SUCCESS

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X EASY MATHS

SLOW LEARNER

82 MARKS

STUDYMATERIAL

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1. RELATIONS AND FUNCTION

5 MARKS

If $f(x) = 2x + 3$, $g(x) = 1 - 2x$ and $h(x) = 3x$. Prove that $f \circ (g \circ h) = (f \circ g) \circ h$.

Solution : $f(x) = 2x + 3\,,\ g(x) = 1 - 2x\,,\ h(x) = 3x$

$$(f \circ g) = 2(1 - 2x) + 3 = 2 - 4x + 3 = 5 - 4x$$

$$(f \circ g) \circ h = 5 - 4(3x) = 5 - 12x \quad ....(1)$$

$$(g \circ h) = 1 - 2(3x) = 1 - 6x$$

$$f \circ (g \circ h) = 2(1 - 6x) + 3 = 5 - 12x \quad ....(2)$$

From (1) and (2) $\Rightarrow f \circ (g \circ h) = (f \circ g) \circ h$.

If $f(x) = \frac{x+6}{3}$ and $g(x) = 3 - x$, find $f \circ g$ and $g \circ f$. Check whether $f \circ g = g \circ f$.

Solution :

$$(f \circ g) = \frac{(3-x)+6}{3} = \frac{9-x}{3}$$

$$(g \circ f) = 3 - \frac{x+6}{3} = \frac{3-x}{3}$$

From (1) and (2) $\Rightarrow f \circ g \neq g \circ f$.

If $f(x) = x - 4$, $g(x) = x^2$ and $h(x) = 3x - 5$, Prove that $f \circ (g \circ h) = (f \circ g) \circ h$.

Solution :

$$(f \circ g) = x^2 - 4$$

$$\therefore ((f \circ g) \circ h) = (3x - 5)^2 - 4 \quad ...(1)$$

$$(g \circ h) = (3x - 5)^2$$

$$\therefore (f \circ (g \circ h) = (3x - 5)^2 - 4 \quad ...(2)$$

From (1) and (2)

$$\Rightarrow (f \circ g) \circ h = f \circ (g \circ h)$$

If $f(x) = x^2$, $g(x) = 2x$ and $h(x) = (x + 4)$, Prove that $f \circ (g \circ h) = (f \circ g) \circ h$.

Solution : $(f \circ g) = (2x)^2 = 4x^2$

$$\therefore ((f \circ g) \circ h) = 4\,(x + 4)^2 \quad ...(1)$$

$$(g \circ h) = 2\,(x + 4)$$

$$\therefore (f \circ (g \circ h) = (2\,(x + 4))^2 = 4(x + 4)^2 \quad ...(2)$$

From (1) and (2) $\Rightarrow (f \circ g) \circ h = f \circ (g \circ h)$

If $f(x) = 3x - 2$, $g(x) = 2x + k$ and if $f \circ g = g \circ f$, then find the value of k.

Solution : $f \circ g = 3(2x + k) - 2 = 6x + 3k - 2$

$$g \circ f = 2(3x - 2) + k = 6x - 4 + k$$

$$f \circ g = g \circ f \Rightarrow 6x + 3k - 2 = 6x - 4 + k$$

$$k = -1$$

Given the function $f : x \to x^2 - 5x + 6$, evaluate (i) f(– 1) (ii) f(2a) (iii) f(2) (iv) f(x – 1)

Solution : $f : x \to x^2 - 5x + 6 \Rightarrow f(x) = x^2 - 5x + 6$

(i) $f(-1) = (-1)^2 - 5(-1) + 6 = 12$

(ii) $f(2a) = (2a)^2 - 5(2a) + 6 = 4a^2 - 10a + 6$

(iii) $f(2) = 2^2 - 5(2) + 6 = 0$

(iv) $f(x - 1) = (x - 1)^2 - 5\,(x - 1) + 6$

The functions f and g are defined by $f(x) = 6x + 8$; $g(x) = \frac{x-2}{3}$ (i) Calculate the value of $gg\left(\frac{1}{2}\right)$ (ii) Write an expression for $gf(x)$ in its simplest form.

Solution :

$$i) \quad gg(x) = \left(\frac{\frac{x-2}{3} - 2}{3}\right) = \left(\frac{x-8}{9}\right)$$

$$\therefore gg\left(\frac{1}{2}\right) = \left(\frac{\frac{1}{2} - 8}{9}\right) = \frac{-15}{18}$$

$$\text{(ii)} \quad gf(x) = \frac{6x+8-2}{3} = \frac{6x+6}{3} = 2x+2$$

If $f(x) = 2x - 1$, $g(x) = \frac{x+1}{2}$, show that $f \circ g = g \circ f = x$.

Solution :

$$f \circ g = 2\left(\frac{x+1}{2}\right) - 1 = x + 1 - 1 = x$$

$$g \circ f = \frac{2x-1+1}{2} = x$$

$$\therefore f \circ g = g \circ f = x$$

If $f(x) = 2x - k$, $g(x) = 4x + 5$ such that $f \circ g = g \circ f$. Find the value of k

Solution :

$$(f \circ g) = (g \circ f)$$

$$2(4x + 5) - k = 4(2x - k) + 5$$

$$8x + 10 - k = 8x - 4k + 5$$

$$\Rightarrow k = \frac{-5}{3}$$

If $f(x) = 3x + 2$, $g(x) = 6x - k$ such that $f \circ g = g \circ f$. Find the value of k

Solution :

$$(f \circ g) = (g \circ f)$$

$$3(6x - k) + 2 = 6(3x + 2) - k$$

$$18x - 3k + 2 = 18x + 12 - k$$

$$k = -5$$

Find x if $gff(x) = fgg(x)$, given $f(x) = 3x + 1$ and $g(x) = x + 3$.

Solution :

$$ff(x) = [\ 3(3x + 1) + 1]$$

$$= (9x + 4)$$

$$gff(x) = [(9x + 4) + 3]$$

$$= 9x + 7$$

$$gg(x) = [(x + 3) + 3]$$

$$= (x + 6)$$

$$fgg(x) = [3(x + 6) + 1\ ]$$

$$= 3x + 19$$

$$\Rightarrow gff(x) = fgg(x)$$

$$9x + 7 = 3x + 19$$

$$x = 2$$

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If the function $f : \mathrm{R} \to \mathrm{R}$ defined by $f(x) = \begin{cases} 2x + 7, x < -2 \\ x^2 - 2, -2 \le x < 3, \\ 3x - 2,\ x \ge 3 \end{cases}$ then find the values of

(i) f(4) (ii) f(– 2) (iii) f(4) + 2f(1) (iv) $\frac{f(1) - 3f(4)}{f(-3)}$

Solution :

(i) $f(4) = 12 - 2 = 10$

(ii) $f(-2) = 4 - 2 = 2$

(iii) $f(4) + 2f(1) = 10 - 2 = 8$

(iv) $\frac{f(1) - 3f(4)}{f(-3)} = -31$

A function $f : [-5, 9] \to \mathrm{R}$ is defined as follows : Find (i) $f(-3) + f(2)$ (ii) $f(7) - f(1)$

$$f(x) = \begin{cases} 6x + 1 & if\ -5 \le x < 2 \\ 5x^2 - 1 & if\ \ 2 \le x < 6 \\ 3x - 4 & if\ \ 6 \le x < 6 \end{cases}$$

(iii) $2f(4) + f(8)$ (iv) $\frac{2f(-2) - f(6)}{f(4) + f(-2)}$

Solution : (i) $f(-3) + f(2) = -17 + 19 = 2$ (ii) $f(7) - f(1) = 17 - 7 = 10$

(iii) $2f(4) + f(8) = 178$ (iv) $\frac{2f(-2) - f(6)}{f(4) + f(-2)} = \frac{-9}{17}$

If the function f is defined by $f(x) = \begin{cases} x + 2 & if\ x > 1 \\ 2 & if\ -1 \le x \le 1 \\ x - 1 & if\ -3 < x < -1 \end{cases}$; find the values of (i) $f(3)$ (ii) $f(0)$ (iii) $f(-1, 5)$ (iv) $f(2) + f(-2)$

Solution :

(i) $f(3) = 3 + 2 = 5$

(ii) $f(0) = 2$ (iii) $f(-1.5) = -1.5 - 1 = -2.5$ (iv) $f(2) + f(-2) = 4 - 3 = 1$

Let A = The set of all natural numbers less than 8, B = The set of all prime numbers less than 8, C = The set of even prime number. Verify that A × (B – C) = (A × B) – (A × C)

Solution : $\mathrm{A} = \{1, 2, 3, 4, 5, 6, 7\}$, $\mathrm{B} = \{2, 3, 5, 7\}$ and $\mathrm{C} = \{2\}$

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$$\mathrm{B} - \mathrm{C} = \{3, 5, 7\} - \{2\} = \{3, 5, 7\}$$

$$\therefore \mathrm{A} \times (\mathrm{B} - \mathrm{C}) = \{1, 2, 3, 4, 5, 6, 7\} \times \{3, 5, 7\}$$

$$= \{(1, 3), (1, 5), (1, 7), (2, 3), (2, 5), (2, 7), (3, 3), (3, 5), (3, 7), (4, 3), (4, 5), (4, 7), (5, 3), (5, 5), (5, 7), (6, 3), (6, 5), (6, 7), (7, 3), (7, 5), (7, 7),\}$$

$$\mathrm{A} \times \mathrm{B} = \{1, 2, 3, 4, 5, 6, 7\} \times \{2, 3, 5, 7\}$$

$$= \{(1, 2), (1, 3), (1, 5), (1, 7), (2, 2), (2, 3), (2, 5), (2, 7), (3, 2), (3, 3), (3, 5), (3, 7), (4, 2), (4, 3), (4, 5), (4, 7), (5, 2), (5, 3), (5, 5), (5, 7), (6, 2), (6, 3), (6, 5), (6, 7), (7, 2), (7, 3), (7, 5), (7, 7),\}$$

$$\mathrm{A} \times \mathrm{C} = \{1, 2, 3, 4, 5, 6, 7\} \times \{2\} = \{(1, 2), (2, 2), (3, 2), (4, 2), (5, 2), (6, 2), (7, 2)\}$$

$$\therefore (\mathrm{A} \times \mathrm{B}) - (\mathrm{A} \times \mathrm{C}) = \{(1, 3), (1, 5), (1, 7), (2, 3), (2, 5), (2, 7), (3, 3), (3, 5), (3, 7), (4, 3), (4, 5), (4, 7), (5, 3), (5, 5), (5, 7), (6, 3), (6, 5), (6, 7), (7, 3), (7, 5), (7, 7),\}$$

The function 't' which maps temperature in Celsius (C) into temperature in Fahrenheit (F) is defined by t(C) = F where F = $\frac{9}{5}$C + 32, Find, (i) t(0) (ii) t(28) (iii) t(– 10) (iv) the value of C when t(C) = 212 (v) the temperature when the Celsius value is equal to the Farenheit value.

Solution : Given $\mathrm{t(C) = F} = \frac{9C}{5} + 32$ (*i*) $t(0) = \frac{9(0)}{5} + 32 = 32^o F$ (*ii*) $t(28) = \frac{9(28)}{5} + 32 = 82.4^o F$

(*iii*) $t(-10) = \frac{9(-10)}{5} + 32 = 14^o F$ (*iv*) When $t(c) = 212 \Rightarrow 212 = \frac{9C}{5} + 32 \Rightarrow C = 100^o C$

(v) When Celsius value = Farenheit value $\Rightarrow C = \frac{9C}{5} + 32 \Rightarrow C = -40^o$

Let A = The set of all natural numbers less than 8, B = The set of all prime numbers less than 8, C = The set of even prime number. Verify that $(A \cap B) \times C = (A \times C) \cap (B \times C)$

Solution : $A = \{1, 2, 3, 4, 5, 6, 7\}$, $B = \{2, 3, 5, 7\}$ and $C = \{2\}$

$A \cap B = \{1, 2, 3, 4, 5, 6, 7\} \cap \{2, 3, 5, 7\} = \{2, 3, 5, 7\}$

$\therefore (A \cap B) \times C = \{2, 3, 5, 7\} \times \{2\} = \{(2, 2), (3, 2), (5, 2), (7, 2)\}$ ... (1)

$A \times C = \{1, 2, 3, 4, 5, 6, 7\} \times \{2\} = \{(1, 2), (2, 2), (3, 2), (4, 2), (5, 2), (6, 2), (7, 2)\}$

$B \times C = \{2, 3, 5, 7\} \times \{2\} = \{(2, 2), (3, 2), (5, 2), (7, 2)\}$

$(A \times C) \cap (B \times C) = \{(2, 2), (3, 2), (5, 2), (7, 2)\}$ ... (2)

$\therefore$ From (1) and (2), $(A \cap B) \times C = (A \times C) \cap (B \times C)$

If $A = \{5, 6\}$, $B = \{4, 5, 6\}$, $C = \{5, 6, 7\}$. Show that $A \times A = (B \times B) \cap (C \times C)$.

Solution : $A = \{5, 6\}$, $B = \{4, 5, 6\}$, $C = \{5, 6, 7\}$

$A \times A = \{5, 6\} \times \{5, 6\} = \{(5, 5), (5, 6), (6, 5), (6, 6)\}$ ...(1)

$B \times B = \{4, 5, 6\} \times \{4, 5, 6\} = \{(4, 4), (4, 5), (4, 6), (5, 4), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6)\}$

$C \times C = \{5, 6, 7\} \times \{5, 6, 7\} = \{(5, 5), (5, 6), (5, 7), (6, 5), (6, 6), (6, 7), (7, 5), (7, 6), (7, 7)\}$

$\therefore \quad (B \times B) \cap (C \times C) = \{(5, 5), (5, 6), (6, 5), (6, 6)\}$ ...(2)

$\therefore$ From (1) and (2). **$A \times A = (B \times B) \cap (C \times C)$.**

Given $A = \{1, 2, 3\}$, $B = \{2, 3, 5\}$, $C = \{3, 4\}$ and $D = \{1, 3, 5\}$, check if $(A \cap C) \times (B \cap D) = (A \times B) \cap (C \times D)$ is true ?

Solution : $A = \{1, 2, 3\}$, $B = \{2, 3, 5\}$, $C = \{3, 4\}$, $D = \{1, 3, 5\}$

$A \cap C = \{1, 2, 3\} \cap \{3, 4\} = \{3\}$, $B \cap D = \{2, 3, 5\} \cap \{1, 3, 5\} = \{3, 5\}$

$\therefore (A \cap C) \times (B \cap D) = \{(3, 3), (3, 5)\}$ ... (1)

$A \times B = \{(1, 2), (1, 3), (1, 5), (2, 2), (2, 3), (2, 5), (3, 2), (3, 3), (3, 5)\}$

$C \times D = \{(3, 1), (3, 3), (3, 5), (4, 1), (4, 3), (4, 5)\}$

$\therefore (A \times B) \cap (C \times D) = \{(3, 3), (3, 5)\}$ ...(2)

$\therefore$ From (1) and (2) **$(A \cap C) \times (B \cap D) = (A \times B) \cap (C \times D)$**

If $A = \{1, 3, 5\}$ and $B = \{2, 3\}$ then (i) find $A \times B$ and $B \times A$. (ii) Is $A \times B = B \times A$? If not why ? (iii) Show that $n(A \times B) = n(B \times A) = n(A) \times n(B)$.

Solution : $A = \{1, 3, 5\}$ and $B = \{2, 3\}$

(i) $A \times B = \{1, 3, 5\} \times \{2, 3\} = \{(1, 2), (1, 3), (3, 2), (3, 3), (5, 2), (5, 3)\}$

$B \times A = \{2, 3\} \times \{1, 3, 5\} = \{(2, 1), (2, 3), (2, 5), (3, 1), (3, 3), (3, 5)\}$

(ii) $(1, 2) \neq (2, 1) \Rightarrow A \times B \neq B \times A$

(iii) $n(A \times B) = n(B \times A) = 6$; $n(B) \times n(A) = 2 \times 3 = 6$

$\therefore n(A \times B) = n(B \times A) = n(A) \times n(B)$.

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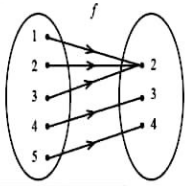
Represent the function $f = \{(1, 2), (2, 2), (3, 2), (4, 3), (5, 4)\}$ through (i) an arrow diagram (ii) a table form (iii) a graph

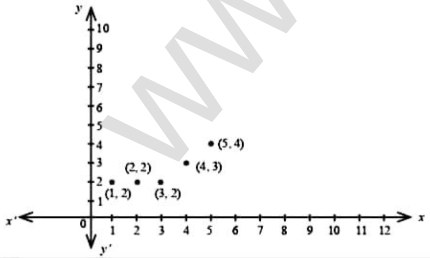
Solution : **(i) Arrow Diagram :**



(ii) Table Form :

x	1	2	3	4	5
$f(x)$	2	2	2	3	4

(iii) Graph :



Let $A = \{x \in W \mid x < 2\}$, $B = \{x \in N \mid 1 < x \leq 4\}$ and $C = \{3, 5\}$. Verify that $A \times (B \cup C) = (A \times B) \cup (A \times C)$

Solution : $A = \{x \in W \mid x < 2\} \Rightarrow A = \{0, 1\}$ $\quad B = \{x \in N \mid 1 < x \leq 4\} \Rightarrow B = \{2, 3, 4\}$ $\quad C = \{3, 5\}$

$B \cup C = \{2, 3, 4\} \cup \{3, 5\} = \{2, 3, 4, 5\}$

$\therefore A \times (B \cup C) = \{0, 1\} \times \{2, 3, 4, 5\} = \{(0, 2), (0, 3), (0, 4), (0, 5), (1, 2), (1, 3), (1, 4), (1, 5)\}$ ...(1)

$A \times B = \{0, 1\} \times \{2, 3, 4\} = \{(0, 2), (0, 3), (0, 4), (1, 2), (1, 3), (1, 4)\}$

$A \times C = \{0, 1\} \times \{3, 5\} = \{(0, 3), (0, 5), (1, 3), (1, 5)\}$

$\therefore (A \times B) \cup (A \times C) = \{(0, 2), (0, 3), (0, 4), (0, 5), (1, 2), (1, 3), (1, 4), (1, 5)\}$ ...(2)

$\therefore$ From (1) and (2) **$A \times (B \cup C) = (A \times B) \cup (A \times C)$**

Let $A = \{x \in W \mid x < 2\}$, $B = \{x \in N \mid 1 < x \leq 4\}$ and $C = \{3, 5\}$. Verify that $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Solution : $A = \{x \in W \mid x < 2\} \Rightarrow A = \{0, 1\}$, $B = \{x \in N \mid 1 < x \leq 4\} \Rightarrow B = \{2, 3, 4\}$ and $C = \{3, 5\}$

$B \cap C = \{2, 3, 4\} \cap \{3, 5\} = \{3\}$

$\therefore A \times (B \cap C) = \{0, 1\} \times \{3\} = \{(0, 3), (1, 3)\}$ ...(1)

$A \times B = \{0, 1\} \times \{2, 3, 4\} = \{(0, 2), (0, 3), (0, 4), (1, 2), (1, 3), (1, 4)\}$

$A \times C = \{0, 1\} \times \{3, 5\} = \{(0, 3), (0, 5), (1, 3), (1, 5)\}$

$\therefore (A \times B) \cap (A \times C) = \{(0, 3), (1, 3)\}$ ...(2)

$\therefore$ From (1) and (2), $A \times (B \cap C) = (A \times B) \cap (A \times C)$

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Let $A = \{x \in W \mid x < 2\}$, $B = \{x \in N \mid 1 < x \leq 4\}$ and $C = \{3, 5\}$. Verify that $(A \cup B) \times C = (A \times C) \cup (B \times C)$

Solution : $A = \{x \in W \mid x < 2\} \Rightarrow A = \{0, 1\}$, $B = \{x \in N \mid 1 < x \leq 4\} \Rightarrow B = \{2, 3, 4\}$ and $C = \{3, 5\}$

$A \cup B = \{0, 1\} \cup \{2, 3, 4\} = \{0, 1, 2, 3, 4\}$

$\therefore (A \cup B) \times C = \{(0, 3), (0, 5), (1, 3), (1, 5), (2, 3), (2, 5), (3, 3), (3, 5), (4, 3), (4, 5)\}$ ...(1)

$A \times C = \{0, 1\} \times \{3, 5\} = \{(0, 3), (0, 5), (1, 3), (1, 5)\}$

$B \times C = \{2, 3, 4\} \times \{3, 5\} = \{(2, 3), (2, 5), (3, 3), (3, 5), (4, 3), (4, 5)\}$

$\therefore (A \times C) \cup (B \times C) = \{(0, 3), (0, 5), (1, 3), (1, 5), (2, 3), (2, 5) (3, 3), (3, 5), (4, 3). (4, 5)\}$

$\therefore$ From (1) and (2) **$(A \cup B) \times C = (A \times C) \cup (B \times C)$**

Let $A = \{x \in N \mid 1 < x < 4\}$, $B = \{x \in W \mid 0 \leq x < 2\}$ and $C = \{x \in N \mid x < 3\}$. verify that $A \times (B \cup C) = (A \times B) \cup (A \times C)$

Solution : $A = \{x \in N \mid 1 < x < 4\} = \{2, 3\}$, $B = \{x \in W \mid 0 \leq x < 2\} = \{0, 1\}$ and $C = \{x \in N \mid x < 3\} = \{1, 2\}$

$B \cup C = \{0, 1\} \cup \{1, 2\} = \{0, 1, 2\}$

$A \times (B \cup C) = \{2, 3\} \times \{0, 1, 2\} = \{(2, 0), (2, 1), (2, 2), (3, 0), (3, 1), (3, 2)\}$ ... (1)

$A \times B = \{2, 3\} \times \{0, 1\} = \{(2, 0), (2, 1), (3, 0), (3, 1)\}$

$A \times C = \{2, 3\} \times \{1, 2\} = \{(2, 1), (2, 2), (3, 1), (3, 2)\}$

$(A \times B) \cup (A \times C) = \{(2, 0), (2, 1), (3, 0), (3, 1)\} \cup \{(2, 1), (2, 2), (3, 1), (3, 2)\}$

$= \{(2, 0), (2, 1), (2, 2), (3, 0), (3, 1), (3, 2)\}$ ... (2)

From (1) and (2), $A \times (B \cup C) = (A \times B) \cup (A \times C)$ is verified.

