Unit Test - 1 (Relations & Functions)

Part-1 [Mark 14]

Answer all 14 questions

1.	c	3
2.	c	$p \times q$
3	c	12
4	b	2
5	c	$\{4, 9, 25, 49, 121\}$
6	c	$2^{mn} - 1$
7	a	$\{8, 6\}$
8	c	one to one function
9	c	$2/9x^2$
10	a	7
11	b	Domain
12	d	$\{0, 1, 2\}$
13	b	$\{2, -1\}$
14	d	quadratic

Part - II

Answer all the questions

15. $A = \{1, 2, 3\}$ $B = \{2, 3, 5, 7\}$

$A \times B = \{1, 2, 3\} \times \{2, 3, 5, 7\}$

$= \{(1,2)(1,3)(1,5)(1,7)(2,2)(2,3)(2,5)$
$(2,7)(3,2)(3,3)(3,5)(3,7)\}$

$B \times A = \{2,3,5,7\} \times \{1,2,3\}$

$= \{(2,1)(2,2)(2,3)(3,1)(3,2)(3,3)(5,1)$
$(5,2)(5,3)(7,1)(7,2)(7,3)\}$

16. $B \times A = \{(-2,3)(-2,4)(0,3)(0,4)(3,3)(3,4)\}$

$A = \{$set of all second coordinates of $B \times A\}$

$B = \{$set of all first coordinates of $B \times A\}$

$A = \{3,4,3\}$

$B = \{-2,0,3\}$

17. $A = \{3,4,7,8\}$ $B = \{1,7,10\}$

$R = \{(3,7),(4,7)(7,10)(8,1)\}$

R is a relation from A to B

18 $A = \{1,2,3,4,\ldots\ldots 45\}$

$R =$ "Square of on A"

$A \times A = \{(1,1)(1,2)\ldots\ldots(1,45)$
$(2,1)(2,2)\ldots\ldots(2,45)$
$(3,1)(3,2)\ldots\ldots(3,45)$
$\vdots$
$(45,1)(45,2)\ldots\ldots(45,45)\}$

