Exercise 1.4

Choose the correct answer

1. If $A=(1\ \ 2\ \ 3)$, then the rank of AA^T is

(a) 0 (b) 2 (c) 3 (d) 1

2. The rank of $m\times n$ matrix whose elements are unity is

(a) 0 (b) 1 (c) m (d) n

3. If $T = \begin{matrix} & A & B \\ A & 0.4 & 0.6 \\ B & 0.2 & 0.8 \end{matrix}$ is a transition probability matrix, then at equilibrium A is equal to

(a) $\frac{1}{4}$ (b) $\frac{1}{5}$ (c) $\frac{1}{6}$ (d) $\frac{1}{8}$

4. If $A=\begin{pmatrix} 2 & 0 \\ 0 & 8 \end{pmatrix}$, then $\rho(A)$ is

(a) 0 (b) 1 (c) 2 (d) n

5. The rank of the matrix $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix}$ is

(a) 0 (b) 1 (c) 2 (d)3

6. The rank of the unit matrix of order n is

(a) $n-1$ (b) n (c) $n+1$ (d) n^2

7. If $\rho(A)=r$ then which of the following is correct?

(a) all the minors of order r which does not vanish

(b) A has at least one minor of order r which does not vanish

(c) A has at least one $(r+1)$ order minor which vanishes

(d) all $(r+1)$ and higher order minors should not vanish

8. If $A=\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ then the rank of AA^T is

(a) 0 (b) 1 (c) 2 (d) 3

9. If the rank of the matrix $\begin{pmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ -1 & 0 & \lambda \end{pmatrix}$ is 2. Then λ is

(a) 1 (b) 2 (c) 3 (d) only real number

10. The rank of the diagonal matrix $\begin{pmatrix} 1 & & & & & \\ & 2 & & & & \\ & & -3 & & & \\ & & & 0 & & \\ & & & & 0 & \\ & & & & & 0 \end{pmatrix}$

(a) 0 (b) 2 (c) 3 (d) 5

11. If $T = \begin{matrix} & A & B \\ A & 0.7 & 0.3 \\ B & 0.6 & x \end{matrix}$ is a transition probability matrix, then the value of x is

(a) 0.2 (b) 0.3 (c) 0.4 (d) 0.7

12. Which of the following is not an elementary transformation?

(a) $R_i \leftrightarrow R_j$ (b) $R_i \rightarrow 2R_i + 2C_j$

(c) $R_i \rightarrow 2R_i - 4R_j$ (d) $C_i \rightarrow C_i + 5C_j$

13. If $\rho(A) = \rho(A, B)$ then the system is

(a) Consistent and has infinitely many solutions

(b) Consistent and has a unique solution

(c) Consistent (d) inconsistent

14. If $\rho(A) = \rho(A, B) =$ the number of unknowns, then the system is

(a) Consistent and has infinitely many solutions

(b) Consistent and has a unique solution

(c) inconsistent (d) consistent

15. If $\rho(A) \neq \rho(A, B)$, then the system is

(a) Consistent and has infinitely many solutions

(b) Consistent and has a unique solution

(c) inconsistent (d) consistent

16. In a transition probability matrix, all the entries are greater than or equal to

(a) 2 (b) 1 (c) 0 (d) 3

17. If the number of variables in a non- homogeneous system $AX = B$ is n, then the system possesses a unique solution only when

(a) $\rho(A) = \rho(A, B) > n$ (b) $\rho(A) = \rho(A, B) = n$

(c) $\rho(A) = \rho(A, B) < n$ (d) none of these

18. The system of equations $4x + 6y = 5, 6x + 9y = 7$ has

(a) a unique solution (b) no solution

(c) infinitely many solutions (d) none of these

19. For the system of equations $x + 2y + 3z = 1, \quad 2x + y + 3z = 2 \; 5x + 5y + 9z = 4$

(a) there is only one solution

(b) there exists infinitely many solutions

(c) there is no solution (d) None of these

20. If $|A| \neq 0$, then A is

(a) non- singular matrix (b) singular matrix

(c) zero matrix (d) none of these

21. The system of linear equations $x + y + z = 2, \; 2x + y - z = 3, \; 3x + 2y + k = 4$ has unique solution, if k is not equal to

(a) 4 (b) 0 (c) –4 (d) 1

22. Cramer's rule is applicable only to get an unique solution when

(a) $\Delta_z \neq 0$ (b) $\Delta_x \neq 0$ (c) $\Delta \neq 0$ (d) $\Delta_y \neq 0$