Let $A = \{x \in N \mid 1 < x < 4\}$, $B = \{x \in W \mid 0 \leq x < 2\}$ and $C = \{x \in N \mid x < 3\}$. verify that $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Solution : $A = \{x \in N \mid 1 < x < 4\} = \{2, 3\}$, $B = \{x \in W \mid 0 \leq x < 2\} = \{0, 1\}$ and $C = \{x \in N \mid x < 3\} = \{1, 2\}$

$B \cap C = \{0, 1\} \cap \{1, 2\} = \{1\}$

$A \times (B \cap C) = \{2, 3\} \times \{1\} = \{(2, 1), (3, 1)\}$ ... (1)

$A \times B = \{2, 3\} \times \{0, 1\} = \{(2, 0), (2, 1), (3, 0), (3, 1)\}$

$A \times C = \{2, 3\} \times \{1, 2\} = \{(2, 1), (2, 2), (3, 1), (3, 2)\}$

$(A \times B) \cap (A \times C) = \{(2, 0), (2, 1), (3, 0), (3, 1)\} \cap \{(2, 1), (2, 2), (3, 1), (3, 2)\} = \{(2, 1), (3, 1)\}$ ... (2)

From (1) and (2), $A \times (B \cap C) = (A \times B) \cap (A \times C)$ is verified.

Let $f : A \to B$ be a function define by. $f(x) = \frac{x}{2} - 1$ where A = {2, 4, 6, 10, 12}, B = {0, 1, 2, 4, 5, 9}. Represent f by (i) set of ordered pairs ; (ii) a table ; (iii) an arrow diagram ; (iv) a graph

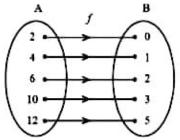
Solution : Given $f(x) = \frac{x}{2} - 1$

$f(2) = 0 \quad f(4) = 1$

$f(6) = 2 \quad f(10) = 4$

$f(12) = 5$

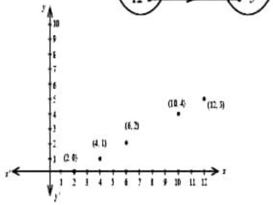
(iii) Arrow diagram :



(i) Set of order pairs :

$f = \{(2, 0), (4, 1), (6, 2), (10, 4), (12, 5)\}$

(iv) Graph



(ii) Table :

x	2	4	6	10	12
f(x)	0	1	2	4	5

Let A = {1, 2, 3, 4} and B = {2, 5, 8, 11, 14} be two sets. Let $f : A \to B$ be a function given by f(x) = 3x – 1. Represent this function (i) by arrow diagram (ii) in a table form (iii) as a set of ordered pairs (iv) in a graphical form

Solution :

A = {1, 2, 3, 4} ; B = {2, 5, 8, 11, 14} ;

$f(x) = 3x - 1$

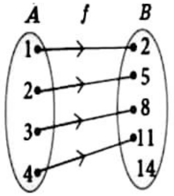
$f(1) = 2$;

$f(2) = 5$

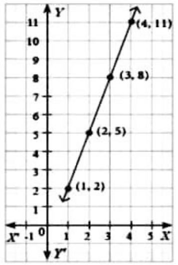
$f(3) = 8$;

$f(4) = 11$

(i) Arrow diagram



(iv) Graphical form



(ii) Table form

x	1	2	3	4
f(x)	2	5	8	11

(iii) Set of ordered pairs

$f = \{(1, 2), (2, 5), (3, 8), (4, 11)\}$

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2 .NUMBERS AND SEQUENCES

5 MARKS

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Find the sum of $9^3 + 10^3 + \ldots.. + 21^3$

Solution : $9^3 + 10^3 + \ldots. + 21^3$

$$\left[\frac{21\times(21+1)}{2}\right]^2 - \left[\frac{8\times(8+1)}{2}\right]^2 = (231)^2 - (36)^2 = 52065 \quad \left[\therefore \sum_{k=1}^{n} K^3 = \left(\frac{n(n+1)}{2}\right)^2\right]$$

Find the sum of $5^2 + 10^2 + 15^2 + \ldots. + 105^2$

Solution : $5^2 + 10^2 + 15^2 + \ldots. + 105^2 = 5^2(1^2 + 2^2 + 3^2 + \ldots. + 21^2)$

$$= \frac{25\times 21\times 22\times 43}{6} = 82775 \quad \left[\therefore \sum_{k=1}^{n} K^2 = \frac{n(n+1)(2n+1)}{6}\right]$$

Find the sum of $15^2 + 16^2 + 17^2 + \ldots. 28^2$

Solution : $15^2 + 16^2 + 17^2 + \ldots. 28^2$

$$\frac{28\times 29\times 57}{6} - \frac{14\times 15\times 29}{6} = 7714 - 1015 = 6699 \left[\therefore \sum_{k=1}^{n} K^2 = \frac{n(n+1)(2n+1)}{6}\right]$$

Rekha has 15 square colour papers of sizes 10 cm, 11 cm, 12 cm,…, 24 cm. How much area can be decorated with these colour papers?

Solution : 10 cm, 11 cm, 12 cm, …….. 24 cm

$$10^2 + 11^2 + 12^2 + \ldots\ldots.. + 24^2 = \frac{24\times 25\times 49}{6} - \frac{9\times 10\times 19}{6} = 4615 \text{ cm}^2 \quad \left[\therefore \sum_{k=1}^{n} K^2 = \frac{n(n+1)(2n+1)}{6}\right]$$

The sum of first, n, 2n and 3n terms of an A.P. are S_1, S_2 and S_3 respectively. Prove that $S_3 = 3(S_2 - S_1)$.

Solution : $S_1 = t_1 = a$, $S_2 = a + a + d = 2a + d$, $S_3 = a + a + d + a + 2d = 3a + 3d$

$\therefore\ 3(S_2 - S_1) = 3a + 3d = S_3$

The houses of a street are numbered from 1 to 49. Senthil's house is numbered such that the sum of numbers of the houses prior to Senthil's house is equal to the sum of numbers of the houses following Senthil's house. Find Senthil's house number ?

Solution : Let Senthil's house number be x. $\frac{x(x-1)}{2} + \frac{x(x+1)}{2} = \frac{49\times 50}{2} \Rightarrow x^2 = 35^2 \quad \left[\therefore S_n = \frac{n}{2}[a+l]\right]$

Senthil's hosue number is 35.

Find the sum of $10^3 + 11^3 + 12^3 + \ldots.. + 20^3$

$$10^3 + 11^3 + 12^3 + \ldots.. + 20^3 = (1^3 + 2^3 + \ldots\ldots. + 20^3) - (1^3 + 2^3 + \ldots\ldots. + 9^3) \quad \left(\sum_{k=1}^{n} K^3 = \left(\frac{n(n+1)}{2}\right)^2\right)$$

$$= \left(\frac{20\times 21}{2}\right)^2 - \left(\frac{9\times 10}{2}\right)^2 = 42075$$

Find the sum to n terms of the series 5 + 55 + 555 + …

Solution : $5 + 55 + 555 + \ldots. + n$ terms $= 5[1 + 11 + 111 + \ldots. + \text{n terms}] = \frac{5}{9}\left[\frac{10(10^n - 1)}{(10-1)} - n\right] \quad \left[S_n = a\,.\frac{r^n - 1}{r-1}\right]$

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Find the sum to n terms of the series 0.4 + 0.44 + 0.444 + ……. to *n* terms

Solution :

0.4 + 0.44 + 0.444 + ……. to *n* terms $= \frac{4}{10} + \frac{44}{100} + \frac{444}{1000} + \ldots\ldots.. \text{to } n \text{ terms} = \frac{4}{9}\left[n - \frac{1}{9}\left(1 - \left(\frac{1}{10}\right)^n\right)\right] \quad \left[\therefore S_n = a\,.\frac{r^n - 1}{r-1}\right]$

Find the sum to n terms of the series 3 + 33 + 333 + …… to *n* terms

Solution $3(1 + 11 + 111 + \ldots\ldots.. + n \text{ terms}) = \frac{3}{9}(9 + 99 + 999 + \ldots\ldots.. + n \text{ terms}) = \frac{3}{9}\left[\frac{10(10^n - 1)}{(10-1)} - n\right] \left[S_n = a\,.\frac{r^n - 1}{r-1}\right]$

Kumar writes a letter to four of his friends. He asks each one of them to copy the letter and mail to four different persons with the instruction that they con-tinue the process similarly. Assuming that the process is unaltered and it costs ₹ 2 to mail one letter, spent on find the amount postage when 8th set of letters is mailed.

Solution : The total cost $= (4 \times 2) + (16 \times 2) + (64 \times 2) + \ldots\ldots\ldots.$ 8th set $= 8\,.\frac{4^8 - 1}{3} =$ ₹ 174760 $\left[\therefore S_n = a\,.\frac{r^n - 1}{r-1}\right]$

If $S_n = (x + y) + (x^2 + xy + y^2) + (x^3 + x^2y + xy^2 + y^3) + \ldots\ldots..$ *n* terms then prove that

$$(x - y)\,S_n = \left|\frac{x^2(x^n - 1)}{x - 1} - \frac{y^2(y^n - 1)}{y - 1}\right|$$

Solution : $(x - y)\,S_n = (x^2 + x^3 + x^4 + \ldots\ldots.. n \text{ terms}) - (y^2 + y^3 + y^4 + \ldots\ldots.. n \text{ terms}) \quad \left(\because S_n = a\,.\frac{r^n - 1}{r-1}\right)$

$$= \frac{x^2(x^n - 1)}{x - 1} - \frac{y^2(y^n - 1)}{y - 1}$$

5 MARKS 3 . ALGEBRA

Find the square root of $(6x^2+x-1)(3x^2+2x-1)(2x^2+3x+1)$

Solution : $\sqrt{(6x^2+x-1)(3x^2+2x-1)(2x^2+3x+1)} = \sqrt{(3x-1)(2x+1)(3x-1)(x+1)(2x+1)(x+1)}$

$= |(3x-1)(2x+1)(x+1)|$

Find the square root of $(4x^2-9x+2)(7x^2-13x-2)(28x^2-3x-1)$

Solution : $\sqrt{(4x^2-9x+2)(7x^2-13x-2)(28x^2-3x-1)} = \sqrt{(4x-1)(x-2)(7x+1)(x-2).(7x+1)(4x-1)}$

$= |(7x+1)(4x-1)(x-2)|$

Find the square root of the following $\left(2x^2+\frac{17}{6}x+1\right)\left(\frac{3}{2}x^2+4x+2\right)\left(\frac{4}{3}x^2+\frac{11}{3}x+2\right)$

Solution :

$\sqrt{(2x^2+{}^{17}/_{6}x+1)({}^{3}/_{2}x^2+4x+2)\ ({}^{4}/_{3}x^2+{}^{11}/_{3}x+2)} = {}^{1}/_{6}\sqrt{(4x+3)^2.(3x+2)^2.(x+2)^2}$

$= {}^{1}/_{6}\,|(4x+3)(3x+2)(x+2)|$

Simplify $\frac{1}{x^2-5x+6}+\frac{1}{x^2-3x+2}-\frac{1}{x^2-8x+15}$

Solution : $\frac{1}{x^2-5x+6}+\frac{1}{x^2-3x+2}-\frac{1}{x^2-8x+15} = \frac{1}{(x-2)(x-3)}+\frac{1}{(x-2)(x-1)}-\frac{1}{(x-5)(x-3)}$

$= \frac{(x-9)(x-2)}{(x-1)(x-2)(x-3)(x-5)} = \frac{x-9}{(x-1)(x-3)(x-5)}$

The number of seats in a row is equal to the total number of rows in a hall. The total number of seats in the hall will increase by 375 if the number of rows is doubled and the number of seats in each row is reduced by 5. Find the number of rows in the hall at the beginning.

Solution : Let the number of rows be x.

∴ Number of seats in each row = x ∴ Total number of seats in the hall = x^2

∴ By the data given, $2x \times (x-5) = x^2+375$

∴ $x = 25, -15$ ∴ No. of rows at the beginning = 25.

From a group of $2x^2$ black bees, square root of half of the group went to a tree. Again eight-ninth of the bees went to the same tree. The remaining two got caught up in a fragrant lotus. How many bees were there in total ?

Solution : Given number of black bees $= 2x^2$

By the data given, $2x^2 - x - {}^{8}/_{9}(2x^2) = 2 \Rightarrow x = 6, {}^{-3}/_{2}$ ∴ $x = 6$

∴ Total number of bees $= 2x^2 = 2\,(36) = 72$

A flock of swans contained x^2 members. As the clouds gathered, 10x went to a lake and one eighth of the members flew away to a garden. The remaining three pairs played about in the water. How many swans were there in total?

Solution : A flock of swans contained x^2 members.

$x^2-10x-\frac{1}{8}x^2 = 6 \Rightarrow x = 12.$ Total number of swans is $x^2 = 144$.

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Find the square root of $\left[\sqrt{15}x^2+(\sqrt{3}+\sqrt{10})x+\sqrt{2}\right]\left[\sqrt{5}x^2+(2\sqrt{5}+1)x+2\right]\left[\sqrt{3}x^2+(\sqrt{2}+2\sqrt{3})x+2\sqrt{2}\right]$

Solution :

$\sqrt{(\sqrt{5}x+1)(\sqrt{3}x+\sqrt{2})(\sqrt{5}x+1)(x+2)(\sqrt{3}x+\sqrt{2})(x+2)} = \left|(\sqrt{5}x+1)(\sqrt{3}x+\sqrt{2})(x+2)\right|$

Find the square root of the expression

$\frac{x^2}{y^2}-\frac{10x}{y}+27-\frac{10y}{x}+\frac{y^2}{x^2}$

Solution :

$$\begin{array}{r|lllll}
 & 1 & -5 & 1 & & \\ \hline
1 & 1 & -10 & 27 & -10 & 1 \\
 & 1 & & & & \\
 & (-) & & & & \\ \hline
2\ \ -5 & & -10 & 27 & & \\
 & & -10 & 25 & & \\
 & & (+) & (-) & & \\ \hline
2\ \ -10\ \ 1 & & & 2 & -10 & 1 \\
 & & & 2 & -10 & 1 \\
 & & & (-) & (+) & (-) \\ \hline
 & & & 0 & & \\ \hline
\end{array}$$

$\therefore \sqrt{\frac{x^2}{y^2}-10\frac{x}{y}+27-10\frac{y}{x}+\frac{y^2}{x^2}} = \left|\frac{x}{y}-5+\frac{y}{x}\right|$

If $4x^4-12x^3+37x^2+bx+a$ is **perfect square, Find the values of a and b**

Solution :

$$\begin{array}{r|lllll}
 & 2 & -3 & 7 & & \\ \hline
2 & 4 & -12 & 37 & b & a \\
 & 4 & & & & \\
 & (-) & & & & \\ \hline
4\ \ -3 & & -12 & 37 & & \\
 & & -12 & 9 & & \\
 & & (+) & (-) & & \\ \hline
4\ \ -6\ \ 7 & & & 28 & b & a \\
 & & & 28 & -42 & 49 \\ \hline
 & & & 0 & & \\ \hline
\end{array}$$

∴ a = 49 , b = – 42

If $ax^4+bx^3+361x^2+220x+100$ is **perfect square, Find the values of a and b**

Solution :

$$\begin{array}{r|lllll}
 & 10 & 11 & 12 & & \\ \hline
10 & 100 & 220 & 361 & b & \\
 & 100 & & & & \\
 & (-) & & & & \\ \hline
20\ \ 11 & & 220 & 361 & & \\
 & & 220 & 121 & & \\
 & & (-) & (-) & & \\ \hline
20\ \ 22\ \ 12 & & & 240 & b & a \\
 & & & 240 & 264 & 144 \\ \hline
 & & & 0 & & \\ \hline
\end{array}$$

∴ a = 144 , b = 264

If $\frac{1}{x^4}-\frac{6}{x^3}+\frac{13}{x^2}+\frac{m}{x}+n$ is **perfect square, Find the values of m and n**

Solution :

$$\begin{array}{r|lllll}
 & 1 & -3 & 2 & & \\ \hline
1 & 1 & -6 & 13 & m & n \\
 & 1 & & & & \\
 & (-) & & & & \\ \hline
2\ \ -3 & & -6 & 13 & & \\
 & & -6 & 9 & & \\
 & & (+) & (-) & & \\ \hline
2\ \ -6\ \ 2 & & & 4 & m & n \\
 & & & 4 & -12 & 4 \\ \hline
 & & & 0 & & \\ \hline
\end{array}$$

∴ m = – 12 n = 4

If $x^4-8x^3+mx^2+nx+16$ is **perfect square, Find the values of m and n**

Solution :

$$\begin{array}{r|lllll}
 & 1 & -4 & 4 & & \\ \hline
1 & 1 & -8 & m & n & 16 \\
 & 1 & & & & \\
 & (-) & & & & \\ \hline
2\ \ -4 & & -8 & m & & \\
 & & -8 & 16 & & \\
 & & (+) & (-) & & \\ \hline
2\ \ -8\ \ 4 & & & (m-16) & n & 16 \\
 & & & 8 & -32 & 16 \\ \hline
 & & & 0 & & \\ \hline
\end{array}$$

∴ m = 24 , n = – 32

Find $\sqrt{64x^4-16x^3+17x^2-2x+1}$

Solution :

$$\begin{array}{r|lllll}
 & 8 & -1 & 1 & & \\ \hline
8 & 64 & -16 & 17 & -2 & 1 \\
 & 64 & & & & \\
 & (-) & & & & \\ \hline
16\ \ -1 & & -16 & 17 & & \\
 & & -16 & 1 & & \\
 & & (+) & (-) & & \\ \hline
16\ \ -2\ \ 1 & & & 16 & -2 & 1 \\
 & & & 16 & -2 & 1 \\
 & & & (-) & (+) & (-) \\ \hline
 & & & 0 & & \\ \hline
\end{array}$$

$\sqrt{64x^4-16x^3+17x^2-2x+1} = |8x^2-x+1|$

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If $9x^4 + 12x^3 + 28x^2 + ax + b$ is a perfect square, find the values of a and b.

Solution :

	3	2	4		
3	9	12	28	a	b
	(−) 9				
6 2		12	28		
		(−) 12	(−) 4		
6 4 4			24	a	b
			24	16	16
			0		

$\therefore$ a = 16, b = 16.

Find the square root by division method $121x^4 - 198x^3 - 183x^2 + 216x + 144$

Solution :

	11	−9	−12		
11	121	−198	−183	216	144
	(−) 121				
22 −9		−198	−183		
		(+) −198	(−) 81		
22 −18 −12			−264	216	144
			(+) −264	(−) 216	144
			0		

$$\therefore \sqrt{121x^4 - 198x^3 - 183x^2 + 216x + 144} = \left| 11x^2 - 9x - 12 \right|$$

Find the square root of $x^4 - 12x^3 + 42x^2 - 36x + 9$ by division method

Solution :

	1	−6	3		
1	1	−12	42	−36	9
	(−) 1				
2 −6		−12	42		
		(+) −12	(−) 36		
2 −12 3			6	−36	9
			(−) 6	(+) −36	(−) 9
			0		

$$\therefore \sqrt{x^4 - 12x^3 + 42x^2 - 36x + 9} = \left| x^2 - 6x + 3 \right|$$

Find the square root of $37x^2 - 28x^3 + 4x^4 + 42x + 9$ by division method

Solution :

	2	−7	−3		
2	4	−28	37	42	9
	(−) 4				
4 −7		−28	37		
		(+) −28	(−) 49	(+)	
4 −14 −3			−12	42	9
			(+) −12	(−) 42	(−) 9
			0		

$$\therefore \sqrt{4x^4 - 28x^3 + 37x^2 + 42x + 9} = \left| 2x^2 - 7x - 3 \right|$$

Find the square root of the expression $\frac{4x^2}{y^2} + \frac{20x}{y} + 13 - \frac{30y}{x} + \frac{9y^2}{x^2}$

Solution :

	2	5	−3		
2	4	20	13	−30	9
	4				
	(−)				
4 5		20	13		
		20	25		
		(−)	(−)		
4 10 −3			−12	−30	9
			−12	−30	9
			(+)	(+)	(−)
			0		

$$\sqrt{\frac{4x^2}{y} + \frac{20x}{y} + 13 - \frac{30y}{x} + \frac{9y^2}{x^2}} = \left| \frac{2x}{y} + 5 - \frac{3y}{x} \right|$$

Find the square root of $289x^4 - 612x^3 + 970x^2 - 684x + 361$

Solution :

	17	−18	19		
17	289	−612	970	−684	361
	(−) 289				
34 −18		−612	970		
		(+) −612	(−) 324		
34 −36 19			646	−684	361
			(−) 646	(+) −684	(−) 361
			0		

$$\sqrt{289x^4 - 612x^3 + 970x^2 - 684x + 361} = \left| 17x^2 - 18x + 19 \right|$$

Find the GCD of the following by division algorithm $2x^4+13x^3+27x^2+23x+7$, $x^3 + 3x^2 + 3x + 1$, $x^2 + 2x + 1$

Solution :

	1	1		
1 2 1	1	3	3	1
	(−) 1	(−) 2	(−) 1	
		1	2	1
		(−) 1	(−) 2	(−) 1
			0	

	2	9	7		
1 2 1	2	13	27	23	7
	(−) 2	(−) 4	(−) 2		
		9	25	23	
		(−) 9	(−) 18	(−) 9	
			7	14	7
			(−) 7	(−) 14	(−) 7
			0		

$\therefore$ G.C.D. = $x^2 + 2x + 1$

Find the GCD of the following by division algorithm $3x^4 + 6x^3 - 12x^2 - 24x$, $4x^4 + 14x^3 + 8x^2 - 8x$

Solution : Let $f(x) = 4x^4 + 14x^3 + 8x^2 - 8x = 2x(2x^3 + 7x^2 + 4x - 4)$

$g(x) = 3x^4 + 6x^3 - 12x^2 - 24x = 3x(1x^3 + 2x^2 - 4x - 8)$ GCD of 2x , 3x = x

	2			
1 2 −4 −8	2	7	4	−4
	2	4	−8	−16
	(−)	(−)	(+)	(+)
		3	12	12

$\therefore 3(x^2 + 4x + 4) \neq 0$

3 is not a divisor of f(x)

	1	−2		
1 4 4	1	2	−4	−8
	(−) 1	(−) 4	(−) 4	
		−2	−8	−8
		(+) −2	(+) −8	(+) −8
		0		

$\therefore$ G.C.D = $x(x^2 + 4x + 4)$

Find the GCD of the following by division algorithm $3x^3 + 3x^2 + 3x + 3$, $6x^3 + 12x^3 + 6x + 12$

Solution : Let $f(x) = 6x^3 + 12x^2 + 6x + 12 = 6(x^3 + 2x^2 + x + 2)$

$g(x) = 3x^3 + 3x^2 + 3x + 3 = 3(x^3 + x^2 + x + 1)$ GCD of 6, 3 = 3

	1			
1 1 1 1	1	2	1	2
	1	1	1	1
	(−)	(−)	(−)	(−)
		1	0	1

$\therefore x^2 + 1 \neq 0$

	1	1		
1 0 1	1	1	1	1
	1	(−) 0	(−) 1	0
		1	0	1
		1	0	1
		(−)	(−)	(−)
		0		

$\therefore$ G.C.D = $3(x^2 + 1)$

Find the GCD of $6x^3 - 30x^2 + 60x - 48$ and $3x^3 - 12x^2 + 21x - 18$

Solution : Let $f(x) = 6x^3 - 30x^2 + 60x - 48 = 6(x^3 - 5x^2 + 10x - 8)$

$g(x) = 3x^3 - 12x^2 + 21x - 18 = 3(x^3 - 4x^2 + 7x - 6)$ GCD of 3 and 6 is 3.

