10 MATHS UNIT-3 (PART-I)

SOLVING LINEAR EQUATIONS

- New formula derived.
- Step by step Method explained in detail.

I. TWO VARIABLES:-

Let the system of equations in TWO Variables be..

$$a_1x+b_1y+c_1=0 \quad \text{—(1)}$$
$$a_2x+b_2y+c_2=0 \quad \text{—(2)}.$$

Then, by **VEDIC METHOD**, [or modified Version of Cross multiplication Method] the solution for the system of linear equations is..

$$\boxed{x=\left[\frac{b_1c_2-b_2c_1}{a_1b_2-a_2b_1}\right]} \quad \& \quad \boxed{y=-\left[\frac{a_1c_2-a_2c_1}{a_1b_2-a_2b_1}\right]}$$

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Application:

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Solve: $2x-3y=6$,
$x+y=1$.

Soln: Step-1: Write down the equation in standard form: (i.e)

$$2x-3y-6=0 \quad \text{—(1)}$$
$$x+y-1=0 \quad \text{—(2)}$$

only coefficients

x	y	1
2	−3	−6
1	1	−1

Step-2: Denominator is common in the solution. Let us Find out.

$$\therefore \text{Denominator} = (2\times 1)-1(-3)$$
$$= 2+3=5$$

x	y	1
2	−3	−6
1	1	−1

Step-3: $x=\dfrac{(-3)(-1)-1(-6)}{5}$

x	y	1
2	−3	−6
1	1	−1

$$x=\frac{3+6}{5}=\frac{9}{5}.$$

Step-4: $y=-\left[\dfrac{2(-1)-1(-6)}{5}\right]$

x	y	1
2	−3	−6
1	1	−1

$$y=-\left[\frac{-2+6}{5}\right]=-\frac{4}{5}$$

Step-5: Verification.

$x = \frac{9}{5}$ and $y = -\frac{4}{5}$ in equation (2)

LHS: $x+y-1 = \frac{9}{5} - \frac{4}{5} - 1 = \frac{9-4-5}{5} = \frac{9-9}{5} = 0 = \text{RHS}$.

$\therefore$ Verified.

$\therefore \boxed{x = \frac{9}{5}}$ & $\boxed{y = -\frac{4}{5}}$

II. THREE VARIABLES:-

Let the system of linear equations in THREE variables be....

$$a_1x + b_1y + c_1z + d_1 = 0 \quad \text{—(1)}$$
$$a_2x + b_2y + c_2z + d_2 = 0 \quad \text{—(2)}$$
$$a_3x + b_3y + c_3z + d_3 = 0 \quad \text{—(3)}$$

only co-efficients.

x	y	z	1
a_1	b_1	c_1	d_1
a_2	b_2	c_2	d_2
a_3	b_3	c_3	d_3

By using the concepts of vedic Method, the solution for the system of linear equations is derived as follows:

$$x = -\left[\frac{b_1[c_2d_3 - c_3d_2] - c_1[b_2d_3 - b_3d_2] + d_1[b_2c_3 - b_3c_2]}{a_1[b_2c_3 - b_3c_2] - b_1[a_2c_3 - a_3c_2] + c_1[a_2b_3 - a_3b_2]}\right]$$

$$y = \left[\frac{a_1[c_2d_3 - c_3d_2] - c_1[a_2d_3 - a_3d_2] + d_1[a_2c_3 - a_3c_2]}{a_1[b_2c_3 - b_3c_2] - b_1[a_2c_3 - a_3c_2] + c_1[a_2b_3 - a_3b_2]}\right]$$

$$z = -\left[\frac{a_1[b_2d_3 - b_3d_2] - b_1[a_2d_3 - a_3d_2] + d_1[a_2b_3 - a_3b_2]}{a_1[b_2c_3 - b_3c_2] - b_1[a_2c_3 - a_3c_2] + c_1[a_2b_3 - a_3b_2]}\right]$$

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This formula seem to be bigger and complicated. But, when we adopt a method of application, then it will be easier & simple.

