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# DEPARTMENT OF MATHEMATICS

ADHIYAMAN MATRIC HR.SEC.SCHOOL,UTHANGARAI.

## 10 - MATHEMATICS

### QUESTION BANK

**NAME OF THE STUDENT :** \_\_\_\_\_

#### Math Quotes :

- Mathematics is the most beautiful and most powerful creation of the human spirit.  
— **Stefan Banach, Polish mathematician**
- Without mathematics, there's nothing you can do. Everything around you is mathematics. Everything around you is numbers.  
— **Shakuntala Devi, Indian writer and mental calculator**
- Life is a math equation. In order to gain the most, you have to know how to convert negatives into positives.  
— **Anonymous**
- I've always been interested in using mathematics to make the world work better.  
— **Alvin E. Roth, American academic**

Prepared By **M. Sakthivel** MathsTeacher.

**ONE MARKS****1. RELATIONS AND FUNCTIONS**

1. If  $n(A \times B) = 6$  and  $A = \{1, 3\}$  then  $n(B)$  is  
 (A) 1 (B) 2 (C) 3 (D) 6
2.  $A = \{a, b, p\}$ ,  $B = \{2, 3\}$ ,  $C = \{p, q, r, s\}$  then  $n[(A \cup C) \times B]$  is  
 (A) 8 (B) 20 (C) 12 (D) 16
3. If  $A = \{1, 2\}$ ,  $B = \{1, 2, 3, 4\}$ ,  $C = \{5, 6\}$  and  $D = \{5, 6, 7, 8\}$  then state which of the following statement is true.  
 (A)  $(A \times C) \subset (B \times D)$  (B)  $(B \times D) \subset (A \times C)$   
 (C)  $(A \times B) \subset (A \times D)$  (D)  $(D \times A) \subset (B \times A)$
4. If there are 1024 relations from a set  $A = \{1, 2, 3, 4, 5\}$  to a set  $B$ , then the number of elements in  $B$  is  
 (A) 3 (B) 2 (C) 4 (D) 8
5. The range of the relation  $R = \{(x, x^2) \mid x \text{ is a prime number less than } 13\}$  is  
 (A)  $\{2, 3, 5, 7\}$  (B)  $\{2, 3, 5, 7, 11\}$   
 (C)  $\{4, 9, 25, 49, 121\}$  (D)  $\{1, 4, 9, 25, 49, 121\}$
6. If the ordered pairs  $(a + 2, 4)$  and  $(5, 2a + b)$  are equal then  $(a, b)$  is  
 (A)  $(2, -2)$  (B)  $(5, 1)$  (C)  $(2, 3)$  (D)  $(3, -2)$
7. Let  $n(A) = m$  and  $n(B) = n$  then the total number of non-empty relations that can be defined from  $A$  to  $B$  is  
 (A)  $m^n$  (B)  $n^m$  (C)  $2^{mn} - 1$  (D)  $2^{mn}$

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**2. NUMBERS AND SEQUENCES**

1. Euclid's division lemma states that for positive integers  $a$  and  $b$ , there exist unique integers  $q$  and  $r$  such that  $a = bq + r$ , where  $r$  must satisfy.  
 (A)  $1 < r < b$  (B)  $0 < r < b$  (C)  $0 \leq r < b$  (D)  $0 < r \leq b$
2. Using Euclid's division lemma, if the cube of any positive integer is divided by 9 then the possible remainders are  
 (A) 0, 1, 8 (B) 1, 4, 8 (C) 0, 1, 3 (D) 1, 3, 5
3. If the HCF of 65 and 117 is expressible in the form of  $65m - 117$ , then the value of  $m$  is  
 (A) 4 (B) 2 (C) 1 (D) 3
4. The sum of the exponents of the prime factors in the prime factorization of 1729 is  
 (A) 1 (B) 2 (C) 3 (D) 4

5. The least number that is divisible by all the numbers from 1 to 10 (both inclusive) is  
 (A) 2025 (B) 5220 (C) 5025 (D) 2520
7. Given  $F_1 = 1$ ,  $F_2 = 3$  and  $F_n = F_{n-1} + F_{n-2}$  then  $F_5$  is  
 (A) 3 (B) 5 (C) 8 (D) 11
8. The first term of an arithmetic progression is unity and the common difference is 4. Which of the following will be a term of this A.P.  
 (A) 4551 (B) 10091 (C) 7881 (D) 13531
9. If 6 times of 6<sup>th</sup> term of an A.P. is equal to 7 times the 7<sup>th</sup> term, then the 13<sup>th</sup> term of the A.P. is  
 (A) 0 (B) 6 (C) 7 (D) 13

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### 3.ALGEBRA

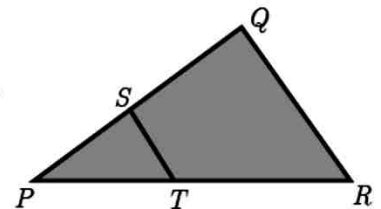
1. A system of three linear equations in three variables is inconsistent if their planes  
 (A) intersect only at a point (B) intersect in a line  
 (C) coincides with each other (D) do not intersect
2. The solution of the system  $x + y - 3z = -6$ ,  $-7y + 7z = 7$ ,  $3z = 9$  is  
 (A)  $x = 1, y = 2, z = 3$  (B)  $x = -1, y = 2, z = 3$   
 (C)  $x = -1, y = -2, z = 3$  (D)  $x = 1, y = -2, z = 3$
3. If  $(x - 6)$  is the HCF of  $x^2 - 2x - 24$  and  $x^2 - kx - 6$  then the value of  $k$  is  
 (A) 3 (B) 5 (C) 6 (D) 8
4.  $\frac{3y - 3}{y} \div \frac{7y - 7}{3y^2}$  is  
 (A)  $\frac{9y}{7}$  (B)  $\frac{9y^3}{(21y - 21)}$  (C)  $\frac{21y^2 - 42y + 21}{3y^3}$  (D)  $\frac{7(y^2 - 2y + 1)}{y^2}$
5.  $y^2 + \frac{1}{y^2}$  is not equal to  
 (A)  $\frac{y^4 + 1}{y^2}$  (B)  $\left(y + \frac{1}{y}\right)^2$  (C)  $\left(y - \frac{1}{y}\right)^2 + 2$  (D)  $\left(y + \frac{1}{y}\right)^2 - 2$
6.  $\frac{x}{x^2 - 25} - \frac{8}{x^2 + 6x + 5}$  gives  
 (A)  $\frac{x^2 - 7x + 40}{(x - 5)(x + 5)}$  (B)  $\frac{x^2 + 7x + 40}{(x - 5)(x + 5)(x + 1)}$   
 (C)  $\frac{x^2 - 7x + 40}{(x^2 - 25)(x + 1)}$  (D)  $\frac{x^2 + 10}{(x^2 - 25)(x + 1)}$

7. The square root of  $\frac{256x^8y^4z^{10}}{25x^6y^6z^6}$  is equal to  
 (A)  $\frac{16}{5} \left| \frac{x^2z^4}{y^2} \right|$  (B)  $16 \left| \frac{y^2}{x^2z^4} \right|$  (C)  $\frac{16}{5} \left| \frac{y}{xz^2} \right|$  (D)  $\frac{16}{5} \left| \frac{xz^2}{y} \right|$
8. Which of the following should be added to make  $x^4 + 64$  a perfect square  
 (A)  $4x^2$  (B)  $16x^2$  (C)  $8x^2$  (D)  $-8x^2$
9. The solution of  $(2x - 1)^2 = 9$  is equal to  
 (A)  $-1$  (B)  $2$  (C)  $-1, 2$  (D) None of these
10. The values of  $a$  and  $b$  if  $4x^4 - 24x^3 + 76x^2 + ax + b$  is a perfect square are  
 (A) 100,120 (B) 10,12 (C)  $-120,100$  (D) 12,10
11. If the roots of the equation  $q^2x^2 + p^2x + r^2 = 0$  are the squares of the roots of the equation  $qx^2 + px + r = 0$ , then  $q, p, r$  are in \_\_\_\_\_  
 (A) A.P (B) G.P (C) Both A.P and G.P (D) none of these
12. Graph of a linear equation is a \_\_\_\_\_  
 (A) straight line (B) circle (C) parabola (D) hyperbola
13. The number of points of intersection of the quadratic polynomial  $x^2 + 4x + 4$  with the X axis is  
 (A) 0 (B) 1 (C) 0 or 1 (D) 2

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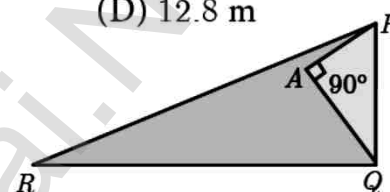
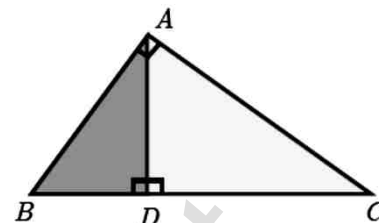
#### 4.GEOMETRY

1. In the figure, if  $BD \perp AC$  and  $CE \perp AB$ , prove that when  
 (A)  $\angle B = \angle E$  (B)  $\angle A = \angle D$  (C)  $\angle B = \angle D$  (D)  $\angle A = \angle F$
2. In  $\triangle LMN$ ,  $\angle L = 60^\circ$ ,  $\angle M = 50^\circ$ . If  $\triangle LMN \sim \triangle PQR$  then the value of  $\angle R$  is  
 (A)  $40^\circ$  (B)  $70^\circ$  (C)  $30^\circ$  (D)  $110^\circ$
3. If  $\triangle ABC$  is an isosceles triangle with  $\angle C = 90^\circ$  and  $AC = 5$  cm, then  $AB$  is  
 (A) 2.5 cm (B) 5 cm (C) 10 cm (D)  $5\sqrt{2}$  cm
4. In a given figure  $ST \parallel QR$ ,  $PS = 2$  cm and  $SQ = 3$  cm. Then the ratio of the area of  $\triangle PQR$  to the area of  $\triangle PST$  is  
 (A) 25 : 4 (B) 25 : 7  
 (C) 25 : 11 (D) 25 : 13
5. The perimeters of two similar triangles  $\triangle ABC$  and  $\triangle PQR$  are 36 cm and 24 cm respectively. If  $PQ = 10$  cm, then the length of  $AB$  is  
 (A)  $6\frac{2}{3}$  cm (B)  $\frac{10\sqrt{6}}{3}$  cm (C)  $66\frac{2}{3}$  cm (D) 15 cm





6. If in  $\triangle ABC$ ,  $DE \parallel BC$ .  $AB = 3.6$  cm,  $AC = 2.4$  cm and  $AD = 2.1$  cm then the length of  $AE$  is  
 (A) 1.4 cm (B) 1.8 cm (C) 1.2 cm (D) 1.05 cm
7. In a  $\triangle ABC$ ,  $AD$  is the bisector of  $\angle BAC$ . If  $AB = 8$  cm,  $BD = 6$  cm and  $DC = 3$  cm. The length of the side  $AC$  is  
 (A) 6 cm (B) 4 cm (C) 3 cm (D) 8 cm
8. In the adjacent figure  $\angle BAC = 90^\circ$  and  $AD \perp BC$  then  
 (A)  $BD \cdot CD = BC^2$  (B)  $AB \cdot AC = BC^2$   
 (C)  $BD \cdot CD = AD^2$  (D)  $AB \cdot AC = AD^2$
9. Two poles of heights 6 m and 11 m stand vertically on a plane ground. If the distance between their feet is 12 m, what is the distance between their tops?  
 (A) 13 m (B) 14 m (C) 15 m (D) 12.8 m
10. In the given figure,  $PR = 26$  cm,  $QR = 24$  cm,  $\angle PAQ = 90^\circ$ ,  $PA = 6$  cm and  $QA = 8$  cm. Find  $\angle PQR$   
 (A)  $80^\circ$  (B)  $85^\circ$  (C)  $75^\circ$  (D)  $90^\circ$



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### 5. COORDINATE GEOMETRY

1. The area of triangle formed by the points  $(-5,0)$ ,  $(0,-5)$  and  $(5,0)$  is  
 (A) 0 sq.units (B) 25 sq.units (C) 5 sq.units (D) none of these
2. A man walks near a wall, such that the distance between him and the wall is 10 units. Consider the wall to be the  $Y$  axis. The path travelled by the man is  
 (A)  $x = 10$  (B)  $y = 10$  (C)  $x = 0$  (D)  $y = 0$
3. The straight line given by the equation  $x = 11$  is  
 (A) parallel to  $X$  axis (B) parallel to  $Y$  axis  
 (C) passing through the origin (D) passing through the point  $(0,11)$
4. If  $(5,7)$ ,  $(3,p)$  and  $(6,6)$  are collinear, then the value of  $p$  is  
 (A) 3 (B) 6 (C) 9 (D) 12
5. The point of intersection of  $3x - y = 4$  and  $x + y = 8$  is  
 (A)  $(5,3)$  (B)  $(2,4)$  (C)  $(3,5)$  (D)  $(4,4)$
6. The slope of the line joining  $(12,3)$ ,  $(4,a)$  is  $\frac{1}{8}$ . The value of ' $a$ ' is  
 (A) 1 (B) 4 (C)  $-5$  (D) 2
7. The slope of the line which is perpendicular to a line joining the points  $(0,0)$  and  $(-8,8)$  is  
 (A)  $-1$  (B) 1 (C)  $\frac{1}{3}$  (D)  $-8$

8. If slope of the line  $PQ$  is  $\frac{1}{\sqrt{3}}$  then slope of the perpendicular bisector of  $PQ$  is  
 (A)  $\sqrt{3}$  (B)  $-\sqrt{3}$  (C)  $\frac{1}{\sqrt{3}}$  (D) 0
9. If  $A$  is a point on the  $Y$  axis whose ordinate is 8 and  $B$  is a point on the  $X$  axis whose abscissae is 5 then the equation of the line  $AB$  is  
 (A)  $8x + 5y = 40$  (B)  $8x - 5y = 40$  (C)  $x = 8$  (D)  $y = 5$
10. The equation of a line passing through the origin and perpendicular to the line  $7x - 3y + 4 = 0$  is  
 (A)  $7x - 3y + 4 = 0$  (B)  $3x - 7y + 4 = 0$  (C)  $3x + 7y = 0$  (D)  $7x - 3y = 0$
11. Consider four straight lines  
 (i)  $l_1; 3y = 4x + 5$  (ii)  $l_2; 4y = 3x - 1$  (iii)  $l_3; 4y + 3x = 7$  (iv)  $l_4; 4x + 3y = 2$   
 Which of the following statement is true ?  
 (A)  $l_1$  and  $l_2$  are perpendicular (B)  $l_1$  and  $l_4$  are parallel  
 (C)  $l_2$  and  $l_4$  are perpendicular (D)  $l_2$  and  $l_3$  are parallel
12. A straight line has equation  $8y = 4x + 21$ . Which of the following is true  
 (A) The slope is 0.5 and the  $y$  intercept is 2.6  
 (B) The slope is 5 and the  $y$  intercept is 1.6  
 (C) The slope is 0.5 and the  $y$  intercept is 1.6  
 (D) The slope is 5 and the  $y$  intercept is 2.6
13. When proving that a quadrilateral is a trapezium, it is necessary to show  
 (A) Two sides are parallel. (B) Two parallel and two non-parallel sides.  
 (C) Opposite sides are parallel. (D) All sides are of equal length.
14. When proving that a quadrilateral is a parallelogram by using slopes you must find  
 (A) The slopes of two sides (B) The slopes of two pair of opposite sides  
 (C) The lengths of all sides (D) Both the lengths and slopes of two sides

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### 6. TRIGONOMETRY

10. If the ratio of the height of a tower and the length of its shadow is  $\sqrt{3} : 1$ , then the angle of elevation of the sun has measure  
 (A)  $45^\circ$  (B)  $30^\circ$  (C)  $90^\circ$  (D)  $60^\circ$
11. The electric pole subtends an angle of  $30^\circ$  at a point on the same level as its foot. At a second point ' $b$ ' metres above the first, the depression of the foot of the pole is  $60^\circ$ . The height of the pole (in metres) is equal to  
 (A)  $\sqrt{3} b$  (B)  $\frac{b}{3}$  (C)  $\frac{b}{2}$  (D)  $\frac{b}{\sqrt{3}}$
12. A tower is 60 m high. Its shadow is  $x$  metres shorter when the sun's altitude is  $45^\circ$  than when it has been  $30^\circ$ , then  $x$  is equal to  
 (A) 41.92 m (B) 43.92 m (C) 43 m (D) 45.6 m



13. The angle of depression of the top and bottom of 20 m tall building from the top of a multistoried building are  $30^\circ$  and  $60^\circ$  respectively. The height of the multistoried building and the distance between two buildings (in metres) is  
 (A)  $20, 10\sqrt{3}$  (B)  $30, 5\sqrt{3}$  (C)  $20, 10$  (D)  $30, 10\sqrt{3}$
14. Two persons are standing ' $x$ ' metres apart from each other and the height of the first person is double that of the other. If from the middle point of the line joining their feet an observer finds the angular elevations of their tops to be complementary, then the height of the shorter person (in metres) is  
 (A)  $\sqrt{2} x$  (B)  $\frac{x}{2\sqrt{2}}$  (C)  $\frac{x}{\sqrt{2}}$  (D)  $2x$
15. The angle of elevation of a cloud from a point  $h$  metres above a lake is  $\beta$ . The angle of depression of its reflection in the lake is  $45^\circ$ . The height of location of the cloud from the lake is  
 (A)  $\frac{h(1 + \tan \beta)}{1 - \tan \beta}$  (B)  $\frac{h(1 - \tan \beta)}{1 + \tan \beta}$  (C)  $h \tan(45^\circ - \beta)$  (D) none of these