$R = 1^2 = 1$ $5^2 = 25$

$2^2 = 4$ $6^2 = 36$

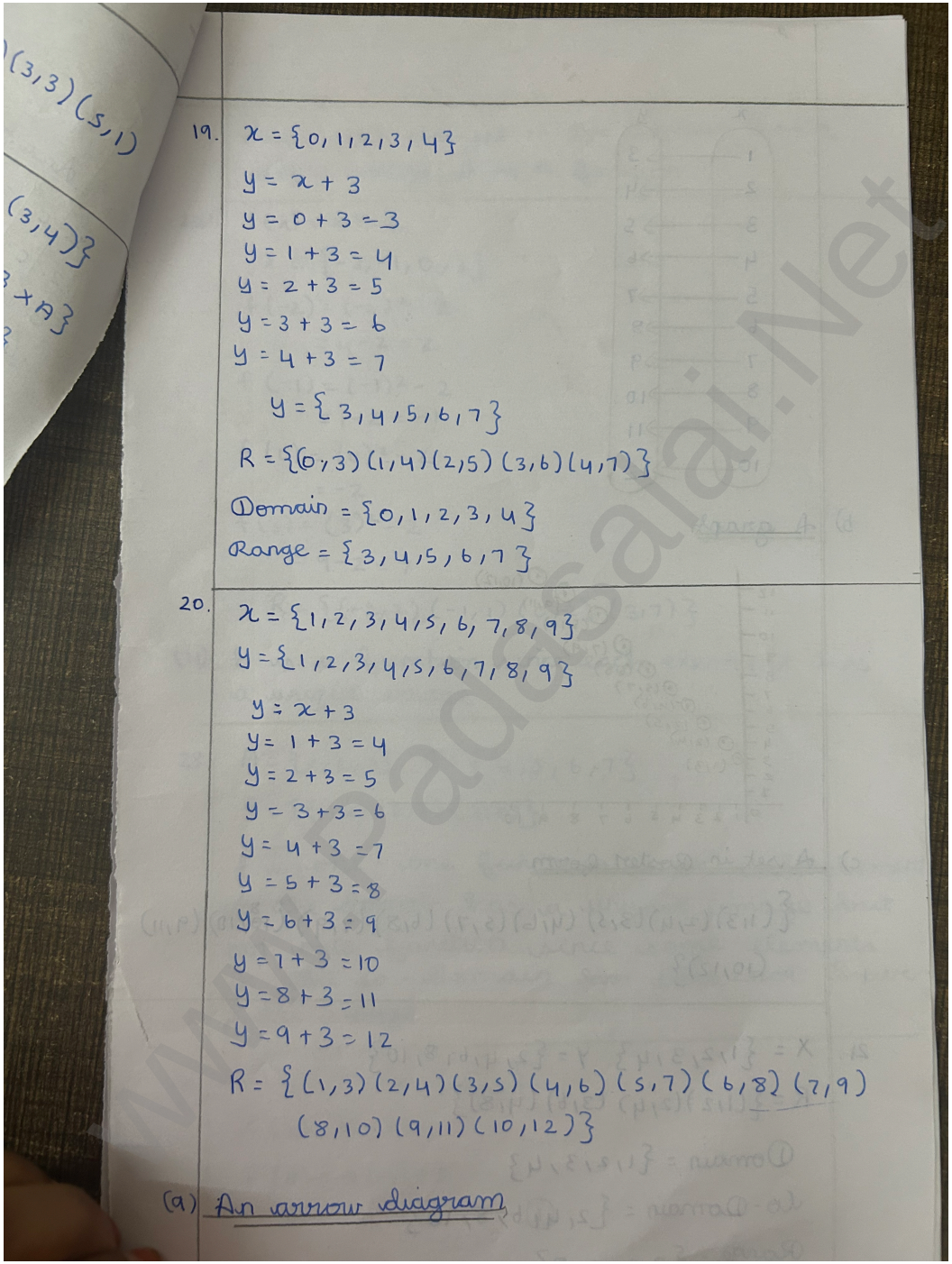
$3^2 = 9$ 7

$4^2 = 16$

$R = \{(1,1)(2,4)(3,9)(4,16)(5,25)(6,36)$

Domain of $R = \{1,2,3,4,5,6\}$

Range of $R = \{1,4,9,16,25,36\}$



19. $x = \{0, 1, 2, 3, 4\}$

$y = x + 3$

$y = 0 + 3 = 3$

$y = 1 + 3 = 4$

$y = 2 + 3 = 5$

$y = 3 + 3 = 6$

$y = 4 + 3 = 7$

$y = \{3, 4, 5, 6, 7\}$

$R = \{(0,3)(1,4)(2,5)(3,6)(4,7)\}$

Domain $= \{0, 1, 2, 3, 4\}$

Range $= \{3, 4, 5, 6, 7\}$

20. $x = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$y = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$y = x + 3$