	1			
1 −5 10 −8	1	−4	7	−6
	1	−5	10	−8
	(−)	(+)	(−)	(+)
		1	−3	2

	1	−2		
1 −3 2	1	−5	10	−8
	(−) 1	(+) −3	(−) 2	
		−2	8	−8
		(+) −2	(−) 6	(+) −4
			2	−4

$2(x - 2) \neq 0$

2 is not a divisor of g(x)

	1	−1	
1 −2	1	−3	2
	(−) 1	(+) −2	
		−1	2
		(+) −1	(−) 2
		0	

GCD = 3(x − 2)

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3.MATRICES
5 MARKS

Find X and Y if $\mathbf{X+Y}=\begin{pmatrix}7&0\\3&5\end{pmatrix}$ **and** $\mathbf{X-Y}=\begin{pmatrix}3&0\\0&4\end{pmatrix}$

Solution :

$$X+Y=\begin{pmatrix}7&0\\3&5\end{pmatrix}\text{ .... (1)}\qquad X-Y=\begin{pmatrix}3&0\\0&4\end{pmatrix}\text{ .... (2)}$$

$$(1)+(2)\Rightarrow 2X=\begin{pmatrix}10&0\\3&9\end{pmatrix}\Rightarrow X=\begin{pmatrix}5&0\\3/2&9/2\end{pmatrix}\qquad (1)-(2)\Rightarrow 2Y=\begin{pmatrix}4&0\\3&1\end{pmatrix}\Rightarrow Y=\begin{pmatrix}2&0\\3/2&1/2\end{pmatrix}$$

If $A=\begin{pmatrix}a&b\\c&d\end{pmatrix}$ and $I=\begin{pmatrix}1&0\\0&1\end{pmatrix}$ **show that** $\mathbf{A^2-(a+d)A=(bc-ad)I_2}$.

Solution :

$$A^2=A.A=\begin{pmatrix}a&b\\c&d\end{pmatrix}\begin{pmatrix}a&b\\c&d\end{pmatrix}=\begin{pmatrix}a^2+bc&ab+bd\\ac+cd&bc+d^2\end{pmatrix}\text{ and }(a+d)A=\begin{pmatrix}a^2+ad&ab+bd\\ca+cd&ad+d^2\end{pmatrix}$$

$$A^2-(a+d)A=\begin{pmatrix}a^2+bc&ab+bd\\ac+cd&bc+d^2\end{pmatrix}-\begin{pmatrix}a^2+ad&ab+bd\\ca+cd&ad+d^2\end{pmatrix}=\begin{pmatrix}bc-ad&0\\0&bc-ad\end{pmatrix}=(bc-ad)\begin{pmatrix}1&0\\0&1\end{pmatrix}$$

$$=\mathbf{(bc-ad)\,I_2}.$$

If $A=\begin{pmatrix}5&2&9\\1&2&8\end{pmatrix},B=\begin{pmatrix}1&7\\1&2\\5&-1\end{pmatrix}$ **verify that** $\mathbf{(AB)^T=B^TA^T}$

Solution :

$$AB=\begin{pmatrix}5&2&9\\1&2&8\end{pmatrix}\begin{pmatrix}1&7\\1&2\\5&-1\end{pmatrix}=\begin{pmatrix}\begin{smallmatrix}5&2&9\end{smallmatrix}\begin{smallmatrix}1\\1\\5\end{smallmatrix}&\begin{smallmatrix}5&2&9\end{smallmatrix}\begin{smallmatrix}7\\2\\-1\end{smallmatrix}\\\begin{smallmatrix}1&2&8\end{smallmatrix}\begin{smallmatrix}1\\1\\5\end{smallmatrix}&\begin{smallmatrix}1&2&8\end{smallmatrix}\begin{smallmatrix}7\\2\\-1\end{smallmatrix}\end{pmatrix}=\begin{pmatrix}5+2+45&35+4-9\\1+2+40&7+4-8\end{pmatrix}=\begin{pmatrix}52&30\\43&3\end{pmatrix}$$

$$\therefore (AB)^T=\begin{pmatrix}52&43\\30&3\end{pmatrix}\text{ .... (1)}$$

$$A^T=\begin{pmatrix}5&1\\2&2\\9&8\end{pmatrix}\text{ and }B^T=\begin{pmatrix}1&1&5\\7&2&-1\end{pmatrix}$$

$$B^TA^T=\begin{pmatrix}1&1&5\\7&2&-1\end{pmatrix}\begin{pmatrix}5&1\\2&2\\9&8\end{pmatrix}=\begin{pmatrix}\begin{smallmatrix}1&1&5\end{smallmatrix}\begin{smallmatrix}5\\2\\9\end{smallmatrix}&\begin{smallmatrix}1&1&5\end{smallmatrix}\begin{smallmatrix}1\\2\\8\end{smallmatrix}\\\begin{smallmatrix}7&2&-1\end{smallmatrix}\begin{smallmatrix}5\\2\\9\end{smallmatrix}&\begin{smallmatrix}7&2&-1\end{smallmatrix}\begin{smallmatrix}1\\2\\8\end{smallmatrix}\end{pmatrix}=\begin{pmatrix}5+2+45&1+2+40\\35+4-9&7+4-8\end{pmatrix}=\begin{pmatrix}52&43\\30&3\end{pmatrix}\text{ ..... (2)}$$

$\therefore$ From (1) & (2), $(AB)^T=B^TA^T$

If $\mathbf{A}=\begin{pmatrix}3&1\\-1&2\end{pmatrix}$ **show that** $\mathbf{A^2-5A+7I_2=0}$

A ZAYAAN ABDUL M.SC.,B.ED.,
JAYANKON DAM,ARIYALUR DT.

Solution : **If** $A=\begin{pmatrix}a&b\\c&d\end{pmatrix}$ and $I=\begin{pmatrix}1&0\\0&1\end{pmatrix}$ $\mathbf{A^2-(a+d)A=(bc-ad)I_2}$.

$$A=\begin{pmatrix}a&b\\c&d\end{pmatrix}=\begin{pmatrix}3&1\\-1&2\end{pmatrix}$$

$$\therefore\ \mathbf{A^2-(3+2)A=((1)(-1)-(3)(2))\,I_2}$$

$$\therefore\ A^2-5A+7I_2$$

If $A=\begin{pmatrix}1&2&1\\2&-1&1\end{pmatrix}$ and $B=\begin{pmatrix}2&-1\\-1&4\\0&2\end{pmatrix}$ show that $(AB)^T=B^TA^T$.

Solution :

$$AB=\begin{pmatrix}1&2&1\\2&-1&1\end{pmatrix}\times\begin{pmatrix}2&-1\\-1&4\\0&2\end{pmatrix}=\begin{pmatrix}\begin{smallmatrix}1&2&1\end{smallmatrix}\begin{smallmatrix}2\\-1\\0\end{smallmatrix}&\begin{smallmatrix}1&2&1\end{smallmatrix}\begin{smallmatrix}-1\\4\\2\end{smallmatrix}\\\begin{smallmatrix}2&-1&1\end{smallmatrix}\begin{smallmatrix}2\\-1\\0\end{smallmatrix}&\begin{smallmatrix}2&-1&1\end{smallmatrix}\begin{smallmatrix}-1\\4\\2\end{smallmatrix}\end{pmatrix}=\begin{pmatrix}2-2+0&-1+8+2\\4+1+0&-2-4+2\end{pmatrix}=\begin{pmatrix}0&9\\5&-4\end{pmatrix}$$

$$(AB)^T=\begin{pmatrix}0&5\\9&-4\end{pmatrix}\text{ .... (1)}$$

$$B^T=\begin{pmatrix}2&-1&0\\-1&4&2\end{pmatrix},A^T=\begin{pmatrix}1&2\\2&-1\\1&1\end{pmatrix}$$

$$B^TA^T=\begin{pmatrix}2&-1&0\\-1&4&2\end{pmatrix}\begin{pmatrix}1&2\\2&-1\\1&1\end{pmatrix}=\begin{pmatrix}\begin{smallmatrix}2&-1&0\end{smallmatrix}\begin{smallmatrix}1\\2\\1\end{smallmatrix}&\begin{smallmatrix}2&-1&0\end{smallmatrix}\begin{smallmatrix}2\\-1\\1\end{smallmatrix}\\\begin{smallmatrix}-1&4&2\end{smallmatrix}\begin{smallmatrix}1\\2\\1\end{smallmatrix}&\begin{smallmatrix}-1&4&2\end{smallmatrix}\begin{smallmatrix}2\\-1\\1\end{smallmatrix}\end{pmatrix}=\begin{pmatrix}2-2+0&4+1+0\\-1+8+2&-2-4+2\end{pmatrix}=\begin{pmatrix}0&5\\9&-4\end{pmatrix}\text{ ..... (2)}$$

From (1) and (2), $(AB)^T=B^TA^T$. Hence proved.

Let $A=\begin{pmatrix}1&2\\1&3\end{pmatrix}, B=\begin{pmatrix}4&0\\1&5\end{pmatrix}$, **Show that** $\mathbf{(A-B)^T=A^T-B^T}$

Solution : $$\mathbf{(A-B)}=\begin{pmatrix}1&2\\1&3\end{pmatrix}-\begin{pmatrix}4&0\\1&5\end{pmatrix}=\begin{pmatrix}-3&2\\0&-2\end{pmatrix}\qquad\therefore (A-B)^T=\begin{pmatrix}-3&0\\2&-2\end{pmatrix}\text{ .... (1)}$$

$$A^T=\begin{pmatrix}1&1\\2&3\end{pmatrix},\ B^T=\begin{pmatrix}4&1\\0&5\end{pmatrix}\qquad\therefore A^T-B^T=\begin{pmatrix}1&1\\2&3\end{pmatrix}-\begin{pmatrix}4&1\\0&5\end{pmatrix}=\begin{pmatrix}-3&0\\2&-2\end{pmatrix}\text{ .... (2)}$$

$\therefore$ From (1) & (2) $(A-B)^T=A^T-B^T$

If $A=\begin{pmatrix}4&3&1\\2&3&-8\\1&0&-4\end{pmatrix},B=\begin{pmatrix}2&3&4\\1&9&2\\-7&1&-1\end{pmatrix}$ and $C=\begin{pmatrix}8&3&4\\1&-2&3\\2&4&-1\end{pmatrix}$ **then verify that** $\mathbf{A+(B+C)=(A+B)+C}$

Solution :

$$B+C=\begin{pmatrix}10&6&8\\2&7&5\\-5&5&-2\end{pmatrix}$$

$$A+(B+C)=\begin{pmatrix}4&3&1\\2&3&-8\\1&0&-4\end{pmatrix}+\begin{pmatrix}10&6&8\\2&7&5\\-5&5&-2\end{pmatrix}=\begin{pmatrix}14&9&9\\4&10&-3\\-4&5&-6\end{pmatrix}\text{ .....(1)}$$

$$A+B=\begin{pmatrix}6&6&5\\3&12&-6\\-6&1&-5\end{pmatrix}$$

A ZAYAAN ABDUL M.SC.,B.ED.,
JAYANKON DAM,ARIYALUR DT.

$$\therefore (A+B)+C=\begin{pmatrix}6&6&5\\3&12&-6\\-6&1&-5\end{pmatrix}+\begin{pmatrix}8&3&4\\1&-2&3\\2&4&-1\end{pmatrix}=\begin{pmatrix}14&9&9\\4&10&-3\\-4&5&-6\end{pmatrix}\text{ ....(2)}$$

$\therefore$ From (1) & (2) $\mathbf{A+(B+C)=(A+B)+C}$

Let $A=\begin{pmatrix}1&2\\1&3\end{pmatrix},\ B=\begin{pmatrix}4&0\\1&5\end{pmatrix},\ C=\begin{pmatrix}2&0\\1&2\end{pmatrix}$ **Show that A(BC) = (AB)C**

Solution : $A=\begin{pmatrix}1&2\\1&3\end{pmatrix},\ B=\begin{pmatrix}4&0\\1&5\end{pmatrix},\ C=\begin{pmatrix}2&0\\1&2\end{pmatrix}$

$$BC=\begin{pmatrix}4&0\\1&5\end{pmatrix}\begin{pmatrix}2&0\\1&2\end{pmatrix}=\begin{pmatrix}(4\ 0)\begin{pmatrix}2\\1\end{pmatrix}&(4\ 0)\begin{pmatrix}0\\2\end{pmatrix}\\(1\ 5)\begin{pmatrix}2\\1\end{pmatrix}&(1\ 5)\begin{pmatrix}0\\2\end{pmatrix}\end{pmatrix}=\begin{pmatrix}8+0&0+0\\2+5&0+10\end{pmatrix}=\begin{pmatrix}8&0\\7&10\end{pmatrix}$$

$$A(BC)=\begin{pmatrix}1&2\\1&3\end{pmatrix}\begin{pmatrix}8&0\\7&10\end{pmatrix}=\begin{pmatrix}(1\ 2)\begin{pmatrix}8\\7\end{pmatrix}&(1\ 2)\begin{pmatrix}0\\10\end{pmatrix}\\(1\ 3)\begin{pmatrix}8\\7\end{pmatrix}&(1\ 3)\begin{pmatrix}0\\10\end{pmatrix}\end{pmatrix}=\begin{pmatrix}8+14&0+20\\8+21&0+30\end{pmatrix}=\begin{pmatrix}22&20\\29&30\end{pmatrix}\ \ldots\ldots\ldots(1)$$

$$AB=\begin{pmatrix}1&2\\1&3\end{pmatrix}\begin{pmatrix}4&0\\1&5\end{pmatrix}=\begin{pmatrix}(1\ 2)\begin{pmatrix}4\\1\end{pmatrix}&(1\ 2)\begin{pmatrix}0\\5\end{pmatrix}\\(1\ 3)\begin{pmatrix}4\\1\end{pmatrix}&(1\ 3)\begin{pmatrix}0\\5\end{pmatrix}\end{pmatrix}=\begin{pmatrix}4+2&0+10\\4+3&0+15\end{pmatrix}=\begin{pmatrix}6&10\\7&15\end{pmatrix}$$

$$(AB)C=\begin{pmatrix}6&10\\7&15\end{pmatrix}\begin{pmatrix}2&0\\1&2\end{pmatrix}=\begin{pmatrix}(6\ 10)\begin{pmatrix}2\\1\end{pmatrix}&(6\ 10)\begin{pmatrix}0\\2\end{pmatrix}\\(7\ 15)\begin{pmatrix}2\\1\end{pmatrix}&(7\ 15)\begin{pmatrix}0\\2\end{pmatrix}\end{pmatrix}=\begin{pmatrix}12+10&0+20\\14+15&0+30\end{pmatrix}=\begin{pmatrix}22&20\\29&30\end{pmatrix}\ \ldots\ldots\ldots(2)$$

$\therefore$ From (1) & (2) **A(BC) = (AB)C**

Let $A=\begin{pmatrix}1&2\\1&3\end{pmatrix},\ B=\begin{pmatrix}4&0\\1&5\end{pmatrix},\ C=\begin{pmatrix}2&0\\1&2\end{pmatrix}$ **Show that (A − B) C = AC − BC**

Solution : $A=\begin{pmatrix}1&2\\1&3\end{pmatrix},\ B=\begin{pmatrix}4&0\\1&5\end{pmatrix},\ C=\begin{pmatrix}2&0\\1&2\end{pmatrix}$

$$(A-B)=\begin{pmatrix}1&2\\1&3\end{pmatrix}-\begin{pmatrix}4&0\\1&5\end{pmatrix}=\begin{pmatrix}-3&2\\0&-2\end{pmatrix}$$

$$(A-B)\,C=\begin{pmatrix}-3&2\\0&-2\end{pmatrix}\begin{pmatrix}2&0\\1&2\end{pmatrix}=\begin{pmatrix}(-3\ 2)\begin{pmatrix}2\\1\end{pmatrix}&(-3\ 2)\begin{pmatrix}0\\2\end{pmatrix}\\(0\ -2)\begin{pmatrix}2\\1\end{pmatrix}&(0\ -2)\begin{pmatrix}0\\2\end{pmatrix}\end{pmatrix}=\begin{pmatrix}-6+2&0+4\\0-2&0-4\end{pmatrix}=\begin{pmatrix}-4&4\\-2&-4\end{pmatrix}\ \ldots\ldots\ldots(1)$$

$$AC=\begin{pmatrix}1&2\\1&3\end{pmatrix}\begin{pmatrix}2&0\\1&2\end{pmatrix}=\begin{pmatrix}(1\ 2)\begin{pmatrix}2\\1\end{pmatrix}&(1\ 2)\begin{pmatrix}0\\2\end{pmatrix}\\(1\ 3)\begin{pmatrix}2\\1\end{pmatrix}&(1\ 3)\begin{pmatrix}0\\2\end{pmatrix}\end{pmatrix}=\begin{pmatrix}2+2&0+4\\2+3&0+6\end{pmatrix}=\begin{pmatrix}4&4\\5&6\end{pmatrix}$$

$$BC=\begin{pmatrix}4&0\\1&5\end{pmatrix}\begin{pmatrix}2&0\\1&2\end{pmatrix}=\begin{pmatrix}(4\ 0)\begin{pmatrix}2\\1\end{pmatrix}&(4\ 0)\begin{pmatrix}0\\2\end{pmatrix}\\(1\ 5)\begin{pmatrix}2\\1\end{pmatrix}&(1\ 5)\begin{pmatrix}0\\2\end{pmatrix}\end{pmatrix}=\begin{pmatrix}8+0&0+0\\2+5&0+10\end{pmatrix}=\begin{pmatrix}8&0\\7&10\end{pmatrix}$$

$$\therefore AC-BC=\begin{pmatrix}4&4\\5&6\end{pmatrix}-\begin{pmatrix}8&0\\7&10\end{pmatrix}=\begin{pmatrix}-4&4\\-2&-4\end{pmatrix}\ \ldots\ldots\ldots(2)$$

$\therefore$ From (1) & (2). (A − B) C = AC − BC

A ZAYAAN ABDUL M.SC.,B.ED.,
JAYANKONDAM,ARIYALUR DT.

Solve for x, y $\begin{pmatrix}x^2\\y^2\end{pmatrix}+2\begin{pmatrix}-2x\\y\end{pmatrix}=\begin{pmatrix}5\\8\end{pmatrix}$

Solution :

$$\begin{pmatrix}x^2\\y^2\end{pmatrix}+2\begin{pmatrix}-2x\\y\end{pmatrix}=\begin{pmatrix}5\\8\end{pmatrix}$$

$\Rightarrow x^2-4x=5$

$\Rightarrow x=5,-1$

$\Rightarrow y^2-2y=8$

$\Rightarrow \therefore y=4, y=-2$

A ZAYAAN ABDUL M.SC.,B.ED.,
JAYANKONDAM,ARIYALUR DT.