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### 7.MENSURATION

1. The curved surface area of a right circular cone of height 15 cm and base diameter 16 cm is  
 (A)  $60\pi \text{ cm}^2$  (B)  $68\pi \text{ cm}^2$  (C)  $120\pi \text{ cm}^2$  (D)  $136\pi \text{ cm}^2$
2. If two solid hemispheres of same base radius  $r$  units are joined together along their bases, then curved surface area of this new solid is  
 (A)  $4\pi r^2$  sq. units (B)  $6\pi r^2$  sq. units (C)  $3\pi r^2$  sq. units (D)  $8\pi r^2$  sq. units
3. The height of a right circular cone whose radius is 5 cm and slant height is 13 cm will be  
 (A) 12 cm (B) 10 cm (C) 13 cm (D) 5 cm
4. If the radius of the base of a right circular cylinder is halved keeping the same height, then the ratio of the volume of the cylinder thus obtained to the volume of original cylinder is  
 (A) 1:2 (B) 1:4 (C) 1:6 (D) 1:8
5. The total surface area of a cylinder whose radius is  $\frac{1}{3}$  of its height is  
 (A)  $\frac{9\pi h^2}{8}$  sq.units (B)  $24\pi h^2$  sq.units (C)  $\frac{8\pi h^2}{9}$  sq.units (D)  $\frac{56\pi h^2}{9}$  sq.units
6. In a hollow cylinder, the sum of the external and internal radii is 14 cm and the width is 4 cm. If its height is 20 cm, the volume of the material in it is  
 (A)  $5600\pi \text{ cm}^3$  (B)  $11200\pi \text{ cm}^3$  (C)  $56\pi \text{ cm}^3$  (D)  $3600\pi \text{ cm}^3$
7. If the radius of the base of a cone is tripled and the height is doubled then the volume is  
 (A) made 6 times (B) made 18 times (C) made 12 times (D) unchanged

8. The total surface area of a hemi-sphere is how much times the square of its radius.  
 (A)  $\pi$  (B)  $4\pi$  (C)  $3\pi$  (D)  $2\pi$
9. A solid sphere of radius  $x$  cm is melted and cast into a shape of a solid cone of same radius. The height of the cone is  
 (A)  $3x$  cm (B)  $x$  cm (C)  $4x$  cm (D)  $2x$  cm
10. A frustum of a right circular cone is of height 16cm with radii of its ends as 8cm and 20cm. Then, the volume of the frustum is  
 (A)  $3328\pi$  cm<sup>3</sup> (B)  $3228\pi$  cm<sup>3</sup> (C)  $3240\pi$  cm<sup>3</sup> (D)  $3340\pi$  cm<sup>3</sup>
11. A shuttle cock used for playing badminton has the shape of the combination of  
 (A) a cylinder and a sphere (B) a hemisphere and a cone  
 (C) a sphere and a cone (D) frustum of a cone and a hemisphere
12. A spherical ball of radius  $r_1$  units is melted to make 8 new identical balls each of radius  $r_2$  units. Then  $r_1 : r_2$  is  
 (A) 2:1 (B) 1:2 (C) 4:1 (D) 1:4
13. The volume (in cm<sup>3</sup>) of the greatest sphere that can be cut off from a cylindrical log of wood of base radius 1 cm and height 5 cm is  
 (A)  $\frac{4}{3}\pi$  (B)  $\frac{10}{3}\pi$  (C)  $5\pi$  (D)  $\frac{20}{3}\pi$
14. The height and radius of the cone of which the frustum is a part are  $h_1$  units and  $r_1$  units respectively. Height of the frustum is  $h_2$  units and radius of the smaller base is  $r_2$  units. If  $h_2 : h_1 = 1 : 2$  then  $r_2 : r_1$  is  
 (A) 1 : 3 (B) 1 : 2 (C) 2 : 1 (D) 3 : 1
15. The ratio of the volumes of a cylinder, a cone and a sphere, if each has the same diameter and same height is  
 (A) 1:2:3 (B) 2:1:3 (C) 1:3:2 (D) 3:1:2

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### 8. STATISTICS AND PROBABILITY

9. Which of the following is incorrect?  
 (A)  $P(A) > 1$  (B)  $0 \leq P(A) \leq 1$  (C)  $P(\phi) = 0$  (D)  $P(A) + P(\bar{A}) = 1$
10. The probability a red marble selected at random from a jar containing  $p$  red,  $q$  blue and  $r$  green marbles is  
 (A)  $\frac{q}{p+q+r}$  (B)  $\frac{p}{p+q+r}$  (C)  $\frac{p+q}{p+q+r}$  (D)  $\frac{p+r}{p+q+r}$
11. A page is selected at random from a book. The probability that the digit at units place of the page number chosen is less than 7 is  
 (A)  $\frac{3}{10}$  (B)  $\frac{7}{10}$  (C)  $\frac{3}{9}$  (D)  $\frac{7}{9}$



12. The probability of getting a job for a person is  $\frac{x}{3}$ . If the probability of not getting the job is  $\frac{2}{3}$  then the value of  $x$  is  
 (A) 2 (B) 1 (C) 3 (D) 1.5
13. Kamalam went to play a lucky draw contest. 135 tickets of the lucky draw were sold. If the probability of Kamalam winning is  $\frac{1}{9}$ , then the number of tickets bought by Kamalam is  
 (A) 5 (B) 10 (C) 15 (D) 20
14. If a letter is chosen at random from the English alphabets  $\{a, b, \dots, z\}$ , then the probability that the letter chosen precedes  $x$   
 (A)  $\frac{12}{13}$  (B)  $\frac{1}{13}$  (C)  $\frac{23}{26}$  (D)  $\frac{3}{26}$
15. A purse contains 10 notes of ₹2000, 15 notes of ₹500, and 25 notes of ₹200. One note is drawn at random. What is the probability that the note is either a ₹500 note or ₹200 note?  
 (A)  $\frac{1}{5}$  (B)  $\frac{3}{10}$  (C)  $\frac{2}{3}$  (D)  $\frac{4}{5}$

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## 2- MARKS AND 5- MARKS

### 1.RELATIONS AND FUNCTIONS

**Example 1.1** If  $A = \{1,3,5\}$  and  $B = \{2,3\}$  then (i) find  $A \times B$  and  $B \times A$ .

(ii) Is  $A \times B = B \times A$ ? If not why? (iii) Show that  $n(A \times B) = n(B \times A) = n(A) \times n(B)$

**Example 1.2** If  $A \times B = \{(3,2), (3,4), (5,2), (5,4)\}$  then find  $A$  and  $B$ .

**Example 1.3** Let  $A = \{x \in \mathbb{N} \mid 1 < x < 4\}$ ,  $B = \{x \in \mathbb{W} \mid 0 \leq x < 2\}$  and  $C = \{x \in \mathbb{N} \mid x < 3\}$ .

### EXERCISE : 1.1

- Find  $A \times B$ ,  $A \times A$  and  $B \times A$   
 (i)  $A = \{2, -2, 3\}$  and  $B = \{1, -4\}$  (ii)  $A = B = \{p, q\}$  (iii)  $A = \{m, n\}$ ;  $B = \phi$
- Let  $A = \{1, 2, 3\}$  and  $B = \{x \mid x \text{ is a prime number less than } 10\}$ . Find  $A \times B$  and  $B \times A$ .
- If  $B \times A = \{(-2, 3), (-2, 4), (0, 3), (0, 4), (3, 3), (3, 4)\}$  find  $A$  and  $B$ .
- If  $A = \{5, 6\}$ ,  $B = \{4, 5, 6\}$ ,  $C = \{5, 6, 7\}$ , Show that  $A \times A = (B \times B) \cap (C \times C)$ .
- Given  $A = \{1, 2, 3\}$ ,  $B = \{2, 3, 5\}$ ,  $C = \{3, 4\}$  and  $D = \{1, 3, 5\}$ , check if  $(A \cap C) \times (B \cap D) = (A \times B) \cap (C \times D)$  is true?
- Let  $A = \{x \in \mathbb{W} \mid x < 2\}$ ,  $B = \{x \in \mathbb{N} \mid 1 < x \leq 4\}$  and  $C = \{3, 5\}$ . Verify that  
 (i)  $A \times (B \cup C) = (A \times B) \cup (A \times C)$  (ii)  $A \times (B \cap C) = (A \times B) \cap (A \times C)$   
 (iii)  $(A \cup B) \times C = (A \times C) \cup (B \times C)$

7. Let  $A =$  The set of all natural numbers less than 8,  $B =$  The set of all prime numbers less than 8,  $C =$  The set of even prime number. Verify that

$$(i) (A \cap B) \times C = (A \times C) \cap (B \times C) \quad (ii) A \times (B - C) = (A \times B) - (A \times C)$$

**Example 1.4** Let  $A = \{3,4,7,8\}$  and  $B = \{1,7,10\}$ . Which of the following sets are relations from  $A$  to  $B$ ?

$$(i) R_1 = \{(3,7), (4,7), (7,10), (8,1)\} \quad (ii) R_2 = \{(3,1), (4,12)\}$$

$$(iii) R_3 = \{(3,7), (4,10), (7,7), (7,8), (8,11), (8,7), (8,10)\}$$

**Example 1.5** The arrow diagram shows (Fig.1.10) a relationship between the sets  $P$  and  $Q$ . Write the relation in (i) Set builder form (ii) Roster form (iii) What is the domain and range of  $R$ .

### EXERCISE : 1.2

- Let  $A = \{1,2,3,7\}$  and  $B = \{3,0,-1,7\}$ , which of the following are relation from  $A$  to  $B$ ?
  - $R_1 = \{(2,1), (7,1)\}$
  - $R_2 = \{(-1,1)\}$
  - $R_3 = \{(2,-1), (7,7), (1,3)\}$
  - $R_4 = \{(7,-1), (0,3), (3,3), (0,7)\}$
- Let  $A = \{1,2,3,4,\dots,45\}$  and  $R$  be the relation defined as "is square of a number" on  $A$ . Write  $R$  as a subset of  $A \times A$ . Also, find the domain and range of  $R$ .
- A Relation  $R$  is given by the set  $\{(x,y) / y = x + 3, x \in \{0,1,2,3,4,5\}\}$ . Determine its domain and range.
- Represent each of the given relations by (a) an arrow diagram, (b) a graph and (c) a set in roster form, wherever possible.
  - $\{(x,y) | x = 2y, x \in \{2,3,4,5\}, y \in \{1,2,3,4\}\}$
  - $\{(x,y) | y = x+3, x, y \text{ are natural numbers } < 10\}$
- A company has four categories of employees given by Assistants ( $A$ ), Clerks ( $C$ ), Managers ( $M$ ) and an Executive Officer ( $E$ ). The company provide ₹10,000, ₹25,000, ₹50,000 and ₹1,00,000 as salaries to the people who work in the categories  $A$ ,  $C$ ,  $M$  and  $E$  respectively. If  $A_1, A_2, A_3, A_4$  and  $A_5$  were Assistants;  $C_1, C_2, C_3, C_4$  were Clerks;  $M_1, M_2, M_3$  were managers and  $E_1, E_2$  were Executive officers and if the relation  $R$  is defined by  $xRy$ , where  $x$  is the salary given to person  $y$ , express the relation  $R$  through an ordered pair and an arrow diagram.

### UNIT EXERCISE : 1

- If the ordered pairs  $(x^2 - 3x, y^2 + 4y)$  and  $(-2,5)$  are equal, then find  $x$  and  $y$ .
- The cartesian product  $A \times A$  has 9 elements among which  $(-1, 0)$  and  $(0,1)$  are found. Find the set  $A$  and the remaining elements of  $A \times A$ .

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## 2.NUMBERS AND SEQUENCES

**Example 2.1** We have 34 cakes. Each box can hold 5 cakes only. How many boxes we need to pack and how many cakes are unpacked?



**Example 2.2** Find the quotient and remainder when  $a$  is divided by  $b$  in the following cases (i)  $a = -12$ ,  $b = 5$  (ii)  $a = 17$ ,  $b = -3$  (iii)  $a = -19$ ,  $b = -4$

**Example 2.3** Show that the square of an odd integer is of the form  $4q + 1$ , for some integer  $q$ .

**Example 2.4** If the Highest Common Factor of 210 and 55 is expressible in the form  $55x - 325$ , find  $x$ .

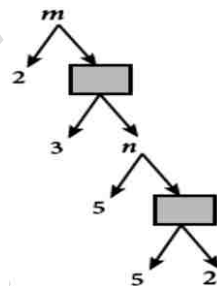
**Example 2.5** Find the greatest number that will divide 445 and 572 leaving remainders 4 and 5 respectively.

**Example 2.6** Find the HCF of 396, 504, 636.

### EXERCISE : 2.1

- Find all positive integers, when divided by 3 leaves remainder 2.
- A man has 532 flower pots. He wants to arrange them in rows such that each row contains 21 flower pots. Find the number of completed rows and how many flower pots are left over.
- Prove that the product of two consecutive positive integers is divisible by 2.
- When the positive integers  $a$ ,  $b$  and  $c$  are divided by 13, the respective remainders are 9, 7 and 10. Show that  $a+b+c$  is divisible by 13.
- Prove that square of any integer leaves the remainder either 0 or 1 when divided by 4.
- Use Euclid's Division Algorithm to find the Highest Common Factor (HCF) of
  - 340 and 412
  - 867 and 255
  - 10224 and 9648
  - 84, 90 and 120
- Find the largest number which divides 1230 and 1926 leaving remainder 12 in each case.
- If  $d$  is the Highest Common Factor of 32 and 60, find  $x$  and  $y$  satisfying  $d = 32x + 60y$ .
- A positive integer when divided by 88 gives the remainder 61. What will be the remainder when the same number is divided by 11?
- Prove that two consecutive positive integers are always coprime.

**Example 2.7** In the given factorisation, find the numbers  $m$  and  $n$ .



**Example 2.8** Can the number  $6^n$ ,  $n$  being a natural number end with the digit 5? Give reason for your answer.

**Example 2.9** Is  $7 \times 5 \times 3 \times 2 + 3$  a composite number? Justify your answer.

**Example 2.10** ' $a$ ' and ' $b$ ' are two positive integers such that  $a^b \times b^a = 800$ . Find ' $a$ ' and ' $b$ '.

**EXERCISE : 2.2**

- For what values of natural number  $n$ ,  $4^n$  can end with the digit 6?
- If  $m, n$  are natural numbers, for what values of  $m$ , does  $2^n \times 5^m$  ends in 5?
- Find the HCF of 252525 and 363636.
- If  $13824 = 2^a \times 3^b$  then find  $a$  and  $b$ .
- If  $p_1^{x_1} \times p_2^{x_2} \times p_3^{x_3} \times p_4^{x_4} = 113400$  where  $p_1, p_2, p_3, p_4$  are primes in ascending order and  $x_1, x_2, x_3, x_4$  are integers, find the value of  $p_1, p_2, p_3, p_4$  and  $x_1, x_2, x_3, x_4$ .
- Find the LCM and HCF of 408 and 170 by applying the fundamental theorem of arithmetic.
- Find the greatest number consisting of 6 digits which is exactly divisible by 24,15,36?
- What is the smallest number that when divided by three numbers such as 35, 56 and 91 leaves remainder 7 in each case?
- Find the least number that is divisible by the first ten natural numbers.

Example 2.19 Find the next three terms of the sequences

$$(i) \frac{1}{2}, \frac{1}{6}, \frac{1}{10}, \frac{1}{14}, \dots \quad (ii) 5, 2, -1, -4, \dots \quad (iii) 1, 0.1, 0.01, \dots$$

Example 2.20 Find the general term for the following sequences

$$(i) 3, 6, 9, \dots \quad (ii) \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \dots \quad (iii) 5, -25, 125, \dots$$

Example 2.21 The general term of a sequence is defined as

$$a_n = \begin{cases} n(n+3); & n \in \mathbb{N} \text{ is odd} \\ n^2 + 1 & ; n \in \mathbb{N} \text{ is even} \end{cases}$$

Example 2.22 Find the first five terms of the following sequence.

$$a_1 = 1, a_2 = 1, a_n = \frac{a_{n-1}}{a_{n-2} + 3}; n \geq 3, n \in \mathbb{N}$$

**EXERCISE : 2.4**

- Find the next three terms of the following sequence.
  - 8, 24, 72, ...
  - 5, 1, -3, ...
  - $\frac{1}{4}, \frac{2}{9}, \frac{3}{16}, \dots$
- Find the first four terms of the sequences whose  $n^{\text{th}}$  terms are given by
  - $a_n = n^3 - 2$
  - $a_n = (-1)^{n+1}n(n+1)$
  - $a_n = 2n^2 - 6$
- Find the  $n^{\text{th}}$  term of the following sequences
  - 2, 5, 10, 17, ...
  - $0, \frac{1}{2}, \frac{2}{3}, \dots$
  - 3, 8, 13, 18, ...
- Find the indicated terms of the sequences whose  $n^{\text{th}}$  terms are given by
  - $a_n = \frac{5n}{n+2}; a_6$  and  $a_{13}$
  - $a_n = -(n^2 - 4); a_4$  and  $a_{11}$



4. Find the indicated terms of the sequences whose  $n^{\text{th}}$  terms are given by

(i)  $a_n = \frac{5n}{n+2}$ ;  $a_6$  and  $a_{13}$       (ii)  $a_n = -(n^2 - 4)$ ;  $a_4$  and  $a_{11}$

5. Find  $a_8$  and  $a_{15}$  whose  $n^{\text{th}}$  term is  $a_n = \begin{cases} \frac{n^2 - 1}{n + 3} & ; n \text{ is even, } n \in \mathbb{N} \\ \frac{n^2}{2n + 1} & ; n \text{ is odd, } n \in \mathbb{N} \end{cases}$

6. If  $a_1 = 1$ ,  $a_2 = 1$  and  $a_n = 2a_{n-1} + a_{n-2}$ ,  $n \geq 3$ ,  $n \in \mathbb{N}$ , then find the first six terms of the sequence.

**Example 2.23** Check whether the following sequences are in A.P. or not?

(i)  $x + 2, 2x + 3, 3x + 4, \dots$       (ii)  $2, 4, 8, 16, \dots$       (iii)  $3\sqrt{2}, 5\sqrt{2}, 7\sqrt{2}, 9\sqrt{2}, \dots$

**Example 2.24** Write an A.P. whose first term is 20 and common difference is 8.

**Example 2.25** Find the  $15^{\text{th}}$ ,  $24^{\text{th}}$  and  $n^{\text{th}}$  term (general term) of an A.P. given by 3, 15, 27, 39, ...

**Example 2.26** Find the number of terms in the A.P. 3, 6, 9, 12, ..., 111.

**Example 2.27** Determine the general term of an A.P. whose  $7^{\text{th}}$  term is  $-1$  and  $16^{\text{th}}$  term is 17.

**Example 2.28** If  $l^{\text{th}}$ ,  $m^{\text{th}}$  and  $n^{\text{th}}$  terms of an A.P. are  $x, y, z$  respectively, then show that

(i)  $x(m - n) + y(n - l) + z(l - m) = 0$       (ii)  $(x - y)n + (y - z)l + (z - x)m = 0$

**Example 2.29** In an A.P., sum of four consecutive terms is 28 and their sum of their squares is 276. Find the four numbers.

**Example 2.30** A mother divides ₹207 into three parts such that the amount are in A.P. and gives it to her three children. The product of the two least amounts that the children had ₹4623. Find the amount received by each child.