$y = 1 + 3 = 4$

$y = 2 + 3 = 5$

$y = 3 + 3 = 6$

$y = 4 + 3 = 7$

$y = 5 + 3 = 8$

$y = 6 + 3 = 9$

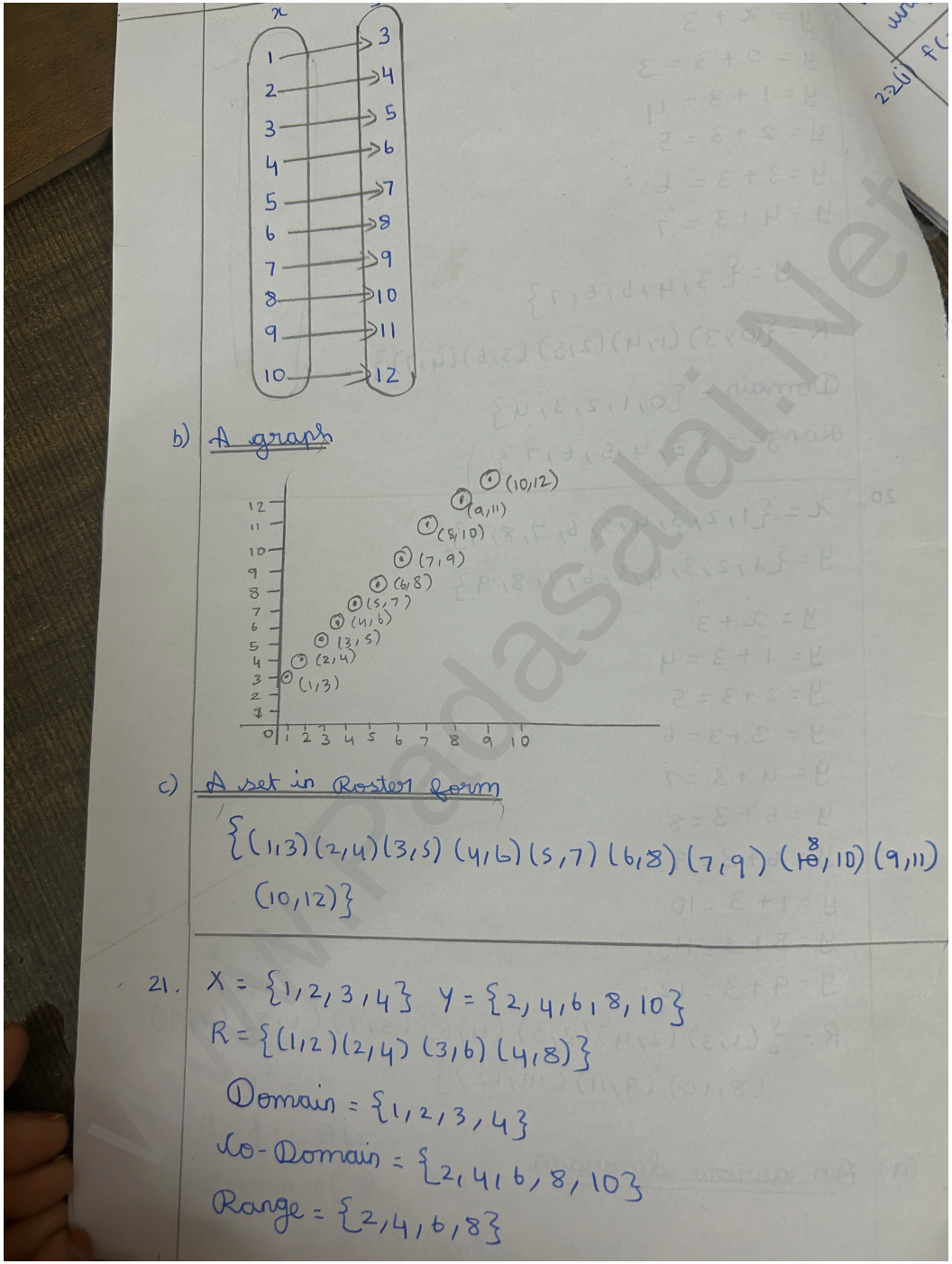
$y = 7 + 3 = 10$

$y = 8 + 3 = 11$

$y = 9 + 3 = 12$

$R = \{(1,3)(2,4)(3,5)(4,6)(5,7)(6,8)(7,9)$
$(8,10)(9,11)(10,12)\}$

(a) An arrow diagram



b) A graph

c) A set in Roster form

$\{(1,3)(2,4)(3,5)(4,6)(5,7)(6,8)(7,9)(8,10)(9,11)(10,12)\}$

21. $X = \{1,2,3,4\}$ $Y = \{2,4,6,8,10\}$

$R = \{(1,2)(2,4)(3,6)(4,8)\}$

Domain $= \{1,2,3,4\}$

Co-Domain $= \{2,4,6,8,10\}$

Range $= \{2,4,6,8\}$

Since each element in the Domain has a unique image, it is a function.

