3.4.24

Relations and Functions

Ex 1) $A = \{1, 2, 3\}$

$B = \{a, b\}$ $\quad A \times B = ?$

Sol:

$$A \times B = \{1, 2, 3\} \times \{a, b\}$$

$$A \times B = \{(1,a), (1,b), (2,a), (2,b), (3,a), (3,b)\}$$

$B \times A = ?$

$$B \times A = \{a, b\} \times \{1, 2, 3\}$$

$$B \times A = \{(a,1), (a,2), (a,3), (b,1), (b,2), (b,3)\}$$

9) $A = \{2, -2, 3\}, B = \{1, -4\}$

$A \times B = ?$

$$A \times B = \{2, -2, 3\} \times B\{1, -4\}$$

$$A \times B = \{(2,1), (2,-4), (-2,1), (-2,-4), (3,1), (3,-4)\}$$

$A \times A = ?$

$$A \times A = \{2, -2, 3\} \times \{2, -2, 3\}$$

$$A \times A = \{(2,2), (2,-2), (2,3), (-2,2), (-2,-2), (-2,3), (3,2), (3,-2), (3,3)\}$$

$B \times A = ?$

$$B \times A = \{1, -4\} \times \{2, -2, 3\}$$

$$= \{(1,2), (1,-2), (1,3), (-4,2), (-4,-2), (-4,3)\}$$

11) $A = B = \{p, q\}$

$A \times B = ?$

$$A \times B = \{p, q\} \times \{p, q\}$$

$$A \times B = \{(p,p), (p,q), (q,p), (q,q)\}$$

$A \times A = ?$

$A \times A = \{P, q\} \times \{P, q\}$

$A \times A = \{(P, P), (P, q), (q, P), (q, q)\}$

$B \times A = ?$

$B \times A = \{P, q\} \times \{P, q\}$

$B \times A = \{(P, P), (P, q), (q, P), (q, q)\}$

iii) $A = \{m, n\}$; $B = \emptyset$

$A \times A = ?$

$A \times A = \{m, n\} \times \{m, n\}$

$A \times A = \{(m, m), (m, n), (n, m), (n, n)\}$

$A \times B = ?$

$A \times B = \{m, n\} \times \{\emptyset\}$

$A \times B = \{\}$

$B \times A = ?$

$B \times A = \{\emptyset\} \times \{m, n\}$

$B \times A = \{\}$

9.4.24

Example 1.20

2. If $A \times B = \{(3, 2), (3, 4), (5, 2), (5, 4)\}$ then find A and B.

Sol:

$A = ?$

$B = ?$

$A = \{3, 5\}$

$B = \{2, 4\}$

Exercise 1.1

3. If $B \times A = \{(-2, 3), (-2, 4), (0, 3), (0, 4), (3, 3), (3, 4)\}$ Find A and B

Find A and B

$B = \{-2, 0, 3\}$

$A = \{3, 4\}$

---

4. Let $A = \{1, 2, 3\}$ and $B = \{x \mid x$ is a prime number less than $10\}$. Find $A \times B$ and $B \times A$.

Sol:

$A = \{1, 2, 3\}$

$B = \{2, 3, 5, 7\}$

$A \times B = \{1, 2, 3\} \times \{2, 3, 5, 7\}$

$A \times B = \{(1,2), (1,3), (1,5), (1,7), (2,2), (2,3), (2,5), (2,7), (3,2), (3,3), (3,5), (3,7)\}$

$B \times A = \{2, 3, 5, 7\} \times \{1, 2, 3\}$

$B \times A = \{(2,1), (2,2), (2,3), (3,1), (3,2), (3,3), (5,1), (5,2), (5,3), (7,1), (7,2), (7,3)\}$

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5. If $A = \{5, 6\}$, $B = \{4, 5, 6\}$, $C = \{5, 6, 7\}$, Show that $A \times A = (B \times B) \cap (C \times C)$

Sol:

$A = \{5, 6\}$ $B = \{4, 5, 6\}$ $C = \{5, 6, 7\}$

$A \times A = \{5, 6\} \times \{5, 6\}$

$A \times A = \{(5,5), (5,6), (6,5), (6,6)\} \rightarrow (1)$

$B \times B = \{4, 5, 6\} \times \{4, 5, 6\}$

$B \times B = \{(4,4), (4,5), (4,6), (5,4), (5,5), (5,6), (6,4), (6,5), (6,6)\} \rightarrow (2)$

$C \times C = \{5, 6, 7\} \times \{5, 6, 7\}$

$C \times C = \{(5,5), (5,6), (5,7), (6,5), (6,6), (6,7), (7,5), (7,6), (7,7)\} \rightarrow (3)$

$(B \times B) \cap (C \times C) = \{(5,5), (5,6), (6,5), (6,6)\} \rightarrow (4)$

(1) = (4)

$A \times A = (B \times B) \cap (C \times C)$

6. Given $A = \{1,2,3\}$, $B = \{2,3,5\}$, $C = \{3,4\}$ and $D = \{1,3,5\}$, Check if $(A \cap C) \times (B \cap D) = (A \times B) \cap (C \times D)$ is true?