### EXERCISE : 2.5

1. Check whether the following sequences are in A.P.

(i)  $a - 3, a - 5, a - 7, \dots$       (ii)  $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \dots$       (iii)  $9, 13, 17, 21, 25, \dots$

(iv)  $\frac{-1}{3}, 0, \frac{1}{3}, \frac{2}{3}, \dots$       (v)  $1, -1, 1, -1, 1, -1, \dots$

2. First term  $a$  and common difference  $d$  are given below. Find the corresponding A.P.

(i)  $a = 5, d = 6$       (ii)  $a = 7, d = -5$       (iii)  $a = \frac{3}{4}, d = \frac{1}{2}$

3. Find the first term and common difference of the Arithmetic Progressions whose  $n^{\text{th}}$  terms are given below

(i)  $t_n = -3 + 2n$       (ii)  $t_n = 4 - 7n$

4. Find the  $19^{\text{th}}$  term of an A.P.  $-11, -15, -19, \dots$

5. Which term of an A.P. 16, 11, 6, 1, ... is  $-54$ ?

6. Find the middle term(s) of an A.P. 9, 15, 21, 27, ..., 183.
7. If nine times ninth term is equal to the fifteen times fifteenth term, show that six times twenty fourth term is zero.
8. If  $3 + k$ ,  $18 - k$ ,  $5k + 1$  are in A.P. then find  $k$ .
9. Find  $x$ ,  $y$  and  $z$ , given that the numbers  $x$ , 10,  $y$ , 24,  $z$  are in A.P.
10. In a theatre, there are 20 seats in the front row and 30 rows were allotted. Each successive row contains two additional seats than its front row. How many seats are there in the last row?
11. The sum of three consecutive terms that are in A.P. is 27 and their product is 288. Find the three terms.
12. The ratio of 6<sup>th</sup> and 8<sup>th</sup> term of an A.P. is 7:9. Find the ratio of 9<sup>th</sup> term to 13<sup>th</sup> term.
13. In a winter season let us take the temperature of Ooty from Monday to Friday to be in A.P. The sum of temperatures from Monday to Wednesday is 0° C and the sum of the temperatures from Wednesday to Friday is 18° C. Find the temperature on each of the five days.
14. Priya earned ₹15,000 in the first month. Thereafter her salary increased by ₹1500 per year. Her expenses are ₹13,000 during the first year and the expenses increases by ₹900 per year. How long will it take for her to save ₹20,000 per month.

### UNIT EXERCISE : 2

1. Prove that  $n^2 - n$  divisible by 2 for every positive integer  $n$ .
2. A milk man has 175 litres of cow's milk and 105 litres of buffalows milk. He wishes to sell the milk by filling the two types of milk in cans of equal capacity. Calculate the following (i) Capacity of a can (ii) Number of cans of cow's milk (iii) Number of cans of buffalows milk.
3. When the positive integers  $a$ ,  $b$  and  $c$  are divided by 13 the respective remainders are 9, 7 and 10. Find the remainder when  $a + 2b + 3c$  is divided by 13.
4. Show that 107 is of the form  $4q + 3$  for any integer  $q$ .
5. If  $(m + 1)^{\text{th}}$  term of an A.P. is twice the  $(n + 1)^{\text{th}}$  term, then prove that  $(3m + 1)^{\text{th}}$  term is twice the  $(m + n + 1)^{\text{th}}$  term.
6. Find the 12<sup>th</sup> term from the last term of the A. P  $-2, -4, -6, \dots -100$ .
7. Two A.P's have the same common difference. The first term of one A.P. is 2 and that of the other is 7. Show that the difference between their 10<sup>th</sup> terms is the same as the difference between their 21<sup>st</sup> terms, which is the same as the difference between any two corresponding terms.

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### 3.ALGEBRA

**Example 3.1** The father's age is six times his son's age. Six years hence the age of father will be four times his son's age. Find the present ages (in years) of the son and father.

**Example 3.2** Solve  $2x - 3y = 6$ ,  $x + y = 1$

**Example 3.3** Solve the following system of linear equations in three variables  $3x - 2y + z = 2$ ,  $2x + 3y - z = 5$ ,  $x + y + z = 6$ .

**Example 3.4** In an interschool athletic meet, with total of 24 individual prizes, securing a total of 56 points, a first place secures 5 points, a second place secures 3 points, and a third place secures 1 point. Having as many third place finishers as first and second place finishers, find how many athletes finished in each place.

**Example 3.5** Solve  $x + 2y - z = 5$ ;  $x - y + z = -2$ ;  $-5x - 4y + z = -11$

**Example 3.6** Solve  $3x + y - 3z = 1$ ;  $-2x - y + 2z = 1$ ;  $-x - y + z = 2$ .

**Example 3.7** Solve  $\frac{x}{2} - 1 = \frac{y}{6} + 1 = \frac{z}{7} + 2$ ;  $\frac{y}{3} + \frac{z}{2} = 13$

**Example 3.8** Solve:  $\frac{1}{2x} + \frac{1}{4y} - \frac{1}{3z} = \frac{1}{4}$ ;  $\frac{1}{x} = \frac{1}{3y}$ ;  $\frac{1}{x} - \frac{1}{5y} + \frac{4}{z} = 2\frac{2}{15}$

**Example 3.9** The sum of thrice the first number, second number and twice the third number is 5. If thrice the second number is subtracted from the sum of first number and thrice the third we get 2. If the third number is subtracted from the sum of twice the first, thrice the second, we get 1. Find the numbers.

#### EXERCISE:3.1

1. Solve the following system of linear equations in three variables

(i)  $x + y + z = 5$ ;  $2x - y + z = 9$ ;  $x - 2y + 3z = 16$

(ii)  $\frac{1}{x} - \frac{2}{y} + 4 = 0$ ;  $\frac{1}{y} - \frac{1}{z} + 1 = 0$ ;  $\frac{2}{z} + \frac{3}{x} = 14$

(iii)  $x + 20 = \frac{3y}{2} + 10 = 2z + 5 = 110 - (y + z)$

2. Discuss the nature of solutions of the following system of equations

(i)  $x + 2y - z = 6$ ;  $-3x - 2y + 5z = -12$ ;  $x - 2z = 3$

(ii)  $2y + z = 3(-x + 1)$ ;  $-x + 3y - z = -4$ ;  $3x + 2y + z = -\frac{1}{2}$

(iii)  $\frac{y+z}{4} = \frac{z+x}{3} = \frac{x+y}{2}$ ;  $x + y + z = 27$

3. Vani, her father and her grand father have an average age of 53. One-half of her grand father's age plus one-third of her father's age plus one fourth of Vani's age is 65. Four years ago if Vani's grandfather was four times as old as Vani then how old are



4. The sum of the digits of a three-digit number is 11. If the digits are reversed, the new number is 46 more than five times the former number. If the hundreds digit plus twice the tens digit is equal to the units digit, then find the original three digit number ?
5. There are 12 pieces of five, ten and twenty rupee currencies whose total value is ₹105. When first 2 sorts are interchanged in their numbers its value will be increased by ₹20. Find the number of currencies in each sort.

**Example 3.10** Find the GCD of the polynomials  $x^3 + x^2 - x + 2$  and  $2x^3 - 5x^2 + 5x - 3$ .

**Example 3.11** Find the GCD of  $6x^3 - 30x^2 + 60x - 48$  and  $3x^3 - 12x^2 + 21x - 18$ .

**Example 3.12** Find the LCM of the following

(i)  $8x^4y^2, 48x^2y^4$

(ii)  $5x - 10, 5x^2 - 20$

(iii)  $x^4 - 1, x^2 - 2x + 1$

(iv)  $x^3 - 27, (x - 3)^2, x^2 - 9$

### EXERCISE:3.2

1. Find the GCD of the given polynomials

(i)  $x^4 + 3x^3 - x - 3, x^3 + x^2 - 5x + 3$

(ii)  $x^4 - 1, x^3 - 11x^2 + x - 11$

(iii)  $3x^4 + 6x^3 - 12x^2 - 24x, 4x^4 + 14x^3 + 8x^2 - 8x$

(iv)  $3x^3 + 3x^2 + 3x + 3, 6x^3 + 12x^2 + 6x + 12$

2. Find the LCM of the given expressions.

(i)  $4x^2y, 8x^3y^2$

(ii)  $9a^3b^2, 12a^2b^2c$

(iii)  $16m, 12m^2n^2, 8n^2$

(iv)  $p^2 - 3p + 2, p^2 - 4$

(v)  $2x^2 - 5x - 3, 4x^2 - 36$

(vi)  $(2x^2 - 3xy)^2, (4x - 6y)^3, 8x^3 - 27y^3$

### EXERCISE:3.3

1. Find the LCM and GCD for the following and verify that  $f(x) \times g(x) = LCM \times GCD$

(i)  $21x^2y, 35xy^2$

(ii)  $(x^3 - 1)(x + 1), (x^3 + 1)$

(iii)  $(x^2y + xy^2), (x^2 + xy)$

2. Find the LCM of each pair of the following polynomials

(i)  $a^2 + 4a - 12, a^2 - 5a + 6$  whose GCD is  $a - 2$

(ii)  $x^4 - 27a^3x, (x - 3a)^2$  whose GCD is  $(x - 3a)$

3. Find the GCD of each pair of the following polynomials

(i)  $12(x^4 - x^3), 8(x^4 - 3x^3 + 2x^2)$  whose LCM is  $24x^3(x - 1)(x - 2)$

(ii)  $(x^3 + y^3), (x^4 + x^2y^2 + y^4)$  whose LCM is  $(x^3 + y^3)(x^2 + xy + y^2)$

4. Given the LCM and GCD of the two polynomials  $p(x)$  and  $q(x)$  find the unknown polynomial in the following table

| S.No. | LCM                               | GCD           | $p(x)$           | $q(x)$                        |
|-------|-----------------------------------|---------------|------------------|-------------------------------|
| (i)   | $a^3 - 10a^2 + 11a + 70$          | $a - 7$       | $a^2 - 12a + 35$ |                               |
| (ii)  | $(x^4 - y^4)(x^4 + x^2y^2 + y^4)$ | $(x^2 - y^2)$ |                  | $(x^4 - y^4)(x^2 + y^2 - xy)$ |

**Example 3.13** Reduce the rational expressions to its lowest form

(i)  $\frac{x - 3}{x^2 - 9}$

(ii)  $\frac{x^2 - 16}{x^2 + 8x + 16}$



**Example 3.14** Find the excluded values of the following expressions (if any).

(i)  $\frac{x+10}{8x}$

(ii)  $\frac{7p+2}{8p^2+13p+5}$

(iii)  $\frac{x}{x^2+1}$

**EXERCISE:3.4**

1. Reduce each of the following rational expressions to its lowest form.

(i)  $\frac{x^2-1}{x^2+x}$

(ii)  $\frac{x^2-11x+18}{x^2-4x+4}$

(iii)  $\frac{9x^2+81x}{x^3+8x^2-9x}$

(iv)  $\frac{p^2-3p-40}{2p^3-24p^2+64p}$

2. Find the excluded values, if any of the following expressions.

(i)  $\frac{y}{y^2-25}$

(ii)  $\frac{t}{t^2-5t+6}$

(iii)  $\frac{x^2+6x+8}{x^2+x-2}$

(iv)  $\frac{x^3-27}{x^3+x^2-6x}$

**Example 3.15** (i) Multiply  $\frac{x^3}{9y^2}$  by  $\frac{27y}{x^5}$  (ii) Multiply  $\frac{x^4b^2}{x-1}$  by  $\frac{x^2-1}{a^4b^3}$

**Example 3.16** Find

(i)  $\frac{14x^4}{y} \sqrt{\frac{7x}{3y^4}}$

(ii)  $\frac{x^2-16}{x+4} \div \frac{x-4}{x+4}$

(iii)  $\frac{16x^2-2x-3}{3x^2-2x-1} \div \frac{8x^2+11x+3}{3x^2-11x-4}$

**EXERCISE:3.5**

1. Simplify

(i)  $\frac{4x^2y}{2z^2} \times \frac{6xz^3}{20y^4}$

(ii)  $\frac{p^2-10p+21}{p-7} \times \frac{p^2+p-12}{(p-3)^2}$

(iii)  $\frac{5t^3}{4t-8} \times \frac{6t-12}{10t}$

2. Simplify

(i)  $\frac{x+4}{3x+4y} \times \frac{9x^2-16y^2}{2x^2+3x-20}$

(ii)  $\frac{x^3-y^3}{3x^2+9xy+6y^2} \times \frac{x^2+2xy+y^2}{x^2-y^2}$

3. Simplify

(i)  $\frac{2a^2+5a+3}{2a^2+7a+6} \div \frac{a^2+6a+5}{-5a^2-35a-50}$

(ii)  $\frac{b^2+3b-28}{b^2+4b+4} \div \frac{b^2-49}{b^2-5b-14}$

(iii)  $\frac{x+2}{4y} \div \frac{x^2-x-6}{12y^2}$

(iv)  $\frac{12t^2-22t+8}{3t} \div \frac{3t^2+2t-8}{2t^2+4t}$

4. If  $x = \frac{a^2+3a-4}{3a^2-3}$  and  $y = \frac{a^2+2a-8}{2a^2-2a-4}$  find the value of  $x^2y^{-2}$ .

5. If a polynomial  $p(x) = x^2 - 5x - 14$  is divided by another polynomial  $q(x)$  we get  $\frac{x-7}{x+2}$ , find  $q(x)$ .

**Example 3.17** Find  $\frac{x^2+20x+36}{x^2-3x-28} - \frac{x^2+12x+4}{x^2-3x-28}$

**Example 3.18** Simplify  $\frac{1}{x^2-5x+6} + \frac{1}{x^2-3x+2} - \frac{1}{x^2-8x+15}$

**EXERCISE:3.6**

- Simplify (i)  $\frac{x(x+1)}{x-2} + \frac{x(1-x)}{x-2}$  (ii)  $\frac{x+2}{x+3} + \frac{x-1}{x-2}$  (iii)  $\frac{x^3}{x-y} + \frac{y^3}{y-x}$
- Simplify (i)  $\frac{(2x+1)(x-2)}{x-4} - \frac{(2x^2-5x+2)}{x-4}$  (ii)  $\frac{4x}{x^2-1} - \frac{x+1}{x-1}$
- Subtract  $\frac{1}{x^2+2}$  from  $\frac{2x^3+x^2+3}{(x^2+2)^2}$
- Which rational expression should be subtracted from  $\frac{x^2+6x+8}{x^3+8}$  to get  $\frac{3}{x^2-2x+4}$
- If  $A = \frac{2x+1}{2x-1}$ ,  $B = \frac{2x-1}{2x+1}$  find  $\frac{1}{A-B} - \frac{2B}{A^2-B^2}$
- If  $A = \frac{x}{x+1}$ ,  $B = \frac{1}{x+1}$ , prove that  $\frac{(A+B)^2 + (A-B)^2}{A+B} = \frac{2(x^2+1)}{x(x+1)^2}$
- Pari needs 4 hours to complete a work. His friend Yuvan needs 6 hours to complete the same work. How long will it take to complete if they work together?
- Iniya bought 50 kg of fruits consisting of apples and bananas. She paid twice as much per kg for the apple as she did for the banana. If Iniya bought ₹ 1800 worth of apples and ₹ 600 worth bananas, then how many kgs of each fruit did she buy?

**Example 3.19** Find the square root of the following expressions

$$(i) 256(x-a)^8(x-b)^4(x-c)^{16}(x-d)^{20} \quad (ii) \frac{144 a^8 b^{12} c^{16}}{81 f^{12} g^4 h^{14}}$$

**Example 3.20** Find the square root of the following expressions

$$(i) 16x^2 + 9y^2 - 24xy + 24x - 18y + 9 \quad (ii) (6x^2 + x - 1)(3x^2 + 2x - 1)(2x^2 + 3x + 1)$$

$$(iii) \left[ \sqrt{15}x^2 + (\sqrt{3} + \sqrt{10})x + \sqrt{2} \right] \left[ \sqrt{5}x^2 + (2\sqrt{5} + 1)x + 2 \right] \left[ \sqrt{3}x^2 + (\sqrt{2} + 2\sqrt{3})x + 2\sqrt{2} \right]$$

**EXERCISE:3.7**

1. Find the square root of the following rational expressions.

$$(i) \frac{400x^4y^{12}z^{16}}{100x^8y^4z^4} \quad (ii) \frac{7x^2 + 2\sqrt{14}x + 2}{x^2 - \frac{1}{2}x + \frac{1}{16}} \quad (iii) \frac{121(a+b)^8(x+y)^8(b-c)^8}{81(b-c)^4(a-b)^{12}(b-c)^4}$$

2. Find the square root of the following

$$(i) 4x^2 + 20x + 25 \quad (ii) 9x^2 - 24xy + 30xz - 40yz + 25z^2 + 16y^2$$

$$(iii) (4x^2 - 9x + 2)(7x^2 - 13x - 2)(28x^2 - 3x - 1)$$

$$(iv) \left( 2x^2 + \frac{17}{6}x + 1 \right) \left( \frac{3}{2}x^2 + 4x + 2 \right) \left( \frac{4}{3}x^2 + \frac{11}{3}x + 2 \right)$$

**Example 3.21** Find the square root of  $64x^4 - 16x^3 + 17x^2 - 2x + 1$

**Example 3.22** If  $9x^4 + 12x^3 + 28x^2 + ax + b$  is a perfect square, find the values of  $a$  and  $b$ .

**EXERCISE:3.8**

- Find the square root of the following polynomials by division method
  - $x^4 - 12x^3 + 42x^2 - 36x + 9$
  - $37x^2 - 28x^3 + 4x^4 + 42x + 9$
  - $16x^4 + 8x^2 + 1$
  - $121x^4 - 198x^3 - 183x^2 + 216x + 144$
- Find the values of  $a$  and  $b$  if the following polynomials are perfect squares
  - $4x^4 - 12x^3 + 37x^2 + bx + a$
  - $ax^4 + bx^3 + 361x^2 + 220x + 100$
- Find the values of  $m$  and  $n$  if the following polynomials are perfect squares
  - $36x^4 - 60x^3 + 61x^2 - mx + n$
  - $x^4 - 8x^3 + mx^2 + nx + 16$

**Example 3.23** Find the zeroes of the quadratic expression  $x^2 + 8x + 12$ .

**Example 3.24** Write down the quadratic equation in general form for which sum and product of the roots are given below.