22(i) $f(x) = x^2 - 2$

$x \in \{-2, -1, 0, 3\}$

$f(-2) = (-2)^2 - 2$

$= 4 - 2 = 2$

$f(-1) = (-1)^2 - 2$

$= 1 - 2 = -1$

$f(0) = (0)^2 - 2$

$= -2$

$f(3) = (3)^2 - 2$

$= 9 - 2 = 7$

$R = \{(-2, 2) (-1, 1) (0, -2) (3, 7)\}$

(ii) f is a function since each element has a unique image

23. $A = \{1, 2, 3\}$ $B = \{4, 5, 6, 7\}$

$f = \{(1, 4) (2, 5) (3, 6)\}$

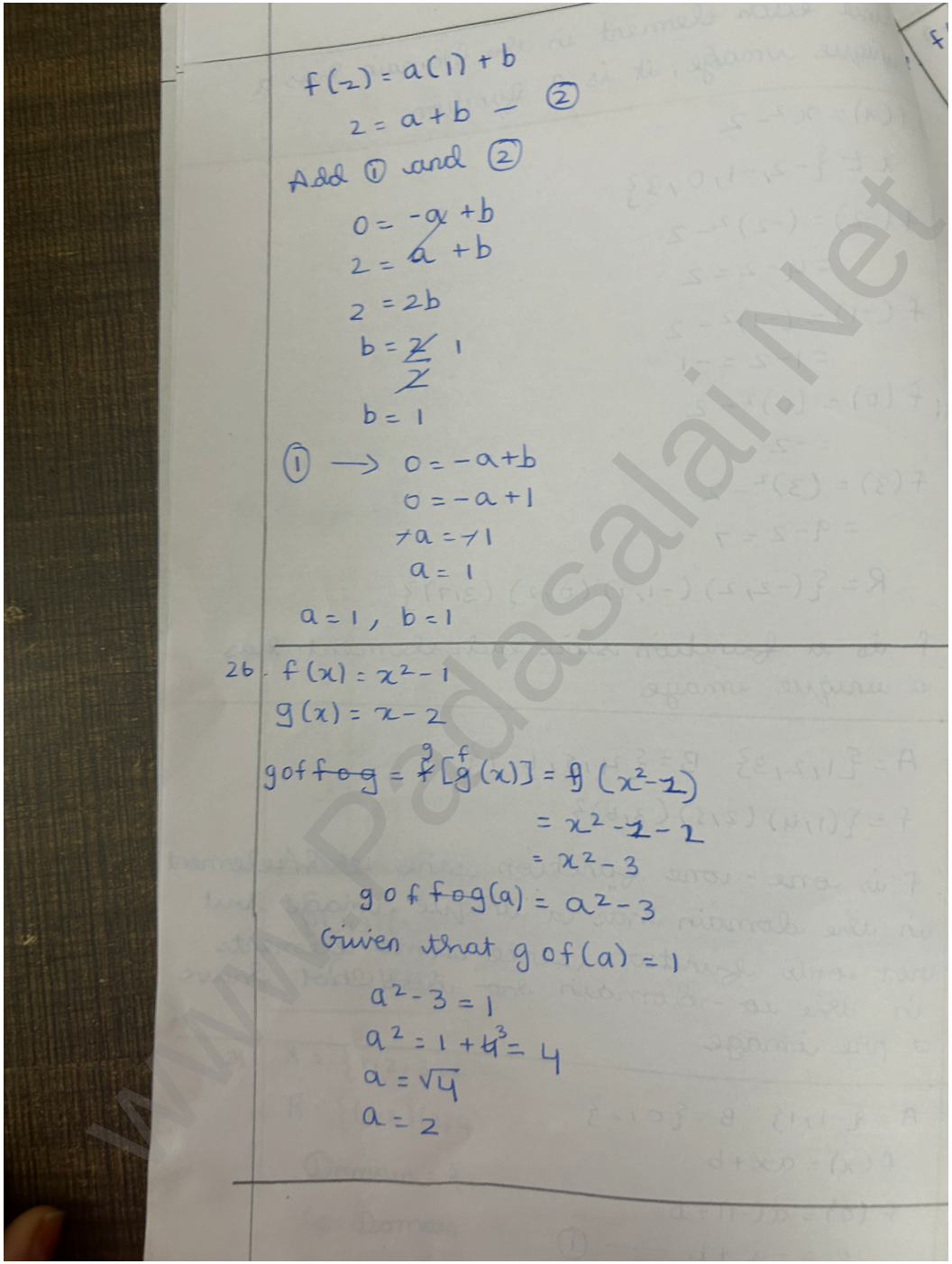
f is one - one function since each element in the domain has a unique image but not onto function since some elements in the co - domain ~~is~~ does not have a pre image.