23. If $\frac{a_1}{x} + \frac{b_1}{y} = c_1, \frac{a_2}{x} + \frac{b_2}{y} = c_2, \Delta_1 = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}; \; \Delta_2 = \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} \; \Delta_3 = \begin{vmatrix} c_1 & a_1 \\ c_2 & a_2 \end{vmatrix}$ then (x, y) is

(a) $\left(\frac{\Delta_2}{\Delta_1}, \frac{\Delta_3}{\Delta_1}\right)$ (b) $\left(\frac{\Delta_3}{\Delta_1}, \frac{\Delta_2}{\Delta_1}\right)$ (c) $\left(\frac{\Delta_1}{\Delta_2}, \frac{\Delta_1}{\Delta_3}\right)$ (d) $\left(\frac{-\Delta_1}{\Delta_2}, \frac{-\Delta_1}{\Delta_3}\right)$

24. $|A_{n\times n}| = 3 \quad |adjA| = 243$ then the value n is

(a) 4 (b) 5 (c) 6 (d) 7

25. Rank of a null matrix is

(a) 0 (b) –1 (c) ∞ (d) 1

Exercise 2.12

Choose the correct answer:

1. $\int \frac{1}{x^3} dx$ is

(a) $\frac{-3}{x^2}+c$ (b) $\frac{-1}{2x^2}+c$ (c) $\frac{-1}{3x^2}+c$ (d) $\frac{-2}{x^2}+c$

2. $\int 2^x dx$ is

(a) $2^x \log 2+c$ (b) 2^x+c (c) $\frac{2^x}{\log 2}+c$ (d) $\frac{\log 2}{2^x}+c$

3. $\int \frac{\sin 2x}{2\sin x} dx$ is

(a) $\sin x+c$ (b) $\frac{1}{2}\sin x+c$ (c) $\cos x+c$ (d) $\frac{1}{2}\cos x+c$

4. $\int \frac{\sin 5x-\sin x}{\cos 3x} dx$ is

(a) $-\cos 2x+c$ (b) $-\cos 2x+c$ (c) $-\frac{1}{4}\cos 2x+c$ (d) $-4\cos 2x+c$

5. $\int \frac{\log x}{x} dx$ $, x>0$ is

(a) $\frac{1}{2}(\log x)^2+c$ (b) $-\frac{1}{2}(\log x)^2$ (c) $\frac{2}{x^2}+c$ (d) $\frac{2}{x^2}+c$

6. $\int \frac{e^x}{\sqrt{1+e^x}} dx$ is

(a) $\frac{e^x}{\sqrt{1+e^x}}+c$ (b) $2\sqrt{1+e^x}+c$ (c) $\sqrt{1+e^x}+c$ (d) $e^x\sqrt{1+e^x}+c$

7. $\int \sqrt{e^x}\, dx$ is

(a) $\sqrt{e^x}+c$ (b) $2\sqrt{e^x}+c$ (c) $\frac{1}{2}\sqrt{e^x}+c$ (d) $\frac{1}{2\sqrt{e^x}}+c$

8. $\int e^{2x}[2x^2+2x]dx$

(a) $e^{2x}x^2+c$ (b) $xe^{2x}+c$ (c) $2x^2e^2+c$ (d) $\frac{x^2e^x}{2}+c$

9. $\int \frac{e^x}{e^x+1} dx$ is

(a) $\log\left|\frac{e^x}{e^x+1}\right|+c$ (b) $\log\left|\frac{e^x+1}{e^x}\right|+c$ (c) $\log\left|e^x\right|+c$ (d) $\log\left|e^x+1\right|+c$

10. $\int\left[\frac{9}{x-3}-\frac{1}{x+1}\right] dx$ is

(a) $\log|x-3|-\log|x+1|+c$ (b) $\log|x-3|+\log|x+1|+c$

(c) $9\log|x-3|-\log|x+1|+c$ (d) $9\log|x-3|+\log|x+1|+c$

11. $\int \frac{2x^3}{4+x^4}\,dx$ is

(a) $\log\left|4+x^4\right|+c$ (b) $\frac{1}{2}\log\left|4+x^4\right|+c$ (c) $\frac{1}{4}\log\left|4+x^4\right|+c$ (d) $\log\left|\frac{2x^3}{4+x^4}\right|+c$