**Example 3.25** Find the sum and product of the roots for each of the following quadratic equations : (i)  $x^2 + 8x - 65 = 0$  (ii)  $2x^2 + 5x + 7 = 0$   
(iii)  $kx^2 - k^2x - 2k^3 = 0$

**EXERCISE:3.9**

- Determine the quadratic equations, whose sum and product of roots are
  - $-9, 20$
  - $\frac{5}{3}, 4$
  - $\frac{-3}{2}, -1$
  - $-(2-a)^2, (a+5)^2$
- Find the sum and product of the roots for each of the following quadratic equations
  - $x^2 + 3x - 28 = 0$
  - $x^2 + 3x = 0$
  - $3 + \frac{1}{a} = \frac{10}{a^2}$
  - $3y^2 - y - 4 = 0$

**Example 3.26** Solve  $2x^2 - 2\sqrt{6}x + 3 = 0$  **Example 3.27** Solve  $2m^2 + 19m + 30 = 0$

**Example 3.28** Solve  $x^4 - 13x^2 + 42 = 0$

**Example 3.29** Solve  $\frac{x}{x-1} + \frac{x-1}{x} = 2\frac{1}{2}$

**EXERCISE:3.10**

- Solve the following quadratic equations by factorization method
  - $4x^2 - 7x - 2 = 0$
  - $3(p^2 - 6) = p(p + 5)$
  - $\sqrt{a(a-7)} = 3\sqrt{2}$
  - $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$
  - $2x^2 - x + \frac{1}{8} = 0$
- The number of volleyball games that must be scheduled in a league with  $n$  teams is given by  $G(n) = \frac{n^2 - n}{2}$  where each team plays with every other team exactly once. A league schedules 15 games. How many teams are in the league?

**Example 3.30** Solve  $x^2 - 3x - 2 = 0$

**Example 3.31** Solve  $2x^2 - x - 1 = 0$



**Example 3.32** Solve  $x^2 + 2x - 2 = 0$  by formula method

**Example 3.33** Solve  $2x^2 - 3x - 3 = 0$  by formula method.

**Example 3.34** Solve  $3p^2 + 2\sqrt{5}p - 5 = 0$  by formula method.

**Example 3.35** Solve  $pqx^2 - (p + q)^2x + (p + q)^2 = 0$

### **EXERCISE:3.11**

1. Solve the following quadratic equations by completing the square method

(i)  $9x^2 - 12x + 4 = 0$

(ii)  $\frac{5x + 7}{x - 1} = 3x + 2$

2. Solve the following quadratic equations by formula method

(i)  $2x^2 - 5x + 2 = 0$

(ii)  $\sqrt{2}f^2 - 6f + 3\sqrt{2} = 0$

(iii)  $3y^2 - 20y - 23 = 0$

(iv)  $36y^2 - 12ay + (a^2 - b^2) = 0$

3. A ball rolls down a slope and travels a distance  $d = t^2 - 0.75t$  feet in  $t$  seconds. Find the time when the distance travelled by the ball is 11.25 feet.

**Example 3.36** The product of Kumaran's age (in years) two years ago and his age four years from now is one more than twice his present age. What is his present age?

**Example 3.37** A ladder 17 feet long is leaning against a wall. If the ladder, vertical wall and the floor from the bottom of the wall to the ladder form a right triangle, find the height of the wall where the top of the ladder meets if the distance between bottom of the wall to bottom of the ladder is 7 feet less than the height of the wall?

**Example 3.38** A flock of swans contained  $x^2$  members. As the clouds gathered,  $10x$  went to a lake and one-eighth of the members flew away to a garden. The remaining three pairs played about in the water. How many swans were there in total?

**Example 3.39** A passenger train takes 1 hr more than an express train to travel a distance of 240 km from Chennai to Virudhachalam. The speed of passenger train is less than that of an express train by 20 km per hour. Find the average speed of both the trains.

### **EXERCISE :3.12**

1. If the difference between a number and its reciprocal is  $\frac{24}{5}$ , find the number.

2. A garden measuring 12m by 16m is to have a pedestrian pathway that is ' $w$ ' meters wide installed all the way around so that it increases the total area to 285 m<sup>2</sup>. What is the width of the pathway?

3. A bus covers a distance of 90 km at a uniform speed. Had the speed been 15 km/hour more it would have taken 30 minutes less for the journey. Find the original speed of the bus.

4. A girl is twice as old as her sister. Five years hence, the product of their ages (in years) will be 375. Find their present ages.

5. A pole has to be erected at a point on the boundary of a circular ground of diameter 20 m in such a way that the difference of its distances from two diametrically opposite fixed gates P and Q on the boundary is 4 m. Is it possible to do so? If answer is yes at what distance from the two gates should the pole be erected?



6. From a group of  $2x^2$  black bees, square root of half of the group went to a tree. Again eight-ninth of the bees went to the same tree. The remaining two got caught up in a fragrant lotus. How many bees were there in total?
7. Music is been played in two opposite galleries with certain group of people. In the first gallery a group of 4 singers were singing and in the second gallery 9 singers were singing. The two galleries are separated by the distance of 70 m. Where should a person stand for hearing the same intensity of the singers voice? (Hint: The ratio of the sound intensity is equal to the square of the ratio of their corresponding distances).
8. There is a square field whose side is 10 m. A square flower bed is prepared in its centre leaving a gravel path all round the flower bed. The total cost of laying the flower bed and gravelling the path at ₹3 and ₹4 per square metre respectively is ₹364. Find the width of the gravel path.
9. The hypotenuse of a right angled triangle is 25 cm and its perimeter 56 cm. Find the length of the smallest side.

**Example 3.40** Determine the nature of roots for the following quadratic equations

(i)  $x^2 - x - 20 = 0$     (ii)  $9x^2 - 24x + 16 = 0$     (iii)  $2x^2 - 2x + 9 = 0$

**Example 3.41** (i) Find the values of 'k', for which the quadratic equation

$$kx^2 - (8k + 4)x + 81 = 0 \text{ has real and equal roots?}$$

(ii) Find the values of 'k' such that quadratic equation

$$(k + 9)x^2 + (k + 1)x + 1 = 0 \text{ has no real roots?}$$

**Example 3.42** Prove that the equation  $x^2(p^2 + q^2) + 2x(pr + qs) + r^2 + s^2 = 0$  has no real roots. If  $ps = qr$ , then show that the roots are real and equal.

### EXERCISE : 3.13

1. Determine the nature of the roots for the following quadratic equations  
 (i)  $15x^2 + 11x + 2 = 0$     (ii)  $x^2 - x - 1 = 0$     (iii)  $\sqrt{2}t^2 - 3t + 3\sqrt{2} = 0$   
 (iv)  $9y^2 - 6\sqrt{2}y + 2 = 0$     (v)  $9a^2b^2x^2 - 24abcdx + 16c^2d^2 = 0, a \neq 0, b \neq 0$
2. Find the value(s) of 'k' for which the roots of the following equations are real and equal. (i)  $(5k - 6)x^2 + 2kx + 1 = 0$     (ii)  $kx^2 + (6k + 2)x + 16 = 0$
3. If the roots of  $(a - b)x^2 + (b - c)x + (c - a) = 0$  are real and equal, then prove that b, a, c are in arithmetic progression.
4. If a, b are real then show that the roots of the equation  $(a - b)x^2 - 6(a + b)x - 9(a - b) = 0$  are real and unequal.
5. If the roots of the equation  $(c^2 - ab)x^2 - 2(a^2 - bc)x + b^2 - ac = 0$  are real and equal prove that either  $a=0$  (or)  $a^3 + b^3 + c^3 = 3abc$

**Example 3.43** If the difference between the roots of the equation  $x^2 - 13x + k = 0$  is 17 find k.

**Example 3.44** If  $\alpha$  and  $\beta$  are the roots of  $x^2 + 7x + 10 = 0$  find the values of

(i)  $(\alpha - \beta)$  (ii)  $\alpha^2 + \beta^2$  (iii)  $\alpha^3 - \beta^3$  (iv)  $\alpha^4 + \beta^4$  (v)  $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$  (vi)  $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

**Example 3.45** If  $\alpha, \beta$  are the roots of the equation  $3x^2 + 7x - 2 = 0$ , find the values of

(i)  $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$  (ii)  $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

**Example 3.46** If  $\alpha, \beta$  are the roots of the equation  $2x^2 - x - 1 = 0$ , then form the equation

whose roots are (i)  $\frac{1}{\alpha}, \frac{1}{\beta}$  (ii)  $\alpha^2\beta, \beta^2\alpha$  (iii)  $2\alpha + \beta, 2\beta + \alpha$

### **EXERCISE : 3.14**

1. Write each of the following expression in terms of  $\alpha + \beta$  and  $\alpha\beta$ .

(i)  $\frac{\alpha}{3\beta} + \frac{\beta}{3\alpha}$  (ii)  $\frac{1}{\alpha^2\beta} + \frac{1}{\beta^2\alpha}$  (iii)  $(3\alpha - 1)(3\beta - 1)$  (iv)  $\frac{\alpha + 3}{\beta} + \frac{\beta + 3}{\alpha}$

2. The roots of the equation  $2x^2 - 7x + 5 = 0$  are  $\alpha$  and  $\beta$ . Without solving for the roots, find

(i)  $\frac{1}{\alpha} + \frac{1}{\beta}$  (ii)  $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$  (iii)  $\frac{\alpha + 2}{\beta + 2} + \frac{\beta + 2}{\alpha + 2}$

3. The roots of the equation  $x^2 + 6x - 4 = 0$  are  $\alpha, \beta$ . Find the quadratic equation whose roots are

(i)  $\alpha^2$  and  $\beta^2$  (ii)  $\frac{2}{\alpha}$  and  $\frac{2}{\beta}$  (iii)  $\alpha^2\beta$  and  $\beta^2\alpha$

4. If  $\alpha, \beta$  are the roots of  $7x^2 + ax + 2 = 0$  and if  $\beta - \alpha = \frac{-13}{7}$ . Find the values of  $a$ .

5. If one root of the equation  $2y^2 - ay + 64 = 0$  is twice the other then find the values of  $a$ .

6. If one root of the equation  $3x^2 + kx + 81 = 0$  (having real roots) is the square of the other then find  $k$ .

### **UNIT EXERCISE : 3**

1. Solve  $\frac{1}{3}(x + y - 5) = y - z = 2x - 11 = 9 - (x + 2z)$

2. One hundred and fifty students are admitted to a school. They are distributed over three sections A, B and C. If 6 students are shifted from section A to section C, the sections will have equal number of students. If 4 times of students of section C exceeds the number of students of section A by the number of students in section B, find the number of students in the three sections.

3. In a three-digit number, when the tens and the hundreds digit are interchanged the new number is 54 more than three times the original number. If 198 is added to the number, the digits are reversed. The tens digit exceeds the hundreds digit by twice as that of the tens digit exceeds the unit digit. Find the original number.

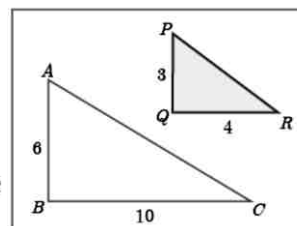
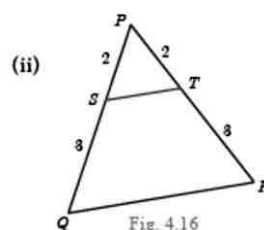
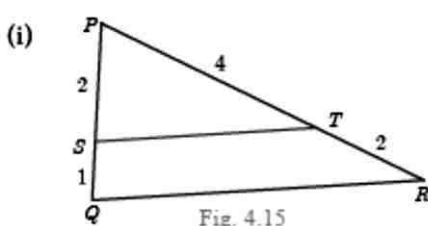


4. Find the least common multiple of  $xy(k^2 + 1) + k(x^2 + y^2)$  and  $xy(k^2 - 1) + k(x^2 - y^2)$
5. Find the GCD of the following by division algorithm  
 $2x^4 + 13x^3 + 27x^2 + 23x + 7$ ,  $x^3 + 3x^2 + 3x + 1$ ,  $x^2 + 2x + 1$
6. Reduce the given Rational expressions to its lowest form  
 (i)  $\frac{x^{3a} - 8}{x^{2a} + 2x^a + 4}$  (ii)  $\frac{10x^3 - 25x^2 + 4x - 10}{-4 - 10x^2}$
7. Simplify  $\frac{\frac{1}{p} + \frac{1}{q+r}}{\frac{1}{p} - \frac{1}{q+r}} \times \left(1 + \frac{q^2 + r^2 - p^2}{2qr}\right)$
8. Arul, Madan and Ram working together can clean a store in 6 hours. Working alone, Madan takes twice as long to clean the store as Arul does. Ram needs three times as long as Arul does. How long would it take each if they are working alone?
9. Find the square root of  $289x^4 - 612x^3 + 970x^2 - 684x + 361$ .
10. Solve  $\sqrt{y+1} + \sqrt{2y-5} = 3$
11. A boat takes 1.6 hours longer to go 36 kms up a river than down the river. If the speed of the water current is 4 km per hr, what is the speed of the boat in still water?
12. Is it possible to design a rectangular park of perimeter 320 m and area  $4800 \text{ m}^2$ ? If so find its length and breadth.
13. At  $t$  minutes past 2 pm, the time needed to 3 pm is 3 minutes less than  $\frac{t^2}{4}$ . Find  $t$ .
14. The number of seats in a row is equal to the total number of rows in a hall. The total number of seats in the hall will increase by 375 if the number of rows is doubled and the number of seats in each row is reduced by 5. Find the number of rows in the hall at the beginning.
15. If  $\alpha$  and  $\beta$  are the roots of the polynomial  $f(x) = x^2 - 2x + 3$ , find the polynomial whose roots are (i)  $\alpha + 2$ ,  $\beta + 2$  (ii)  $\frac{\alpha - 1}{\alpha + 1}$ ,  $\frac{\beta - 1}{\beta + 1}$ .
16. If  $-4$  is a root of the equation  $x^2 + px - 4 = 0$  and if the equation  $x^2 + px + q = 0$  has equal roots, find the values of  $p$  and  $q$ .

\*\*\*\*\*

#### 4. GEOMETRY

Example 4.1 Show that  $\Delta PST \sim \Delta PQR$  Example 4.2 Is  $\Delta ABC \sim \Delta PQR$ ?





**Example 4.3** Observe Fig.4.18 and find  $\angle P$ .

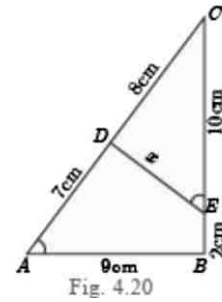
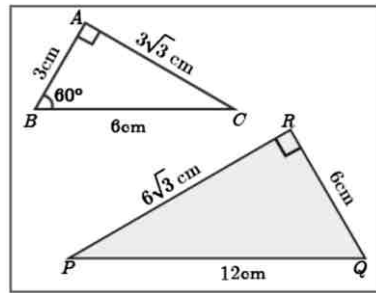


Fig. 4.20

**Example 4.5** In Fig.4.20  $\angle A = \angle CED$  prove that  $\triangle CAB \sim \triangle CED$ .

Also find the value of  $x$ .

**Example 4.4** A boy of height 90cm is walking away from the base of a lamp post at a speed of 1.2m/sec. If the lamppost is 3.6m above the ground, find the length of his shadow cast after 4 seconds.

**Example 4.6** In Fig.4.21,  $QA$  and  $PB$  are perpendiculars to  $AB$ . If  $AO = 10$  cm,  $BO = 6$  cm and  $PB = 9$  cm. Find  $AQ$ .

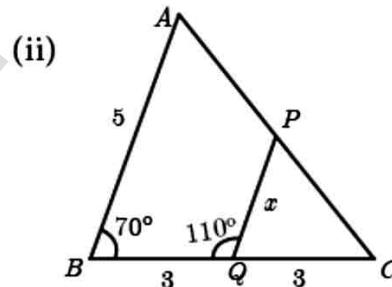
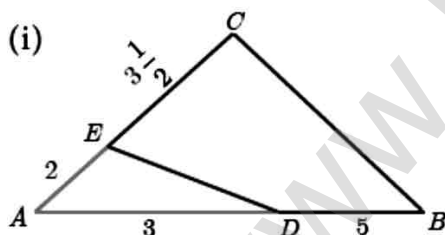
**Example 4.7** The perimeters of two similar triangles  $ABC$  and  $PQR$  are respectively 36 cm and 24 cm. If  $PQ = 10$  cm, find  $AB$ .

**Example 4.8** If  $\triangle ABC$  is similar to  $\triangle DEF$  such that  $BC = 3$  cm,  $EF = 4$  cm and area of  $\triangle ABC = 54$  cm<sup>2</sup>. Find the area of  $\triangle DEF$ .

**Example 4.9** Two poles of height ' $a$ ' metres and ' $b$ ' metres are ' $p$ ' meters apart. Prove that the height of the point of intersection of the lines joining the top of each pole to the foot of the opposite pole is given by  $\frac{ab}{a+b}$  metres.

### EXERCISE : 4.1

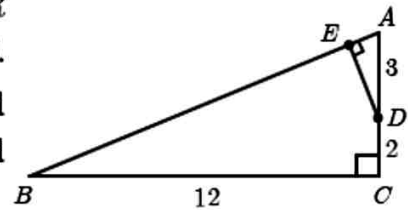
1. Check whether the which triangles are similar and find the value of  $x$ .



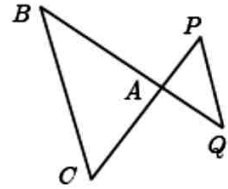
- A girl looks the reflection of the top of the lamp post on the mirror which is 6.6 m away from the foot of the lamppost. The girl whose height is 1.25 m is standing 2.5 m away from the mirror. Assuming the mirror is placed on the ground facing the sky and the girl, mirror and the lamppost are in a same line, find the height of the lamp post.
- A vertical stick of length 6 m casts a shadow 400 cm long on the ground and at the same time a tower casts a shadow 28 m long. Using similarity, find the height of the tower.

4. Two triangles  $QPR$  and  $QSR$ , right angled at  $P$  and  $S$  respectively are drawn on the same base  $QR$  and on the same side of  $QR$ . If  $PR$  and  $SQ$  intersect at  $T$ , prove that  $PT \times TR = ST \times TQ$ .

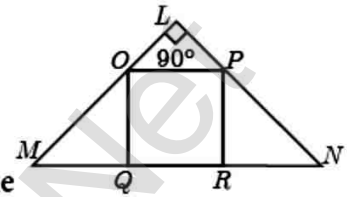
5. In the adjacent figure,  $\triangle ABC$  is right angled at  $C$  and  $DE \perp AB$ . Prove that  $\triangle ABC \sim \triangle ADE$  and hence find the lengths of  $AE$  and  $DE$ .



