DIRECTORATE OF GOVERNEMENT EXAMINATION, CHENNAI-6
HSE (FIRST YEAR) EXAMINATIONS – MARCH - 2024
BUSINESS MATHEMATICS AND STATISTICS - ANSWER KEY

Maximum Marks : 90

GENERAL INSTRUCTIONS :

1. Answers written only in **BLACK** or **BLUE** should be evaluated.
2. For objective type questions, award 1 mark for "writing the correct option's code and the corresponding option's answer".
3. Award "0 marks" for one who wrote both "option's code" and "option's answer" with one of them is not correct.
4. Marks should be awarded for suitable alternative method also.
5. Mark(s) should not be reduced for the correct answer / stage, if it is written without formula / properties also, 2* means award one mark for the formula.
6. Award full mark directly, if the solution is arrived with no mistakes without giving weightage for the stages.
7. The stage mark is essential, only if the part of the solution is incorrect.
8. Award marks, if the answer is in decimal value and also approximately equal to the key answer
9. **Important Note for Part II, Part III and Part IV**

   For a particular stage in which the stage mark is greater than 1 and one who begins with correct step but reaches with incorrect solution, for such suitable credits should be given by breaking the stage marks.

PART – I

I. **Answer All the Questions**

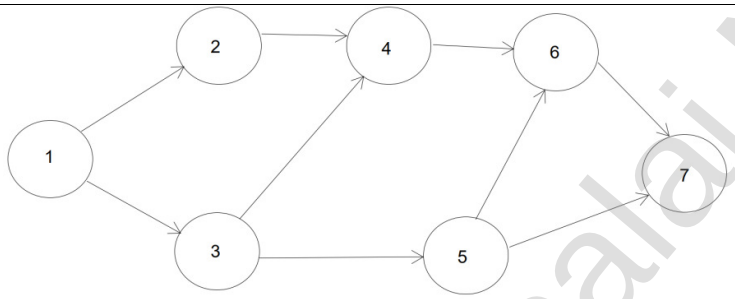
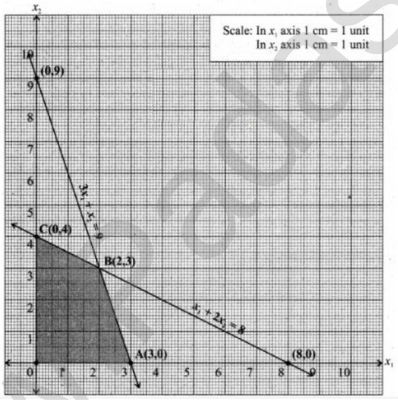
II. **Choose the most appropriate from the given FOUR alternatives and write the option code and the corresponding answer.** **20×1=20**

Q.No	Option code	Answer	Marks
1	(a)	Prof. Wassily W. Leontief	1
2	(d)	n^r	1
3	(b)	4	1
4	(b)	$r = \pm\sqrt{b_{xy} \times b_{yx}}$	1
5	(b)	$\frac{3}{25}$	1
6	(a)	3	1
7	(a)	± 3	1
8	(d)	$A = \frac{a}{i}\ [(1+i)^n-1]$	1
9	(c)	$\frac{16}{5}$	1
10	(c)	$16ae^{4x}$	1
11	(d)	$-\frac{\sqrt{3}}{2}$	1
12	(d)	$-\frac{2h}{b}$	1
13	(b)	$E_j - E_i = L_j - L_i = t_{ij}$	1
14	(d)	8	1
15	(b)	₹1600	1
16	(d)	$\frac{(n-1)!}{2}$	1
17	(a)	$\frac{P(A \cap B)}{P(A)}$	1
18	(d)	-1 to 1	1
19	(a)	$x^2+y^2 -8x-10y+32 = 0$	1
20	(b)	100	1