Observe that,

1) A negative sign is there, in the overall solution of x and z. (y is positive)

2) There are three terms in both numerator and denominator of each formula. Out of which <u>SECOND</u> term has negative sign.

APPLICATION:

Solve the following system of linear equations in three variables:

$3x-2y+z=2, \quad 2x+3y-z=5, \quad x+y+z=6.$

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Soln:

STEP-1: Write down the equation in standard form. Note down its co-efficients.

$3x-2y+z-2=0 \quad \text{—(1)}$

$2x+3y-z-5=0 \quad \text{—(2)}$

$x+y+z-6=0 \quad \text{—(3)}$

x	y	z	1
3	-2	1	-2
2	3	-1	-5
1	1	1	-6

STEP-2: Denominator is common in the solution of x, y and z. While calculating the value of denominator, just ignore the last column (constant d_1, d_2, d_3). First row is the co-efficient of each term. That is...

x	y	z
3	-2	1
2	3	-1
1	1	1

x	y	z
3	-2	1
2	3	-1
1	1	1

x	y	z
3	-2	1
2	3	-1
1	1	1

$$\text{Denominator} = 3[3\times1-1(-1)]-(-2)[2\times1-1(-1)]+1[2\times1-1\times3]$$

$$= 3[3+1]+2[2+1]+1[2-3]$$

$$= (3\times4)+(2\times3)+(-1) = 12+6-1 = 18-1 = \boxed{17}$$

STEP-3: While calculating the numerator part of x value, just ignore the first column. (i.e co-efficient of x). Hence,

y	z	1
-2	1	-2
3	-1	-5
1	1	-6

y	z	1
-2	1	-2
3	-1	-5
1	1	-6

y	z	1
-2	1	-2
3	-1	-5
1	1	-6

$$x = -\left[\frac{-2[(-1)(-6)-1(-5)]-1[3(-6)-1(-5)]+(-2)[3\times1-1(-1)]}{17}\right]$$

$$= -\left[\frac{-2[6+5]-[-18+5]-2[3+1]}{17}\right] = -\left[\frac{(-2\times11)-(-13)-(2\times4)}{17}\right]$$

$$= -\left[\frac{-22+13-8}{17}\right] = -\left[\frac{-30+13}{17}\right] = -\left[\frac{-17}{17}\right] = 1 \qquad \boxed{\therefore x = 1}$$

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STEP-4: While calculating the numerator part of y value, just ignore the second column (i.e. co-efficient of y). Hence,

x	z	1
3	1	−2
2	−1	−5
1	1	−6

x	z	1
3	1	−2
2	−1	−5
1	1	−6

x	z	1
3	1	−2
2	−1	−5
1	1	−6

$$y = \left[\frac{3[(-1)(-6)-1(-5)] - 1[2(-6)-1(-5)] + (-2)[2\times1 - 1(-1)]}{17}\right]$$

$$= \frac{3[6+5]-[-12+5]-2[2+1]}{17} = \frac{(3\times11)-(-7)-(2\times3)}{17}$$

$$= \frac{33+7-6}{17} = \frac{40-6}{17} = \frac{34}{17} = 2 \qquad \boxed{\therefore y = 2}$$

STEP-5: Instead of applying the formula, it is easier to obtain the value of z, by applying the value of x and y in any equation.

$\therefore$ Apply $x=1$, $y=2$ in equation (1).

$$3x-2y+z-2=0 \Rightarrow (3\times1)-2\times2+z-2=0$$
$$3-4+z-2=0$$
$$z-3=0 \Rightarrow \boxed{z=3}$$

STEP-6: Verify: In equation (2),

$$\text{LHS: } 2x+3y-z-5 = (2\times1)+(3\times2)-3-5$$
$$= 2+6-3-5 = 8-8 = 0 = \text{RHS}$$

$\therefore$ Verified.

NOTE: When the denominator value is ZERO (0), then the system of equation has no solution or have infinitely many solution.