Given that $A=\begin{pmatrix}1&3\\5&-1\end{pmatrix},\ B=\begin{pmatrix}1&-1&2\\3&5&2\end{pmatrix},\ C=\begin{pmatrix}1&3&2\\-4&1&3\end{pmatrix}$ **verify that A(B + C) = AB + AC**

Solution :

$$A=\begin{pmatrix}1&3\\5&-1\end{pmatrix},B=\begin{pmatrix}1&-1&2\\3&5&2\end{pmatrix},C=\begin{pmatrix}1&3&2\\-4&1&3\end{pmatrix}$$

$$(B+C)=\begin{pmatrix}1&-1&2\\3&5&2\end{pmatrix}+\begin{pmatrix}1&3&2\\-4&1&3\end{pmatrix}=\begin{pmatrix}2&2&4\\-1&6&5\end{pmatrix}$$

$$A(B+C)=\begin{pmatrix}1&3\\5&-1\end{pmatrix}\begin{pmatrix}2&2&4\\-1&6&5\end{pmatrix}=\begin{pmatrix}(1\ 3)\begin{pmatrix}2\\-1\end{pmatrix}&(1\ 3)\begin{pmatrix}2\\6\end{pmatrix}&(1\ 3)\begin{pmatrix}4\\5\end{pmatrix}\\(5\ -1)\begin{pmatrix}2\\-1\end{pmatrix}&(5\ -1)\begin{pmatrix}2\\6\end{pmatrix}&(5\ -1)\begin{pmatrix}4\\5\end{pmatrix}\end{pmatrix}=\begin{pmatrix}2-3&2+18&4+15\\10+1&10-6&20-5\end{pmatrix}$$

$$=\begin{pmatrix}-1&20&19\\11&4&15\end{pmatrix}\ \ldots.(1)$$

$$AB=\begin{pmatrix}1&3\\5&-1\end{pmatrix}\begin{pmatrix}1&-1&2\\3&5&2\end{pmatrix}=\begin{pmatrix}(1\ 3)\begin{pmatrix}1\\3\end{pmatrix}&(1\ 3)\begin{pmatrix}-1\\5\end{pmatrix}&(1\ 3)\begin{pmatrix}2\\2\end{pmatrix}\\(5\ -1)\begin{pmatrix}1\\3\end{pmatrix}&(5\ -1)\begin{pmatrix}-1\\5\end{pmatrix}&(5\ -1)\begin{pmatrix}2\\2\end{pmatrix}\end{pmatrix}$$

$$=\begin{pmatrix}1+9&-1+15&2+6\\5-3&-5-5&10-2\end{pmatrix}=\begin{pmatrix}10&14&8\\2&-10&8\end{pmatrix}$$

$$AB=\begin{pmatrix}1&3\\5&-1\end{pmatrix}\begin{pmatrix}1&-1&2\\3&5&2\end{pmatrix}=\begin{pmatrix}(1\ 3)\begin{pmatrix}1\\3\end{pmatrix}&(1\ 3)\begin{pmatrix}-1\\5\end{pmatrix}&(1\ 3)\begin{pmatrix}2\\2\end{pmatrix}\\(5\ -1)\begin{pmatrix}1\\3\end{pmatrix}&(5\ -1)\begin{pmatrix}-1\\5\end{pmatrix}&(5\ -1)\begin{pmatrix}2\\2\end{pmatrix}\end{pmatrix}=\begin{pmatrix}1+9&-1+15&2+6\\5-3&-5-5&10-2\end{pmatrix}=\begin{pmatrix}10&14&8\\2&-10&8\end{pmatrix}$$

$$AC=\begin{pmatrix}1&3\\5&-1\end{pmatrix}\begin{pmatrix}1&3&2\\-4&1&3\end{pmatrix}=\begin{pmatrix}(1\ 3)\begin{pmatrix}1\\-4\end{pmatrix}&(1\ 3)\begin{pmatrix}3\\1\end{pmatrix}&(1\ 3)\begin{pmatrix}2\\3\end{pmatrix}\\(5\ -1)\begin{pmatrix}1\\-4\end{pmatrix}&(5\ -1)\begin{pmatrix}3\\1\end{pmatrix}&(5\ -1)\begin{pmatrix}2\\3\end{pmatrix}\end{pmatrix}=\begin{pmatrix}1-12&3+3&2+9\\5+4&15-1&10-3\end{pmatrix}$$

$$AB+AC=\begin{pmatrix}10&14&8\\2&-10&8\end{pmatrix}+\begin{pmatrix}-11&6&11\\9&14&7\end{pmatrix}=\begin{pmatrix}-1&20&19\\11&4&15\end{pmatrix}\ \ldots\ldots\ldots\ldots(2)$$

$\therefore$ From (1) & (2) **A(B + C) = AB + AC**

If $A=\begin{pmatrix}1 & -1 & 2\end{pmatrix}$, $B=\begin{pmatrix}1 & -1\\ 2 & 1\\ 1 & 3\end{pmatrix}$ and $C=\begin{pmatrix}1 & 2\\ 2 & -1\end{pmatrix}$ show that(AB)C = A(BC).

Solution :

$$(AB)=\begin{pmatrix}1 & -1 & 2\end{pmatrix}\times\begin{pmatrix}1 & -1\\ 2 & 1\\ 1 & 3\end{pmatrix}=\begin{pmatrix}(1\ -1\ 2)\begin{pmatrix}1\\2\\1\end{pmatrix} & (1\ -1\ 2)\begin{pmatrix}-1\\1\\3\end{pmatrix}\end{pmatrix}=(1-2+2\quad -1-1+6)=(1\ \ 4)$$

$$(AB)\,C=(1\ \ 4)\times\begin{pmatrix}1 & 2\\ 2 & -1\end{pmatrix}=\begin{pmatrix}(1\ 4)\begin{pmatrix}1\\2\end{pmatrix} & (1\ 4)\begin{pmatrix}2\\-1\end{pmatrix}\end{pmatrix}=(1+8\ \ 2-4)=(9\ \ -2)\ \ldots\ldots(1)$$

$$BC=\begin{pmatrix}1 & -1\\ 2 & 1\\ 1 & 3\end{pmatrix}\times\begin{pmatrix}1 & 2\\ 2 & -1\end{pmatrix}=\begin{pmatrix}(1\ -1)\begin{pmatrix}1\\2\end{pmatrix} & (1\ -1)\begin{pmatrix}2\\-1\end{pmatrix}\\ (2\ 1)\begin{pmatrix}1\\2\end{pmatrix} & (2\ 1)\begin{pmatrix}2\\-1\end{pmatrix}\\ (1\ 3)\begin{pmatrix}1\\2\end{pmatrix} & (1\ 3)\begin{pmatrix}2\\-1\end{pmatrix}\end{pmatrix}=\begin{pmatrix}1-2 & 2+1\\ 2+2 & 4-1\\ 1+6 & 2-3\end{pmatrix}=\begin{pmatrix}-1 & 3\\ 4 & 3\\ 7 & -1\end{pmatrix}$$

$$A(BC)=\begin{pmatrix}1 & -1 & 2\end{pmatrix}\times\begin{pmatrix}-1 & 3\\ 4 & 3\\ 7 & -1\end{pmatrix}=\begin{pmatrix}(1\ -1\ 2)\begin{pmatrix}-1\\4\\7\end{pmatrix} & (1\ -1\ 2)\begin{pmatrix}3\\3\\-1\end{pmatrix}\end{pmatrix}$$

$$=(-1-4+14\qquad 3-3-2)\ =\ (9\ \ -2)\ldots\ldots\ldots\ldots(2)$$

From (1) and (2), (AB)C = A(BC).

If $A=\begin{pmatrix}1 & 1\\ -1 & 3\end{pmatrix}, B=\begin{pmatrix}1 & 2\\ -4 & 2\end{pmatrix}, C=\begin{pmatrix}-7 & 6\\ 3 & 2\end{pmatrix}$ verify that A (B + C) = AB + AC.

Solution :

$$B+C=\begin{pmatrix}1 & 2\\ -4 & 2\end{pmatrix}+\begin{pmatrix}-7 & 6\\ 3 & 2\end{pmatrix}=\begin{pmatrix}-6 & 8\\ -1 & 4\end{pmatrix}$$

$$A(B+C)=\begin{pmatrix}1 & 1\\ -1 & 3\end{pmatrix}\times\begin{pmatrix}-6 & 8\\ -1 & 4\end{pmatrix}=\begin{pmatrix}(1\ 1)\begin{pmatrix}-6\\-1\end{pmatrix} & (1\ 1)\begin{pmatrix}8\\4\end{pmatrix}\\ (-1\ 3)\begin{pmatrix}-6\\-1\end{pmatrix} & (-1\ 3)\begin{pmatrix}8\\4\end{pmatrix}\end{pmatrix}=\begin{pmatrix}-6-1 & 8+4\\ 6-3 & -8+12\end{pmatrix}=\begin{pmatrix}-7 & 12\\ 3 & 4\end{pmatrix}\ \ldots(1)$$

$$AB=\begin{pmatrix}1 & 1\\ -1 & 3\end{pmatrix}\times\begin{pmatrix}1 & 2\\ -4 & 2\end{pmatrix}=\begin{pmatrix}(1\ 1)\begin{pmatrix}1\\-4\end{pmatrix} & (1\ 1)\begin{pmatrix}2\\2\end{pmatrix}\\ (-1\ 3)\begin{pmatrix}1\\-4\end{pmatrix} & (-1\ 3)\begin{pmatrix}2\\2\end{pmatrix}\end{pmatrix}=\begin{pmatrix}1-4 & 2+2\\ -1-12 & -2+6\end{pmatrix}=\begin{pmatrix}-3 & 4\\ -13 & 4\end{pmatrix}$$

$$AC=\begin{pmatrix}1 & 1\\ -1 & 3\end{pmatrix}\times\begin{pmatrix}-7 & 6\\ 3 & 2\end{pmatrix}=\begin{pmatrix}(1\ 1)\begin{pmatrix}-7\\3\end{pmatrix} & (1\ 1)\begin{pmatrix}6\\2\end{pmatrix}\\ (-1\ 3)\begin{pmatrix}-7\\3\end{pmatrix} & (-1\ 3)\begin{pmatrix}6\\2\end{pmatrix}\end{pmatrix}=\begin{pmatrix}-7+3 & 6+2\\ 7+9 & -6+6\end{pmatrix}=\begin{pmatrix}-4 & 8\\ 16 & 0\end{pmatrix}$$

$$AB+AC=\begin{pmatrix}-3 & 4\\ -13 & 4\end{pmatrix}+\begin{pmatrix}-4 & 8\\ 16 & 0\end{pmatrix}=\begin{pmatrix}-7 & 12\\ 3 & 4\end{pmatrix}\quad\ldots\ldots(2)$$

From (1) and (2), A (B + C) = AB + AC.

Hence proved.

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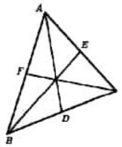
4 . GEOMETRY
5 MARKS

Show that in a triangle, the medians are concurrent.

Solution : The medians are the cevians where D, E, F are midpoints of BC, CA and AB



D is a mid point of $\frac{BD}{DC}=1\ldots.(1)$

E is a midpoint of $\frac{CE}{EA}=1\cdots(2)$

F is a midpoint of $\frac{AF}{FB}=1\ \ldots.(3)$

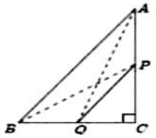
(1), (2) and (3) we get, $\frac{BD}{DC}\times\frac{CE}{EA}\times\frac{AF}{FB}=1\times1\times1=1$

Ceva's theorem is satisfied.

Hence the Medians are concurrent.

P and Q are the mid-points of the sides CA and CB respectively of a ΔABC, right angled at C. Prove that $4\,(AQ^2+BP^2)=54\,AB^2$.

Solution :



ΔAQC is a right triangle at C, ⇒ $AQ^2=AC^2+QC^2\ldots\ldots..(1)$

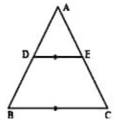
ΔBPC is a right triangle at C, ⇒ $BP^2=BC^2+CP^2\ldots..(2)$

From (1) and (2), $AQ^2+BP^2=\ AC^2+QC^2+BC^2+CP^2$

$4\,(AQ^2+BP^2)\ =4AC^2+4QC^2+4BC^2+4CP^2=5\,AB^2$ (By Pythagoras Theorem)

In ΔABC if DE | | BC, **AD = 8x – 7, DB = 5x – 3, AE = 4x – 3 and EC = 3x – 1, find the value of x.**

Solution : Given AD = 8x – 7, DB = 5x – 3 , AE = 4x – 3 **and** EC = 3x – 1



DE | | BC, By Thales theorem $\frac{AD}{DB}=\frac{AE}{EC}$

$$\frac{8x-7}{5x-3}=\frac{4x-3}{3x-1}$$

$$\Rightarrow x=1,\ {-1}/{2}\qquad \therefore\ x=1$$

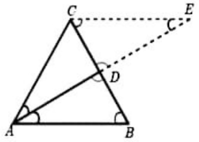
Statement and prove **Angle Bisector Theorem**

Statement The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the corresponding sides containing the angle.

Given : In ΔABC, AD is the internal bisector

To Prove : $\frac{AB}{AC}=\frac{BD}{CD}$

Construction : Draw CE parallel to AB. Extend AD to E



Proof : In ΔABD ~ ΔECD , AB | | CE

∠AEC = ∠BAE Alternate angles equal.

In ΔACE is isosceles

∠CAE = ∠CEA , AC = CE ....(1)

By AA Similarity ΔABD ~ ΔECD

$$\frac{AB}{CE}=\frac{BD}{CD}$$

$$\Rightarrow\ \frac{AB}{AC}=\frac{BD}{CD}$$

From (1) AC = CE.

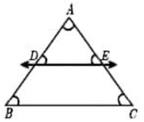
Hence proved.

A ZAYAAN ABDUL M.SC.,B.ED.,
JAYANKONDAM,ARIYALUR DT.

Statement and prove Basic Proportionality Theorem or Thales theorem

Statement

A straight line drawn parallel to a side of triangle intersecting the other two sides, divides the sides in the same ratio.



Given: In ΔABC, D is a point on AB and E is a point on AC.

To Prove : $\frac{AD}{DB}=\frac{AE}{EC}$

Construction : Draw a line $DE \,||\, BC$

Proof : In ΔADE and ΔABC, $DE \,||\, BC$

$\angle A$ common angle

$\angle ADE = \angle ABC$ Corresponding angles are equal

By AA similarity $\Delta ADE \sim \Delta ABC$

$$\frac{AD}{AB}=\frac{AE}{AC}$$

$$\frac{AD}{AD+DB}=\frac{AE}{AE+EC}$$

$$\frac{AD}{AD}+\frac{AD}{DB}=\frac{AE}{AE}+\frac{AE}{EC}$$

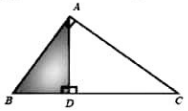
$$1+\frac{AD}{DB}=1+\frac{AE}{EC} \Rightarrow \frac{AD}{DB}=\frac{AE}{EC}$$

Hence proved

Statement and prove Pythagoras Theorem

Statement

In a right angle triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.



Given : In ΔABC, $A = 90^0$

To prove : $AB^2 + AC^2 = BC^2$

Construction : Draw $AD \perp BC$

Proof : In ΔABC and ΔABD $\angle B$ is common

$\angle BAC = \angle BDA = 90°$

By AA similarity, $\Delta ABC \sim \Delta ABD$

$$\frac{AB}{BD}=\frac{BC}{AB} \Rightarrow AB^2 = BC \times BD \quad .... (1)$$

In ΔABC and ΔADC $\angle C$ is common

$\angle BAC = \angle ADC = 90°$

By AA similarity, $\Delta ABC \sim \Delta ADC$ $\frac{BC}{AC}=\frac{AC}{DC} \Rightarrow AC^2 = BC \times DC$ .... (2)

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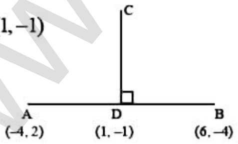
Adding, (1) and (2) $AB^2 + AC^2 = BC \times BD + BC \times DC = BC(BD+DC) = BC \times BC = BC^2$

Hence proved.

5 MARKS 5 . COORDINATE GEOMETRY

Find the equation of the perpendicular bisector of the line joining the points A(-4,2) and B(6,-4)

Solution :



D is the midpoint of AB $\therefore D=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{-4+6}{2},\frac{2-4}{2}\right)=(1,-1)$

Slope of AB $=\frac{y_2-y_1}{x_2-x_1}=\frac{(-4)-(2)}{(6)-(-4)}=\frac{-4-2}{6+4}=\frac{-6}{10}=\frac{-3}{5}$

$\therefore$ Slope of $CD=\frac{5}{3}$ $(\because CD \perp AB)$

$\therefore$ Equation of perpendicular bisector CD is $\Rightarrow y-y_1=m(x-x_1)$ here $m=\frac{5}{3}$, $(x_1,y_1)=(1,-1)$

$$y+1=\frac{5}{3}(x-1) \Rightarrow 3y+3=5x-5 \Rightarrow 5x-3y-8=0$$

Find the area of the quadrilateral whose vertices are (–9, 0), (–8, 6), (–1, –2) and (–6, –3)

Solution :

Area of **quadrilateral** $=\frac{1}{2}\left\{\begin{matrix} x_1 & & x_2 & & x_3 & & x_4 & & x_1 \\ & \times & & \times & & \times & & \times & \\ y_1 & & y_2 & & y_3 & & y_4 & & y_1 \end{matrix}\right\}$

$$=\frac{1}{2}\begin{bmatrix} -8 & -9 & -6 & -1 & -8 \\ 6 & 0 & -3 & -2 & 6 \end{bmatrix}$$

$$=\frac{1}{2}[(0+27+12-6)-(-54+0+3+16)]$$

$$=\frac{1}{2}[33-(-35)] = \frac{1}{2}[68] = 34 \text{ sq. units}$$

Find the area of the quadrilateral whose vertices are (–9, 0), (–8, 6), (–1, –2) and (–6, –3)

Solution :

Area of **quadrilateral** $=\frac{1}{2}\left\{\begin{matrix} x_1 & & x_2 & & x_3 & & x_4 & & x_1 \\ & \times & & \times & & \times & & \times & \\ y_1 & & y_2 & & y_3 & & y_4 & & y_1 \end{matrix}\right\} = \frac{1}{2}\begin{bmatrix} -8 & -9 & -6 & -1 & -8 \\ 6 & 0 & -3 & -2 & 6 \end{bmatrix}$

$$=\frac{1}{2}[(0+27+12-6)-(-54+0+3+16)]$$

$$=\frac{1}{2}[33-(-35)] = \frac{1}{2}[68] = 34 \text{ sq. units}$$

Find the area of the quadrilateral whose vertices are (–9, –2), (–8, –4), (2, 2) and (1, –3)

Solution :

Area of **quadrilateral** $=\frac{1}{2}\left\{\begin{matrix} x_1 & & x_2 & & x_3 & & x_4 & & x_1 \\ & \times & & \times & & \times & & \times & \\ y_1 & & y_2 & & y_3 & & y_4 & & y_1 \end{matrix}\right\}$

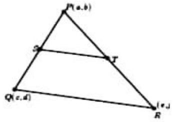
$$=\frac{1}{2}\begin{bmatrix} -9 & -8 & 1 & 2 & -9 \\ -2 & -4 & -3 & 2 & -2 \end{bmatrix}$$

$$=\frac{1}{2}[(36+24+2-4)-(16-4-6-18)]$$

$$=\frac{1}{2}[58-(-12)] = \frac{1}{2}[70] = 35 \text{ sq. units}$$

Prove analytically that the line segment joining the mid-points of two sides of a triangle is parallel to the third side and is equal to half of its length.

Solution :



$S=\left(\frac{a+c}{2},\frac{b+d}{2}\right)$ and $T=\left(\frac{a+e}{2},\frac{b+f}{2}\right)$

slope of $ST=\frac{f-d}{e-c}$ and slope of $QR=\frac{f-d}{e-c}$ $\therefore$ ST is parallel to QR.

$$ST=\sqrt{\left(\frac{a+e}{2}-\frac{a+c}{2}\right)^2+\left(\frac{b+f}{2}-\frac{b+d}{2}\right)^2}$$

$$=\frac{1}{2}\sqrt{(e-c)^2+(f-d)^2} \qquad ST=\frac{1}{2}QR$$

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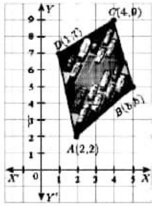
The given diagram shows a plan for constructing a new parking lot at a campus. It is estimated that such construction would cost ₹1300 per square feet. What will be the total cost for making the parking lot?



Solution :

$$\text{Area of parking lot} = \frac{1}{2}\begin{Bmatrix} x_1 & \times & x_2 & \times & x_3 & \times & x_4 & \times & x_1 \\ y_1 & & y_2 & & y_3 & & y_4 & & y_1 \end{Bmatrix}$$

$$= \frac{1}{2}\begin{Bmatrix} 2 & \times & 5 & \times & 4 & \times & 1 & \times & 2 \\ 2 & & 5 & & 9 & & 7 & & 2 \end{Bmatrix}$$

$$= \frac{1}{2}\{(10+45+28+2)-(10+20+9+14)\}$$

$$= \frac{1}{2}\{85-53\} = \frac{1}{2}(32) = 16 \text{ sq. units.}$$

Total cost for constructing the parking lot $= 16 \times 1300 =$ ₹20800

A triangular shaped glass with vertices at A(-5,-4) , B(1,6) and C(7,-4) has to be painted. If one bucket of paint covers 6 square feet, how many buckets of paint will be required to paint the whole glass, if only one coat of paint is applied.

Solution : $\therefore$ Area of triangle $= \frac{1}{2}\begin{Bmatrix} x_1 & \times & x_2 & \times & x_3 & \times & x_1 \\ y_1 & & y_2 & & y_3 & & y_1 \end{Bmatrix}$

$$= \frac{1}{2}\begin{bmatrix} -5 & 1 & 7 & -5 \\ -4 & 6 & -4 & -4 \end{bmatrix}$$

$$= \frac{1}{2}[(-30-4-28)-(-4+42+20)] = \frac{1}{2}[-120] = 60 \text{ sq. units}$$

$$\therefore \text{No. of paint cans needed} = \frac{60}{6} = 10$$

Find the area of a triangle formed by the lines 3x +y −2 = 0 , 5x +2y −3 = 0 and 2x − y −3 = 0.

Solution :

$3x + y - 2 = 0$ .... (1)

$5x + 2y - 3 = 0$ .... (2)

$2x - y - 3 = 0$ .... (3)

$$\begin{aligned} (1)\times 2 &\Rightarrow 6x + 2y = 4 \\ (2) &\Rightarrow \underline{5x + 2y = 3} \\ & \quad x \quad = 1 \end{aligned}$$

Sub. in (1) $3 + y - 2 = 0 \Rightarrow y = -1$

$\therefore$ A (1, −1)

$$\begin{aligned} 3x + y &= 2 \\ \underline{2x - y} &\underline{= 3} \\ 5x \quad &= 5 \end{aligned}$$

$x = 1 \quad \therefore y = -1$

$\therefore$ B is (1, −1)

$$\begin{aligned} (2) &\Rightarrow 5x + 2y = 3 \\ (3)\times 2 &\Rightarrow \underline{4x - 2y = 6} \\ & \quad 9x \quad = 9 \\ & \quad x = 1 \end{aligned}$$

$\therefore (3) \Rightarrow 2 - y - 3 = 0$

$\Rightarrow \quad -y = 1 \quad \therefore \quad y = -1$

$\therefore$ C is (1, −1)

$\therefore$ A (1, − 1), B (1, − 1), C (1, − 1) $\quad\therefore$ All point line on the same line $\quad\therefore$ Area of $\Delta = 0$ sq. units

---

Find the area of the quadrilateral whose vertices are (–9, –2), (–8, –4), (2, 2) and (1, –3)

Solution : Area of quadrilateral

$$= \frac{1}{2}\begin{Bmatrix} x_1 & \times & x_2 & \times & x_3 & \times & x_4 & \times & x_1 \\ y_1 & & y_2 & & y_3 & & y_4 & & y_1 \end{Bmatrix} = \frac{1}{2}\begin{bmatrix} -9 & -8 & 1 & 2 & -9 \\ -2 & -4 & -3 & 2 & -2 \end{bmatrix}$$

$$= \frac{1}{2}[(36+24+2-4)-(16-4-6-18)]$$

$$= \frac{1}{2}[58-(-12)] = \frac{1}{2}[70] = 35 \text{ sq. units}$$

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8. STATISTICS AND PROBABILITY
5 MARKS

Find the mean and variance of the first n natural numbers.