PART – II

Q.No.	Answer Any **SEVEN** Questions. Q.No. 30 is Compulsory.	7×2=14	
21	$6\begin{vmatrix} 1 & 3 & 4 \\ 17 & 3 & 6 \\ 17 & 3 & 6 \end{vmatrix}$	1	**2**
	= 6 (0) = 0	1	
22	n = 4+6=10	1	2
	$12C_{10} = 66$	1	
23	$f(\frac{1}{x}) = \frac{1}{x^3} - x^3$	1	2
	$f(x) + f(\frac{1}{x}) = x^3 - \frac{1}{x^3} + \frac{1}{x^3} - x^3 = 0$	1	
24	n = 11, Ascending order 2,4,6,8,10,12,14,16,18,20,22		
	$D_2 = 2^{nd}$ Term = 4, $D_6 = 7^{th}$ Term = 14	1+1	2
25	Income from 20% of shares $= \frac{20}{140}$ x (140 x 70) = 1400	1	2
	Income from 10% of shares = $\frac{10}{70}$ x (140 x 70) = 1400		
	They are equivalent shares	1	
26	$R = 4000$, $C_1 = 50$, $C_3 = 160$ $EOQ = \sqrt{\frac{2RC_3}{C_1}}$ $EOQ = 160$	1+1	2
27	Centre = (-1,3)	1	2
	=> a(-1)+2(3)+2 = 0 => a = 8	1	
28	$T_{r+1} = nC_r x^{n-r} a^r$ (or) n = 13, r = 4, x = x, a = -2y $t_{4+1} = 13C_4 x^{13-4} (-2y)^4$	1	2
	$t_5 = 13C_4 x^9 (16y^4)$ (or) $t_5 = 11440x^9y^4$	1	
29	$f(x+y) = 2^{x+y} = 2^x.2^y$	1	2
	f(x+y) = f(x).f(y) **Hence proved**	1	
30	Alphabetical Order A, B, E, L, T	1	2
	Number of words starting with A = 4! = 24 Number of words starting with B = 4! = 24 Number of words starting with E = 4! = 24 Number of words starting with L = 4! = 24 Number of words starting with TABE = 1! = 1 Number of words starting with TABLE = 0! = 1 **RANK OF THE WORD TABLE = 98** **(ANY OTHER ALTERNATE METHOD)**	1	

PART – III

Q.No	Answer Any SEVEN Questions. Q.NO. 40 is Compulsory.		7×3=21
31	coefficient of xy = 0	1	3
	b=8, a=9	1+1	
32	$\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}\times\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}$	1	3
	$\lim_{x\to 0}\frac{2x}{x(\sqrt{1+x}+\sqrt{1-x})}$	1	
	= 1	1	
33			3
34		1	3
	Corner points / **Z=40x_1+50x_2**: O(0,0) → 0; A(3,0) → 120; B(2,3) → 230; C(0,4) → 200	1	
	Z_{max} = 230 at (2,3)	1	

<table>
<tr><td>35</td><td>$f'(x) = 2x-4$
$f'(x) = 0$
x=2
<table><tr><th>Interval</th><th>Sign of $f'(x)$</th><th>Nature of the function</th></tr><tr><td>(-∞,2)</td><td><0</td><td>Strictly decreasing in (-∞,2)</td></tr><tr><td>(2,∞)</td><td>>0</td><td>Strictly increasing in (2,∞)</td></tr></table></td><td>1
1
1</td><td>3</td></tr>
<tr><td>36</td><td>tan75°= tan(45°+30°)
tan(A+B) = $\frac{tanA+tanB}{1-tanAtanB}$ (or)
tan(45°+30°) = $\frac{tan45°+tan30°}{1-tan45°tan30°}$
$=\frac{\sqrt{3}+1}{\sqrt{3}-1}$ (or) $2+\sqrt{3}$</td><td>1
1
1</td><td>3</td></tr>
<tr><td>37</td><td>H.M = $\frac{n}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}}$
$=\frac{4}{\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}$
= 1.92 kg per rupee</td><td>1
1
1</td><td>3</td></tr>
<tr><td>38</td><td>I-B = $\begin{pmatrix} 0.2 & -0.2 \\ -0.9 & 0.3 \end{pmatrix}$
$|I - B|$ = -0.12 <0
Hawkins simon conditions are not satisfied</td><td>1
1
1</td><td>3</td></tr>
<tr><td>39</td><td>a = Rs. 2000, i = 0.02 per quarterly, n=10x4 =40
(or)
a=Rs.2000, k=4 $\frac{i}{k} = 0.02$ n=10
A= $\frac{a}{i}((1+i)^n - 1)$ (or) $A = \frac{a}{i}\left[(1+\frac{i}{k})^{nk} - 1\right]$
(or) $=\frac{2000}{0.02}((1+0.02)^{40} - 1)$
= Rs. 1,20,800</td><td>1
1
1</td><td>3</td></tr>
</table>