In such case, the numerator of x is also ZERO (0), then the equations "<u>have infinitely many solutions</u>". [i.e x is in the form of $\frac{0}{0}$]

Otherwise, the solution of x is in the form of $\frac{N}{0}$ (N-any number), then the equations are inconsistent and "have no Solution".

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TEACHERS' NOTE

DERIVATION OF NEW FORMULA.

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PROOF

I. TWO VARIABLE: Let k is a constant.

Let $x=k$. Then the system of linear equation

$a_1x+b_1y+c_1=0 \Rightarrow a_1k+b_1y+c_1=0$ — (1)

$a_2x+b_2y+c_2=0 \Rightarrow a_2k+b_2y+c_2=0$ — (2)

(1) $\Rightarrow b_1y=-a_1k-c_1 \Rightarrow y=\frac{-a_1k}{b_1}-\frac{c_1}{b_1}$ — (3)

(2) $\Rightarrow b_2y=-a_2k-c_2 \Rightarrow y=\frac{-a_2k}{b_2}-\frac{c_2}{b_2}$ — (4)

from (3) & (4) $\frac{-a_1k}{b_1}-\frac{c_1}{b_1}=\frac{-a_2k}{b_2}-\frac{c_2}{b_2}$

$$\frac{a_2k}{b_2}-\frac{a_1k}{b_1}=\frac{c_1}{b_1}-\frac{c_2}{b_2}.$$

$$k\left[\frac{a_2b_1-a_1b_2}{b_1b_2}\right]=\frac{b_2c_1-b_1c_2}{b_1b_2}$$

$$\therefore k=\frac{b_2c_1-b_1c_2}{a_2b_1-a_1b_2} \Rightarrow \boxed{x=\frac{b_1c_2-b_2c_1}{a_1b_2-a_2b_1}}$$

Now, let $y=k$. Then

$a_1x+b_1k+c_1=0$ — (5) $\Rightarrow x=\frac{-b_1k}{a_1}-\frac{c_1}{a_1}$ — (7)

$a_2x+b_2k+c_2=0$ — (6) $\Rightarrow x=-\frac{b_2k}{a_2}-\frac{c_2}{a_2}$ — (8)

from (7) & (8) $\frac{b_2k}{a_2}-\frac{b_1k}{a_1}=\frac{c_1}{a_1}-\frac{c_2}{a_2}$

$$k\left[\frac{a_1b_2-a_2b_1}{a_1a_2}\right]=\frac{a_2c_1-a_1c_2}{a_1a_2}$$

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$$K=\frac{a_2c_1-a_1c_2}{a_1b_2-a_2b_1} \Rightarrow \boxed{y=-\left[\frac{a_1c_2-a_2c_1}{a_1b_2-a_2b_1}\right]}$$

$$\therefore x=\left[\frac{b_1c_2-b_2c_1}{a_1b_2-a_2b_1}\right] \text{ and } y=-\left[\frac{a_1c_2-a_2c_1}{a_1b_2-a_2b_1}\right]$$

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II. THREE VARIABLES:

Let $x = k$. (k is a constant). Then,

$$a_1x+b_1y+c_1z+d_1=0 \Rightarrow a_1k+b_1y+c_1z+d_1=0 \quad \text{---} (1)$$

$$a_2x+b_2y+c_2z+d_2=0 \Rightarrow a_2k+b_2y+c_2z+d_2=0 \quad \text{---} (2)$$

$$a_3x+b_3y+c_3z+d_3=0 \Rightarrow a_3k+b_3y+c_3z+d_3=0 \quad \text{---} (3)$$

$$(1) \Rightarrow b_1y+c_1z+(a_1k+d_1)=0 \quad \text{---} (4)$$

$$(2) \Rightarrow b_2y+c_2z+(a_2k+d_2)=0 \quad \text{---} (5)$$

$$(3) \Rightarrow b_3y+c_3z+(a_3k+d_3)=0 \quad \text{---} (6)$$

NOTE: (4), (5), (6) are Linear equations in TWO variable (y and z)