Solution :

$$\text{Mean } \bar{x} = \frac{\sum x_i}{n} = \frac{\cancel{n}(n+1)}{2\times\cancel{n}} = \frac{n+1}{2}$$

$$\text{Variance } \sigma^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2 = \frac{\cancel{n}(n+1)(2n+1)}{6\times\cancel{n}} - \left[\frac{\cancel{n}(n+1)}{2\times\cancel{n}}\right]^2$$

$$= \frac{(n+1)(2n+1)}{6} - \left[\frac{n(n+1)}{2}\right]^2 = \frac{n^2-1}{12}$$

---

48 students were asked to write the total number of hours per week they spent on watching television. With this information find the standard deviation of hours spent for watching television.

x	6	7	8	9	10	11	12
f	3	6	9	13	8	5	4

Solution :

x	f	$d = x - 9$	d^2	$f.d$	$f.d^2$
6	3	–3	9	− 9	27
7	6	–2	4	−12	24
8	9	–1	1	− 9	9
9	13	0	0	0	0
10	8	1	1	8	8
11	5	2	4	10	20
12	4	3	9	12	36
	48			0	124

$$\sigma = \sqrt{\frac{\sum f d^2}{\sum f} - \left(\frac{\sum f d}{\sum f}\right)^2}$$

$$= \sqrt{\frac{124}{48}}$$

$$= 1.6$$

The marks scored by the students in a slip test are given below.

x	4	6	8	10	12
f	7	3	5	9	5

Find the standard deviation of their marks.

Solution :

x	f	$d = x - 8$	d^2	$f.d$	$f.d^2$
4	7	− 4	16	−28	112
6	3	− 2	4	− 6	12
8	5	0	0	0	0
10	9	2	4	18	36
12	5	4	16	20	80
	29			4	240

$$\sigma = \sqrt{\frac{\sum f d^2}{\sum f} - \left(\frac{\sum f d}{\sum f}\right)^2}$$

$$= \sqrt{\frac{240}{29} - \left(\frac{4}{29}\right)^2} = 2.87$$

Three unbiased coins are tossed once. Find the probability of getting atmost 2 tails or atleast 2 heads.

Solution : S = {(HHH), (HHT), (HTH), (THH), (HTT), (THT), (TTH), (TTT)} n(S) = 8

Let A - at most 2 tails

A = {(HHT), (HTH), (THH), (HTT), (THT), (TTH), (HHH)} $\quad n(A) = 7 \Rightarrow P(A) = \frac{7}{8}$

Let B - atleast 2 heads

B = {(HHH), (HHT), (HTH), (THH)} $\quad n(B) = 4 \Rightarrow P(B) = \frac{4}{8}$

$\therefore A\cap B$ = {(HHH), (HHT), (HTH), (THH)} $\quad n(A\cap B) = 4 \Rightarrow P(A\cap B) = \frac{4}{8}$

$$\therefore P(A\cup B) = P(A) + P(B) - P(A\cap B) = \frac{7}{8} + \frac{4}{8} - \frac{4}{8} = \frac{7}{8}$$

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A card is drawn from a pack of 52 cards. Find the probability of getting a king or a heart or a red card.

Solution : n (S) = 52

Let A king card, Let B heart card, Let C red card.

$$P(A)=\frac{4}{52},\ P(B)=\frac{13}{52},\ P(C)=\frac{26}{52},\ P(A\cap C)=\frac{2}{52},\ P(A\cap B)=\frac{1}{52},\ P(B\cap C)=\frac{13}{52}$$

$$P(A\cap B\cap C)=\frac{1}{52}$$

$$P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(B\cap C) - P(C\cap A) + P(A\cap B\cap C)$$

$$=\frac{4}{52}+\frac{13}{52}+\frac{26}{52}-\frac{1}{52}-\frac{13}{52}-\frac{2}{52}+\frac{1}{52}=\frac{28}{52}=\frac{7}{13}$$

A bag contains 5 red balls, 6 white balls, 7 green balls, 8 black balls. One ball is drawn at random from the bag. Find the probability that the ball drawn is (i) white (ii) black or red (iii) not white (iv) neither white nor black

Solution : S = {5R, 6W, 7G, 8B}

i) Let A - White ball n(A) = 6 $\Rightarrow P(A)=\frac{6}{26}=\frac{3}{13}$

ii) Let B - Black (or) red n(B) = 5 + 8 = 13 $\Rightarrow P(B)=\frac{13}{26}=\frac{1}{2}$

iii) Let C - not white n(C) = 20 $\Rightarrow P(C)=\frac{20}{26}=\frac{10}{13}$

iv) Let D - Neither white nor black n(D) = 12 $\Rightarrow P(D)=\frac{12}{26}=\frac{6}{13}$

What is the probability of drawing either a king or a queen in a single draw from a well shuffled pack of 52 cards?

Solution : Total number of cards = 52

Let A king card $n(A)=4 \Rightarrow P(A)=\frac{4}{52}$

Let B queen card $n(B)=4 \Rightarrow P(B)=\frac{4}{52}$ $\therefore P(A\cap B)=\frac{0}{52}$

$$P(A\cup B) = P(A) + P(B) - P(A\cap B) = \frac{4}{52}+\frac{4}{52}-\frac{0}{52}=\frac{2}{13}$$

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Marks of the students in a particular subject of a class are given below.

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	-
Number of students	8	12	17	14	9	7	4	-

Solution :

Marks	Mid value (x)	f	d = $x-$A	$d=\frac{x-A}{c}$	fd	fd^2
0-10	5	8	−30	−3	−24	72
10-20	15	12	−20	−2	−24	48
20-30	25	17	−10	−1	−17	17
30-40	35	14	0	0	0	0
40-50	45	9	10	1	9	9
50-60	55	7	20	2	14	28
60-70	65	4	30	3	12	36
		Σf =71			Σfd = −30	Σf d² = 210

Standard deviation $\sigma = c\times\sqrt{\frac{\Sigma fd^2}{\Sigma f}-\left(\frac{\Sigma fd}{\Sigma f}\right)^2}$

$$\sigma = 10\times\sqrt{\frac{210}{71}-\left(\frac{-30}{71}\right)^2}$$

$$=16.67$$

Two unbiased dice are rolled once. Find the probability of getting (i) a doublet (equal numbers on both dice) (ii) the product as a prime number (iii) the sum as a prime number (iv) the sum as 1

Solution : S = {(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)
(3,1),(3,2),(3,3),(3,4),(3,5),(3,6), (4,1),(4,2),(4,3),(4,4),(4,5),(4,6)
(5,1),(5,2),(5,3),(5,4),(5,5),(5,6), (6,1),(6,2),(6,3),(6,4),(6,5),(6,6)}

n(S) = 36

i) Let A a doublet
A = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)} n(A) = 6 $\therefore P(A)=\frac{6}{36}=\frac{1}{6}$

ii) Let B the product as a prime number.
B = {(1, 2), (1, 3), (1, 5), (2, 1), (3, 1), (5, 1)} n(B) = 6 $\therefore P(B)=\frac{6}{36}=\frac{1}{6}$

iii) Let C be the sum of numbers on the dice is prime.
C = {(1, 1), (1, 2), (1, 4), (1,6), (2, 1), (2, 3), (2, 5), (3, 2),
(3, 4), (4, 1), (4, 3), (5, 2), (5, 6), (6, 1), (6, 5)} n(C) = 15 $\therefore P(C)=\frac{15}{36}=\frac{5}{12}$

iv) Let D be the sum of numbers is 1. n(D) = 0 ∴ P(D) = 0

If two dice are rolled, then find the probability of getting the product of face value 6 or the difference of face values 5.

Solution : S = {(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)
(3,1),(3,2),(3,3),(3,4),(3,5),(3,6), (4,1),(4,2),(4,3),(4,4),(4,5),(4,6)
(5,1),(5,2),(5,3),(5,4),(5,5),(5,6), (6,1),(6,2),(6,3),(6,4),(6,5),(6,6)}
∴ n(S) = 36

Let A - Product of face value is 6.
A = {(1, 6), (2, 3), (3, 2), (6, 1)} ⇒ n(A) = 4 ⇒ P(A) = $\frac{4}{36}$

Let B - Difference of face value is 5. B = {(6, 1)} ⇒ n(B) = 1 ⇒ P(B) = $\frac{1}{36}$

A∩B = {(6, 1)} ⇒ n(A∩B) = 1 ⇒ P(A∩B) = $\frac{1}{36}$

$$\therefore P(A\cup B) = P(A) + P(B) - P(A\cap B) = \frac{4}{36}+\frac{1}{36}-\frac{1}{36}=\frac{4}{36}=\frac{1}{9}$$

In a class of 50 students, 28 opted for NCC, 30 opted for NSS and 18 opted both NCC and NSS. One of the students is selected at random. Find the probability that
(i) The student opted for NCC but not NSS. (ii) The student opted for NSS but not NCC.
(iii) The student opted for exactly one of them.

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Solution: Let A and B be NCC and NSS
Total number of students n(S)= 50.

n(A) = 28 , n(B) = 30 , n(A∩B) = 18 $\Rightarrow P(A)=\frac{28}{50}$, $P(B)=\frac{30}{50}$ and $P(A\cap B)=\frac{18}{50}$

(i) Probability of the students opted for NCC but not NSS $P(A\cap\overline{B}) = P(A) - P(A\cap B) = \frac{28}{50}-\frac{18}{50}=\frac{1}{5}$

(ii) Probability of the students opted for NSS but not NCC. $P(A\cap\overline{B}) = P(B) - P(A\cap B) = \frac{30}{50}-\frac{18}{50}=\frac{6}{25}$

(iii) Probability of the students opted for exactly one of them $P(A\cap\overline{B})+P(\overline{A}\cap B)=\frac{1}{5}+\frac{6}{25}=\frac{11}{25}$

From a well-shuffled pack of 52 cards, a card is drawn at random. Find the probability of it being either a red king or a black queen.

Solution : n(S) = 52

Let A - Red King n(A) = 2 ⇒ P(A) = $\frac{2}{52}$

Let B - Black Queen $n(B) = 2 \Rightarrow P(B) = \frac{2}{52}$

$n(A\cap B) = 0 \Rightarrow P(A\cap B) = \frac{0}{52}$

$\therefore P(A\cup B) = P(A) + P(B) - P(A\cap B) = \frac{2}{52} + \frac{2}{52} - \frac{0}{52} = \frac{4}{52} = \frac{1}{13}$

Two dice are rolled. Find the probability that the sum of outcomes is (i) equal to 4 (ii) greater than 10 (iii) less than 13

Solution : S = {(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)
(3,1),(3,2),(3,3),(3,4),(3,5),(3,6), (4,1),(4,2),(4,3),(4,4),(4,5),(4,6)
(5,1),(5,2),(5,3),(5,4),(5,5),(5,6), (6,1),(6,2),(6,3),(6,4),(6,5),(6,6)} n(S) = 36

(i) Let A the sum of outcome values equal to 4.
A = {(1,3),(2,2),(3,1)}; n (A) = 3 . $P(A) = \frac{3}{36} = \frac{1}{12}$

(ii) Let B the sum of outcome values greater than 10.
B = {(5,6),(6,5),(6,6)}; n (B) = 3 $P(B) = \frac{3}{36} = \frac{1}{12}$

(iii) Let C the sum of outcomes less than 13.
n(C) = n(S) = 36 $P(C) = \frac{36}{36} = 1$

From a well shuffled pack of 52 cards, one card is drawn at random. Find the probability of getting (i) red card (ii) heart card (iii) red king (iv) face card (v) number card

Solution : n (S) = 52

(i) Let A red card. $n(A) = 26 \Rightarrow P(A) = \frac{26}{52} = \frac{1}{2}$

(ii) Let B heart card. $n(B) = 13 \Rightarrow P(B) = \frac{13}{52} = \frac{1}{4}$

(iii) Let C red king card. $n(C) = 2 \Rightarrow P(C) = \frac{2}{52} = \frac{1}{26}$

A ZAYAAN ABDUL M.SC.,B.ED.,
JAYANKONDAM, ARIYALUR DT.

(iv) Let D face card.
The face cards are Jack (J), Queen (Q), and King (K). $n(D) = 4 \times 3 = 12 \Rightarrow P(D) = \frac{12}{52} = \frac{3}{13}$

(v) Let E a number card.
The number cards are 2, 3, 4, 5, 6, 7, 8, 9 and 10. $n(E) = 4 \times 9 = 36 \Rightarrow P(E) = \frac{36}{52} = \frac{9}{13}$

Three fair coins are tossed together. Find the probability of getting (i) all heads (ii) atleast one tail (iii) atmost one head (iv) atmost two tails

Solution : When 3 fair coins are tossed,
S = {(HHH), (HHT), (HTH), (HTT), (THH), (THT), (TTH), (TTT)}
∴ n(S) = 8

i) Let A all heads. A = {(HHH)} n(A) = 1 $\therefore P(A) = \frac{1}{8}$

ii) Let B atleast one tail.
B = {(HHT), (HTH), (HTT), (THH), (THT), (TTH), (TTT)} $n(B) = 7 \Rightarrow P(B) = \frac{7}{8}$

iii) Let C at most one head.
C = {(HTT), (THT), (TTH), (TTT)} $n(C) = 4 \Rightarrow P(C) = \frac{4}{8} = \frac{1}{2}$

iv) Let D - atmost 2 tails
D = {(HHH), (HHT), (HTT), (HTH), (THH), (THT), (TTH)} $n(D) = 7 \Rightarrow P(D) = \frac{7}{8}$

The king and queen of diamonds, queen and jack of hearts, jack and king of spades are removed from a deck of 52 playing cards and then well shuffled. Now one card is drawn at random from the remaining cards. Determine the probability that the card is (i) a clavor (ii) a queen of red card (iii) a king of black card

Solution : By the data given, n(S) = 52 − 2 − 2 − 2 = 46

i) Let A a clavor card. $n(A) = 13 \Rightarrow P(A) = \frac{13}{46}$

ii) Let B - queen of red card. $n(B) = 0 \Rightarrow P(B) = 0$ (queen diamond and heart are included in S)

iii) Let C - King of black cards n(C) = 1 (encluding spade king) $\Rightarrow P(C) = \frac{1}{46}$

Two dice are rolled together. Find the probability of getting a doublet or sum of faces as 4.

Solution : S = {(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)
(3,1),(3,2),(3,3),(3,4),(3,5),(3,6), (4,1),(4,2),(4,3),(4,4),(4,5),(4,6)
(5,1),(5,2),(5,3),(5,4),(5,5),(5,6), (6,1),(6,2),(6,3),(6,4),(6,5),(6,6)} n(S) = 36

Let A a doublet
A = {(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)} $n(A) = 6 \Rightarrow P(A) = \frac{6}{36}$

Let B face sum 4.
B = {(1,3),(2,2),(3,1)} $n(B) = 3 \Rightarrow P(B) = \frac{3}{36}$

$A\cap B = \{(2,2)\} \Rightarrow n(A\cap B) = 1 \Rightarrow P(A\cap B) = \frac{1}{36}$

$P(A\cup B) = P(A) + P(B) - P(A\cap B) = \frac{6}{36} + \frac{3}{36} - \frac{1}{36} = \frac{8}{36} = \frac{2}{9}$

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A coin is tossed thrice. Find the probability of getting exactly two heads or atleast one tail or two consecutive heads.

Solution : S = {(HHH), (HHT), (HTH), (THH), (TTH), (THT), (HTT), (TTT)}
n(S) = 8

Let A - exactly 2 heads , A = {(HHT), (HTH), (THH)} $n(A) = 3 \Rightarrow P(A) = \frac{3}{8}$

Let B - atleast one tail , B = {(HHT), (HTH), (THH), (TTH), (THT), (HTT), (TTT)} $n(B) = 7 \Rightarrow P(B) = \frac{7}{8}$

Let C - Consecutively 2 heads , C = {(HHH), (HHT), (THH)} $n(C) = 3 \Rightarrow P(C) = \frac{3}{8}$

$A\cap B = \{(HHT), (HTH), (THH)\}$, $n(A\cap B) = 3 \Rightarrow P(A\cap B) = \frac{3}{8}$

$B\cap C = \{(HHT), (THH)\}$, $n(B\cap C) = 2 \Rightarrow P(B\cap C) = \frac{2}{8}$

$C\cap A = \{(HHT), (THH)\}$,

$n(C\cap A) = 2 \Rightarrow P(C\cap A) = \frac{2}{8}$

$P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(B\cap C) - P(C\cap A) + P(A\cap B\cap C)$

$= \frac{3}{8} + \frac{7}{8} + \frac{3}{8} - \frac{3}{8} - \frac{2}{8} - \frac{2}{8} + \frac{2}{8} = \frac{8}{8} = 1$

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In a town of 8000 people, 1300 are over 50 years and 3000 are females. It is known that 30% of the females are over 50years. What is the probability that a chosen individual from the town is either a female or over 50 years?

Solution : Let A - Female , B - Over 50 years

$n(S) = 8000, n(A) = 3000, n(B) = 1300 \quad n(A \cap B) = \frac{30}{100} \times 3000 = 900$

$\therefore P(A) = \frac{3000}{8000}, P(B) = \frac{1300}{8000}, P(A \cap B) = \frac{900}{8000}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{3000+1300-900}{8000} = \frac{3400}{8000} = \frac{34}{80} = \frac{17}{40}$

Two dice are rolled once. Find the probability of getting an even number on the first die or a total of face sum 8.

Solution : S = {(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)
(3,1),(3,2),(3,3),(3,4),(3,5),(3,6), (4,1),(4,2),(4,3),(4,4),(4,5),(4,6)
(5,1),(5,2),(5,3),(5,4),(5,5),(5,6), (6,1),(6,2),(6,3),(6,4),(6,5),(6,6)}

n(S) = 36

Let A even number on the 1st die.

A = {(2, 1),(2, 2),(2, 3),(2, 4),(2, 5),(2, 6),(4, 1),(4, 2),(4, 3),(4, 4),(4, 5),(4, 6), (6, 1),(6, 2),(6, 3),(6, 4), (6, 5),(6, 6)} $\quad n(A) = 18 \Rightarrow P(A) = \frac{18}{36}$

Let B - Total of face sum as 8.

B = {(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)} $\quad n(B) = 5 \Rightarrow P(B) = \frac{5}{36}$

A∩B = {(2, 6), (4, 4), (6, 2)} $\quad n(A \cap B) = 3 \Rightarrow P(A \cap B) = \frac{3}{36}$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{18}{36} + \frac{5}{36} - \frac{3}{36} = \frac{20}{36} = \frac{5}{9}$

The King, Queen and Jack of the suit spade are removed from a deck of 52 cards. One card is selected from the remaining cards. Find the probability of getting (i) a diamond (ii) a queen (iii) a spade (iv) a heart card bearing the number 5.

Solution : n(S) = 52 – 3 = 49

i) Let A - a diamond card $\quad n(A) = 13 \quad \therefore P(A) = \frac{13}{49}$

ii) Let B - a queen card $\quad n(B) = 3$ (except spade queen out of 4) $\quad \therefore P(B) = \frac{3}{49}$

iii) Let C - a spade card $\quad n(C) = 10 \; (13 - 3 = 10) \quad \therefore P(C) = \frac{10}{49}$

iv) Let D - 5 of heart $\quad n(D) = 1 \quad \therefore P(D) = \frac{1}{49}$

A box contains cards numbered 3, 5, 7, 9, … 35, 37. A card is drawn at random from the box. Find the probability that the drawn card have either multiples of 7 or a prime number.

Solution : S = {3, 5, 7, 9, ... 35, 37} ⇒ n(S) = 18

Let A - multiple of 7. A = {7, 14, 21, 28, 35} $\quad n(A) = 5 \Rightarrow P(A) = \frac{5}{18}$

Let B - a prime number

B = {3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37} $\quad n(B) = 11 \Rightarrow P(B) = \frac{11}{18}$

A∩B = {7} $\quad n(A \cap B) = 1 \Rightarrow P(A \cap B) = \frac{1}{18}$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cup B) = \frac{5}{18} + \frac{11}{18} - \frac{1}{18} = \frac{15}{18} = \frac{5}{6}$

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Find the variance and standard deviation of the wages of 9 workers given below:
₹310, ₹290, ₹320, ₹280, ₹300, ₹290, ₹320, ₹310, ₹280.

Solution :

x	$d = x - 300$	d²
280	–20	400
280	–20	400
290	–10	100
290	–10	100
300	0	0
310	10	100
310	10	100
320	20	400
320	20	400
	Σd – 0	Σd² = 2000

variance $\sigma^2 = \frac{\sum d^2}{n} - \left(\frac{\sum d}{n}\right)^2$

$= \frac{2000}{9} - \left(\frac{0}{9}\right)^2$

$= \frac{2000}{9} = 222.2$

$S.D = \sqrt{222.2} = 14.91$

Find the coefficient of variation of 24, 26, 33, 37, 29, 31.

Solution : Given data is 24, 26, 33, 37, 29, 31. $\bar{x} = \frac{24+26+33+37+29+31}{6} = \frac{180}{6} = 30$

x	d = x – 30	d²
24	– 6	36
26	–4	16
29	– 1	1
31	1	1
33	3	9
37	7	49
	0	112

$\therefore \sigma = \sqrt{\frac{\sum d^2}{n} - \left(\frac{\sum d}{n}\right)^2} = \sqrt{\frac{112}{6} - \left(\frac{0}{6}\right)^2} = 4.31$

$\therefore C.V = \frac{\sigma}{\bar{x}} \times 100 = \frac{4.31}{30} \times 10 = 14.36$

The time taken (in minutes) to complete a homework by 8 students in a day are given by 38, 40, 47, 44, 46, 43, 49, 53. Find the coefficient of variation.

Solution : Given data is 38, 40, 47, 44, 46, 43, 49, 53.