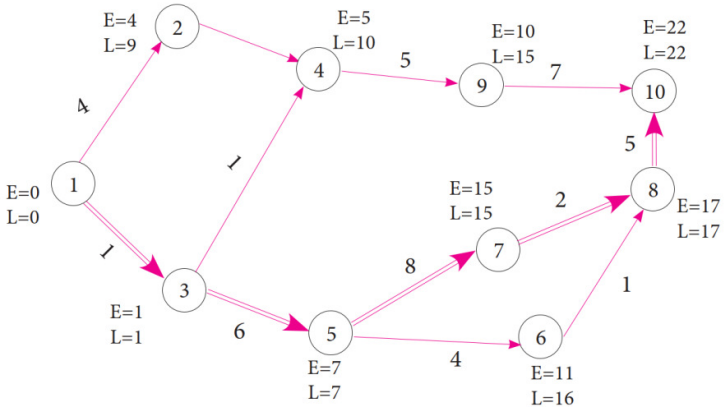
40	$tan^{-1}\ x + tan^{-1}\ y = tan^{-1}(\frac{x+y}{1-xy})$	1	3
	$tan^{-1}(\frac{2}{11}) + tan^{-1}(\frac{7}{24}) = tan^{-1}(\frac{\frac{2}{11}+\frac{7}{24}}{1-\frac{2}{11}X\frac{7}{24}})$	1	
	$= tan^{-1}(\frac{1}{2})$	1	

PART – IV

Q.No	Answer All the Questions.		7×5=35
41(a)	Technology matrix B = $\begin{bmatrix}0.4 & 0.1\\0.7 & 0.6\end{bmatrix}$	1	5
	$I - B = \begin{bmatrix}0.6 & -0.1\\-0.7 & 0.4\end{bmatrix}$ & $\lvert I - B\rvert = 0.17 > 0$	1	
	$X = (I - B)^{-1}D$ (or) $X = \frac{1}{0.17}\begin{pmatrix}0.4 & 0.1\\0.7 & 0.6\end{pmatrix}\begin{pmatrix}30\\95\end{pmatrix} = \begin{pmatrix}176.5\\558.8\end{pmatrix}$	1	
	Steel output = 176.5 tonnes Coal output = 558.8 tonnes	1	
	Total labour = 2000 labour days **(OR)**	1	
41 (b)	U is a homogeneous function of degree 3.(or) n = 3	1	5
	By Euler's theorem: $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3U$	1	
	Verification : $\frac{\partial u}{\partial x} = 3x^2 + 3y^2$	1	
	$\frac{\partial u}{\partial y} = 3y^2 + 6xy$	1	
	$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3U$ $\therefore$ Euler's theorem is verified.	1	

42 (a)	$a = 2,\ b = 3,\ h = \frac{7}{2}\ ,\ g = \frac{5}{2}\ ,\ f = \frac{5}{2},\ \ c = 2.$	1	5
	$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0,\quad \begin{vmatrix} 2 & \frac{7}{2} & \frac{5}{2} \\ \frac{7}{2} & 3 & \frac{5}{2} \\ \frac{5}{2} & \frac{5}{2} & 2 \end{vmatrix} = 0$	1	
	Hence the given equation represents a pair of straight lines.	1	
	$2x^2 + 7xy + 3y^2 + 5x + 5y + 2$ $= (x + 3y + l)(2x + y + m),$ $m = 1,\quad l = 2$	1	
	The Separate equations are $x + 3y + 2 = 0\ ,\ \ 2x + y + 1 = 0$ **(OR)**	1	
42 (b)	$\tan^{-1}\left(\frac{x+1+x-1}{1-(x+1)(x-1)}\right) = \tan^{-1}\frac{4}{7}$	2*	5 Marks
	$\frac{2x}{2-x^2} = \frac{4}{7}$	1	
	$2x^2 + 7x - 4 = 0$	1	
	$(2x-1)(x+4) = 0$ $x = \frac{1}{2}, -4\ \ but\ x \neq -4$ $\therefore x = \frac{1}{2}$	1	