12. $\int \frac{dx}{\sqrt{x^2-36}}$ is

(a) $\sqrt{x^2-36}+c$ (b) $\log\left|x+\sqrt{x^2-36}\right|+c$

(c) $\log\left|x-\sqrt{x^2-36}\right|+c$ (d) $\log\left|x^2+\sqrt{x^2-36}\right|+c$

13. $\int \frac{2x+3}{\sqrt{x^2+3x+2}}\,dx$ is

(a) $\sqrt{x^2+3x+2}+c$ (b) $2\sqrt{x^2+3x+2}+c$

(c) $\log\left(x^2+3x+2\right)+c$ (d) $\frac{2}{3}\left(x^2+3x+2\right)^{\frac{3}{2}}+c$

14. $\int_0^1 (2x+1)\,dx$ is

(a) 1 (b) 2 (c) 3 (d) 4

15. $\int_2^4 \frac{dx}{x}$ is

(a) log 4 (b) 0 (c) log 2 (d) log 8

16. $\int_0^\infty e^{-2x}\,dx$ is

(a) 0 (b) 1 (c) 2 (d) $\frac{1}{2}$

17. $\int_{-1}^{1} x^3 e^{x^4}\,dx$ is

(a) 1 (b) $2\int_0^1 x^3 e^{x^4}\,dx$ (c) 0 (d) e^{x^4}

18. If $f(x)$ is a continuous function and $a<c<b$, then $\int_a^c f(x)\,dx+\int_c^b f(x)\,dx$ is

(a) $\int_a^b f(x)\,dx-\int_a^c f(x)\,dx$ (b) $\int_a^c f(x)\,dx-\int_a^b f(x)\,dx$

(c) $\int_a^b f(x)\,dx$ (d) 0

19. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x\,dx$ is

(a) 0 (b) 2 (c) 1 (d) 4

20. $\int_0^1 \sqrt{x^4(1-x)^2}\, dx$ is

(a) $\frac{1}{12}$ (b) $\frac{-7}{12}$ (c) $\frac{7}{12}$ (d) $\frac{-1}{12}$

21. If $\int_0^1 f(x)\,dx = 1$, $\int_0^1 x\, f(x)\,dx = a$ and $\int_0^1 x^2\, f(x)\,dx = a^2$, then $\int_0^1 (a-x)^2\, f(x)\,dx$ is

(a) $4a^2$ (b) 0 (c) $2a^2$ (d) 1

22. The value of $\int_2^3 f(5-x)\,dx - \int_2^3 f(x)\,dx$ is

(a) 1 (b) 0 (c) -1 (d) 5

23. $\int_0^4 \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) dx$ is

(a) $\frac{20}{3}$ (b) $\frac{21}{3}$ (c) $\frac{28}{3}$ (d) $\frac{1}{3}$

24. $\int_0^{\frac{\pi}{3}} \tan x\, dx$ is

(a) log 2 (b) 0 (c) $\log \sqrt{2}$ (d) 2 log 2

25. Using the factorial representation of the gamma function, which of the following is the solution for the gamma function $\Gamma(n)$ when $n = 8$

(a) 5040 (b) 5400 (c) 4500 (d) 5540

26. $\Gamma(n)$ is

(a) $(n-1)!$ (b) $n!$ (c) $n\Gamma(n)$ (d) $(n-1)\Gamma(n)$

27. $\Gamma(1)$ is

(a) 0 (b) 1 (c) n (d) $n!$

28. If $n > 0$, then $\Gamma(n)$ is

(a) $\int_0^1 e^{-x} x^{n-1} dx$ (b) $\int_0^1 e^{-x} x^{n} dx$ (c) $\int_0^\infty e^{x} x^{-n} dx$ (d) $\int_0^\infty e^{-x} x^{n-1} dx$

29. $\Gamma\left(\frac{3}{2}\right)$

(a) $\sqrt{\pi}$ (b) $\frac{\sqrt{\pi}}{2}$ (c) $2\sqrt{\pi}$ (d) $\frac{3}{2}$

30. $\int_0^\infty x^4 e^{-x} dx$ is

(a) 12 (b) 4 (c) 4! (d) 64

Exercise 3.4

Choose the best answer form the given alternatives

1. Area bounded by the curve $y=x(4-x)$ between the limits 0 and 4 with $x-$ axis is

   (a) $\frac{30}{3}$ sq.units (b) $\frac{31}{2}$ sq.units (c) $\frac{32}{3}$ sq.units (d) $\frac{15}{2}$ sq.units