25 $A = \{-1, 1\}$ $B = \{0, 2\}$

$f(x) = ax + b$

$f(0) = a(-1) + b$

$0 = -a + b$ —— (1)



$f(-2) = a(1) + b$

$2 = a + b$ — (2)

Add (1) and (2)

$0 = -a + b$

$2 = a + b$

$2 = 2b$

$b = \frac{2}{2} = 1$

$b = 1$

(1) $\longrightarrow$ $0 = -a + b$

$0 = -a + 1$

$+a = +1$

$a = 1$

$a = 1, \ b = 1$

26. $f(x) = x^2 - 1$

$g(x) = x - 2$

$gof = f[g(x)] = f(x^2 - 2)$

$= x^2 - 2 - 2$

$= x^2 - 3$

$gof\ fog(a) = a^2 - 3$

Given that $gof(a) = 1$

$a^2 - 3 = 1$

$a^2 = 1 + 3 = 4$

$a = \sqrt{4}$

$a = 2$

27. $f(x) = 2x+1 \quad g(x) = x^2-1$

$fog = f[g(x)] = f(x^2-1)$

$= 2(x^2-1)+1$

$= 2x^2-2+1$

$fog = 2x^2-1$

$gof = g[f(x)] = g[2x+1]$

$= (2x+1)^2-1$

$= 2x^2+4x+1^2-1$

$gof = 4x^2+4x$

$fog = 2x^2-1 \quad gof = 4x^2+4x$