$\bar{x} = \frac{38+40+47+44+46+43+49+53}{8} = \frac{360}{8} = 45$

x	d = x – 45	d²
38	– 7	49
40	– 5	25
43	– 2	4
44	– 1	1
46	1	1
47	2	4
49	4	16
53	8	64
8	0	164

$\therefore \sigma = \sqrt{\frac{\sum d^2}{n} - \left(\frac{\sum d}{n}\right)^2} = \sqrt{\frac{164}{8} - \left(\frac{0}{8}\right)^2} = 4.53$

$\therefore C.V = \frac{\sigma}{\bar{x}} \times 100 = \frac{4.53}{45} \times 100 = 10.07$

The marks scored by 10 students in a class test are 25, 29, 30, 33, 35, 37, 38, 40, 44, 48. Find the standard deviation.

Solution :

x	d = x – 35	d²
25	–10	100
29	6	36
30	–5	25
33	–2	4
35	0	0
37	2	4
38	3	9
40	5	25
44	9	81
48	13	169
n = 10	9	453

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JAYANKONDAM,ARIYALUR DT.

$\therefore \sigma = \sqrt{\frac{\sum d^2}{n} - \left(\frac{\sum d}{n}\right)^2}$

$= \sqrt{\frac{453}{10} - \left(\frac{9}{10}\right)^2}$

$= 6.67$

8 MARKS GRAPH

1. Determine the nature of roots for the following quadratic equation $2x^2 - 2x + 9 = 0$
2. Determine the nature of roots for the following quadratic equation $x^2 - x - 20 = 0$
3. Determine the nature of roots for the following quadratic equation $9x^2 - 24x + 16 = 0$
4. Determine the nature of roots for the following quadratic equation $x^2 - 9x + 20 = 0$
5. Determine the nature of roots for the following quadratic equation $x^2 - 4x + 4 = 0$
6. Determine the nature of roots for the following quadratic equation $x^2 + x + 7 = 0$
7. Determine the nature of roots for the following quadratic equation $x^2 - 9 = 0$
8. Determine the nature of roots for the following quadratic equation $x^2 - 6x + 9 = 0$
9. Determine the nature of roots for the following quadratic equation $(2x-3)(x+2) = 0$

8 MARKS GEOMETRY

1. Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{2}{3}$ of the corresponding sides of the triangle PQR (scale factor $\frac{2}{3}$).
2. Construct a triangle similar to a given triangle LMN with its sides equal to $\frac{4}{5}$ of the corresponding sides of the triangle LMN (scale factor $\frac{4}{5}$).
3. Construct a triangle similar to a given triangle ABC with its sides equal to $\frac{6}{5}$ of the corresponding sides of the triangle ABC (scale factor $\frac{6}{5}$).
4. Construct a triangle similar to a given triangle PQR with its sides equal to $\frac{7}{3}$ of the corresponding sides of the triangle PQR (scale factor $\frac{7}{3}$).
5. Draw a circle of radius 4 cm. At a point L on it draw a tangent to the circle using the alternate segment.
6. Draw a circle of diameter 6 cm from a point P, which is 8 cm away from its centre. Draw the two tangents PA and PB to the circle and measure their lengths.
7. Draw a circle of radius 4.5 cm. Take a point on the circle. Draw the tangent at that point using the alternate segment theorem.
8. Draw the two tangents from a point which is 10 cm away from the centre of a circle of radius 5 cm. Also, measure the lengths of the tangents.
9. Take a point which is 11 cm away from the centre of a circle of radius 4 cm and draw the two tangents to the circle from that point.
10. Draw the two tangents from a point which is 5 cm away from the centre of a circle of diameter 6 cm. Also, measure the lengths of the tangents.
11. Draw a tangent to the circle from the point P having radius 3.6 cm, and centre at O. Point P is at a distance 7.2 cm from the centre.

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1. RELATIONS AND FUNCTION

Let A = {1, 2, 3, 4} and B = N. Let $f : A \to B$ be defined by $f(x) = x^3$ then, (i) find the range of f (ii) identify the type of function

Solution : A = {1, 2, 3, 4}, B = N $f(x) = x^3$

$x = 1 \Rightarrow f(1) = 1$ $x = 3 \Rightarrow f(3) = 27$

$x = 2 \Rightarrow f(2) = 8$ $x = 4 \Rightarrow f(4) = 64$

(i) Range of f = {1, 8, 27, 64} (ii) f is one-one

A function f is defined by $f(x) = 3 - 2x$. Find x such that $f(x^2) = (f(x))^2$.

Solution : $f(x) = 3 - 2x$ and $f(x^2) = (f(x))^2$

$\Rightarrow 3 - 2x^2 = (3 - 2x)^2 \Rightarrow (x-1)^2 = 0$

$\Rightarrow x = 1$ (twice)

The Cartesian product A × A has 9 elements among which (–1, 0) and (0, 1) are found. Find the set A and the remaining elements of A × A.

Solution : n(A × A) = 9 and (–1, 0), (0, 1) ∈ A × A

∴ A = {–1, 0, 1}

The **remaining elements of A × A** = {(–1, –1), (–1, 1), (0, –1), (0, 0), (1, –1), (1, 0), (1, 1)}

Find the domain of the function $f(x) = \sqrt{1+\sqrt{1-\sqrt{1-x^2}}}$.

Solution : If $x > 1$ and $x < -1$, $f(x)$ leads to unreal

∴ The domain of $f(x)$ = {–1, 0, 1}

If the ordered pairs $(x^2 - 3x, y^2 + 4y)$ and $(-2, 5)$ are equal, then find x and y.

Solution : Given $(x^2 - 3x, y^2 + 4y) = (-2, 5)$

$x^2 - 3x = -2$ | $y^2 + 4y = 5$

$x = 2,1$ | $y = -5,1$

Let X = {3, 4, 6, 8}. Determine whether the relation $R = \{(x, f(x)) \mid x \in X, f(x) = x^2 + 1\}$ is a function from X to N ?

Solution : X = {3, 4, 6, 8} $R = \{(x, f(x)) \mid x \in X, f(x) = x^2 + 1\}$

$x = 3 \Rightarrow f(3) = 9 + 1 = 10$ $x = 6 \Rightarrow f(6) = 36 + 1 = 37$

$x = 4 \Rightarrow f(4) = 16 + 1 = 17$ $x = 8 \Rightarrow f(8) = 64 + 1 = 65$

R = {(3, 10), (4, 17), (6, 37), (8, 65)} ∴ The relation R : X → N is a function.

A relation 'f' is defined by $f(x) = x^2 - 2$ where, $x \in \{-2, -1, 0, 3\}$ (i) List the elements of f (ii) If f a function ?

Solution : $f(x) = x^2 - 2$

$x \in \{-2, -1, 0, 3\}$

(i) List the elements of f $f(-2) = (-2)^2 - 2 = 2$; $f(-1) = (-1)^2 - 2 = -1$

$f(0) = (0)^2 - 2 = -2$; $f(3) = (3)^2 - 2 = 7$

$f = \{(-2, 2), (-1, -1), (0, -2), (3, 7)\}$

(ii) Since all the elements has unique image. f is a function.

Let $f(x) = 2x + 5$. If $x \neq 0$ then find $\frac{f(x+2)-f(2)}{x}$.

Solution :

$$f(x) = 2x + 5$$

$$f(x+2) = 2(x+2) + 5 = 2x + 9$$

$$f(2) = 2(2) + 5 = 9$$

$$\therefore \frac{f(x+2) - f(x)}{x} = \frac{2x+9-9}{x} = 2$$

Let A = {9, 10, 11, 12, 13, 14, 15, 16, 17} and let $f : A \to N$ be defined by $f(n)$ = the highest prime factor of $n \in A$. Write f as a set of ordered pairs and find the range of f.

Solution :

f = {(9, 3), (10, 5), (11, 11), (12, 3), (13, 13),(14, 7), (15, 5), (16, 2), (17, 17)}

Range of f = {2, 3, 5, 7, 11, 13, 17}

Let A = {1, 2} and B = {1, 2, 3, 4}, C = {5, 6} and D = {5, 6, 7, 8}. Verify whether A × C is a subset of B × D =?

Solution : A = {1, 2}, B = {1, 2, 3, 4}, C = {5, 6}, D = {5, 6, 7, 8}

A × C = {(1, 5), (1, 6), (2, 5), (2, 6)}

B × D = {**(1, 5), (1, 6)**, (1, 7), (1, 8), **(2, 5), (2, 6)**, (2,7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8).}

∴ A × C is a subset of B × D.

If $f : R \to R$ and $g : R \to R$ are defined by $f(x) = x^5$ and $g(x) = x^4$ then check if f, g are one-one and $f \circ g$ is one-one ?

Solution : Let A be the domain. B be the co-domain.

For every element A, there is a unique image in B.

Since f is an odd function

∴ f is 1 – 1.

$g(x)$ is an even function.

∴ Two elements of domain will have image in co-domain. ∴ g is not 1 – 1.

Let $f(x) = x^2 - 1$. Find $f \circ f$

Solution : $(f \circ f) = (x^2 - 1)(x^2 - 1) = (x^2 - 1)^2 - 1 = x^4 - 2x^2 + 1 - 1 = x^4 - 2x^2$

Let A = {3, 4, 7, 8} and B = {1, 7, 10}. R_3 = {(3, 7), (4, 10), (7, 7), (7, 8), (8, 11),(8, 7), (8, 10)} are relations from A to B ?

Solution : A×B = {(3, 1), (3, 7), (3, 10), (4, 1), (4, 7),(4,10), (7,1), (7,7), (7,10), (8, 1), (8, 7), (8, 10)}

R_3 = {(3, 7), (4, 10), (7, 7), (7, 8), (8, 11),(8, 7), (8, 10)}

(7, 8) ∈ R_3, but (7, 8) ∉ A × B. So R_3 is not a relation from A to B.

If $f(x) = x^2 - 1$, **find** $f \circ f \circ f$

Solution :

$(f \circ f) = ((x^2 - 1)^2 - 1)$

$(f \circ f \circ f)(x) = (x^4 - 2x^2)^2 - 1$

$f : R \to R$ defined by $f(x) = 2x + 1$ whether the function is bijective or not. Justify your answer.

Solution : $f : R \to R$ defined by $f(x) = 2x + 1$

Let $f(x_1) = f(x_2)$

$\Rightarrow 2x_1 + 1 = 2x_2 + 1 \Rightarrow 2x_1 = 2x_2 \Rightarrow x_1 = x_2 \quad \therefore \quad f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

∴ f is 1 – 1 function.

$y = 2x + 1 \Rightarrow \therefore 2x = y - 1 \Rightarrow x = \frac{y-1}{2} \quad \therefore f(x) = 2\left(\frac{y-1}{2}\right) + 1 = y$ ∴ f is onto.

∴ f is one-one and onto ⇒ f is bijective.

Find k if $f \circ f(k) = 5$ where $f(k) = 2k - 1$.

Solution : $f \circ f(k) = (2k - 1)(2k - 1) = 2(2k - 1) - 1 = 4k - 3$.

$f \circ f(k) = 5 \Rightarrow 4k - 3 = 5 \Rightarrow 4k = 5 + 3 \Rightarrow 4k = 8 \Rightarrow k = 2.$

Represent the function $f(x) = \sqrt{2x^2 - 5x + 3}$ as a composition of two functions.

Solution : $f_2(x) = 2x^2 - 5x + 3$ and $f_1(x) = \sqrt{x}$

$f(x) = \sqrt{2x^2 - 5x + 3} = \sqrt{f_2(x)} = f_1 f_2(x)$

If A × B = {(3, 2), (3, 4), (5, 2), (5, 4)} then find A and B.

Solution : A × B = {(3, 2), (3, 4), (5, 2), (5, 4)}

A = {3, 5} and B = {2, 4}.

If **A = B = {p, q} find A × B, A × A and B × A**

Solution: A × B = A × A = B × A = {p, q} × {p, q} = {(p, p), (p, q), (q, p), (q, q)}

If **A = {m, n} and B = ϕ** then find **A × B, A × A and B × A**

Solution: A × B = B × A = ϕ **and**

A × A = {(m, m), (m, n), (n, m), (n, n)}

Let A = {1, 2, 3} and B = {x | x is a prime number less than 10}. Find A × B and B × A.

Solution : A = {1, 2, 3}, B = {x | x is a prime number less than 10} = {2, 3, 5, 7}

A × B = {(1, 2), (1, 3), (1, 5), (1, 7), (2, 2),(2, 3), (2, 5), (2, 7), (3, 2), (3, 3), (3, 5), (3, 7)}

B × A = {(2, 1), (2, 2), (2, 3), (3, 1), (3, 2),(3, 3), (5, 1), (5, 2), (5, 3), (7, 1), (7, 2), (7, 3)}

If B × A = {(– 2, 3), (– 2, 4), (0, 3), (0, 4),(3, 3), (3, 4)} find A and B.

Solution : B × A = {(–2, 3), (– 2, 4), (0, 3),(0, 4), (3, 3), (3, 4)}

∴ B = {– 2, 0, 3}, A = {3, 4}

If A × B = {(3, 2), (3, 4), (5, 2), (5, 4)} then find A and B.

Solution : A × B = {(3, 2), (3, 4), (5, 2), (5, 4)}

A = {3, 5} and B = {2, 4}.

If **A = {2, – 2, 3} and B = {1, – 4}** then find **A × B and B × A**

Solution: Given A = {2, – 2, 3}, B = {1, – 4}.

A × B = **{2, – 2, 3}** × **{1, – 4}** = {(2, 1), (2, – 4), (– 2, 1), (–2, – 4), (3, 1), (3, – 4)}

B × A = **{1, – 4}** × **{2, – 2, 3}** = {(1, 2), (1, – 2), (1, 3), (– 4, 2), (– 4, – 2), (– 4, 3)}

If **A = B = {p, q} find A × B, A × A and B × A**

Solution: A × B = A × A = B × A = {p, q} × {p, q} = {(p, p), (p, q), (q, p), (q, q)}

If **A = {m, n} and B = ϕ** then find **A × B, A × A and B × A**

Solution: A × B = B × A = ϕ **and** A × A = {(m, m), (m, n), (n, m), (n, n)}

Let A = {1, 2, 3} and B = {x | x is a prime number less than 10}. Find A × B and B × A.

Solution : A = {1, 2, 3}, B = {x | x is a prime number less than 10} = {2, 3, 5, 7}

A × B = {(1, 2), (1, 3), (1, 5), (1, 7), (2, 2),(2, 3), (2, 5), (2, 7), (3, 2), (3, 3), (3, 5), (3, 7)}

B × A = {(2, 1), (2, 2), (2, 3), (3, 1), (3, 2),(3, 3), (5, 1), (5, 2), (5, 3), (7, 1), (7, 2), (7, 3)}

If B × A = {(– 2, 3), (– 2, 4), (0, 3), (0, 4),(3, 3), (3, 4)} find A and B.

Solution : B × A = {(–2, 3), (– 2, 4), (0, 3),(0, 4), (3, 3), (3, 4)}

∴ B = {– 2, 0, 3}, A = {3, 4}

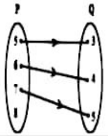
Define - Cartesian Product.

If A and B are two non-empty sets, then the set of all ordered pairs (a, b) such that $a \in A, b \in B$ is called the Cartesian Product of A and B, and is denoted by A × B. Thus, $A \times B = \{(a, b) \mid a \in A, b \in B\}$

Define - Relation

A relation f between two non-empty sets X and Y is called a function from X to Y if, for each $x \in X$ there exists only one $y \in Y$ such that $(x, y) \in f$. That is, $f = \{(x, y) \mid \text{for all } x \in X, y \in Y\}$.

The arrow diagram shows a relationship between the sets P and Q. Write the relation in (i) Set builder form (ii) Roster form (iii) What is the domain and range of R.



Solution : (i) Set builder form $R = \{(x, y) \mid y = x - 2, x \in P, y \in Q\}$

(ii) Roster form $R = \{(5, 3), (6, 4), (7, 5)\}$

(iii) Domain $= \{5, 6, 7\}$ and Range $= \{3, 4, 5\}$

Let A = {1, 2, 3, 4 ...., 45} and R be the relation defined as "is square of" on A. Write R as a subset of A × A. Also, find the domain and range of R.

Solution : $A = \{1, 2, 3, 4, \ldots\ldots 45\}$ **and** R **"is square of" on A.** $\Rightarrow R = \{1, 4, 9, 16, 25, 36\}$

R is a subset of A.

$\therefore$ Domain $= \{1, 2, 3, 4, 5, 6\}$ $\therefore$ Range $= \{1, 4, 9, 16, 25, 36\}$

Let A = {1, 2, 3, 7} and B = {3, 0, – 1.7}, R_4 = {(7, –1), (0, 3), (3, 3), (0, 7)} are relation from A to B ?

Solution : $\therefore A \times B = \{(1, 3), (1, 0), (1, -1), (1, 7), (2, 3), (2, 0), (2, -1), (2, 7), (3, 3), (3, 0), (3, -1), (3, 7), (7, 3), (7, 0), (7, -1), (7, 7)\}$

$R_4 = \{(7, -1), (0, 3), (3, 3), (0, 7)\}$ **and** $(0, 3), (0, 7) \in R_4$ but not in $A \times B$.

$\therefore R_4$ is not a relation.

2 .NUMBERS AND SEQUENCES

Find the 12th term from the last term of the A.P – 2, – 4, – 6, ... –100.

Solution : Given A.P is – 2, – 4, – 6, ... – 100 $a = -100, d = 2$

$t_{12} = a + 11d = -100 + 11(2) = -100 + 22 = -78$

Find the 8th term of the G.P. 9, 3, 1, ...

Solution : First term $a = 9$, common ratio $r = \frac{t_2}{t_1} = \frac{3}{9} = \frac{1}{3}$

$$t_8 = 9 \times \left(\frac{1}{3}\right)^{8-1} = 9 \times \left(\frac{1}{3}\right)^{7} = \frac{1}{243}$$

Write the Fundamental theorem of arithmetic

Every natural number except 1 can be factorized as a product of primes and this factorization is unique except for the order in which the prime factors are written.

In a G.P. 729, 243, 81,... find t_7.

Solution : 729, 243, 21, ... $a = 729$, $r = {}^{8}\!/_{243} = {}^{1}\!/_{3}$

$\therefore t_n = a \, . \, r^{n-1}$

$$\Rightarrow t_7 = a \, . \, r^6 = 729 \times \left({}^{1}\!/_{3}\right)^6 = 729 \times \left({}^{1}\!/_{729}\right) = 1$$

Find the sum of all odd positive integers less than 450.

Solution :

$$1 + 3 + 5 + 7 + \ldots\ldots + 449 = \left[\frac{(l+1)}{2}\right]^2 = \left[\frac{449+1}{2}\right]^2 = \left[\frac{450}{2}\right]^2 = [225]^2 = 50,625$$

3 . ALGEBRA

Define – Matrix.

A matrix is a rectangular array of elements. The horizontal arrangements are called rows and vertical arrangements are called columns. **Example:**

$$A = \begin{pmatrix} 23 & 18 \\ 47 & 36 \\ 15 & 16 \end{pmatrix}$$

Write the three conditions of nature of roots.

Solution :

Values of Discriminant $\Delta = b^2 - 4ac$

Real and Unequal roots	$\Delta > 0$
Real and Equal roots	$\Delta = 0$
No Real root	$\Delta < 0$

If $\begin{pmatrix} 5 & 4 & 3 \\ 1 & -7 & 9 \\ 3 & 8 & 2 \end{pmatrix}$ **then find the transpose of A.** ***Solution :*** $A = \begin{pmatrix} 5 & 4 & 3 \\ 1 & -7 & 9 \\ 3 & 8 & 2 \end{pmatrix}$ $\therefore A^T = \begin{pmatrix} 5 & 1 & 3 \\ 4 & -7 & 8 \\ 3 & 9 & 2 \end{pmatrix}$

If a matrix has 18 elements, what are the possible orders it can have? What if it has 6 elements?

Solution : Given, a matrix has 18 elements . The possible orders 18 × 1, 1 × 18, 9 × 2, 2 × 9, 6 × 3, 3 × 6 The matrix has 6 elements . The order are 1 × 6, 6 × 1, 3 × 2, 2 × 3

If A = $\begin{pmatrix} \sqrt{7} & -3 \\ -\sqrt{5} & 2 \\ \sqrt{3} & -5 \end{pmatrix}$ **then find the transpose of-A.**

Solution : $-A = \begin{pmatrix} -\sqrt{7} & 3 \\ \sqrt{5} & -2 \\ -\sqrt{3} & 5 \end{pmatrix}$ $\therefore$ Transpose of $-A = \begin{pmatrix} -\sqrt{7} & \sqrt{5} & -\sqrt{3} \\ 3 & -2 & 5 \end{pmatrix}$

Define – diagonal matrix.

A square matrix, all of whose elements, except those in the leading diagonal are zero is called a **diagonal matrix.** **Example:**

$$A = \begin{pmatrix} 5 & 0 & 0 \\ 0 & -7 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

Define – scalar matrix.

A diagonal matrix in which all the leading diagonal elements are equal is called a **scalar matrix.**

Example:

$$A = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

Simplify $\frac{x^3}{x-y} + \frac{y^3}{y-x}$

Solution :

$$\frac{x^3}{x-y} + \frac{y^3}{y-x} = \frac{x^3}{x-y} - \frac{y^3}{x-y}$$

$$= \frac{x^3 - y^3}{x-y} = \frac{\cancel{(x-y)}(x^2 + xy + y^2)}{\cancel{x-y}} = x^2 + xy + y^2$$

Subtract $\frac{1}{x^2+2}$ **from** $\frac{2x^3+x^2+3}{(x^2+2)^2}$

Solution :

$$\frac{2x^3+x^2+3}{(x^2+2)^2}-\frac{1}{x^2+2}=\frac{(2x^3+x^2+3)-(x^2+2)}{(x^2+2)^2}=\frac{2x^3+x^2+3-x^2-2}{(x^2+2)^2}=\frac{2x^3+1}{(x^2+2)^2}$$

Find the square root of $16x^2+9y^2-24xy+24x-18y+9$

Solution : $\sqrt{16x^2+9y^2-24xy+24x-18y+9}=\sqrt{(4x-3y+3)^2}=|4x-3y+3|$

Solve $x^4-13x^2+42=0$

Solution :

$$(x^2)^2-13x^2+42=0$$

$$(x^2-7)(x^2-6)=0 \quad \Rightarrow x=\pm\sqrt{7} \text{ or } x=\pm\sqrt{6}$$

If the difference between a number and its reciprocal is $\frac{24}{5}$, find the number.