43 (a)



2

Activity	Duration (Months)	EST	EFT	LFT	LST
1 - 2	4	0	4	9	5
1 - 3	1	0	1	1	0
2 - 4	1	4	5	10	9
3 - 4	1	1	2	10	9
3 - 5	6	1	7	7	1
4 - 9	5	5	10	15	10
5 - 6	4	7	11	16	12
5 - 7	8	7	15	15	7
6 - 8	1	11	12	17	16
7 - 8	2	15	17	17	15
8 - 10	5	17	22	22	17
9 - 10	7	10	17	22	15

2

5

Critical path is 1 - 3 – 5 – 7–8 – 10

Project completion time is 22.

1

(OR)

43 (b)			
		1	
	$Q_1 =$ size of $(\frac{N}{4})^{th}$ value = 16.75		5
	$Q_1 = l + \left(\frac{\frac{N}{4} - pcf}{f}\right) \times c$	1	
	$= 20 + \left(\frac{16.75 - 12}{19}\right) \times 10 = 22.5$	1	
	$Q_3 =$ size of $(\frac{3N}{4})^{th}$ value = 50.25		
	$Q_3 = l + \left(\frac{\frac{3N}{4} - pcf}{f}\right) \times c$		
	$= 50 + \left(\frac{50.25 - 46}{9}\right) \times 10 = 54.72$	1	
	$QD = \frac{1}{2}(Q_3 - Q_1) = \frac{1}{2}(54.72 - 22.5) = 16.11$	1	
44. (a)	P(n) = $1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$	1	5
	n=1 ∴P(1) is true.	1	
	n=k ∴ P(k) is true.	1	
	P(k+1) = $\frac{(k+1)(k+2)(2k+3)}{6}$ ∴ P(k+1) is true.	1	
	$\therefore 1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$ (or) P(n) is true for all nϵ N **(OR)**	1	

<table>
<tr><td>44. (b)</td><td>$x = \frac{\sin y}{\sin(a+y)}$</td><td>1</td><td rowspan="4">5</td></tr>
<tr><td></td><td>$\frac{dx}{dy} = \frac{sin(a+y).cos\,y - siny.cos(a+y)}{\left(sin(a+y)\right)^2}$</td><td>2*</td></tr>
<tr><td></td><td>$\frac{dx}{dy} = \frac{sin(a+y-y)}{sin^2(a+y)} = \frac{\sin a}{sin^2(a+y)}$</td><td>1</td></tr>
<tr><td></td><td>$\frac{dy}{dx} = \frac{sin^2(a+y)}{\sin a}$</td><td>1</td></tr>
<tr><td>45 (a)</td><td>
<table>
<tr><th>R_X</th><th>R_Y</th><th>$d = R_X - R_Y$</th><th>d^2</th></tr>
<tr><td>6</td><td>4</td><td>2</td><td>4</td></tr>
<tr><td>4</td><td>1</td><td>3</td><td>9</td></tr>
<tr><td>3</td><td>6</td><td>-3</td><td>9</td></tr>
<tr><td>1</td><td>7</td><td>-6</td><td>36</td></tr>
<tr><td>2</td><td>5</td><td>-3</td><td>9</td></tr>
<tr><td>7</td><td>8</td><td>-1</td><td>1</td></tr>
<tr><td>9</td><td>10</td><td>-1</td><td>1</td></tr>
<tr><td>8</td><td>9</td><td>-1</td><td>1</td></tr>
<tr><td>10</td><td>3</td><td>7</td><td>49</td></tr>
<tr><td>5</td><td>2</td><td>3</td><td>9</td></tr>
<tr><td></td><td></td><td></td><td>$\sum d^2 = 128$</td></tr>
</table>
</td><td>TABLE 1
1</td><td rowspan="4">5</td></tr>
<tr><td></td><td>$\rho = 1 - \frac{6\sum d^2}{N(N^2-1)} = 1 - \frac{6 \times 128}{10(100-1)}$</td><td>2*</td></tr>
<tr><td></td><td>$= 1 - \frac{768}{990} = 1 - 0.7758$</td><td></td></tr>
<tr><td></td><td>$\rho = 0.2242$
(OR)</td><td>1</td></tr>
</table>