28. $f(k) = 2k-1$ and $fof(k) = 5$

$f(k) = 2k-1$

$f(f(k) = f(2k-1)$

$= 2(2k-1)-1$

$= 4k-2-1$

$fof(k) = 4k-3$

$5 = 4k-3$

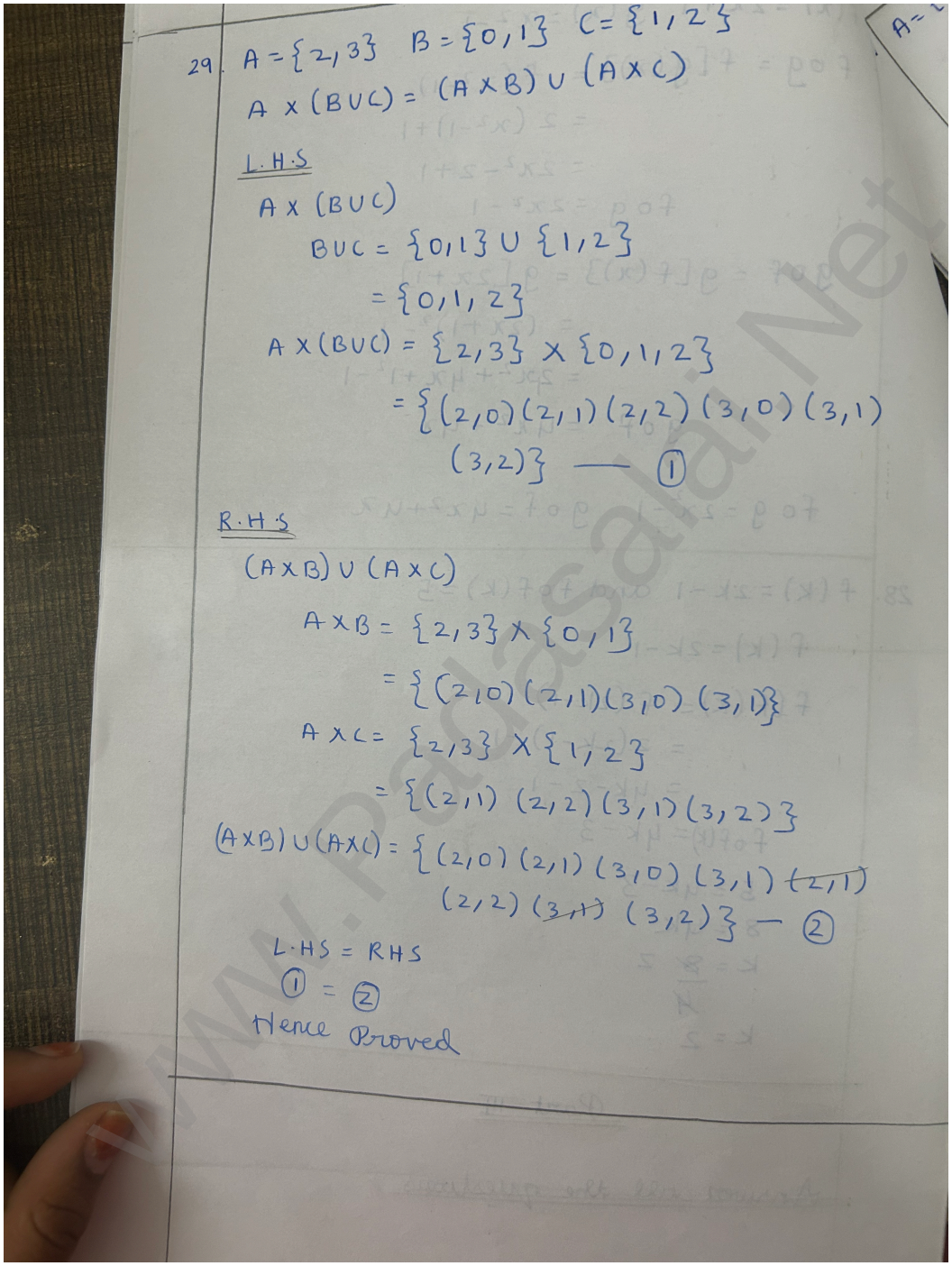
$8 = 4k$

$k = \frac{8}{4} \; 2$

$k = 2$

Part - III

Answer all the questions



29. $A = \{2,3\}$ $B = \{0,1\}$ $C = \{1,2\}$

$A \times (B \cup C) = (A \times B) \cup (A \times C)$

L.H.S

$A \times (B \cup C)$

$B \cup C = \{0,1\} \cup \{1,2\}$

$= \{0,1,2\}$

$A \times (B \cup C) = \{2,3\} \times \{0,1,2\}$

$= \{(2,0)(2,1)(2,2)(3,0)(3,1)(3,2)\}$ — ①

R.H.S

$(A \times B) \cup (A \times C)$

$A \times B = \{2,3\} \times \{0,1\}$

$= \{(2,0)(2,1)(3,0)(3,1)\}$

$A \times C = \{2,3\} \times \{1,2\}$

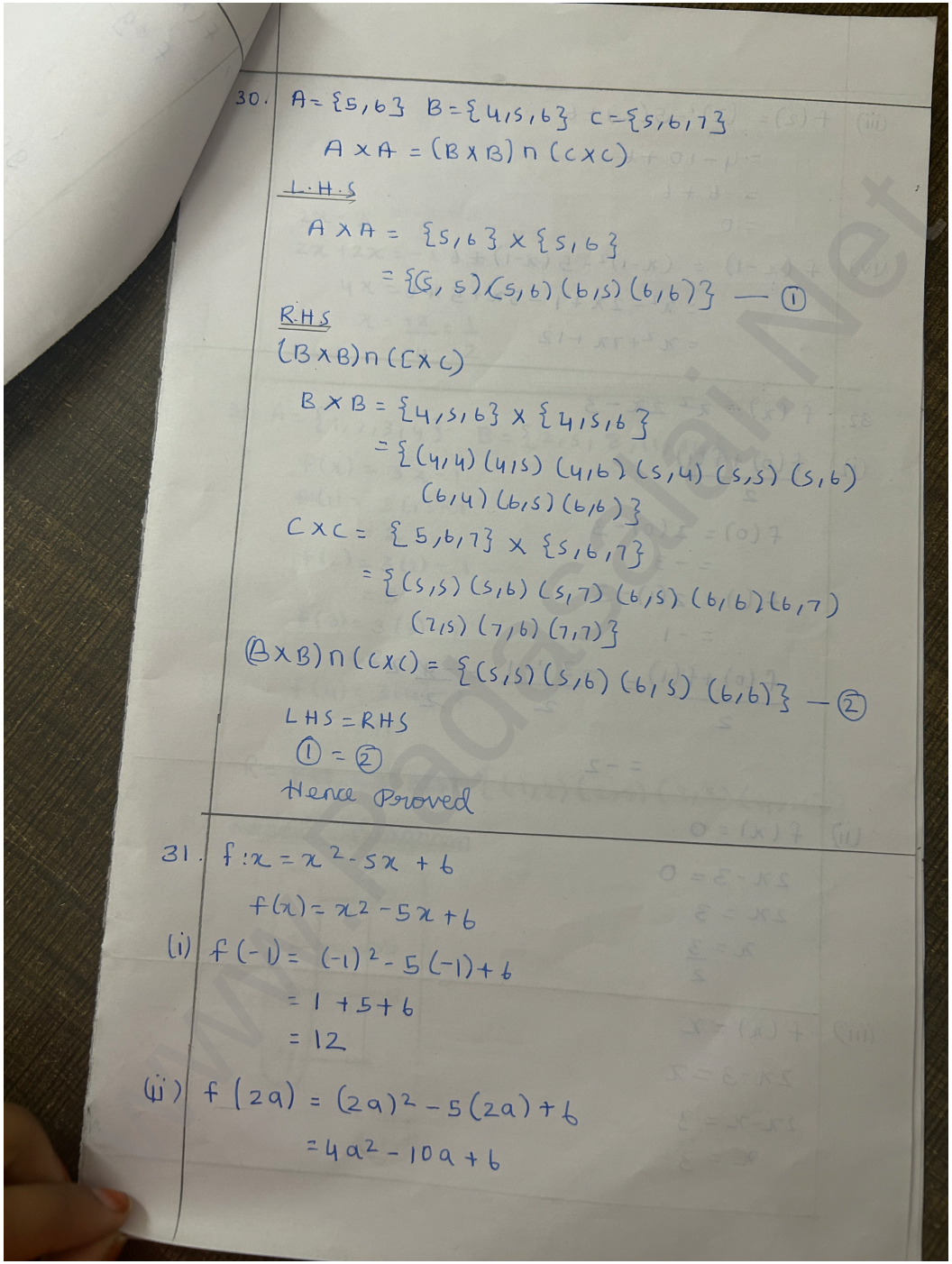
$= \{(2,1)(2,2)(3,1)(3,2)\}$