6. In the adjacent figure,  $\triangle ACB \sim \triangle APQ$ . If  $BC = 8$  cm,  $PQ = 4$  cm,  $BA = 6.5$  cm and  $AP = 2.8$  cm, find  $CA$  and  $AQ$ .

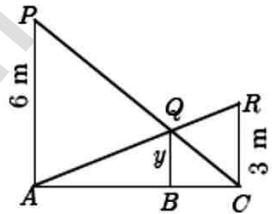


7. If figure  $OPRQ$  is a square and  $\angle MLN = 90^\circ$ . Prove that  
(i)  $\triangle LOP \sim \triangle QMO$  (ii)  $\triangle LOP \sim \triangle RPN$  (iii)  
 $\triangle QMO \sim \triangle RPN$  (iv)  $QR^2 = MQ \times RN$



8. If  $\triangle ABC \sim \triangle DEF$  such that area of  $\triangle ABC$  is  $9\text{cm}^2$  and the area of  $\triangle DEF$  is  $16\text{cm}^2$  and  $BC = 2.1$  cm. Find the length of  $EF$ .

9. Two vertical poles of heights 6 m and 3 m are erected above a horizontal ground  $AC$ . Find the value of  $y$ .



10. Construct a triangle similar to a given triangle  $PQR$  with its

**Theorem 1: Basic Proportionality Theorem (BPT) or Thales theorem**

**Theorem 2: Converse of Basic Proportionality Theorem**

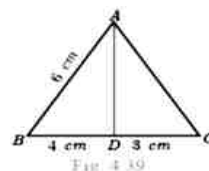
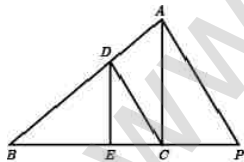
**Theorem 3: Angle Bisector Theorem**

**Theorem 4: Converse of Angle Bisector Theorem**

**Example 4.12** In  $\triangle ABC$ , if  $DE \parallel BC$ ,  $AD = x$ ,  $DB = x - 2$ ,  $AE = x + 2$  and  $EC = x - 1$  then find the lengths of the sides  $AB$  and  $AC$ .

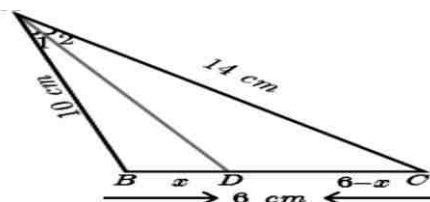
**Example 4.13**  $D$  and  $E$  are respectively the points on the sides  $AB$  and  $AC$  of a  $\triangle ABC$  such that  $AB = 5.6$  cm,  $AD = 1.4$  cm,  $AC = 7.2$  cm and  $AE = 1.8$  cm, show that  $DE \parallel BC$ .

**Example 4.14** In the Fig.4.38,  $DE \parallel AC$  and  $DC \parallel AP$ . Prove that  $\frac{BE}{EC} = \frac{BC}{CP}$ .



**Example 4.15** In the Fig.4.39,  $AD$  is the bisector of  $\angle A$ . If  $BD = 4$  cm,  $DC = 3$  cm and  $AB = 6$  cm, find  $AC$ .

**Example 4.16** In the Fig. 4.40,  $AD$  is the bisector of  $\angle BAC$ , if  $AB = 10$  cm,  $AC = 14$  cm and  $BC = 6$  cm. Find  $BD$  and  $DC$ .



**EXERCISE : 4.2**

1. In  $\triangle ABC$ ,  $D$  and  $E$  are points on the sides  $AB$  and  $AC$  respectively such that  $DE \parallel BC$  (i) If  $\frac{AD}{DB} = \frac{3}{4}$  and  $AC = 15$  cm find  $AE$ .

(ii) If  $AD = 8x - 7$ ,  $DB = 5x - 3$ ,  $AE = 4x - 3$  and  $EC = 3x - 1$ , find the value of  $x$ .

2.  $ABCD$  is a trapezium in which  $AB \parallel DC$  and  $P, Q$  are points on  $AD$  and  $BC$  respectively, such that  $PQ \parallel DC$  if  $PD = 18$  cm,  $BQ = 35$  cm and  $QC = 15$  cm, find  $AD$ .

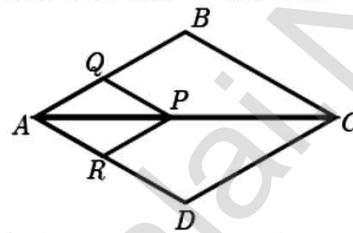
3. In  $\triangle ABC$ ,  $D$  and  $E$  are points on the sides  $AB$  and  $AC$  respectively. For each of the following cases show that  $DE \parallel BC$

(i)  $AB = 12$  cm,  $AD = 8$  cm,  $AE = 12$  cm and  $AC = 18$  cm.

(ii)  $AB = 5.6$  cm,  $AD = 1.4$  cm,  $AC = 7.2$  cm and  $AE = 1.8$  cm.

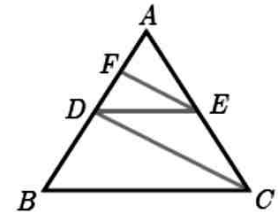
4. In fig. if  $PQ \parallel BC$  and  $PR \parallel CD$  prove that

(i)  $\frac{AR}{AD} = \frac{AQ}{AB}$  (ii)  $\frac{QB}{AQ} = \frac{DR}{AR}$ .



5. Rhombus  $PQRB$  is inscribed in  $\triangle ABC$  such that  $\angle B$  is one of its angle.  $P, Q$  and  $R$  lie on  $AB, AC$  and  $BC$  respectively. If  $AB = 12$  cm and  $BC = 6$  cm, find the sides  $PQ, RB$  of the rhombus.

6. In trapezium  $ABCD$ ,  $AB \parallel DC$ ,  $E$  and  $F$  are points on non-parallel sides  $AD$  and  $BC$  respectively, such that  $EF \parallel AB$ . Show that  $\frac{AE}{ED} = \frac{BF}{FC}$ .



7. In figure  $DE \parallel BC$  and  $CD \parallel EF$ . Prove that  $AD^2 = AB \times AF$ .

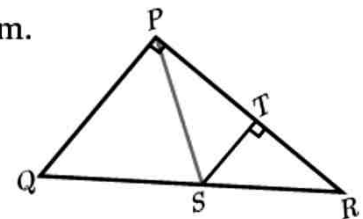
8. Check whether  $AD$  is bisector of  $\angle A$  of  $\triangle ABC$  in each of the following

(i)  $AB = 5$  cm,  $AC = 10$  cm,  $BD = 1.5$  cm and  $CD = 3.5$  cm.

(ii)  $AB = 4$  cm,  $AC = 6$  cm,  $BD = 1.6$  cm and  $CD = 2.4$  cm.

9. In figure  $\angle QPR = 90^\circ$ ,  $PS$  is its bisector.

If  $ST \perp PR$ , prove that  $ST \times (PQ + PR) = PQ \times PR$ .



10.  $ABCD$  is a quadrilateral in which  $AB = AD$ , the bisector of  $\angle BAC$  and  $\angle CAD$  intersect the sides  $BC$  and  $CD$  at the points  $E$  and  $F$  respectively. Prove that  $EF \parallel BD$ .

**Theorem 5 : Pythagoras Theorem and Converse of Pythagoras Theorem.**

**Example 4.20** An insect 8 m away initially from the foot of a lamp post which is 6 m tall, crawls towards it moving through a distance. If its distance from the top of the lamp post is equal to the distance it has moved, how far is the insect away from the foot of the lamp post?



**Example 4.21**  $P$  and  $Q$  are the mid-points of the sides  $CA$  and  $CB$  respectively of a  $\triangle ABC$ , right angled at  $C$ . Prove that  $4(AQ^2 + BP^2) = 5AB^2$ .

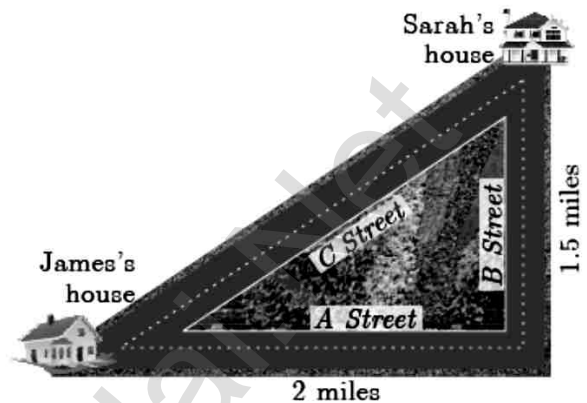
**Example 4.22** What length of ladder is needed to reach a height of 7 ft along the wall when the base of the ladder is 4 ft from the wall? Round off your answer to the next tenth place.

**Example 4.23** An Aeroplane after take off from an airport and flies due north at a speed of 1000 km/hr. At the same time, another aeroplane take off from the same airport and flies due west at a speed of 1200 km/hr. How far apart will be the two planes after  $1\frac{1}{2}$  hours?

### EXERCISE : 4.3

1. A man goes 18 m due east and then 24 m due north. Find the distance of his current position from the starting point?

2. There are two paths that one can choose to go from Sarah's house to James house. One way is to take  $C$  street, and the other way requires to take  $B$  street and then  $A$  street. How much shorter is the direct path along  $C$  street? (Using figure).



3. To get from point  $A$  to point  $B$  you must avoid walking through a pond. You must walk 34 m south and 41 m east. To the nearest meter, how many meters would be saved if it were possible to make a way through the pond?

4. In the rectangle  $WXYZ$ ,  $XY + YZ = 17$  cm, and  $XZ + YW = 26$  cm. Calculate the length and breadth of the rectangle?

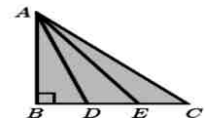


5. The hypotenuse of a right triangle is 6 m more than twice of the shortest side. If the third side is 2 m less than the hypotenuse, find the sides of the triangle.

6. 5 m long ladder is placed leaning towards a vertical wall such that it reaches the wall at a point 4m high. If the foot of the ladder is moved 1.6 m towards the wall, then find the distance by which the top of the ladder would slide upwards on the wall.

7. The perpendicular  $PS$  on the base  $QR$  of a  $\triangle PQR$  intersects  $QR$  at  $S$ , such that  $QS = 3SR$ . Prove that  $2PQ^2 = 2PR^2 + QR^2$

8. In the adjacent figure,  $ABC$  is a right angled triangle with right angle at  $B$  and points  $D, E$  trisect  $BC$ . Prove that  $8AE^2 = 3AC^2 + 5AD^2$



### Theorem 6 : Alternate Segment theorem

**Example 4.24** : Find the length of the tangent drawn from a point whose distance from the centre of a circle is 5 cm and radius of the circle is 3 cm.

**Example 4.25** :  $PQ$  is a chord of length 8 cm to a circle of radius 5 cm. The tangents at  $P$  and  $Q$  intersect at a point  $T$ . Find the length of the tangent  $TP$ .

**Example 4.26 :** In Fig.4.64,  $O$  is the centre of a circle.  $PQ$  is a chord and the tangent  $PR$  at  $P$  makes an angle of  $50^\circ$  with  $PQ$ . Find  $\angle POQ$ .

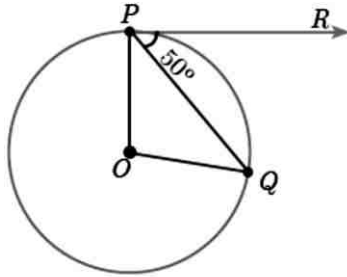


Fig. 4.64

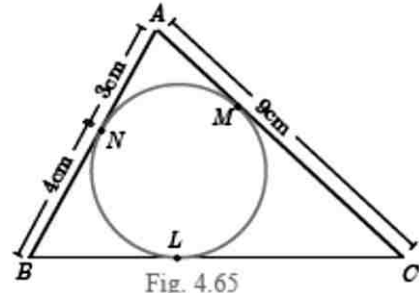


Fig. 4.65

**Example 4.27** In Fig.4.65,  $\triangle ABC$  is circumscribing a circle. Find the length of  $BC$ .

**Example 4.28** If radii of two concentric circles are 4 cm and 5 cm then find the length of the chord of one circle which is a tangent to the other circle.

**Example 4.32** Show that in a triangle, the medians are concurrent.

**Example 4.33** Suppose  $AB$ ,  $AC$  and  $BC$  have lengths 13, 14 and 15 respectively. If

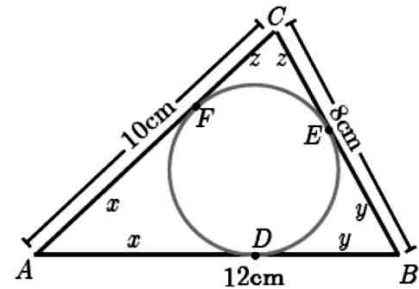
$$\frac{AF}{FB} = \frac{2}{5} \text{ and } \frac{CE}{EA} = \frac{5}{8}. \text{ Find } BD \text{ and } DC.$$

**Example 4.34** In a garden containing several trees, three particular trees  $P, Q, R$  are located in the following way,  $BP = 2$  m,  $CQ = 3$  m,  $RA = 10$  m,  $PC = 6$  m,  $QA = 5$  m,  $RB = 2$  m, where  $A, B, C$  are points such that  $P$  lies on  $BC, Q$  lies on  $AC$  and  $R$  lies on  $AB$ . Check whether the trees  $P, Q, R$  lie on a same straight line.

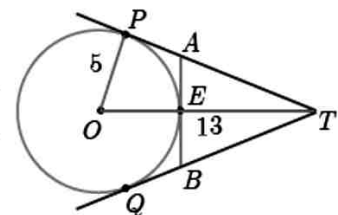
#### EXERCISE : 4.4

- The length of the tangent to a circle from a point  $P$ , which is 25 cm away from the centre is 24 cm. What is the radius of the circle?
- $\triangle LMN$  is a right angled triangle with  $\angle L = 90^\circ$ . A circle is inscribed in it. The lengths of the sides containing the right angle are 6 cm and 8 cm. Find the radius of the circle.

- A circle is inscribed in  $\triangle ABC$  having sides 8 cm, 10 cm and 12 cm as shown in figure, Find  $AD, BE$  and  $CF$ .



- $PQ$  is a tangent drawn from a point  $P$  to a circle with centre  $O$  and  $QOR$  is a diameter of the circle such that  $\angle POR = 120^\circ$ . Find  $\angle OPQ$ .
- A tangent  $ST$  to a circle touches it at  $B$ .  $AB$  is a chord such that  $\angle ABT = 65^\circ$ . Find  $\angle AOB$ , where " $O$ " is the centre of the circle.
- In figure,  $O$  is the centre of the circle with radius 5 cm.  $T$  is a point such that  $OT = 13$  cm and  $OT$  intersects the circle  $E$ , if  $AB$  is the tangent to the circle at  $E$ , find the length of  $AB$ .

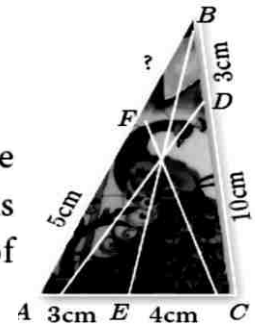


- In two concentric circles, a chord of length 16 cm of larger circle becomes a tangent to the smaller circle whose radius is 6 cm. Find the radius of the larger circle.
- Two circles with centres  $O$  and  $O'$  of radii 3 cm and 4 cm, respectively intersect at two points  $P$  and  $Q$ , such that  $OP$  and  $O'P$  are tangents to the two circles. Find the



length of the common chord  $PQ$ .

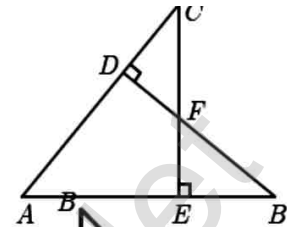
9. Show that the angle bisectors of a triangle are concurrent.
10. An artist has created a triangular stained glass window and has one strip of small length left before completing the window. She needs to figure out the length of left out portion based on the lengths of the other sides as shown in the figure.



**UNIT EXERCISE : 4**

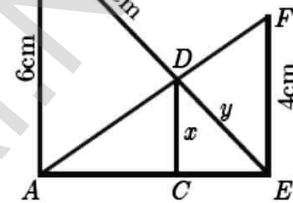
1. In the figure, if  $BD \perp AC$  and  $CE \perp AB$ , prove that

(i)  $\triangle AEC \sim \triangle ADB$       (ii)  $\frac{CA}{AB} = \frac{CE}{DB}$



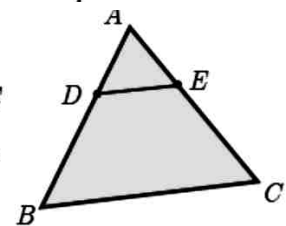
2. In the given figure  $AB \parallel CD \parallel EF$ .

If  $AB = 6$  cm,  $CD = x$  cm,  $EF = 4$  cm,  $BD = 5$  cm and  $DE = y$  cm. Find  $x$  and  $y$ .



3. O is any point inside a triangle  $ABC$ . The bisector of  $\angle AOB$ ,  $\angle BOC$  and  $\angle COA$  meet the sides  $AB$ ,  $BC$  and  $CA$  in point  $D$ ,  $E$  and  $F$  respectively. Show that  $AD \times BE \times CF = DB \times EC \times FA$

4. In the figure,  $ABC$  is a triangle in which  $AB = AC$ . Points  $D$  and  $E$  are points on the side  $AB$  and  $AC$  respectively such that  $AD = AE$ . Show that the points  $B, C, E$  and  $D$  lie on a same circle.



5. Two trains leave a railway station at the same time. The first train travels due west and the second train due north. The first train travels at a speed of 20 km/hr and the second train travels at 30 km/hr. After 2 hours, what is the distance between them?

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**5. COORDINATE GEOMETRY**

**Example 5.1** Find the area of the triangle whose vertices are  $(-3,5)$ ,  $(5,6)$  and  $(5,-2)$

**Example 5.2** Show that the points  $P(-1.5,3)$ ,  $Q(6,-2)$ ,  $R(-3,4)$  are collinear.

**Example 5.3** If the area of the triangle formed by the vertices  $A(-1,2)$ ,  $B(k,-2)$  and  $C(7,4)$  (taken in order) is 22 sq. units, find the value of  $k$ .