from (4), (5), by using vedic Method,

$$y = \frac{c_1[a_2k+d_2]-c_2[a_1k+d_1]}{b_1c_2-b_2c_1} \quad \text{---} (7)$$

from (5), (6) solution is

$$y = \frac{c_2[a_3k+d_3]-c_3[a_2k+d_2]}{b_2c_3-b_3c_2} \quad \text{---} (8)$$

from (7) and (8)

$$\frac{c_1a_2k+c_1d_2-c_2a_1k-c_2d_1}{b_1c_2-b_2c_1} = \frac{c_2a_3k+c_2d_3-c_3a_2k-c_3d_2}{b_2c_3-b_3c_2}$$

$$k\left[\frac{c_1a_2-c_2a_1}{b_1c_2-b_2c_1}\right]+\frac{c_1d_2-c_2d_1}{b_1c_2-b_2c_1} = k\left[\frac{c_2a_3-c_3a_2}{b_2c_3-b_3c_2}\right]+\frac{c_2d_3-c_3d_2}{b_2c_3-b_3c_2}$$

$$k\left[\frac{c_1a_2-c_2a_1}{b_1c_2-b_2c_1}-\frac{c_2a_3-c_3a_2}{b_2c_3-b_3c_2}\right] = \left[\frac{c_2d_3-c_3d_2}{b_2c_3-b_3c_2}-\frac{c_1d_2-c_2d_1}{b_1c_2-b_2c_1}\right]$$

$$k\left[\frac{(c_1a_2-c_2a_1)(b_2c_3-b_3c_2)-(b_1c_2-b_2c_1)(c_2a_3-c_3a_2)}{\cancel{(b_1c_2-b_2c_1)(b_2c_3-b_3c_2)}}\right]$$

$$= \left[\frac{(c_2d_3-c_3d_2)(b_1c_2-b_2c_1)-(b_2c_3-b_3c_2)(c_1d_2-c_2d_1)}{\cancel{(b_1c_2-b_2c_1)(b_2c_3-b_3c_2)}}\right]$$

$$\therefore k = \frac{\left[c_2d_3b_1c_2-c_2d_3b_2c_1-c_3d_2b_1c_2+\cancel{c_3d_2b_2c_1} - \cancel{b_2c_3c_1d_2}+b_2c_3c_2d_1+b_3c_2c_1d_2-b_3c_2c_2d_1\right]}{\left[c_1a_2\cancel{b_2c_3}-c_1a_2b_3c_2-c_2a_1b_2c_3+c_2a_1b_3c_2-b_1c_2c_2a_3 + b_1c_2c_3a_2+b_2c_1c_2a_3-\cancel{b_2c_1c_3a_2}\right]}$$

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$$k = \frac{\cancel{2}\left[b_1c_2d_3 - b_2c_1d_3 - b_1c_3d_2 + b_2c_3d_1 + b_3c_1d_2 - b_3c_2d_1\right]}{\cancel{2}\left[-a_2b_3c_1 - a_1b_2c_3 + a_1b_3c_2 - a_3b_1c_2 + a_2b_1c_3 + a_3b_2c_1\right]}$$

$$k = \frac{b_1[c_2d_3 - c_3d_2] - c_1[b_2d_3 - b_3d_2] + d_1[b_2c_3 - b_3c_2]}{a_1[b_3c_2 - b_2c_3] - b_1[a_3c_2 - a_2c_3] + c_1[a_3b_2 - a_2b_3]}$$

So as to facilitate the application of this formula, all the terms in denominator is interchanged, thus obtained a negative sign for overall value.

$$\therefore k = x = -\left[\frac{b_1[c_2d_3 - c_3d_2] - c_1[b_2d_3 - b_3d_2] + d_1[b_2c_3 - b_3c_2]}{a_1[b_2c_3 - b_3c_2] - b_1[a_2c_3 - a_3c_2] + c_1[a_2b_3 - a_3b_2]}\right]$$

Similarly, when $y = k$ (a constant) we can derive the formula of y. And, to derive the formula for z, let $z = k$ (a constant).

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