variation.

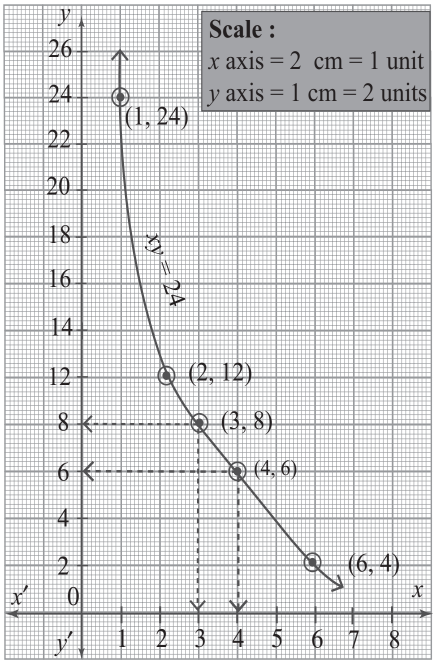
$y \alpha \frac{1}{x}$ or $xy = k$ where k is a constant of proportionality.

Also from the table we find that,

$1 \times 24 = 2 \times 12 = 3 \times 8 = 4 \times 6 = 6 \times 4 = 24 = k$.

∴ We get $k = 24$

Plot the points (1, 24), (2, 12), (3, 8), (4, 6) and (6, 4) and join them.



∴ The relation $xy = 24$ is a rectangular hyperbola as exhibited in the graph. From the graph, we find

(i) when $x = 3$, $y = 8$ (ii) when $y = 6$, $x = 4$

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