Solution :

Let x be the required number $\frac{1}{x}$ be its reciprocal Given $x-\frac{1}{x}=\frac{24}{5}$

$\Rightarrow \frac{x^2-1}{x}=\frac{24}{5} \Rightarrow 5x^2-24x-5=0 \Rightarrow$ ∴ The required numbers are $5, -\frac{1}{5}$

Find the excluded values of $\frac{x}{x^2+1}$ expressions

Solution : $x^2+1\neq 0$ for any x. Therefore, no real excluded values

---

Reduce to lowest form. $\frac{x^2-1}{x^2+x}$ ***Solution :*** $\frac{x^2-1}{x^2+x}=\frac{\cancel{(x+1)}(x-1)}{x\cancel{(x+1)}}=\frac{x-1}{x}$

Simplify $\frac{x^3-y^3}{3x^2+9xy+6y^2}\times\frac{x^2+2xy+y^2}{x^2-y^2}$

Solution :

$$\frac{x^3-y^3}{3x^2+9xy+6y^2}\times\frac{x^2+2xy+y^2}{x^2-y^2}=\frac{\cancel{(x-y)}(x^2+xy+y^2)}{3(x^2+3xy+2y^2)}\times\frac{\cancel{(x+y)}\,\cancel{(x+y)}}{\cancel{(x+y)}\,\cancel{(x-y)}}$$

$$=\frac{(x^2+xy+y^2)\cancel{(x+y)}}{3(x+2y)\cancel{(x+y)}}$$

$$=\frac{x^2+xy+y^2}{3(x+2y)}$$

Simplify $\frac{x^2-16}{x+4}+\frac{x-4}{x+4}$

Solution : $\frac{x^2-16}{x+4}+\frac{x-4}{x+4}=\frac{\cancel{(x+4)}\cancel{(x-4)}}{\cancel{(x+4)}}\times\left(\frac{x+4}{\cancel{x-4}}\right)=x+4$

Reduce to lowest form $\frac{x^{3a}-8}{x^2a+2xa+4}$

Solution : $\frac{x^{3a}-8}{x^2a+2xa+4}=\frac{(x^a-2)\cancel{(x^{2a}+2x^a+4)}}{\cancel{x^{2a}+2x^a+4}}=x^a-2$

Construct a 3 × 3 matrix whose elements are $a_{ij}=i^2j^2$

Solution : The general 3 × 3 matrix is given by

$A=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ whose elements are $a_{ij}=i^2j^2$

$a_{11}=1$; $a_{12}=4$; $a_{13}=9$

$a_{21}=4$; $a_{22}=16$; $a_{23}=36$

$a_{31}=9$; $a_{32}=36$; $a_{33}=81$

$$A=\begin{pmatrix} 1 & 4 & 9 \\ 4 & 16 & 36 \\ 9 & 36 & 81 \end{pmatrix}$$

Find the values of x, y and z from the fol·lowing equations $\begin{pmatrix} x+y+z \\ x+z \\ y+z \end{pmatrix}=\begin{pmatrix} 9 \\ 5 \\ 7 \end{pmatrix}$

Solution :

$$\begin{pmatrix} x+y+z \\ x+z \\ y+z \end{pmatrix}=\begin{pmatrix} 9 \\ 5 \\ 7 \end{pmatrix}$$

$\Rightarrow x+y+z=0$ | $x+z=5$ | $y+z=7$

$\Rightarrow 5+y=9$ | $\Rightarrow x+3=5$ | $\Rightarrow 4+z=7$

$\Rightarrow y=4$ | $\Rightarrow x=2$ | $\Rightarrow z=3$

If $A=\begin{pmatrix} 5 & 4 & -2 \\ \frac{1}{2} & \frac{3}{4} & \sqrt{2} \\ 1 & 9 & 4 \end{pmatrix}$ $B=\begin{pmatrix} -7 & 4 & -3 \\ \frac{1}{4} & \frac{7}{2} & 3 \\ 5 & -6 & 9 \end{pmatrix}$ then Find $4A-3B$

Solution :

$$4A-3B=4\begin{pmatrix} 5 & 4 & -2 \\ \frac{1}{2} & \frac{3}{4} & \sqrt{2} \\ 1 & 9 & 4 \end{pmatrix}-3\begin{pmatrix} -7 & 4 & -3 \\ \frac{1}{4} & \frac{7}{2} & 3 \\ 5 & -6 & 9 \end{pmatrix}=\begin{pmatrix} 20 & 6 & -8 \\ 2 & 3 & 4\sqrt{2} \\ 4 & 36 & 16 \end{pmatrix}+\begin{pmatrix} 21 & -12 & 9 \\ -\frac{3}{4} & -\frac{21}{2} & -9 \\ -15 & 18 & -27 \end{pmatrix}=\begin{pmatrix} 41 & 4 & 1 \\ \frac{5}{4} & -\frac{15}{2} & 4\sqrt{2}-9 \\ -11 & 54 & -11 \end{pmatrix}$$

If $A=\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$, $B=\begin{pmatrix} 1 & 7 & 0 \\ 1 & 3 & 1 \\ 2 & 4 & 0 \end{pmatrix}$, find A + B

Solution :

$$A+B=\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}+\begin{pmatrix} 1 & 7 & 0 \\ 1 & 3 & 1 \\ 2 & 4 & 0 \end{pmatrix}=\begin{pmatrix} 1+1 & 2+7 & 3+0 \\ 4+1 & 5+3 & 6+1 \\ 7+2 & 8+4 & 9+0 \end{pmatrix}=\begin{pmatrix} 2 & 9 & 3 \\ 5 & 8 & 7 \\ 9 & 12 & 9 \end{pmatrix}$$

If $A=\begin{pmatrix} 0 & 4 & 9 \\ 8 & 3 & 7 \end{pmatrix}, B=\begin{pmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{pmatrix}$ **find the value of 3A − 9B**

Solution :

$$3A-9B=3\begin{pmatrix} 0 & 4 & 9 \\ 8 & 3 & 7 \end{pmatrix}-9\begin{pmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{pmatrix}$$

$$=\begin{pmatrix} 0 & 12 & 27 \\ 24 & 9 & 21 \end{pmatrix}-\begin{pmatrix} 63 & 27 & 72 \\ 9 & 36 & 81 \end{pmatrix}=\begin{pmatrix} 0 & 12 & 27 \\ 24 & 9 & 21 \end{pmatrix}+\begin{pmatrix} -63 & -27 & -72 \\ -9 & -36 & -81 \end{pmatrix}$$

$$=\begin{pmatrix} -63 & -65 & -45 \\ 15 & -27 & -60 \end{pmatrix}$$

If $A=\begin{pmatrix}\cos\theta & 0\\ 0 & \cos\theta\end{pmatrix}, B=\begin{pmatrix}\sin\theta & 0\\ 0 & \sin\theta\end{pmatrix}$ **then show that** $A^2+B^2=I$.

Solution : $A=\begin{pmatrix}\cos\theta & 0\\ 0 & \cos\theta\end{pmatrix}, B=\begin{pmatrix}\sin\theta & 0\\ 0 & \sin\theta\end{pmatrix}$

$$A^2=\begin{pmatrix}\cos\theta & 0\\ 0 & \cos\theta\end{pmatrix}\begin{pmatrix}\cos\theta & 0\\ 0 & \cos\theta\end{pmatrix}=\begin{pmatrix}\cos^2\theta & 0\\ 0 & \cos^2\theta\end{pmatrix}$$

$$B^2=\begin{pmatrix}\sin\theta & 0\\ 0 & \sin\theta\end{pmatrix}\begin{pmatrix}\sin\theta & 0\\ 0 & \sin\theta\end{pmatrix}=\begin{pmatrix}\sin^2\theta & 0\\ 0 & \sin^2\theta\end{pmatrix}$$

$$\therefore A^2+B^2=\begin{pmatrix}\cos^2\theta+\sin^2\theta & 0\\ 0 & \cos^2\theta+\sin^2\theta\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}=I.$$

If $A=\begin{pmatrix}1 & 3 & -2\\ 5 & -4 & 6\\ -3 & 2 & 9\end{pmatrix}$ $B=\begin{pmatrix}1 & 8\\ 3 & 4\\ 9 & 6\end{pmatrix}$ find A+B. ***Solution :*** A is of order 3 × 3 B is of order 3 × 2

It is not possible to add A and B because different orders.

Given $A=\begin{pmatrix}p & 0\\ 0 & 2\end{pmatrix}, B=\begin{pmatrix}0 & -q\\ 1 & 0\end{pmatrix}, C=\begin{pmatrix}2 & -2\\ 2 & 2\end{pmatrix}$ **and if** $BA=C^2$**, find p and q.**

Solution :

Given $BA=C^2 \Rightarrow \begin{pmatrix}0 & -q\\ 1 & 0\end{pmatrix}\begin{pmatrix}p & 0\\ 0 & 2\end{pmatrix}=\begin{pmatrix}2 & -2\\ 2 & 2\end{pmatrix}\begin{pmatrix}2 & -2\\ 2 & 2\end{pmatrix}\Rightarrow\begin{pmatrix}0 & -2q\\ p & 0\end{pmatrix}=\begin{pmatrix}0 & -8\\ 8 & 0\end{pmatrix}$

$\therefore p=8,\quad -2q=-8,$

$q=4$

If $A=\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}$ **prove that** $AA^T=I$

Solution : $AA^T=\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}=\begin{pmatrix}\cos^2\theta+\sin^2\theta & -\cos\theta\sin\theta+\sin\theta\cos\theta\\ -\sin\theta\cos\theta+\cos\theta\sin\theta & \sin^2\theta+\cos^2\theta\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}=I$

Hence proved.

If $A=\begin{pmatrix}5 & 2 & 2\\ -\sqrt{17} & 0.7 & \frac{5}{2}\\ 8 & 3 & 1\end{pmatrix}$ **then verify** $(A^T)^T=A$

Solution :

$$A^T=\begin{pmatrix}5 & -\sqrt{17} & 8\\ 2 & 0.7 & 3\\ 2 & 5/2 & 1\end{pmatrix}$$

$$(A^T)^T=\begin{pmatrix}5 & 2 & 2\\ -\sqrt{17} & 0.7 & 5/2\\ 8 & 3 & 1\end{pmatrix}=A$$

Construct a 3 × 3 matrix whose elements are given by $a_{ij}=|i-2j|$

Solution :

Given $a_{ij}=|i-2j|$, 3 × 3 $\quad A=\begin{pmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{pmatrix}$

$$\therefore A=\begin{pmatrix}1 & 3 & 5\\ 0 & 2 & 4\\ 1 & 1 & 3\end{pmatrix}$$

Find the square root $\dfrac{7x^2+2\sqrt{14}x+2}{x^2-\frac{1}{2}x+\frac{1}{16}}$

Solution : $\sqrt{\dfrac{7x^2+2\sqrt{14}x+2}{x^2-\frac{1}{2}x+\frac{1}{16}}}=\sqrt{\dfrac{(\sqrt{7}x+\sqrt{2})^2}{\left(x-\frac{1}{4}\right)^2}}=4\left|\dfrac{\sqrt{7}x+\sqrt{2}}{(4x-1)}\right|$

Find the square root of $4x^2+20x+25$

Solution :

$\sqrt{4x^2+20x+25}=\sqrt{(2x+5)^2}=|2x+5|$

Find the square root of $9x^2-24xy+30xz-40yz+25z^2+16y^2$

Solution : $\sqrt{9x^2-24xy+30xz-40yz+25z^2+16y^2}=\sqrt{(3x-4y+5z)^2}=|3x-4y+5z|$

Find the square root of $1+\dfrac{1}{x^6}+\dfrac{2}{x^3}$

Solution : $\sqrt{1+1/x^6+2/x^3}=\sqrt{\left(1+1/x^3\right)^2}=\left|1+1/x^3\right|$

Find the square root of $16x^4+8x^2+1$ **by division method**

Solution : $\sqrt{16x^4+8x^2+1}=\sqrt{(4x^2+1)^2}=|4x^2+1|$

Verify that $A^2=I$ **when** $A=\begin{pmatrix}5 & -4\\ 6 & -5\end{pmatrix}$

Solution : $A=\begin{pmatrix}5 & -4\\ 6 & -5\end{pmatrix}\quad A^2=AA=\begin{pmatrix}5 & -4\\ 6 & -5\end{pmatrix}\begin{pmatrix}5 & -4\\ 6 & -5\end{pmatrix}=\begin{pmatrix}25-24 & -20+20\\ 30-30 & -24+25\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}=I$

If $\cos\theta\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}+\sin\theta\begin{pmatrix}x & -\cos\theta\\ \cos\theta & x\end{pmatrix}=I_2$ **find x.**

Solution :

$\cos\theta\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}+\sin\theta\begin{pmatrix}x & -\cos\theta\\ \cos\theta & x\end{pmatrix}=I_2\Rightarrow\begin{pmatrix}\cos^2\theta+x\sin\theta & 0\\ 0 & \cos^2\theta+x\sin\theta\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

$\Rightarrow\quad \cos^2\theta+x\sin\theta=1\ \Rightarrow\ x=\sin\theta$

Show that the matrices $A=\begin{pmatrix}1 & 2\\ 3 & 1\end{pmatrix}, B=\begin{pmatrix}1 & -2\\ -3 & 1\end{pmatrix}$ **satisfy commutative property** $AB=BA$

Solution :

$$AB=\begin{pmatrix}1 & 2\\ 3 & 1\end{pmatrix}\begin{pmatrix}1 & -2\\ -3 & 1\end{pmatrix}=\begin{pmatrix}\begin{smallmatrix}1 & 2\end{smallmatrix}\begin{smallmatrix}1\\ -3\end{smallmatrix} & \begin{smallmatrix}1 & 2\end{smallmatrix}\begin{smallmatrix}-2\\ 1\end{smallmatrix}\\ \begin{smallmatrix}3 & 1\end{smallmatrix}\begin{smallmatrix}1\\ -3\end{smallmatrix} & \begin{smallmatrix}3 & 1\end{smallmatrix}\begin{smallmatrix}-2\\ 1\end{smallmatrix}\end{pmatrix}=\begin{pmatrix}1-6 & -2+2\\ 3-3 & -6+1\end{pmatrix}=\begin{pmatrix}-5 & 0\\ 0 & -5\end{pmatrix}$$

$$BA=\begin{pmatrix}1 & -2\\ -3 & 1\end{pmatrix}\begin{pmatrix}1 & 2\\ 3 & 1\end{pmatrix}=\begin{pmatrix}\begin{smallmatrix}1 & -2\end{smallmatrix}\begin{smallmatrix}1\\ 3\end{smallmatrix} & \begin{smallmatrix}1 & -2\end{smallmatrix}\begin{smallmatrix}2\\ 1\end{smallmatrix}\\ \begin{smallmatrix}-3 & 1\end{smallmatrix}\begin{smallmatrix}1\\ 3\end{smallmatrix} & \begin{smallmatrix}-3 & 1\end{smallmatrix}\begin{smallmatrix}2\\ 1\end{smallmatrix}\end{pmatrix}=\begin{pmatrix}1-6 & 2-2\\ -3+3 & -6+1\end{pmatrix}=\begin{pmatrix}-5 & 0\\ 0 & -5\end{pmatrix}$$

$\therefore AB=BA$

$\therefore$ **commutative property** is true.

Find the values of x, y, z if $\begin{pmatrix} x-3 & 3x-z \\ x+y+7 & x+y+z \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 6 \end{pmatrix}$

Solution : $\Rightarrow x-3=1 \;\; \therefore x=4$ | $3x-z=0$, $12-z=0 \Rightarrow z=12$ | $x+y+7=1 \Rightarrow x+y=-6 \Rightarrow 4+y=-6 \Rightarrow y=-10$

If $A=\begin{pmatrix} 0 & 4 & 9 \\ 8 & 3 & 7 \end{pmatrix}, B=\begin{pmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{pmatrix}$ **find the value of** $B-5A$

Solution :

$$B-5A=\begin{pmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{pmatrix} - 5\begin{pmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{pmatrix} = \begin{pmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{pmatrix} - \begin{pmatrix} 0 & 20 & 45 \\ 40 & 15 & 35 \end{pmatrix} = \begin{pmatrix} 7 & 3 & 8 \\ 1 & 4 & 9 \end{pmatrix} + \begin{pmatrix} 0 & -20 & -45 \\ -40 & -15 & -35 \end{pmatrix}$$

$$= \begin{pmatrix} 7 & -17 & -37 \\ -39 & -11 & -26 \end{pmatrix}$$

If a matrix has 16 elements, what are the possible orders it can have?

Solution : The matrix has 16 elements . Hence possible orders are 1 × 16, 16 × 1, 4 × 4, 2 × 8, 8 × 2.

If $A=\begin{pmatrix} 1 & 2 & 0 \\ 3 & 1 & 5 \end{pmatrix}, B=\begin{pmatrix} 8 & 3 & 1 \\ 2 & 4 & 1 \\ 5 & 3 & 1 \end{pmatrix}$ find AB.

Solution :

$$AB=\begin{pmatrix} 1 & 2 & 0 \\ 3 & 1 & 5 \end{pmatrix} \times \begin{pmatrix} 8 & 3 & 1 \\ 2 & 4 & 1 \\ 5 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 1\,2\,0 \begin{smallmatrix} 8 \\ 2 \\ 5 \end{smallmatrix} & 1\,2\,0 \begin{smallmatrix} 3 \\ 4 \\ 3 \end{smallmatrix} & 1\,2\,0 \begin{smallmatrix} 1 \\ 1 \\ 1 \end{smallmatrix} \\ 3\,1\,5 \begin{smallmatrix} 8 \\ 2 \\ 5 \end{smallmatrix} & 3\,1\,5 \begin{smallmatrix} 3 \\ 4 \\ 3 \end{smallmatrix} & 3\,1\,5 \begin{smallmatrix} 1 \\ 1 \\ 1 \end{smallmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 8+4+0 & 3+8+0 & 1+2+0 \\ 24+2+25 & 9+4+15 & 3+1+5 \end{pmatrix} = \begin{pmatrix} 12 & 11 & 3 \\ 51 & 28 & 9 \end{pmatrix}$$

If $A=\begin{pmatrix} 2 & -2\sqrt{2} \\ \sqrt{2} & 2 \end{pmatrix}$ and $B=\begin{pmatrix} 2 & 2\sqrt{2} \\ -\sqrt{2} & 2 \end{pmatrix}$ Show that A and B satisfy commutative property with respect to matrix multiplication.

Solution :

$$AB=\begin{pmatrix} 2 & -2\sqrt{2} \\ \sqrt{2} & 2 \end{pmatrix} \times \begin{pmatrix} 2 & 2\sqrt{2} \\ -\sqrt{2} & 2 \end{pmatrix} = \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix}$$

$$BA=\begin{pmatrix} 2 & 2\sqrt{2} \\ -\sqrt{2} & 2 \end{pmatrix} \times \begin{pmatrix} 2 & -2\sqrt{2} \\ \sqrt{2} & 2 \end{pmatrix} = \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix}$$ A and B satisfy commutative property

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Construct a 3 × 3 matrix whose elements are given by $a_{ij}=\frac{(i+j)^3}{3}$

Solution :

$a_{11}=8/3$, $a_{12}=27/3=9$, $a_{13}=64/3$ $a_{21}=27/3=9$, $a_{22}=64/3$,

$a_{23}=125/3$ $a_{31}=64/3$, $a_{32}=125/3$, $a_{33}=216/3=72$

$$\therefore A=\begin{pmatrix} 8/3 & 9 & 64/3 \\ 9 & 64/3 & 125/3 \\ 64/3 & 125/3 & 72 \end{pmatrix}$$

If $A=\begin{pmatrix} 1 & 8 & 3 \\ 3 & 5 & 0 \\ 8 & 7 & 6 \end{pmatrix}, B=\begin{pmatrix} 8 & -6 & -4 \\ 2 & 11 & -3 \\ 0 & 1 & 5 \end{pmatrix}, C=\begin{pmatrix} 5 & 3 & 0 \\ -1 & -7 & 2 \\ 1 & 4 & 3 \end{pmatrix}$ compute $3A+2B-C$

Solution :

$$3A+2B-C = 3\begin{pmatrix} 1 & 8 & 3 \\ 3 & 5 & 0 \\ 8 & 7 & 6 \end{pmatrix} + 2\begin{pmatrix} 8 & -6 & -4 \\ 2 & 11 & -3 \\ 0 & 1 & 5 \end{pmatrix} - \begin{pmatrix} 5 & 3 & 0 \\ -1 & -7 & 2 \\ 1 & 4 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & 24 & 9 \\ 9 & 15 & 0 \\ 24 & 21 & 18 \end{pmatrix} + \begin{pmatrix} 16 & -12 & -8 \\ 4 & 22 & -6 \\ 0 & 2 & 10 \end{pmatrix} + \begin{pmatrix} -5 & -3 & 0 \\ 1 & 7 & -2 \\ -1 & -4 & -3 \end{pmatrix} = \begin{pmatrix} 14 & 9 & 1 \\ 14 & 44 & -8 \\ 23 & 19 & 25 \end{pmatrix}$$

If $A=\begin{pmatrix} 7 & 8 & 6 \\ 1 & 3 & 9 \\ -4 & 3 & -1 \end{pmatrix}$ $B=\begin{pmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{pmatrix}$ then Find 2A + B

Solution : $$2A+B=2\begin{pmatrix} 7 & 8 & 6 \\ 1 & 3 & 9 \\ -4 & 3 & -1 \end{pmatrix} + \begin{pmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{pmatrix} = \begin{pmatrix} 14 & 16 & 12 \\ 2 & 6 & 18 \\ -8 & 6 & -2 \end{pmatrix} + \begin{pmatrix} 4 & 11 & -3 \\ -1 & 2 & 4 \\ 7 & 5 & 0 \end{pmatrix} = \begin{pmatrix} 18 & 27 & 9 \\ 1 & 8 & 22 \\ -1 & 11 & -2 \end{pmatrix}$$

Find the values of x, y and z from the following equations $\begin{pmatrix} x+y & 2 \\ 5+z & xy \end{pmatrix} = \begin{pmatrix} 6 & 2 \\ 5 & 8 \end{pmatrix}$

Solution : $\begin{pmatrix} x+y & 2 \\ 5+z & xy \end{pmatrix} = \begin{pmatrix} 6 & 2 \\ 5 & 8 \end{pmatrix} \Rightarrow x+y=6, \quad xy=8,$

$x=2$ (or) 4, $y=4$ (or) 2 $\quad 5+z=5 \Rightarrow z=0$

Find the values of x, y and z from the following equations $\begin{pmatrix} 12 & 3 \\ x & \frac{3}{2} \end{pmatrix} = \begin{pmatrix} y & z \\ 3 & 5 \end{pmatrix}$

Solution : $\begin{pmatrix} 12 & 3 \\ x & \frac{3}{2} \end{pmatrix} = \begin{pmatrix} y & z \\ 3 & 5 \end{pmatrix} \Rightarrow x=3, \; y=12, \; z=3$

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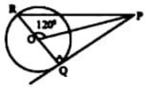
4 . GEOMETRY

Write the Ceva's Theorem

Let ABC be a triangle and let D,E,F be points on lines BC, CA, AB respectively. Then the cevians AD, BE, CF are concurrent if and only if $\frac{BD}{DC} \times \frac{CE}{EA} \times \frac{AF}{FB} = 1$ where the lengths are directed.