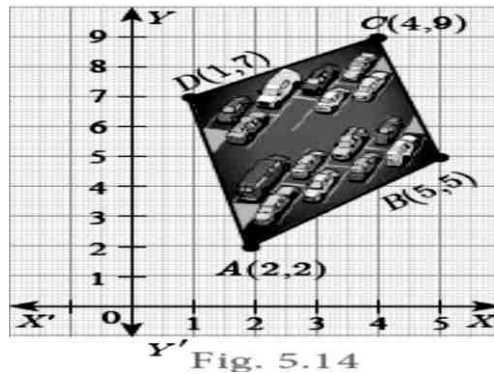
**Example 5.4** If the points  $P(-1,-4)$ ,  $Q(b,c)$  and  $R(5,-1)$  are collinear and if  $2b + c = 4$ , then find the values of  $b$  and  $c$ .

**Example 5.5** The floor of a hall is covered with identical tiles which are in the shapes of triangles. One such triangle has the vertices at  $(-3,2)$ ,  $(-1,-1)$  and  $(1,2)$ . If the floor of the hall is completely covered by 110 tiles, find the area of the floor.

**Example 5.6** Find the area of the quadrilateral formed by the points  $(8,6)$ ,  $(5,11)$ ,  $(-5,12)$  and  $(-4,3)$ .

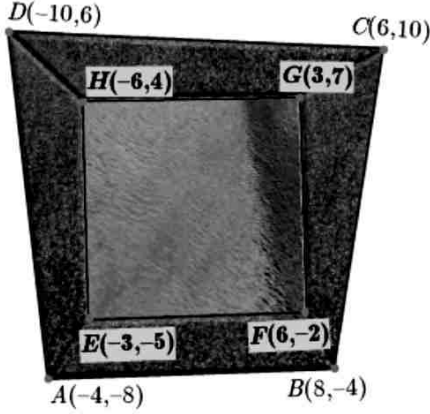


**Example 5.7** The given diagram shows a plan for constructing a new parking lot at a campus. It is estimated that such construction would cost Rs.1300 per square feet. What will be the total cost for making the parking lot?



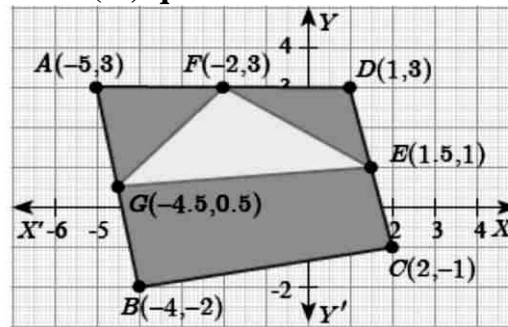
### EXERCISE : 5.1

- Find the area of the triangle formed by the points
  - $(1, -1)$ ,  $(-4, 6)$  and  $(-3, -5)$
  - $(-10, -4)$ ,  $(-8, -1)$  and  $(-3, -5)$
- Determine whether the sets of points are collinear?
  - $\left(-\frac{1}{2}, 3\right)$ ,  $(-5, 6)$  and  $(-8, 8)$
  - $(a, b+c)$ ,  $(b, c+a)$  and  $(c, a+b)$
- Vertices of given triangles are taken in order and their areas are provided aside. In each case, find the value of 'p' :
 

| S.No. | Vertices                        | Area (sq.units) |
|-------|---------------------------------|-----------------|
| (i)   | $(0, 0)$ , $(p, 8)$ , $(6, 2)$  | 20              |
| (ii)  | $(p, p)$ , $(5, 6)$ , $(5, -2)$ | 32              |
- In each of the following, find the value of 'a' for which the given points are collinear.
  - $(2, 3)$ ,  $(4, a)$  and  $(6, -3)$
  - $(a, 2-2a)$ ,  $(-a+1, 2a)$  and  $(-4-a, 6-2a)$
- Find the area of the quadrilateral whose vertices are at
  - $(-9, -2)$ ,  $(-8, -4)$ ,  $(2, 2)$  and  $(1, -3)$
  - $(-9, 0)$ ,  $(-8, 6)$ ,  $(-1, -2)$  and  $(-6, -3)$
- Find the value of  $k$ , if the area of a quadrilateral is 28 sq.units, whose vertices are  $(-4, -2)$ ,  $(-3, k)$ ,  $(3, -2)$  and  $(2, 3)$
- If the points  $A(-3, 9)$ ,  $B(a, b)$  and  $C(4, -5)$  are collinear and if  $a + b = 1$ , then find  $a$  and  $b$ .
- Let  $P(11, 7)$ ,  $Q(13.5, 4)$  and  $R(9.5, 4)$  be the mid-points of the sides  $AB$ ,  $BC$  and  $AC$  respectively of  $\triangle ABC$ . Find the coordinates of the vertices  $A$ ,  $B$  and  $C$ . Hence find the area of  $\triangle ABC$  and compare this with area of  $\triangle PQR$ .
- In the figure, the quadrilateral swimming pool shown is surrounded by concrete patio. Find the area of the patio.
 
- A triangular shaped glass with vertices at  $A(-5, -4)$ ,  $B(1, 6)$  and  $C(7, -4)$  has to be painted. If one bucket of paint covers 6 square feet, how many

buckets of paint will be required to paint the whole glass, if only one coat of paint is applied.

11. In the figure, find the area of (i) triangle AGF (ii) triangle FED  
(iii) quadrilateralBCEG.



Example 5.8 (i) What is the slope of a line whose inclination is  $30^\circ$ ?

Example 5.9 Find the slope of a line joining the given points

Example 5.10 The line  $r$  passes through the points  $(-2, 2)$  and  $(5, 8)$  and the line  $s$  passes through the points  $(-8, 7)$  and  $(-2, 0)$ . Is the line  $r$  perpendicular to  $s$ ?

Example 5.11 The line  $p$  passes through the points  $(3, -2)$ ,  $(12, 4)$  and the line  $q$  passes through the points  $(6, -2)$  and  $(12, 2)$ . Is  $p$  parallel to  $q$ ?

Example 5.12 Show that the points  $(-2, 5)$ ,  $(6, -1)$  and  $(2, 2)$  are collinear.

Example 5.13 Let  $A(1, -2)$ ,  $B(6, -2)$ ,  $C(5, 1)$  and  $D(2, 1)$  be four points

- (i) Find the slope of the line segments (a)  $AB$  (b)  $CD$
- (ii) Find the slope of the line segments (a)  $BC$  (b)  $AD$
- (iii) What can you deduce from your answer.

Example 5.14 Consider the graph representing growth of population (in crores). Find the slope of the line  $AB$  and hence estimate the population in the year 2030?

Example 5.15 Without using Pythagoras theorem, show that the points  $(1, -4)$ ,  $(2, -3)$  and  $(4, -7)$  form a right angled triangle.

Example 5.16 Prove analytically that the line segment joining the mid-points of two sides of a triangle is parallel to the third side and is equal to half of its length.

### EXERCISE :5.2

1. What is the slope of a line whose inclination with positive direction of  $x$ -axis is  
(i)  $90^\circ$  (ii)  $0^\circ$
2. What is the inclination of a line whose slope is (i) 0 (ii) 1
3. Find the slope of a line joining the points  
(i)  $(5, \sqrt{5})$  with the origin (ii)  $(\sin \theta, -\cos \theta)$  and  $(-\sin \theta, \cos \theta)$
4. What is the slope of a line perpendicular to the line joining  $A(5, 1)$  and  $P$  where  $P$  is the mid-point of the segment joining  $(4, 2)$  and  $(-6, 4)$ .
5. Show that the given points are collinear:  $(-3, -4)$ ,  $(7, 2)$  and  $(12, 5)$
6. If the three points  $(3, -1)$ ,  $(a, 3)$  and  $(1, -3)$  are collinear, find the value of  $a$ .



7. The line through the points  $(-2, a)$  and  $(9, 3)$  has slope  $-\frac{1}{2}$ . Find the value of  $a$ .
8. The line through the points  $(-2, 6)$  and  $(4, 8)$  is perpendicular to the line through the points  $(8, 12)$  and  $(x, 24)$ . Find the value of  $x$ .
9. Show that the given points form a right angled triangle and check whether they satisfies pythagoras theorem  
(i)  $A(1, -4)$ ,  $B(2, -3)$  and  $C(4, -7)$  (ii)  $L(0, 5)$ ,  $M(9, 12)$  and  $N(3, 14)$
10. Show that the given points form a parallelogram :  
 $A(2.5, 3.5)$ ,  $B(10, -4)$ ,  $C(2.5, -2.5)$  and  $D(-5, 5)$
11. If the points  $A(2, 2)$ ,  $B(-2, -3)$ ,  $C(1, -3)$  and  $D(x, y)$  form a parallelogram then find the value of  $x$  and  $y$ .
12. Let  $A(3, -4)$ ,  $B(9, -4)$ ,  $C(5, -7)$  and  $D(7, -7)$ . Show that  $ABCD$  is a trapezium.
13. A quadrilateral has vertices at  $A(-4, -2)$ ,  $B(5, -1)$ ,  $C(6, 5)$  and  $D(-7, 6)$ . Show that the mid-points of its sides form a parallelogram.

**Example 5.18** Find the equation of a straight line whose

- (i) Slope is 5 and  $y$  intercept is  $-9$  (ii) Inclination is  $45^\circ$  and  $y$  intercept is 11

**Example 5.19** Calculate the slope and  $y$  intercept of the straight line  $8x - 7y + 6 = 0$

**Example 5.20** The graph relates temperatures  $y$  (in Fahrenheit degree) to temperatures  $x$  (in Celsius degree) (a) Find the slope and  $y$  intercept (b) Write an equation of the line (c) What is the mean temperature of the earth in Fahrenheit degree if its mean temperature is  $25^\circ$  Celsius?

**Example 5.21** Find the equation of a line passing

through the point  $(3, -4)$  and having slope  $\frac{-5}{7}$

**Example 5.22** Find the equation of a line passing through the point  $A(1, 4)$  and perpendicular to the line joining points  $(2, 5)$  and  $(4, 7)$ .

**Example 5.23** Find the equation of a straight line passing through  $(5, -3)$  and  $(7, -4)$ .

**Example 5.24** Two buildings of different heights are located at opposite sides of each other. If a heavy rod is attached joining the terrace of the buildings from  $(6, 10)$  to  $(14, 12)$ , find the equation of the rod joining the buildings ?

**Example 5.25** Find the equation of a line which passes through  $(5, 7)$  and makes intercepts on the axes equal in magnitude but opposite in sign.

**Example 5.26** Find the intercepts made by the line  $4x - 9y + 36 = 0$  on the coordinate axes.

**Example 5.27** A mobile phone is put to use when the battery power is 100%. The percent of battery power ' $y$ ' (in decimal) remaining after using the mobile phone for  $x$  hours is assumed as  $y = -0.25x + 1$

**Example 5.28** A line makes positive intercepts on coordinate axes whose sum is 7 and it passes through  $(-3, 8)$ . Find its equation.



**Example 5.29** A circular garden is bounded by East Avenue and Cross Road. Cross Road intersects North Street at  $D$  and East Avenue at  $E$ .  $AD$  is tangential to the circular garden at  $A(3, 10)$ . Using the figure.

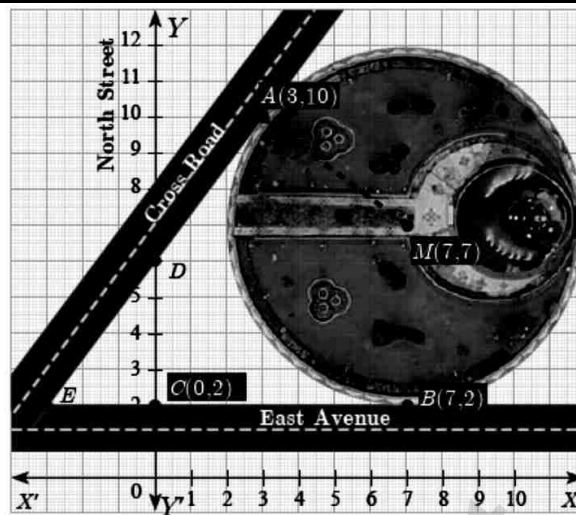


Fig. 5.51

- (a) Find the equation of
- East Avenue.
  - North Street
  - Cross Road
- (b) Where does the Cross Road intersect?
- North Street
  - East Avenue

### EXERCISE :5.3

- Find the equation of a straight line passing through the mid-point of a line segment joining the points  $(1, -5)$ ,  $(4, 2)$  and parallel to (i)  $X$  axis (ii)  $Y$  axis
- The equation of a straight line is  $2(x - y) + 5 = 0$ . Find its slope, inclination and intercept on the  $Y$  axis.
- Find the equation of a line whose inclination is  $30^\circ$  and making an intercept  $-3$  on the  $Y$  axis.
- Find the slope and  $y$  intercept of  $\sqrt{3}x + (1 - \sqrt{3})y = 3$ .
- Find the value of ' $a$ ', if the line through  $(-2, 3)$  and  $(8, 5)$  is perpendicular to  $y = ax + 2$
- The hill in the form of a right triangle has its foot at  $(19, 3)$ . The inclination of the hill to the ground is  $45^\circ$ . Find the equation of the hill joining the foot and top.
- Find the equation of a line through the given pair of points
  - $\left(2, \frac{2}{3}\right)$  and  $\left(\frac{-1}{2}, -2\right)$
  - $(2, 3)$  and  $(-7, -1)$
- A cat is located at the point  $(-6, -4)$  in  $xy$  plane. A bottle of milk is kept at  $(5, 11)$ . The cat wish to consume the milk travelling through shortest possible distance. Find the equation of the path it needs to take its milk.
- Find the equation of the median and altitude of  $\triangle ABC$  through  $A$  where the vertices are  $A(6, 2)$ ,  $B(-5, -1)$  and  $C(1, 9)$
- Find the equation of a straight line which has slope  $\frac{-5}{4}$  and passing through the point  $(-1, 2)$ .
- You are downloading a song. The percent  $y$  (in decimal form) of mega bytes remaining to get downloaded in  $x$  seconds is given by  $y = -0.1x + 1$ .
  - find the total MB of the song.
  - after how many seconds will 75% of the song gets downloaded?
  - after how many seconds the song will be downloaded completely?

12. Find the equation of a line whose intercepts on the  $x$  and  $y$  axes are given below.  
 (i) 4, -6                      (ii)  $-5, \frac{3}{4}$
13. Find the intercepts made by the following lines on the coordinate axes.  
 (i)  $3x - 2y - 6 = 0$     (ii)  $4x + 3y + 12 = 0$
14. Find the equation of a straight line  
 (i) passing through (1, -4) and has intercepts which are in the ratio 2:5  
 (ii) passing through (-8,4) and making equal intercepts on the coordinate axes

### UNIT EXERCISE : 5

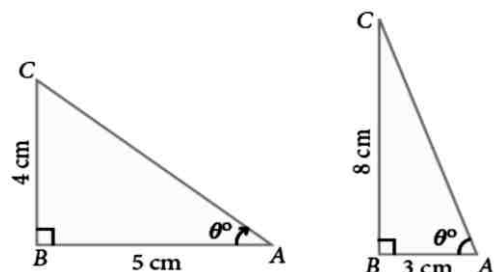
- PQRS is a rectangle formed by joining the points  $P(-1,-1)$ ,  $Q(-1,4)$ ,  $R(5,4)$  and  $S(5,-1)$ .  $A$ ,  $B$ ,  $C$  and  $D$  are the mid-points of  $PQ$ ,  $QR$ ,  $RS$  and  $SP$  respectively. Is the quadrilateral  $ABCD$  a square, a rectangle or a rhombus? Justify your answer.
- The area of a triangle is 5 sq.units. Two of its vertices are (2,1) and (3, -2). The third vertex is  $(x, y)$  where  $y = x + 3$ . Find the coordinates of the third vertex.
- Find the area of a triangle formed by the lines  $3x + y - 2 = 0$ ,  $5x + 2y - 3 = 0$  and  $2x - y - 3 = 0$
- If vertices of a quadrilateral are at  $A(-5,7)$ ,  $B(-4,k)$ ,  $C(-1,-6)$  and  $D(4,5)$  and its area is 72 sq.units. Find the value of  $k$ .
- Without using distance formula, show that the points  $(-2,-1)$ ,  $(4,0)$ ,  $(3,3)$  and  $(-3,2)$  are vertices of a parallelogram.
- Find the equations of the lines, whose sum and product of intercepts are 1 and -6 respectively.
- The owner of a milk store finds that, he can sell 980 litres of milk each week at ₹14/litre and 1220 litres of milk each week at ₹16/litre. Assuming a linear relationship between selling price and demand, how many litres could he sell weekly at ₹17/litre?
- Find the image of the point (3,8) with respect to the line  $x + 3y = 7$  assuming the line to be a plane mirror.

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### 6. TRIGONOMETRY

#### Example 6.18

Calculate the size of  $\angle BAC$  in the given triangles. ( $\tan 38.7^\circ = 0.8011$ ,  $\tan 69.4^\circ = 2.6604$ )



**Example 6.19** A tower stands vertically on the ground. From a point on the ground, which is 48 m away from the foot of the tower, the angle of elevation of the top of the tower is  $30^\circ$ . Find the height of the tower.



**Example 6.20** A kite is flying at a height of 75 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is  $60^\circ$ . Find the length of the string, assuming that there is no slack in the string.

**Example 6.21** Two ships are sailing in the sea on either sides of a lighthouse. The angle of elevation of the top of the lighthouse as observed from the ships are  $30^\circ$  and  $45^\circ$  respectively. If the lighthouse is 200 m high, find the distance between the two ships. ( $\sqrt{3} = 1.732$ )

**Example 6.22** From a point on the ground, the angles of elevation of the bottom and top of a tower fixed at the top of a 30 m high building are  $45^\circ$  and  $60^\circ$  respectively. Find the height of the tower. ( $\sqrt{3} = 1.732$ )

**Example 6.23** A TV tower stands vertically on a bank of a canal. The tower is watched from a point on the other bank directly opposite to it. The angle of elevation of the top of the tower is  $58^\circ$ . From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is  $30^\circ$ . Find the height of the tower and the width of the canal. ( $\tan 58^\circ = 1.6003$ )

**Example 6.24** An aeroplane sets off from  $G$  on a bearing of  $24^\circ$  towards  $H$ , a point 250 km away. At  $H$  it changes course and heads towards  $J$  deviates further by  $55^\circ$  and a distance of 180 km away.

(i) How far is  $H$  to the North of  $G$ ?      (ii) How far is  $H$  to the East of  $G$ ?