$(A \times B) \cup (A \times C) = \{(2,0)(2,1)(3,0)(3,1)$ ~~$(2,1)$~~ $(2,2)$ ~~$(3,1)$~~ $(3,2)\}$ — ②

L.H.S = RHS

① = ②

Hence Proved



30. $A=\{5,6\}\ B=\{4,5,6\}\ C=\{5,6,7\}$

$A\times A=(B\times B)\cap(C\times C)$

L.H.S

$A\times A=\{5,6\}\times\{5,6\}$

$=\{(5,5)(5,6)(6,5)(6,6)\}$ — ①

R.H.S

$(B\times B)\cap(C\times C)$

$B\times B=\{4,5,6\}\times\{4,5,6\}$

$=\{(4,4)(4,5)(4,6)(5,4)(5,5)(5,6)$
$(6,4)(6,5)(6,6)\}$

$C\times C=\{5,6,7\}\times\{5,6,7\}$

$=\{(5,5)(5,6)(5,7)(6,5)(6,6)(6,7)$
$(7,5)(7,6)(7,7)\}$

$(B\times B)\cap(C\times C)=\{(5,5)(5,6)(6,5)(6,6)\}$ — ②

LHS = RHS

① = ②

Hence Proved

31. $f:x=x^2-5x+6$

$f(x)=x^2-5x+6$

(i) $f(-1)=(-1)^2-5(-1)+6$

$=1+5+6$

$=12$

(ii) $f(2a)=(2a)^2-5(2a)+6$

$=4a^2-10a+6$