2. Area bounded by the curve $y=e^{-2x}$ between the limits $0\le x\le\infty$ is

   (a) 1 sq.units (b) $\frac{1}{2}$ sq.unit (c) 5 sq.units (d) 2 sq.units

3. Area bounded by the curve $y=\frac{1}{x}$ between the limits 1 and 2 is

   (a) log2 sq.units (b) log5 sq.units (c) log3 sq.units (d) log 4 sq.units

4. If the marginal revenue function of a firm is MR=$e^{\frac{-x}{10}}$, then revenue is

   (a) $-10e^{\frac{-x}{10}}$ (b) $1-e^{\frac{-x}{10}}$ (c) $10\left(1-e^{\frac{-x}{10}}\right)$ (d) $e^{\frac{-x}{10}}+10$

5. If MR and MC denotes the marginal revenue and marginal cost functions, then the profit functions is

   (a) $P=\int(MR-MC)\,dx+k$ (b) $P=\int(MR+MC)\,dx+k$

   (c) $P=\int(MR)(MC)dx+k$ (d) $P=\int(R-C)dx+k$

6. The demand and supply functions are given by $D(x)=16-x^2$ *and* $S(x)=2x^2+4$ are under perfect competition, then the equilibrium price x is

   (a) 2 (b) 3 (c) 4 (d) 5

7. The marginal revenue and marginal cost functions of a company are $MR=30-6x$ and $MC=-24+3x$ where x is the product, then the profit function is

   (a) $9x^2+54x$ (b) $9x^2-54x$ (c) $54x-\frac{9x^2}{2}$ (d) $54x-\frac{9x^2}{2}+k$

8. The given demand and supply function are given by $D(x)=20-5x$ *and* $S(x)=4x+8$ if they are under perfect competition then the equilibrium demand is

   (a) 40 (b) $\frac{41}{2}$ (c) $\frac{40}{3}$ (d) $\frac{41}{5}$

9. If the marginal revenue $MR=35+7x-3x^2$, then the average revenue AR is

   (a) $35x+\frac{7x^2}{2}-x^3$ (b) $35+\frac{7x}{2}-x^2$

   (c) $35+\frac{7x}{2}+x^2$ (d) $35+7x+x^2$

10. The profit of a function $p(x)$ is maximum when

(a) $MC-MR=0$ (b) $MC=0$ (c) $MR=0$ (d) $MC+MR=0$

11. For the demand function $p(x)$, the elasticity of demand with respect to price is unity then

(a) revenue is constant (b) cost function is constant

(c) profit is constant (d) none of these

12. The demand function for the marginal function $MR=100-9x^2$ is

(a) $100-3x^2$ (b) $100x-3x^2$ (c) $100x-9x^2$ (d) $100+9x^2$

13. When $x_0=5$ *and* $p_0=3$ the consumer's surplus for the demand function $p_d=28-x^2$ is

(a) 250 units (b) $\frac{250}{3}$ *units* (c) $\frac{251}{2}$ *units* (d) $\frac{251}{3}$ *units*

14. When $x_0=2$ *and* $P_0=12$ the producer's surplus for the supply function $P_s=2x^2+4$ is

(a) $\frac{31}{5}$ *units* (b) $\frac{31}{2}$ *units* (c) $\frac{32}{3}$ *units* (d) $\frac{30}{7}$ *units*

15. Area bounded by $y=x$ between the lines $y=1, y=2$ with $y=axis$ is

(a) $\frac{1}{2}$ sq.units (b) $\frac{5}{2}$ sq.units (c) $\frac{3}{2}$ sq.units (d) 1 sq.unit

16. The producer's surplus when the supply function for a commodity is $P=3+x$ *and* $x_0=3$ is

(a) $\frac{5}{2}$ (b) $\frac{9}{2}$ (c) $\frac{3}{2}$ (d) $\frac{7}{2}$

17. The marginal cost function is $MC=100\sqrt{x}$. find AC given that TC =0 when the out put is zero is

(a) $\frac{200}{3}x^{\frac{1}{2}}$ (b) $\frac{200}{3}x^{\frac{3}{2}}$ (c) $\frac{200}{3x^{\frac{3}{2}}}$ (d) $\frac{200}{3x^{\frac{1}{2}}}$

18. The demand and supply function of a commodity are $P(x)=(x-5)^2$ and $S(x)=x^2+x+3$ then the equilibrium quantity x_0 is