(iii) How far is  $J$  to the North of  $H$ ?      (iv) How far is  $J$  to the East of  $H$ ?

$$\left( \begin{array}{ll} \sin 24^\circ = 0.4067 & \sin 11^\circ = 0.1908 \\ \cos 24^\circ = 0.9135 & \cos 11^\circ = 0.9816 \end{array} \right)$$

**Example 6.25** Two trees are standing on flat ground. The angle of elevation of the top of both the trees from a point  $X$  on the ground is  $40^\circ$ . If the horizontal distance between  $X$  and the smaller tree is 8 m and the distance of the top of the two trees is 20 m, calculate

(i) the distance between the point  $X$  and the top of the smaller tree.

(ii) the horizontal distance between the two trees.

$$(\cos 40^\circ = 0.7660)$$

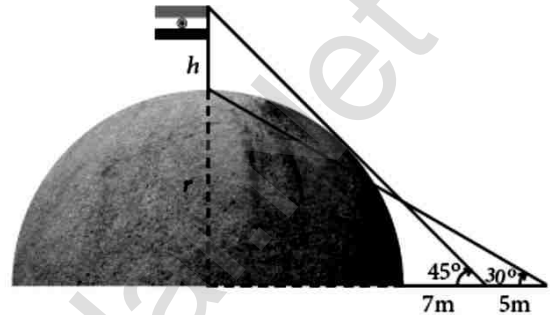
### EXERCISE : 6.2

- Find the angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of a tower of height  $10\sqrt{3}$  m.
- A road is flanked on either side by continuous rows of houses of height  $4\sqrt{3}$  m with no space in between them. A pedestrian is standing on the median of the road facing a row



house. The angle of elevation from the pedestrian to the top of the house is  $30^\circ$ . Find the width of the road.

3. To a man standing outside his house, the angles of elevation of the top and bottom of a window are  $60^\circ$  and  $45^\circ$  respectively. If the height of the man is 180 cm and if he is 5 m away from the wall, what is the height of the window? ( $\sqrt{3} = 1.732$ )
4. A statue 1.6 m tall stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is  $60^\circ$  and from the same point the angle of elevation of the top of the pedestal is  $40^\circ$ . Find the height of the pedestal. ( $\tan 40^\circ = 0.8391$ ,  $\sqrt{3} = 1.732$ )
5. A flag pole ' $h$ ' metres is on the top of the hemispherical dome of radius ' $r$ ' metres. A man is standing 7 m away from the dome. Seeing the top of the pole at an angle  $45^\circ$  and moving 5 m away from the dome and seeing the bottom of the pole at an angle  $30^\circ$ . Find (i) the height of the pole (ii) radius of the dome. ( $\sqrt{3} = 1.732$ )
6. The top of a 15 m high tower makes an angle of elevation of  $60^\circ$  with the bottom of an electronic pole and angle of elevation of  $30^\circ$  with the top of the pole. What is the height of the electric pole?



**Example 6.26** A player sitting on the top of a tower of height 20 m observes the angle of depression of a ball lying on the ground as  $60^\circ$ . Find the distance between the foot of the tower and the ball. ( $\sqrt{3} = 1.732$ )

**Example 6.27** The horizontal distance between two buildings is 140 m. The angle of depression of the top of the first building when seen from the top of the second building is  $30^\circ$ . If the height of the first building is 60 m, find the height of the second building. ( $\sqrt{3} = 1.732$ )

**Example 6.28** From the top of a tower 50 m high, the angles of depression of the top and bottom of a tree are observed to be  $30^\circ$  and  $45^\circ$  respectively. Find the height of the tree. ( $\sqrt{3} = 1.732$ )

**Example 6.29** As observed from the top of a 60 m high lighthouse from the sea level, the angles of depression of two ships are  $28^\circ$  and  $45^\circ$ . If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships. ( $\tan 28^\circ = 0.5317$ )

**Example 6.30** A man is watching a boat speeding away from the top of a tower. The boat makes an angle of depression of  $60^\circ$  with the man's eye when at a distance of 200 m from the tower. After 10 seconds, the angle of depression becomes  $45^\circ$ . What is the approximate speed of the boat (in km / hr), assuming that it is sailing in still water? ( $\sqrt{3} = 1.732$ )

**EXERCISE :6.3**

1. From the top of a rock  $50\sqrt{3}$  m high, the angle of depression of a car on the ground is observed to be  $30^\circ$ . Find the distance of the car from the rock.
2. The horizontal distance between two buildings is 70 m. The angle of depression of the top of the first building when seen from the top of the second building is  $45^\circ$ . If the height of the second building is 120 m, find the height of the first building.
3. From the top of the tower 60 m high the angles of depression of the top and bottom of a vertical lamp post are observed to be  $38^\circ$  and  $60^\circ$  respectively. Find the height of the lamp post. ( $\tan 38^\circ = 0.7813$ ,  $\sqrt{3} = 1.732$ )
1. From the top of a rock  $50\sqrt{3}$  m high, the angle of depression of a car on the ground is observed to be  $30^\circ$ . Find the distance of the car from the rock.
2. The horizontal distance between two buildings is 70 m. The angle of depression of the top of the first building when seen from the top of the second building is  $45^\circ$ . If the height of the second building is 120 m, find the height of the first building.
3. From the top of the tower 60 m high the angles of depression of the top and bottom of a vertical lamp post are observed to be  $38^\circ$  and  $60^\circ$  respectively. Find the height of the lamp post. ( $\tan 38^\circ = 0.7813$ ,  $\sqrt{3} = 1.732$ )
4. An aeroplane at an altitude of 1800 m finds that two boats are sailing towards it in the same direction. The angles of depression of the boats as observed from the aeroplane are  $60^\circ$  and  $30^\circ$  respectively. Find the distance between the two boats. ( $\sqrt{3} = 1.732$ )
5. From the top of a lighthouse, the angle of depression of two ships on the opposite sides of it are observed to be  $30^\circ$  and  $60^\circ$ . If the height of the lighthouse is  $h$  meters and the line joining the ships passes through the foot of the lighthouse, show that the distance between the ships is  $\frac{4h}{\sqrt{3}}$  m.
6. A lift in a building of height 90 feet with transparent glass walls is descending from the top of the building. At the top of the building, the angle of depression to a fountain in the garden is  $60^\circ$ . Two minutes later, the angle of depression reduces to  $30^\circ$ . If the fountain is  $30\sqrt{3}$  feet from the entrance of the lift, find the speed of the lift which is descending.

**Example 6.31** From the top of a 12 m high building, the angle of elevation of the top of a cable tower is  $60^\circ$  and the angle of depression of its foot is  $30^\circ$ . Determine the height of the tower.

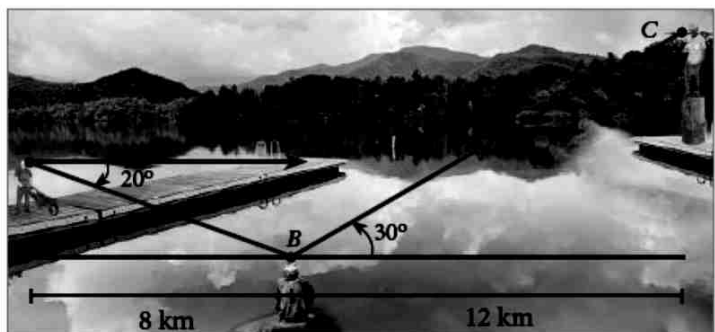
**Example 6.32** A pole 5 m high is fixed on the top of a tower. The angle of elevation of the top of the pole observed from a point 'A' on the ground is  $60^\circ$  and the angle of depression to the point 'A' from the top of the tower is  $45^\circ$ . Find the height of the tower. ( $\sqrt{3} = 1.732$ )



**Example 6.33** From a window ( $h$  metres high above the ground) of a house in a street, the angles of elevation and depression of the top and the foot of another house on the opposite side of the street are  $\theta_1$  and  $\theta_2$  respectively. Show that the height of the opposite house is  $h \left( 1 + \frac{\cot \theta_2}{\cot \theta_1} \right)$ .

### EXERCISE :6.4

1. From the top of a tree of height 13 m the angle of elevation and depression of the top and bottom of another tree are  $45^\circ$  and  $30^\circ$  respectively. Find the height of the second tree. ( $\sqrt{3} = 1.732$ )
2. A man is standing on the deck of a ship, which is 40 m above water level. He observes the angle of elevation of the top of a hill as  $60^\circ$  and the angle of depression of the base of the hill as  $30^\circ$ . Calculate the distance of the hill from the ship and the height of the hill. ( $\sqrt{3} = 1.732$ )
3. If the angle of elevation of a cloud from a point ' $h$ ' metres above a lake is  $\theta_1$  and the angle of depression of its reflection in the lake is  $\theta_2$ . Prove that the height that the cloud is located from the ground is  $\frac{h(\tan \theta_1 + \tan \theta_2)}{\tan \theta_2 - \tan \theta_1}$ .
4. The angle of elevation of the top of a cell phone tower from the foot of a high apartment is  $60^\circ$  and the angle of depression of the foot of the tower from the top of the apartment is  $30^\circ$ . If the height of the apartment is 50 m, find the height of the cell phone tower. According to radiations control norms, the minimum height of a cell phone tower should be 120 m. State if the height of the above mentioned cell phone tower meets the radiation norms.
5. The angles of elevation and depression of the top and bottom of a lamp post from the top of a 66 m high apartment are  $60^\circ$  and  $30^\circ$  respectively. Find
  - (i) The height of the lamp post.
  - (ii) The difference between height of the lamp post and the apartment.
  - (iii) The distance between the lamp post and the apartment. ( $\sqrt{3} = 1.732$ )
6. Three villagers  $A$ ,  $B$  and  $C$  can see each other using telescope across a valley. The horizontal distance between  $A$  and  $B$  is 8 km and the horizontal distance between  $B$  and  $C$  is 12 km. The angle of depression of  $B$  from  $A$  is  $20^\circ$  and the angle of elevation of  $C$  from  $B$  is  $30^\circ$ . Calculate : (i) the vertical height between  $A$  and  $B$ . (ii) the vertical height between  $B$  and  $C$ . ( $\tan 20^\circ = 0.3640$ ,  $\sqrt{3} = 1.732$ )





**UNIT EXERCISE : 6**

5. A bird is sitting on the top of a 80 m high tree. From a point on the ground, the angle of elevation of the bird is  $45^\circ$ . The bird flies away horizontally in such a way that it remained at a constant height from the ground. After 2 seconds, the angle of elevation of the bird from the same point is  $30^\circ$ . Determine the speed at which the bird flies. ( $\sqrt{3} = 1.732$ )
6. An aeroplane is flying parallel to the Earth's surface at a speed of 175 m/sec and at a height of 600 m. The angle of elevation of the aeroplane from a point on the Earth's surface is  $37^\circ$ . After what period of time does the angle of elevation increase to  $53^\circ$ ? ( $\tan 53^\circ = 1.3270$ ,  $\tan 37^\circ = 0.7536$ )
7. A bird is flying from  $A$  towards  $B$  at an angle of  $35^\circ$ , a point 30 km away from  $A$ . At  $B$  it changes its course of flight and heads towards  $C$  on a bearing of  $48^\circ$  and distance 32 km away.
- (i) How far is  $B$  to the North of  $A$ ?                      (ii) How far is  $B$  to the West of  $A$ ?  
 (iii) How far is  $C$  to the North of  $B$ ?                      (iv) How far is  $C$  to the East of  $B$ ?
- ( $\sin 55^\circ = 0.8192$ ,  $\cos 55^\circ = 0.5736$ ,  $\sin 42^\circ = 0.6691$ ,  $\cos 42^\circ = 0.7431$ )
8. Two ships are sailing in the sea on either side of the lighthouse. The angles of depression of two ships as observed from the top of the lighthouse are  $60^\circ$  and  $45^\circ$  respectively. If the distance between the ships is  $200 \left( \frac{\sqrt{3} + 1}{\sqrt{3}} \right)$  metres, find the height of the lighthouse.
9. A building and a statue are in opposite side of a street from each other 35 m apart. From a point on the roof of building the angle of elevation of the top of statue is  $24^\circ$  and the angle of depression of base of the statue is  $34^\circ$ . Find the height of the statue. ( $\tan 24^\circ = 0.4452$ ,  $\tan 34^\circ = 0.6745$ )

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**7. MENSURATION**

**Example 7.1** A cylindrical drum has a height of 20 cm and base radius of 14 cm. Find its curved surface area and the total surface area.

**Example 7.2** The curved surface area of a right circular cylinder of height 14 cm is  $88 \text{ cm}^2$ . Find the diameter of the cylinder.

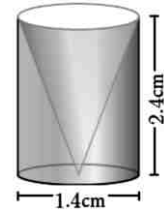
**Example 7.3** A garden roller whose length is 3 m long and whose diameter is 2.8 m is rolled to level a garden. How much area will it cover in 8 revolutions?

**Example 7.4** If one litre of paint covers  $10 \text{ m}^2$ , how many litres of paint is required to paint the internal and external surface areas of a cylindrical tunnel whose thickness is 2 m, internal radius is 6 m and height is 25 m.

**Example 7.5** The radius of a conical tent is 7 m and the height is 24 m. Calculate the length of the canvas used to make the tent if the width of the rectangular canvas is 4 m?

**Example 7.6** If the total surface area of a cone of radius 7cm is  $704 \text{ cm}^2$ , then find its slant height.

**Example 7.7** From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and base is hollowed out (Fig.7.13). Find the total surface area of the remaining solid.



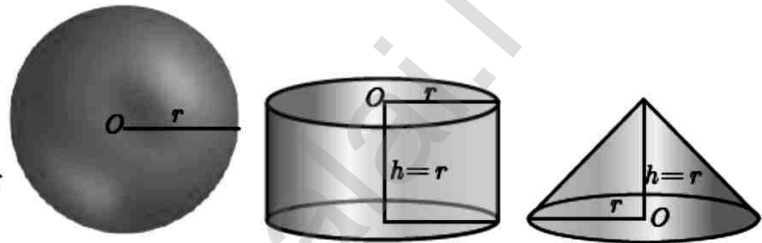
**Example 7.8** Find the diameter of a sphere whose surface area is  $154 \text{ m}^2$ .

**Example 7.9** The radius of a spherical balloon increases from 12 cm to 16 cm as air being pumped into it. Find the ratio of the surface area of the balloons in the two cases.

**Example 7.10** If the base area of a hemispherical solid is 1386 sq. metres, then find its total surface area?

**Example 7.11** The internal and external radii of a hollow hemispherical shell are 3 m and 5 m respectively. Find the T.S.A. and C.S.A. of the shell.

**Example 7.12** A sphere, a cylinder and a cone (Fig.7.20) are of the same radius, where as cone and cylinder are of same height. Find the ratio of their curved surface areas.



**Example 7.13** The slant height of a frustum of a cone is 5 cm and the radii of its ends are 4 cm and 1 cm. Find its curved surface area.

**Example 7.14** An industrial metallic bucket is in the shape of the frustum of a right circular cone whose top and bottom diameters are 10 m and 4 m and whose height is 4 m. Find the curved and total surface area of the bucket.

### EXERCISE :7.1

1. The radius and height of a cylinder are in the ratio 5:7 and its curved surface area is  $5500 \text{ sq.cm}$ . Find its radius and height.
2. A solid iron cylinder has total surface area of  $1848 \text{ sq.m}$ . Its curved surface area is five – sixth of its total surface area. Find the radius and height of the iron cylinder.
3. The external radius and the length of a hollow wooden log are 16 cm and 13 cm respectively. If its thickness is 4 cm then find its T.S.A.
4. A right angled triangle  $PQR$  where  $\angle Q = 90^\circ$  is rotated about  $QR$  and  $PQ$ . If  $QR = 16 \text{ cm}$  and  $PR = 20 \text{ cm}$ , compare the curved surface areas of the right circular cones so formed by the triangle.
5. 4 persons live in a conical tent whose slant height is 19 m. If each person require  $22 \text{ m}^2$  of the floor area, then find the height of the tent.
6. A girl wishes to prepare birthday caps in the form of right circular cones for her birthday party, using a sheet of paper whose area is  $5720 \text{ cm}^2$ , how many caps can be made with radius 5 cm and height 12 cm.



7. The ratio of the radii of two right circular cones of same height is 1:3. Find the ratio of their curved surface area when the height of each cone is 3 times the radius of the smaller cone.
8. The radius of a sphere increases by 25%. Find the percentage increase in its surface area.
9. The internal and external diameters of a hollow hemispherical vessel are 20 cm and 28 cm respectively. Find the cost to paint the vessel all over at ₹ 0.14 per  $\text{cm}^2$ .
10. The frustum shaped outer portion of the table lamp has to be painted including the top part. Find the total cost of painting the lamp if the cost of painting 1 sq.cm is ₹ 2.



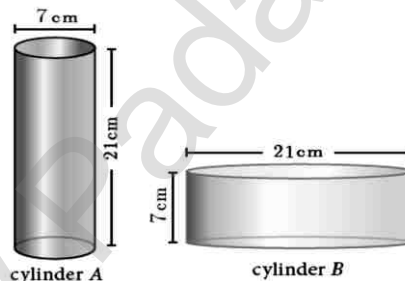
**Example 7.15** Find the volume of a cylinder whose height is 2 m and whose base area is  $250 \text{ m}^2$ .

**Example 7.16** The volume of a cylindrical water tank is  $1.078 \times 10^6$  litres. If the diameter of the tank is 7 m, find its height.

**Example 7.17** Find the volume of the iron used to make a hollow cylinder of height 9 cm and whose internal and external radii are 21 cm and 28 cm respectively.

**Example 7.18** For the cylinders *A* and *B* (Fig. 7.27),

- (i) find out the cylinder whose volume is greater.
- (ii) verify whether the cylinder with greater volume has greater total surface area.
- (iii) find the ratios of the volumes of the cylinders *A* and *B*.



**Example 7.19** The volume of a solid right circular cone is  $11088 \text{ cm}^3$ . If its height is 24 cm then find the radius of the cone.

**Example 7.20** The ratio of the volumes of two cones is 2:3. Find the ratio of their radii if the height of second cone is double the height of the first.

**Example 7.21** The volume of a solid hemisphere is  $29106 \text{ cm}^3$ . Another hemisphere whose volume is two-third of the above is carved out. Find the radius of the new hemisphere.

**Example 7.22** Calculate the weight of a hollow brass sphere if the inner diameter is 14 cm and thickness is 1mm, and whose density is  $17.3 \text{ g/cm}^3$ .

**Example 7.23** If the radii of the circular ends of a frustum which is 45 cm high are 28 cm and 7 cm, find the volume of the frustum.