Sol:

$A = \{1,2,3\}$ $B = \{2,3,5\}$

$C = \{3,4\}$ $D = \{1,3,5\}$

$(A \cap D) = \{1,2,3\} \cap \{3,4\}$

$(A \cap C) = \{3\}$

$(B \cap D) = \{2,3,5\} \cap \{1,3,5\}$

$(B \cap D) = \{3,5\}$

$(A \cap C) \times (B \cap D) = \{3\} \times \{3,5\}$

$(A \cap C) \times (B \cap D) = \{(3,3),(3,5)\}$ → (1)

$(A \times B) = \{1,2,3\} \times \{2,3,5\}$

$A \times B) = \{(1,2),(1,3),(1,5),(2,2),(2,3),(2,5),$
$(3,2),(3,3),(3,5)\}$

$(C \times D) = \{3,4\} \times \{1,3,5\}$

$(C \times D) = \{(3,1),(3,3),(3,5),(4,1),(4,3),(4,5)\}$

$(A \times B) \cap (C \times D) = \{(1,2),(1,3),(1,5),(2,2),(2,3)$
$(2,5),(3,2),(3,3),(3,5)\} \cap$
$\{(3,1),(3,3),(3,5),(4,1),(4,3),$
$(4,5)\}$

$(A \times B) \cap (C \times D) = \{(3,3),(3,5)\}$ → (2)

(1) = (2)

$(A \cap C) \times (B \cap D) = (A \times B) \cap (C \times D)$

7. Let $A=\{x \in W / x<2\}$, $B=\{x \in N / 1<x \leq 4\}$ and $C=\{3,5\}$. Verify that

i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$ ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

iii) $(A \cup B) \times C = (A \times C) \cup (B \times C)$

Sol: $A=\{0,1\}$ $B=\{2,3,4\}$ $C=\{3,5\}$

i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

LHS $= A \times (B \cup C)$

$B \cup C = \{2,3,4\} \cup \{3,5\}$

$B \cup C = \{2,3,4,5\}$

$A \times (B \cup C) = \{0,1\} \times \{2,3,4,5\}$

$A \times (B \cup C) = \{(0,2),(0,3),(0,4),(0,5),(1,2),(1,3),(1,4),(1,5)\} \rightarrow (1)$

RHS $= (A \times B) \cup (A \times C)$

$A \times B = \{0,1\} \times \{2,3,4\}$

$A \times B = \{(0,2),(0,3),(0,4),(1,2),(1,3),(1,4)\}$

$A \times C = \{0,1\} \times \{3,5\}$

$A \times C = \{(0,3),(0,5),(1,3),(1,5)\}$

$(A \times B) \cup (A \times C)$

$(A \times B) \cup (A \times C) = \{(0,2),(0,3),(0,4),(1,2),(1,3),(1,4)\} \cup \{(0,3),(0,5),(1,3),(1,5)\}$

$(A \times B) \cup (A \times C) = \{(0,2),(0,3),(0,4),(0,5),(1,2),(1,3),(1,4),(1,5)\} \rightarrow (2)$

$A \times (B \cup C) = (A \times B) \cup (A \times C)$ (1) = (2)

ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Sol:

$A=\{0,1\}$

$B=\{2,3,4\}$

$C=\{3,5\}$

$B \cap C = \{2,3,4\} \cap \{3,5\}$

$B \cap C = \{3\}$

$A \times (B \cap C) = \{0,1\} \times \{3\}$

$A \times (B \cap C) = \{(0,3),(1,3)\} \rightarrow ①$

RHS $= (A \times B) \cap (A \times C)$

$\{0,1\} \times \{2,3,4\}$

$\{(0,2),(0,3),(0,4),(1,2),(1,3),(1,4)\}$

$A \times C = \{0,1\} \times \{3,5\}$

$A \times C = \{(0,3),(0,5),(1,3),(1,5)\}$

$(A \times B) \cap (A \times C) = \{(0,2),(0,3),(0,4),(0,2),(1,3),(1,4)\}$,
$\{(0,3),(0,5),(1,3),(1,5)\}$

$(A \times B) \cap (A \times C) = \{(0,3),(1,3)\} \rightarrow ②$

$① = ②$ $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(iii) $(A \cup B) \times C = (A \times C) \cup (B \times C)$

Sol:

LHS $= (A \cup B) \times C$

$A \cup B = \{0,1\} \cup \{2,3,4\}$

$A \cup B = \{0,1,2,3,4\}$

$(A \cup B) \times C = \{0,1,2,3,4\} \times \{3,5\}$

$(A \cup B) \times C = \{(0,3),(0,5),(1,3),(1,5),(2,3),(2,5),(3,3),$
$(3,5),(4,3),(4,5)\} \rightarrow ①$

RHS $= (A \times C) \cup (B \times C)$

$A \times C = \{0,1\} \times \{3,5\}$

$A \times C = \{(0,3),(0,5),(1,3),(1,5)\}$

$B \times C = \{2,3,4\} \times \{3,5\}$

$B \times C = \{(2,3),(2,5),(3,3),(3,5),(4,3),(4,5)\}$

(A×C)∪(B×C) = {(0,3),(0,5),(1,3),(1,5),(2,3),
(2,5),(3,3),(3,5),(4,3),(4,5)}

(A×C)∪(B×C) = {(0,3),(0,5),(1,3),(1,5),(2,3),(2,5),
(3,3),(3,5),(4,3),(4,5)} → (2)

(1) ⇒ (2)

(A∪B)×C = (A×C)∪(B×C)

11.4.24

8. Let A = The Set of all natural numbers less than 8, B = The Set of all prime numbers less than 8, C = The Set of even prime number. Verify that

i) (A∩B)×C = (A×C)∩(B×C) ii) A×(B−C) = (A×B)−(A×C)

Sol:

A = {1,2,3,4,5,6,7}

B = {2,3,5,7}

C = {2}

i) (A∩B)×C = (A×C)∩(B×C)

LHS = (A∩B)×C

A∩B = {1,2,3,4,5,6,7} ∩ {2,3,5,7}

A∩B = {2,3,5,7}

(A∩B)×C = {2,3,5,7} × {2}

(A∩B)×C = {(2,2),(3,2),(5,2),(7,2)} → (1)

RHS = (A×C)∩(B×C)

A×C = {1,2,3,4,5,6,7} × {2}

A×C = {(1,2),(2,2),(3,2),(4,2),(5,2),(6,2),(7,2)}

B×C = {2,3,5,7} × {2}

B×C = {(2,2),(3,2),(5,2),(7,2)}

$(A \times C) \cap (B \times C) = \{(1,2),(3,2),(5,2),(7,2)\}$ → ②

① = ②

$(A \cap B) \times C = (A \times C) \cap (B \times C)$

---

11) $A \times (B - C) = (A \times B) - (A \times C)$

$A = \{1,2,3,4,5,6,7\}$

$B = \{2,3,5,7\}$ $C = \{2\}$

LHS = $A \times (B - C)$

$B - C = \{2,3,5,7\} - \{2\}$

$B - C = \{3,5,7\}$

$A \times (B - C) = \{1,2,3,4,5,6,7\} \times \{3,5,7\}$

$A \times (B - C) = \{(1,3),(1,5),(1,7),(2,3),(2,5),(2,7),(3,3),$
$(3,5),(3,7),(4,3),(4,5),(4,7),(5,3),(5,5),$
$(5,7),(6,3),(6,5),(6,7),(7,3),(7,5),(7,7)\}$ → ①

RHS = $(A \times B) - (A \times C)$

$A \times B = \{1,2,3,4,5,6,7\} \times \{2,3,5,7\}$

$A \times B = \{(1,2),(1,3),(1,5),(1,7),(2,2),(2,3),(2,5),(2,7),$
$(3,2),(3,3),(3,5),(3,7),(4,2),(4,3),(4,5),(4,7),$
$(5,2),(5,3),(5,5),(5,7),(6,2),(6,3),(6,5),$
$(6,7),(7,2),(7,3),(7,5),(7,7)\}$

$A \times C = \{1,2,3,4,5,6,7\} \times \{2\}$

$A \times C = \{(1,2),(2,2),(3,2),(4,2),(5,2),(6,2),(7,2)\}$

$(A \times B) - (A \times C) = \{(1,2),(1,3),(1,5),(1,7),(2,2),(2,3),$
$(2,5),(2,7),(3,2),(3,3),(3,5),(3,7),$
$(4,2),(4,3),(4,5),(4,7),(5,2),(5,3),$
$(5,5),(5,7),(6,2),(6,3),(6,5),(6,7),$
$(7,2),(7,3),(7,5),(7,7)\} - \{(1,2),$
$(2,2),(3,2),(4,2),(5,2),(6,2),(7,2)\}$

$(A \times B) - (A \times C) = \{(1,2),(1,3),(1,5),(1,7),(2,3),(2,5),$
$(2,7),(3,3),(3,5),(3,7),(4,3),(4,5),$
$(4,7),(5,3),(5,5),(5,7),(6,3),(6,5),$
$(6,7),(7,3),(7,5),(7,7)\}$ → ②

① = ②

$A \times (B - C) = (A \times B) - (A \times C)$

09/2034