45. (b)	**(i) For 20% stocks:** Income = $\frac{20}{100} \times 10{,}000 = ₹\,2000$	1	5
	Invest. =10,000, F.V.=100, M.V.=100+42-2 = 140 No. of shares = $\frac{10{,}000}{100} = 100$ Sale proceeds = $100 \times 140 = ₹\,14{,}000$	1	
	(ii) For 15%stocks: Market value = 100-22+2=80 No. of shares = $\frac{14000}{80} = 175$	1	
	Income = $175 \times \frac{15}{100} = ₹\,2625$	1	
	(iii) Change in Income = ₹ 2625 – ₹ 2000 = ₹ 625	1	
46. (a)	Let the three numbers be x, y and z $x + y + z = 20, \quad 2x + y - z = 23, \quad 3x + y + z = 46$	1	5
	$\lvert A \rvert = -4 \neq 0$	1	
	$A^{-1} = \frac{-1}{4}\begin{pmatrix} 2 & 0 & -2 \\ -5 & -2 & 3 \\ -1 & 2 & -1 \end{pmatrix}$	1	
	$X = A^{-1}B$ (or) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{-1}{4}\begin{pmatrix} 2 & 0 & -2 \\ -5 & -2 & 3 \\ -1 & 2 & -1 \end{pmatrix}\begin{pmatrix} 20 \\ 23 \\ 46 \end{pmatrix} = \frac{-1}{4}\begin{pmatrix} -52 \\ -8 \\ -20 \end{pmatrix}$	1	
	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 13 \\ 2 \\ 5 \end{pmatrix}$ ∴The three numbers are 13, 2, 5 **(OR)**	1	

46. (b)	$P(A_1) = \frac{1000}{6000} = \frac{1}{6}, \quad P(A_2) = \frac{2000}{6000} = \frac{1}{3},$ $P(A_3) = \frac{3000}{6000} = \frac{1}{2}$	1	5
	$P\left(B/A_1\right) = 1\% = 0.01, P\left(B/A_2\right) = 1.5\% = 0.015,$ $P\left(B/A_1\right) = 2\% = 0.02$	1	
	By Baye's theorem $P\left(A_1/B\right) = \frac{\frac{1}{6} \times 0.01}{\frac{1}{6} \times 0.01 + \frac{1}{3} \times 0.015 + \frac{1}{2} \times 0.02}$	2*	
	$= \frac{1}{10} (or)\ 0.1$	1	
47. (a)	$x^2 + y^2 + 2gx + 2fy + c = 0$ $2g + c = -1, \quad -2g + c = -1, \quad 2f + c = -1$	1 1	5
	$c = -1,\ g = 0,\ f = 0$	2	
	The Equation of the circle is $x^2 + y^2 - 1 = 0$ **(any Alternate Method award Full Marks)** **(OR)**	1	
47. (b)	$\frac{x-2}{(x+2)(x-1)^2} = \frac{A}{x+2} + \frac{B}{x-1} + \frac{c}{(x-1)^2}$	1	5
	$x - 2 = A(x-1)^2 + B(x+2)(x-1) + C(x+2)$ $A = \frac{-4}{9}, \quad B = \frac{4}{9}, \quad C = \frac{-1}{3}$	3	
	$\frac{x-2}{(x+2)(x-1)^2} = \frac{-4}{9(x+2)} + \frac{4}{9(x-1)} - \frac{1}{3(x-1)^2}$	1	