**EXERCISE :7.2**

1. A 14 m deep well with inner diameter 10 m is dug and the earth taken out is evenly spread all around the well to form an embankment of width 5 m. Find the height of the embankment.
2. A cylindrical glass with diameter 20 cm has water to a height of 9 cm. A small cylindrical metal of radius 5 cm and height 4 cm is immersed it completely. Calculate the raise of the water in the glass?
3. If the circumference of a conical wooden piece is 484 cm then find its volume when its height is 105 cm.
4. A conical container is fully filled with petrol. The radius is 10m and the height is 15 m. If the container can release the petrol through its bottom at the rate of 25 cu. meter per minute, in how many minutes the container will be emptied. Round off your answer to the nearest minute.
5. A right angled triangle whose sides are 6 cm, 8 cm and 10 cm is revolved about the sides containing the right angle in two ways. Find the difference in volumes of the two solids so formed.
6. The volumes of two cones of same base radius are  $3600 \text{ cm}^3$  and  $5040 \text{ cm}^3$ . Find the ratio of heights.
7. If the ratio of radii of two spheres is 4:7, find the ratio of their volumes.
8. A solid sphere and a solid hemisphere have equal total surface area. Prove that the ratio of their volume is  $3\sqrt{3} : 4$ .
9. The outer and the inner surface areas of a spherical copper shell are  $576\pi \text{ cm}^2$  and  $324\pi \text{ cm}^2$  respectively. Find the volume of the material required to make the shell.
10. A container open at the top is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends are 8 cm and 20 cm respectively. Find the cost of milk which can completely fill a container at the rate of ₹40 per litre.

**Example 7.24** A toy is in the shape of a cylinder surmounted by a hemisphere. The height of the toy is 25 cm. Find the total surface area of the toy if its common diameter is 12 cm.

**Example 7.25** A jewel box (Fig. 7.39) is in the shape of a cuboid of dimensions  $30 \text{ cm} \times 15 \text{ cm} \times 10 \text{ cm}$  surmounted by a half part of a cylinder as shown in the figure. Find the volume of the box.

**Solution** Let  $l$ ,  $b$  and  $h_1$  be the length, breadth and height of the cuboid. Also let us take  $r$  and  $h_2$  be the radius and height of the cylinder.

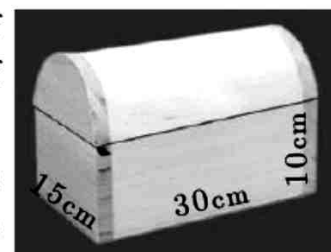
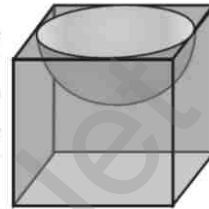


Fig. 7.39

**Example 7.26** Arul has to make arrangements for the accommodation of 150 persons for his family function. For this purpose, he plans to build a tent which is in the shape of cylinder surmounted by a cone. Each person occupies 4 sq. m of the space on ground and 40 cu. meter of air to breathe. What should be the height of the conical part of the tent if the height of cylindrical part is 8 m?

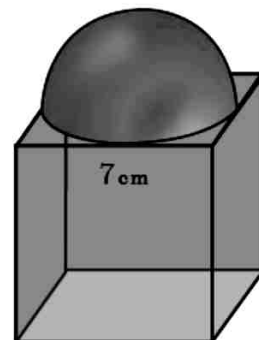
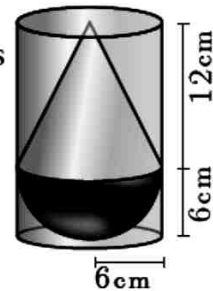
**Example 7.27** A funnel consists of a frustum of a cone attached to a cylindrical portion 12 cm long attached at the bottom. If the total height be 20 cm, diameter of the cylindrical portion be 12 cm and the diameter of the top of the funnel be 24 cm. Find the outer surface area of the funnel.

**Example 7.28** A hemispherical section is cut out from one face of a cubical block (Fig.7.42) such that the diameter  $l$  of the hemisphere is equal to side length of the cube. Determine the surface area of the remaining solid.



### **EXERCISE :7.3**

1. A vessel is in the form of a hemispherical bowl mounted by a hollow cylinder. The diameter is 14 cm and the height of the vessel is 13 cm. Find the capacity of the vessel.
2. Nathan, an engineering student was asked to make a model shaped like a cylinder with two cones attached at its two ends. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of the model that Nathan made.
3. From a solid cylinder whose height is 2.4 cm and the diameter 1.4 cm, a cone of the same height and same diameter is carved out. Find the volume of the remaining solid to the nearest  $cm^3$ .
4. A solid consisting of a right circular cone of height 12 cm and radius 6 cm standing on a hemisphere of radius 6 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of the water displaced out of the cylinder, if the radius of the cylinder is 6 cm and height is 18 cm.
5. A capsule is in the shape of a cylinder with two hemisphere stuck to each of its ends. If the length of the entire capsule is 12 mm and the diameter of the capsule is 3 mm, how much medicine it can hold?
6. As shown in figure a cubical block of side 7 cm is surmounted by a hemisphere. Find the surface area of the solid.
7. A right circular cylinder just enclose a sphere of radius  $r$  units. Calculate
  - (i) the surface area of the sphere
  - (ii) the curved surface area of the cylinder
  - (iii) the ratio of the areas obtained in (i) and (ii).





**UNIT EXERCISE : 7**

- The barrel of a fountain-pen cylindrical in shape, is 7 cm long and 5 mm in diameter. A full barrel of ink in the pen will be used for writing 330 words on an average. How many words can be written using a bottle of ink containing one fifth of a litre?
- A hemi-spherical tank of radius 1.75 m is full of water. It is connected with a pipe which empties the tank at the rate of 7 litre per second. How much time will it take to empty the tank completely?
- Find the maximum volume of a cone that can be carved out of a solid hemisphere of radius  $r$  units.
- An oil funnel of tin sheet consists of a cylindrical portion 10 cm long attached to a frustum of a cone. If the total height is 22 cm, the diameter of the cylindrical portion be 8 cm and the diameter of the top of the funnel be 18 cm, then find the area of the tin sheet required to make the funnel.
- Find the number of coins, 1.5 cm in diameter and 2 mm thick, to be melted to form a right circular cylinder of height 10 cm and diameter 4.5 cm.
- A hollow metallic cylinder whose external radius is 4.3 cm and internal radius is 1.1 cm and whole length is 4 cm is melted and recast into a solid cylinder of 12 cm long. Find the diameter of solid cylinder.
- The slant height of a frustum of a cone is 4 m and the perimeter of circular ends are 18 m and 16 m. Find the cost of painting its curved surface area at ₹100 per sq. m.
- A hemi-spherical hollow bowl has material of volume  $\frac{436\pi}{3}$  cubic cm. Its external diameter is 14 cm. Find its thickness.
- The volume of a cone is  $1005\frac{5}{7}$  cu. cm. The area of its base is  $201\frac{1}{7}$  sq. cm. Find the slant height of the cone.
- A metallic sheet in the form of a sector of a circle of radius 21 cm has central angle of  $216^\circ$ . The sector is made into a cone by bringing the bounding radii together. Find the volume of the cone formed.

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**8. STATISTICS AND PROBABILITY**

**Example 8.17** Express the sample space for rolling two dice using tree diagram.

**Example 8.18** A bag contains 5 blue balls and 4 green balls. A ball is drawn at random from the bag. Find the probability that the ball drawn is (i) blue (ii) not blue.

**Example 8.19** Two dice are rolled. Find the probability that the sum of outcomes is (i) equal to 4 (ii) greater than 10 (iii) less than 13

**Example 8.20** Two coins are tossed together. What is the probability of getting different faces on the coins?

**Example 8.21** From a well shuffled pack of 52 cards, one card is drawn at random. Find the probability of getting (i) red card (ii) heart card (iii) red king (iv) face card (v) number card



**Example 8.22** What is the probability that a leap year selected at random will contain 53 saturdays.

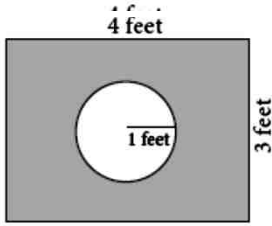
**Example 8.23** A die is rolled and a coin is tossed simultaneously. Find the probability that the die shows an odd number and the coin shows a head.

**Example 8.24** A bag contains 6 green balls, some black and red balls. Number of black balls is as twice as the number of red balls. Probability of getting a green ball is thrice the probability of getting a red ball. Find (i) number of black balls (ii) total number of balls.

**Example 8.25** A game of chance consists of spinning an arrow which is equally likely to come to rest pointing to one of the numbers 1, 2, 3, ...12. What is the probability that it will point to (i) 7 (ii) a prime number (iii) a composite number?

### EXERCISE : 8.3

1. Write the sample space for tossing three coins using tree diagram.
2. Write the sample space for selecting two balls from a bag containing 6 balls numbered 1 to 6 (using tree diagram).
3. If  $A$  is an event of a random experiment such that  $P(A) : P(\bar{A}) = 17:15$  and  $n(S) = 640$  then find (i)  $P(\bar{A})$  (ii)  $n(A)$ .
4. A coin is tossed thrice. What is the probability of getting two consecutive tails?
5. At a fete, cards bearing numbers 1 to 1000, one number on one card are put in a box. Each player selects one card at random and that card is not replaced. If the selected card has a perfect square number greater than 500, the player wins a prize. What is the probability that (i) the first player wins a prize (ii) the second player wins a prize, if the first has won?
6. A bag contains 12 blue balls and  $x$  red balls. If one ball is drawn at random (i) what is the probability that it will be a red ball? (ii) If 8 more red balls are put in the bag, and if the probability of drawing a red ball will be twice that of the probability in (i), then find  $x$ .
7. Two unbiased dice are rolled once. Find the probability of getting
  - (i) a doublet (equal numbers on both dice)
  - (ii) the product as a prime number
  - (iii) the sum as a prime number
  - (iv) the sum as 1
8. Three fair coins are tossed together. Find the probability of getting
  - (i) all heads
  - (ii) atleast one tail
  - (iii) atmost one head
  - (iv) atmost two tails
9. A bag contains 5 red balls, 6 white balls, 7 green balls, 8 black balls. One ball is drawn at random from the bag. Find the probability that the ball drawn is
  - (i) white
  - (ii) black or red
  - (iii) not white
  - (iv) neither white nor black
10. In a box there are 20 non-defective and some defective bulbs. If the probability that a bulb selected at random from the box found to be defective is  $\frac{3}{8}$  then, find the number of defective bulbs.

11. The king and queen of diamonds, queen and jack of hearts, jack and king of spades are removed from a deck of 52 playing cards and then well shuffled. Now one card is drawn at random from the remaining cards. Determine the probability that the card is (i) a clavor (ii) a queen of red card (iii) a king of black card
12. Some boys are playing a game, in which the stone thrown by them landing in a circular region (given in the figure) is considered as win and landing other than the circular region is considered as loss. What is the probability to win the game? ( $\pi = 3.14$ )
- 
13. Two customers Priya and Amuthan are visiting a particular shop in the same week (Monday to Saturday). Each is equally likely to visit the shop on any one day as on another day. What is the probability that both will visit the shop on (i) the same day (ii) different days (iii) consecutive days?
14. In a game, the entry fee is ₹150. The game consists of tossing a coin 3 times. Dhana bought a ticket for entry . If one or two heads show, she gets her entry fee back. If she throws 3 heads, she receives double the entry fees. Otherwise she will lose. Find the probability that she (i) gets double entry fee (ii) just gets her entry fee (iii) loses the entry fee.

### UNIT EXERCISE : 8

8. If two dice are rolled, then find the probability of getting the product of face value 6 or the difference of face values 5.
9. In a two children family, find the probability that there is at least one girl in a family.
10. A bag contains 5 white and some black balls. If the probability of drawing a black ball from the bag is twice the probability of drawing a white ball then find the number of black balls.
11. The probability that a student will pass the final examination in both English and Tamil is 0.5 and the probability of passing neither is 0.1. If the probability of passing the English examination is 0.75, what is the probability of passing the Tamil examination?
12. The King, Queen and Jack of the suit spade are removed from a deck of 52 cards. One card is selected from the remaining cards. Find the probability of getting (i) a diamond (ii) a queen (iii) a spade (iv) a heart card bearing the number 5.

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### 8 MARKS

#### PRACTICAL GEOMETRY:

**Example 4.10** Construct a triangle similar to a given triangle  $PQR$  with its sides equal to  $\frac{3}{5}$  of the corresponding sides of the triangle  $PQR$  (scale factor  $\frac{3}{5} < 1$ )



**Example 4.11** Construct a triangle similar to a given triangle  $PQR$  with its sides equal to  $\frac{7}{4}$  of the corresponding sides of the triangle  $PQR$  (scale factor  $\frac{7}{4} > 1$ )

### **EXERCISE : 4.1**

10. Construct a triangle similar to a given triangle  $PQR$  with its sides equal to  $\frac{2}{3}$  of the corresponding sides of the triangle  $PQR$  (scale factor  $\frac{2}{3} < 1$ ).
11. Construct a triangle similar to a given triangle  $LMN$  with its sides equal to  $\frac{4}{5}$  of the corresponding sides of the triangle  $LMN$  (scale factor  $\frac{4}{5} < 1$ ).
12. Construct a triangle similar to a given triangle  $ABC$  with its sides equal to  $\frac{6}{5}$  of the corresponding sides of the triangle  $ABC$  (scale factor  $\frac{6}{5} > 1$ ).
13. Construct a triangle similar to a given triangle  $PQR$  with its sides equal to  $\frac{7}{3}$  of the corresponding sides of the triangle  $PQR$  (scale factor  $\frac{7}{3} > 1$ ).

**Example 4.17** Construct a  $\Delta PQR$  in which  $PQ = 8$  cm,  $\angle R = 60^\circ$  and the median  $RG$  from  $R$  to  $PQ$  is 5.8 cm. Find the length of the altitude from  $R$  to  $PQ$ .

**Example 4.18** Construct a triangle  $\Delta PQR$  such that  $QR = 5$  cm,  $\angle P = 30^\circ$  and the altitude from  $P$  to  $QR$  is of length 4.2 cm.

**Example 4.19** Draw a triangle  $ABC$  of base  $BC = 8$  cm,  $\angle A = 60^\circ$  and the bisector of  $\angle A$  meets  $BC$  at  $D$  such that  $BD = 6$  cm.

### **EXERCISE : 4.2**

10.  $ABCD$  is a quadrilateral in which  $AB = AD$ , the bisector of  $\angle BAC$  and  $\angle CAD$  intersect the sides  $BC$  and  $CD$  at the points  $E$  and  $F$  respectively. Prove that  $EF \parallel BD$ .
11. Construct a  $\Delta PQR$  which the base  $PQ = 4.5$  cm,  $\angle R = 35^\circ$  and the median from  $R$  to  $PQ$  is 6 cm.
12. Construct a  $\Delta PQR$  in which  $QR = 5$  cm,  $\angle P = 40^\circ$  and the median  $PG$  from  $P$  to  $QR$  is 4.4 cm. Find the length of the altitude from  $P$  to  $QR$ .
13. Construct a  $\Delta PQR$  such that  $QR = 6.5$  cm,  $\angle P = 60^\circ$  and the altitude from  $P$  to  $QR$  is of length 4.5 cm.
14. Construct a  $\Delta ABC$  such that  $AB = 5.5$  cm,  $\angle C = 25^\circ$  and the altitude from  $C$  to  $AB$  is 4 cm.

**Example 4.29** Draw a circle of radius 3 cm. Take a point  $P$  on this circle and draw a tangent at  $P$ .

**Example 4.30** Draw a circle of radius 4 cm. At a point  $L$  on it draw a tangent to the circle using the alternate segment.

**Example 4.31** Draw a circle of diameter 6 cm from a point  $P$ , which is 8 cm away from its centre. Draw the two tangents  $PA$  and  $PB$  to the circle and measure their lengths.

### EXERCISE : 4.4

- Draw a tangent at any point  $R$  on the circle of radius 3.4 cm and centre at  $P$ ?
- Draw a circle of radius 4.5 cm. Take a point on the circle. Draw the tangent at that point using the alternate segment theorem.
- Draw the two tangents from a point which is 10 cm away from the centre of a circle of radius 5 cm. Also, measure the lengths of the tangents.
- Take a point which is 11 cm away from the centre of a circle of radius 4 cm and draw the two tangents to the circle from that point.
- Draw the two tangents from a point which is 5 cm away from the centre of a circle of diameter 6 cm. Also, measure the lengths of the tangents.

### GRAPHS

**Example 3.51** Discuss the nature of solutions of the following quadratic equations.

(i)  $x^2 + x - 12 = 0$       (ii)  $x^2 - 8x + 16 = 0$       (iii)  $x^2 + 2x + 5 = 0$

**Example 3.52** Draw the graph of  $y = 2x^2$  and hence solve  $2x^2 - x - 6 = 0$   
 $y = x^2 + 4x + 3$  and hence find the roots of

**Example 3.53** Draw the graph of  $x^2 + x + 1 = 0$

**Example 3.54** Draw the graph of  $y = x^2 + x - 2$  and hence solve  $x^2 + x - 2 = 0$

**Example 3.55** Draw the graph of  $y = x^2 - 4x + 3$  and use it to solve  $x^2 - 6x + 9 = 0$

### EXERCISE : 3.16

- Graph the following quadratic equations and state their nature of solutions.
  - $x^2 - 9x + 20 = 0$
  - $x^2 - 4x + 4 = 0$
  - $x^2 + x + 7 = 0$
  - $x^2 - 9 = 0$
  - $x^2 - 6x + 9 = 0$
  - $(2x - 3)(x + 2) = 0$
- Draw the graph of  $y = x^2 - 4$  and hence solve  $x^2 - x - 12 = 0$
- Draw the graph of  $y = x^2 + x$  and hence solve  $x^2 + 1 = 0$
- Draw the graph of  $y = x^2 + 3x + 2$  and use it to solve  $x^2 + 2x + 1 = 0$
- Draw the graph of  $y = x^2 + 3x - 4$  and hence use it to solve  $x^2 + 3x - 4 = 0$
- Draw the graph of  $y = x^2 - 5x - 6$  and hence solve  $x^2 - 5x - 14 = 0$
- Draw the graph of  $y = 2x^2 - 3x - 5$  and hence solve  $2x^2 - 4x - 6 = 0$
- Draw the graph of  $y = (x - 1)(x + 3)$  and hence solve  $x^2 - x - 6 = 0$

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*HARD WORK NEVER FAILS.*

*BEST WISHES By M. Sakthivel Teacher, Adhiyaman Matric.Hr. Sec. School, Uthangarai.*

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