(a) 5 (b) 2 (c) 3 (d) 19

19. The demand and supply function of a commodity are $D(x)=25-2x$ and $S(x)=\frac{10+x}{4}$ then the equilibrium price P_0 is

(a) 5 (b) 2 (c) 3 (d) 10

20. If MR and MC denote the marginal revenue and marginal cost and $MR - MC = 36x - 3x^2 - 81$, then the maximum profit at x is equal to

(a) 3 (b) 6 (c) 9 (d)5

21. If the marginal revenue of a firm is constant, then the demand function is

(a) MR (b) MC (c) $C(x)$ (d) AC

22. For a demand function p, if $\int \frac{dp}{p} = k\int \frac{dx}{x}$ then k is equal to

(a) η_d (b) $-\eta_d$ (c) $\frac{-1}{\eta_d}$ (d) $\frac{1}{\eta_d}$

23. Area bounded by $y = e^x$ between the limits 0 to 1 is

(a) $(e-1)$ sq.units (b) $(e+1)$ sq.units (c) $\left(1-\frac{1}{e}\right)$ sq.units (d) $\left(1+\frac{1}{e}\right)$ sq.units

24. The area bounded by the parabola $y^2 = 4x$ bounded by its latus rectum is

(a) $\frac{16}{3}$ *sq.units* (b) $\frac{8}{3}$ *sq.units* (c) $\frac{72}{3}$ *sq.units* (d) $\frac{1}{3}$ *sq.units*

25. Area bounded by $y = |x|$ between the limits 0 and 2 is

(a) 1sq.units (b) 3 sq.units (c) 2 sq.units (d) 4 sq.units

Exercise 4.6

Choose the Correct answer

1. The degree of the differential equation $\frac{d^4y}{dx^4}-\left(\frac{d^2y}{dx^2}\right)^4+\frac{dy}{dx}=3$

   (a) 1 (b) 2 (c) 3 (d) 4

2. The order and degree of the differential equation $\sqrt{\frac{d^2y}{dx^2}}=\sqrt{\frac{dy}{dx}+5}$ are respectively

   (a) 2 and 3 (b) 3 and 2 (c) 2 and 1 (d) 2 and 2

3. The order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^{\frac{3}{2}}-\sqrt{\left(\frac{dy}{dx}\right)}-4=0$ are respectively.

   (a) 2 and 6 (b) 3 and 6 (c) 1 and 4 (d) 2 and 4

4. The differential equation $\left(\frac{dx}{dy}\right)^3+2y^{\frac{1}{2}}=x$ is

   (a) of order 2 and degree 1 (b) of order 1 and degree 3
   (c) of order 1 and degree 6 (d) of order 1 and degree 2

5. The differential equation formed by eliminating a and b from $y=ae^x+be^{-x}$ is

   (a) $\frac{d^2y}{dx^2}-y=0$ (b) $\frac{d^2y}{dx^2}-\frac{dy}{dx}=0$ (c) $\frac{d^2y}{dx^2}=0$ (d) $\frac{d^2y}{dx^2}-x=0$

6. If $y=cx+c-c^3$ then its differential equation is

   (a) $y=x\frac{dy}{dx}+\frac{dy}{dx}-\left(\frac{dy}{dx}\right)^3$ (b) $y+\left(\frac{dy}{dx}\right)^3=x\frac{dy}{dx}-\frac{dy}{dx}$
   (c) $\frac{dy}{dx}+y=\left(\frac{dy}{dx}\right)^3-x\frac{dy}{dx}$ (d) $\frac{d^3y}{dx^3}=0$

7. The integrating factor of the differential equation $\frac{dx}{dy}+Px=Q$ is

   (a) $e^{\int Pdx}$ (b) $e^{\int Pdx}$ (c) $\int Pdy$ (d) $e^{\int Pdy}$

8. The complementary function of $(D^2+4)y=e^{2x}$ is

   (a) $(Ax+B)e^{2x}$ (b) $(Ax+B)e^{-2x}$ (c) $A\cos 2x+B\sin 2x$ (d) $Ae^{-2x}+Be^{2x}$

9. The differential equation of $y=mx+c$ is (m and c are arbitrary constants)

   (a) $\frac{d^2y}{dx^2}=o$ (b) $y=x\frac{dy}{dx}+c$ (c) $xdy+ydx=0$ (d) $ydx-xdy=0$