PQ is a tangent drawn from a point P to a circle with centre O and QOR is a diameter of the circle such that ∠POR = 120°. Find ∠OPQ .



Solution: Given $\angle POR = 120^0 \Rightarrow \angle POQ = 60^0$ (linear pair)

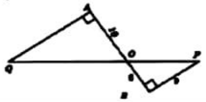
$\angle OQP = 90^0$ (Radius ⊥ tangent)

$\therefore \angle OPQ = 90^0 - 60^0 = 30^0$

If ΔABC is similar to ΔDEF such that BC = 3 cm, EF = 4 cm and area of ΔABC = 54 cm². Find the area of ΔDEF.

Solution : $\frac{Area\,(\Delta ABC)}{Area\,(\Delta DEF)} = \frac{BC^2}{EF^2} \Rightarrow \frac{54}{Area\,(\Delta DEF)} = \frac{3^2}{4^2} \Rightarrow \frac{54}{Area\,(\Delta DEF)} = \frac{9}{16} \Rightarrow Area\,(\Delta DEF) = \frac{16 \times 54}{9} = 96\,cm^2$

In Fig. QA and PB are perpendiculars to AB. If AO = 10 cm, BO = 6 cm and PB = 9 cm. Find AQ.



Solution : $AQ=\frac{10 \times 9}{6} = 15cm$

Write the Menelaus Theorem

A necessary and sufficient condition for points P, Q, R on the respective sides BC, CA, AB of a triangle ABC to be collinear is that $\frac{BP}{PC}\times\frac{CQ}{QA}\times\frac{AR}{RB}=1$ where all segments in the formula are directed segments.

Write the **Alternate Segment theorem**

The angles between the tangent and the chord are equal to the angles in the corresponding alternate segments.

If $\Delta ABC \sim \Delta DEF$ such that area of ΔABC is 9cm² and the area of ΔDEF is 16cm² and BC = 2.1 cm. Find the length of EF.

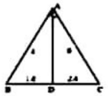
Solution : $\therefore \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF}=\frac{BC^2}{EF^2}$

$$\Rightarrow \frac{9}{16}=\frac{(2.1)^2}{EF^2}$$

$$\therefore EF = \frac{4\times 2.1}{3}=2.8\,cm$$

Check whether AD is bisector of ∠A of ΔABC, AB = 4 cm, AC = 6 cm. BD = 1.6. cm and CD = 2.4 cm.

Solution : $\frac{AB}{AC}=\frac{4}{6}=\frac{2}{3}$, $\frac{BD}{DC}=\frac{1.6}{2.4}=\frac{2}{3}$

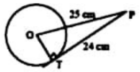


∴ By Converse of ABT, $\therefore \frac{AB}{AC}=\frac{BD}{DC}$ AD is the bisector of ∠A.

The length of the tangent to a circle from a point P, which is 25 cm away from the centre is 24 cm. What is the radius of the circle?

Solution: $\therefore OT=\sqrt{25^2-24^2}=\sqrt{625-576}=\sqrt{49}=7$ cm

∴ Radius = 7 cm



In Figure O is the centre of a circle. PQ is a chord and the tangent PR at P makes an angle of 50° with PQ. Find ∠POQ

Solution : $\angle OPQ=90^0-50^0=40^0$ OP = OQ (Radii of a circle are equal)

$\angle OPQ=\angle OQP=40^0$ (ΔOPQ is isosceles)

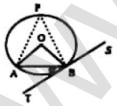
$\angle POQ=180^0-\angle OPQ-\angle OQP$

$\angle POQ=180^0-40^0-40^0=100^0$

A tangent ST to a circle touches it at B. AB is a chord such that ∠ABT = 65°. Find ∠AOB, where "O" is the centre of the circle.

Solution: $\angle TBA=65^0\Rightarrow\angle APB=65^0$ (angles in alternate segment).

$\therefore \angle AOB=2\angle APB=2(65^0)=130^0$



Find the volume of a cylinder whose height is 2 m and whose base area is 250 m².

Solution : Let r and h be the radius and height of the cylinder respectively.

height h = 2 m, base area = 250 m²

volume of a cylinder = πr²h cu. units

= base area × h

= 250 × 2 = 500 m³

5. COORDINATE GEOMETRY

Find the intercepts made by the line $4x-9y+36=0$ on the coordinate axes.

Solution : **put** $x=0 \Rightarrow 4x=-36$ x intercept $a=-9$

put $y=0 \Rightarrow -9y+36=0.\ -9y=-36\Rightarrow y$ intercept $b=4$

Show that the straight lines 2x + 3y − 8 = 0 and 4x + 6y + 18 = 0 are parallel.

Solution :

Slope of the straight line 2x + 3y − 8 = 0 is $m_1=\frac{-\text{coefficient of } x}{\text{coefficient of } y}=\frac{-2}{3}$

Slope of the straight line 4x + 6y + 18 = 0 is $m_2=\frac{-4}{6}=\frac{-2}{3}$ Here, $m_1=m_2$

That is, slopes are equal. Hence, the two straight lines are parallel.

Find the equation of a straight line which is parallel to the line 3x − 7y = 12 and passing through the point (6, 4).

Solution : Equation of the straight line, parallel to $3x-7y-12=0$ is $3x-7y+k=0$

$3(6)-7(4)+k=0 \Rightarrow k=28-18=10$

The required straight line is $3x-7y+10=0$.

Find the equation of a straight line perpendicular to the line $y=\frac{4}{3}x-7$ and passing through the point (7, − 1).

Solution : The equation $y=\frac{4}{3}x-7 \Rightarrow 4x-3y-21=0$.

Equation of a straight line perpendicular to $4x-3y-21=0$ is $3x+4y+k=0$

it is passes through the point (7, −1), $21-4+k=0 \Rightarrow k=-17$

The required straight line is $3x+4y-17=0$.

Find the slope and y intercept of $\sqrt{3}x+(1-\sqrt{3})y=3$.

Solution : $a=\sqrt{3}$ $b=(1-\sqrt{3})$ $c=-3$

Slope of the line $=\frac{-a}{b}=\frac{-\sqrt{3}}{(1-\sqrt{3})}=\frac{3+\sqrt{3}}{2}$

Interecept on y-axis $=\frac{-c}{b}=\frac{-(-3)}{1-\sqrt{3}}=\frac{3+3\sqrt{3}}{-2}$

Find the equation of a line whose inclination is 30° and making an intercept -3 on the Y axis.

Solution : Given $\theta=30^\circ\Rightarrow m=\tan 30^\circ=\frac{1}{\sqrt{3}}$ and y - intercept = − 3

The required equation of the line is $y=mx+c \Rightarrow y=\frac{1}{\sqrt{3}}x-3 \Rightarrow \sqrt{3}y=x-3\sqrt{3}$

$$\Rightarrow x-\sqrt{3}y-3\sqrt{3}=0$$

Find the equation of a line through the given pair of points $\left(2,\frac{2}{3}\right)$ and $\left(\frac{-1}{2},-2\right)$

Solution : Given points are $\left(2,\frac{2}{3}\right),\left(\frac{-1}{2},-2\right)$ two-point form $\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$

$$\frac{y-\frac{2}{3}}{-2-\frac{2}{3}}=\frac{x-2}{\frac{-1}{2}-2}\Rightarrow\frac{3y-2}{-8}=\frac{x-2}{\frac{-5}{2}}\Rightarrow\frac{3y-2}{-8}=\frac{2x-4}{-5}\Rightarrow$$

$$15y-10=16x-32$$

$$16x-15y-22=0$$

Find the equation of a straight line whose Slope is 5 and x intercept is -9

Solution : Given, Slope $= 5$, x intercept, $d = -9$

The equation of a straight line is $y = m(x - d)$ $\quad y = 5(x + 9)$ $\quad y = 5x + 45$

Find the equation of a line passing through the point $(3, -4)$ and having slope $\frac{-5}{7}$.

Solution : Given slope of the line is $-\frac{5}{7}$ and $(3, -4)$ is a point on the line.

$$y - y_1 = m(x - x_1) \Rightarrow y + 4 = \frac{-5}{7}(x - 3) \Rightarrow 5x + 7y + 13 = 0.$$

Find the equation of a straight line which has slope $\frac{-5}{4}$ and passing through the point $(-1, 2)$.

Solution : slope of the line is $\frac{-5}{4}$ and $(-1, 2)$ is a point on the line.

$$y - y_1 = m(x - x_1) \Rightarrow y - 2 = \frac{-5}{4}(x + 1) \Rightarrow 4y - 8 = -5x - 5 \Rightarrow 5x + 4y - 3 = 0$$

Find the equation of a straight line passing through the mid-point of a line segment joining the points (1,-5), (4,2) and parallel to (i) X axis (ii) Y axis

Solution : Mid point of the line joining the points $(1, -5), (4, 2)$ is

$$= \left[\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right] = \left(\frac{1+4}{2}, \frac{-5+2}{2}\right) = \left(\frac{5}{2}, \frac{-3}{2}\right)$$

(i) Parallel to x-axis is $y = {-3}/{2}$ $\qquad$ (ii) Parallel to y-axis is $x = {5}/{2}$

Find the slope of a line joining the given points $(14, 10)$ and $(14, -6)$

Solution : The slope $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-6) - (10)}{(14) - (14)} = \frac{-6 - 10}{14 - 14} = \frac{-16}{0}$. The slope is undefined.

7.MENSURATION

If the base area of a hemispherical solid is 1386 sq. metres, then find its total surface area?

Solution : Let r be the radius of the hemisphere. $\quad \pi r^2 = 1386$ sq. m

$$\text{T.S.A.} = 3\pi r^2 \text{ sq.m} = 3 \times 1386 = 4158 \text{ m}^2.$$

Find the diameter of a sphere whose surface area is 154 m^2.

Solution : Let r be the radius of the sphere.

Given that, $4\pi r^2 = 154 \Rightarrow 4 \times \frac{22}{7} \times r^2 = 154 \Rightarrow r = \frac{7}{2} \Rightarrow$ Diameter $= 7$ m

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The curved surface area of a right circular cylinder of height 14 cm is 88 cm^2. Find the diameter of the cylinder.

Solution : C.S.A. of the cylinder $= 88$ sq. cm $\Rightarrow 2\pi rh = 88 \text{ cm}^2$.

$$2 \times \frac{22}{7} \times r \times 14 = 88 \text{ (given } h = 14 \text{ cm)}$$

$$2r = \frac{88 \times 7}{22 \times 14} = 2 \Rightarrow \text{Diameter} = 2 \text{ cm}$$

If the total surface area of a cone of radius 7cm is 704 cm^2, then find its slant height.

Solution : Given that, radius $r = 7$ cm

$$\text{T.S.A.} = \pi r(l + r) \text{ sq. units} \Rightarrow 704 = \frac{22}{7} \times 7(l + 7) \Rightarrow 32 = l + 7 \Rightarrow l = 25 \text{ cm}$$

The slant height of the cone is 25 cm.

An aluminium sphere of radius 12 cm is melted to make a cylinder of radius 8 cm. Find the height of the cylinder.

Solution: Given radius of sphere $= 12$ cm $= R$ & radius of cylinder $= 8$ cm $= r$

Volume of sphere = Volume of Cylinder

$$\frac{4}{3}\pi R^3 = \pi r^2 h$$

$$\frac{4}{3} \times 12 \times 12 \times 12 = 8 \times 8 \times h \Rightarrow h = 36 \text{ cm}$$

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8. STATISTICS AND PROBABILITY

Find the range and coefficient of range of the following data: 25, 67, 48, 53, 18, 39, 44.

Solution : $L = 67; \; S = 18$

$R = L - S = 67 - 18 = 49$ **and** Coefficient of range $= \frac{L - S}{L + S} = \frac{67 - 18}{67 + 18} = \frac{49}{85}$ (or) 0.576

The range of a set of data is 13.67 and the largest value is 70.08. Find the smallest value.

Solution : $R = 13.67; \; L = 70.08$

$R = L - S \Rightarrow 13.67 = 70.08 - S \Rightarrow S = 70.08 - 13.67 = 56.41 \Rightarrow$ The smallest value is 56.41.

Find the range and coefficient of range of 63, 89, 98, 125, 79, 108, 117, 68

Solution:

Range $= L - S = 125 - 63 = 62$ **and** Coefficient of range $= \frac{L - S}{L + S} = \frac{125 - 63}{125 + 63} = \frac{62}{185}$ (or) 0.33

Find the range and coefficient of range of 43.5, 13.6, 18.9, 38.4, 61.4, 29.8

Solution:

Range $= L - S = 61.4 - 13.6 = 47.8$ $\qquad$ Coefficient of range $= \frac{L - S}{L + S} = \frac{61.4 - 13.6}{61.4 + 13.6} = \frac{47.8}{75}$ (or) 0.64

If the range and the smallest value of a set of data are 36.8 and 13.4 respectively, then find the largest value.

Solution : $R = 36.8 \; ; \; S = 13.4 \quad \therefore R = L - S \Rightarrow 36.8 = L - 13.4 \quad \therefore L = 36.8 + 13.4 = 50.2$

Calculate the range of the following data.

Income	400-450	450-500	500-550	550-600	600-650
Number of workers	8	12	30	21	6

Solution : $L = 650 \; ; \; S = 450$

$\therefore$ Range $= L - S = 650 - 400 = 250$

A wall clock strikes the bell once at 1 o' clock, 2 times at 2 o' clock, 3 times at 3 o' clock and so on. How many times will it strike in a particular day. Find the standard deviation of the number of strikes the bell make a day.

Solution : $\therefore$ Number of times it strikes in a particular day $= 2(1 + 2 + 3 + \dots\dots\dots 12) = 2\left(\frac{12 \times 13}{2}\right) = 156$ times

$$\text{S.D of } 2(1, 2, 3, \dots\dots 12) = 2\left[\sqrt{\frac{n^2 - 1}{12}}\right] = 2\left[\sqrt{\frac{144 - 1}{12}}\right] = 2\sqrt{\frac{143}{12}} = 6.9$$

Find the standard deviation of first 21 natural numbers.

Solution :

$$\text{SD of first 21 natural numbers} = \sqrt{\frac{n^2 - 1}{12}} = \sqrt{\frac{441 - 1}{12}} = \sqrt{\frac{440}{12}} = 6.05$$

Find the standard deviation of first 13 natural numbers.

Solution :

$$\text{SD of first 21 natural numbers} = \sqrt{\frac{n^2 - 1}{12}} = \sqrt{\frac{169 - 1}{12}} = \sqrt{\frac{168}{12}} = 3.74$$

If $n = 5$, $\bar{x} = 6$, $\sum x^2 = 765$, then calculate the coefficient of variation.

Solution : Given $n = 5,\ \bar{x} = 6,\ \sum x^2 = 765,\ CV = ?$

$$\sigma = \sqrt{\frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2} = \sqrt{\frac{765}{5} - (6)^2} = 10.82$$

$$\therefore C.V = \frac{\sigma}{\bar{x}} \times 100 = \frac{10.82}{6} \times 100 = 180.33\%$$

If the range and coefficient of range of the data are 20 and 0.2 respectively, then find the largest and smallest values of the data.

Solution : $L - S = 20$ ...(1) $L + S = 100$ ...(2)

Solviong (1) and (2) $L = 60,\quad S = 40$

What is the probability that a leap year selected at random will contain 53 saturdays. (Hint: 366 = 52×7 +2)

Solution : S = {(Sun-Mon, Mon-Tue, Tue-Wed, Wed-Thu, Thu-Fri, Fri-Sat, Sat-Sun)} $\Rightarrow$ n(S) = 7

Let A be the event of getting 53rd Saturday.

A = {Fri-Sat, Sat-Sun}; n(A) = 2 $\Rightarrow P(A) = \frac{2}{7}$

A die is rolled and a coin is tossed simultaneously. Find the probability that the die shows an odd number and the coin shows a head.

Solution : S = {1H,1T,2H,2T,3H,3T,4H,4T,5H,5T,6H,6T}; n (S) = 12

Let A be the event of getting an odd number and a head.

A = {1H, 3H, 5H}; n (A)= 3 $\Rightarrow P(A) = \frac{3}{12} = \frac{1}{4}$

A bag contains 6 green balls, some black and red balls. Number of black balls is as twice as the number of red balls. Probability of getting a green ball is thrice the probability of getting a red ball. Find (i) number of black balls (ii) total number of balls.

Solution : It is given that, P(G) = 3 × P(R) $\Rightarrow \frac{6}{6+3x} = 3 \times \frac{x}{6+3x}$

$3x = 6 \Rightarrow x = 2.$

(i) Number of black balls = 2×2 = 4

(ii) Total number of balls = 6 +(3×2) = 12

Two coins are tossed together. What is the probability of getting different faces on the coins?

Solution : S = {HH,HT,TH,TT}; n(S) = 4

Let A be the event of getting different faces on the coins.

A = {HT,TH}; n(A) = 2 $\Rightarrow P(A) = \frac{2}{4} = \frac{1}{2}$

A bag contains 5 blue balls and 4 green balls. A ball is drawn at random from the bag. Find the probability that the ball drawn is (i) blue (ii) not blue.

Solution : Total number of possible outcomes

n(S) = 5 + 4 = 9

(i) Let A be the event of getting a blue ball. $P(A) = \frac{5}{9}$

(ii) $\bar{A}$ will be the event of not getting a blue ball. $P(\bar{A}) = \frac{4}{9}$

A bag contains 5 white and some black balls. If the probability of drawing a black ball from the bag is twice the probability of drawing a white ball then find the number of black balls.

Solution : Given n(S) = 5 + x, 5 white balls and x black balls

By daa given, P(B) = 2. P(W) $\Rightarrow \frac{x}{5+x} = 2.\left(\frac{5}{5+x}\right) \Rightarrow x = 10$ $\therefore$ No. of black balls = 10

A coin is tossed thrice. What is the probability of getting two consecutive tails?

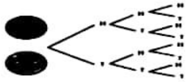
Solution : S = {(HHH), (HHT), (HTH), (HTT), (THH), (THT), (TTH), (TTT)} $\Rightarrow n(S) = 8$

Let A bet the event of getting 2 tails A = {(HTT), (TTH), (TTT)} $\Rightarrow$ n(A) = 3 $\Rightarrow P(A) = \frac{3}{8}$

Write the sample space for tossing three coins using tree diagram.

Solution :

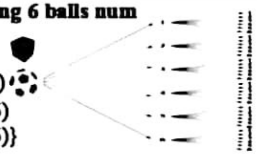
S = {(HHH), (HHT), (HTH), (HTT), (THH), (THT), (TTH), (TTT)}



Write the sample space for selecting two balls from a bag containing 6 balls numbered 1 to 6 (using tree diagram). **Example 8.18 (same answer also)**

Solution :

S = {(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)
(3,1),(3,2),(3,3),(3,4),(3,5),(3,6) ,(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)
(5,1),(5,2),(5,3),(5,4),(5,5),(5,6) ,(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)}



If the standard deviation of a data is 4.5 and if each value of the data is decreased by 5, then find the new standard deviation.

Solution : S.D of a data = **4.5** $\Rightarrow$ new S.D of a data = **4.5**

($\because$ S.D will not be changed when we add (or) subtract fixed constant to all the values of the data).

If the standard deviation of a data is 3.6 and each value of the data is divided by 3, then find the new variance and new standard deviation.

Solution : S.D of a data = 3.6 $\Rightarrow$ new S.D = $\frac{3.6}{3}$ = 1.2

New Variance = $(1.2)^2$ = 1.44

The mean of a data is 25.6 and its coefficient of variation is 18.75. Find the standard deviation.

Solution : Mean $\bar{x}$ = 25.6 , Coefficient of variation, C.V. = 18.75

Coefficient of variation, C.V. = $\frac{\sigma}{\bar{x}} \times 100\%$ $\Rightarrow 18.75 = \frac{\sigma}{25.6} \times 100 \Rightarrow \sigma = 4.8$

The standard deviation and mean of a data are 6.5 and 12.5 respectively. Find the coefficient of variation.

Solution : $\sigma = 6.5, \bar{x} = 12.5$ $\therefore C.V = \frac{\sigma}{\bar{x}} \times 100 = \frac{6.5}{12.5} \times 100 = 52\%$

The standard deviation and coefficient of variation of a data are 1.2 and 25.6 respectively. Find the value of mean.

Solution : Given $\sigma = 1.2$, C.V = 25.6 $\therefore C.V = \frac{\sigma}{\bar{x}} \times 100 \Rightarrow 25.6 = \frac{1.2}{\bar{x}} \times 100 \Rightarrow \bar{x} = \frac{120}{25.6} = 4.69$

If the mean and coefficient of variation of a data are 15 and 48 respectively, then find the value of standard deviation.

Solution : Given $\bar{x}$ = 15, CV = 48, σ = ? $\therefore C.V = \frac{\sigma}{\bar{x}} \times 100 \Rightarrow 48 = \frac{\sigma}{15} \times 100 \Rightarrow \sigma = \frac{15 \times 48}{100} = 7.2$

In a two children family, find the probability that there is at least one girl in a family.

Solution : S = {(BB), (BG), (GB), (GG)} $\Rightarrow$ n(S) = 4

Let A be the event of getting atleast one girl. A = {(BG), (GB), (GG)} $\Rightarrow$ $\therefore$ n(A) = 3

$\Rightarrow$ P(A) = $\frac{3}{4}$