10. The particular integral of the differential equation is $\frac{d^2y}{dx^2}-8\frac{dy}{dx}+16y=2e^{4x}$

(a) $\frac{x^2e^{4x}}{2!}$ (b) $\frac{e^{4x}}{2!}$ (c) x^2e^{4x} (d) xe^{4x}

11. Solution of $\frac{dx}{dy}+Px=0$

`(a) $x=ce^{py}$ (b) $x=ce^{-py}$ (c) $x=py+c$ (d) $x=cy$

12. If $\sec^2 x$ is an integrating factor of the differential equation $\frac{dy}{dx}+Py=Q$ then $P=$

(a) $2\tan x$ (b) $\sec x$ (c) $\cos^2 x$ (d) $\tan^2 x$

13. The integrating factor of $x\frac{dy}{dx}-y=x^2$ is

(a) $\frac{-1}{x}$ (b) $\frac{1}{x}$ (c) $\log x$ (d) x

14. The solution of the differential equation $\frac{dy}{dx}+Py=Q$ where P and Q are the function of x is

(a) $y=\int Qe^{\int Pdx}dx+c$ (b) $y=\int Qe^{-\int Pdx}dx+c$

(c) $ye^{\int Pdx}=\int Qe^{\int Pdx}dx+c$ (d) $ye^{\int Pdx}=\int Qe^{-\int Pdx}dx+C$

15. The differential equation formed by eliminating A and B from $y=e^{-2x}(A\cos x+B\sin x)$ is

(a) $y_2-4y_1+5=0$ (b) $y_2+4y-5=0$

(c) $y_2-4y_1-5=0$ (d) $y_2+4y_1+5=0$

16. The particular integral of the differential equation $f(D)y=e^{ax}$ where $f(D)=(D-a)^2$

(a) $\frac{x^2}{2}e^{ax}$ (b) xe^{ax} (c) $\frac{x}{2}e^{ax}$ (d) x^2e^{ax}

17. The differential equation of $x^2+y^2=a^2$

(a) $xdy+ydx=0$ (b) $ydx-xdy=0$ (c) $xdx-ydx=0$ (d) $xdx+ydy=0$

18. The complementary function of $\frac{d^2y}{dx^2}-\frac{dy}{dx}=0$ is

(a) $A+Be^x$ (b) $(A+B)e^x$ (c) $(Ax+B)e^x$ (d) Ae^x+B

19. The P.I of $(3D^2+D-14)y=13e^{2x}$ is

(a) $\frac{x}{2}e^{2x}$ (b) xe^{2x} (c) $\frac{x^2}{2}e^{2x}$ (d) $13xe^{2x}$

20. The general solution of the differential equation $\frac{dy}{dx} = \cos x$ is

(a) $y = \sin x + 1$ (b) $y = \sin x - 2$

(c) $y = \cos x + c$, c is an arbitrary constant

(d) $y = \sin x + c$, c is an arbitrary constant

21. A homogeneous differential equation of the form $\frac{dy}{dx} = f\left(\frac{y}{x}\right)$ can be solved by making substitution,

(a) $y = v\,x$ (b) $v = y\,x$ (c) $x = v\,y$ (d) $x = v$

22. A homogeneous differential equation of the form $\frac{dx}{dy} = f\left(\frac{x}{y}\right)$ can be solved by making substitution,

(a) $x = v\,y$ (b) $y = v\,x$ (c) $y = v$ (d) $x = v$

23. The variable separable form of $\frac{dy}{dx} = \frac{y(x-y)}{x(x+y)}$ by taking $y = vx$ *and* $\frac{dy}{dx} = v + x\frac{dv}{dx}$ is

(a) $\frac{2v^2}{1+v}dv = \frac{dx}{x}$ (b) $\frac{2v^2}{1+v}dv = -\frac{dx}{x}$

(c) $\frac{2v^2}{1-v}dv = \frac{dx}{x}$ (d) $\frac{1+v}{2v^2}dv = -\frac{dx}{x}$

24. Which of the following is the homogeneous differential equation?

(a) $(3x-5)\,dx = (4y-1)\,dy$ (b) $xy\,dx - (x^3 + y^3)\,dy = 0$

(c) $y^2\,dx + (x^2 - xy - y^2)\,dy = 0$ (d) $(x^2 + y)\,dx = (y^2 + x)\,dy$

25. The solution of the differential equation $\frac{dy}{dx} = \frac{y}{x} + \frac{f\left(\frac{y}{x}\right)}{f'\left(\frac{y}{x}\right)}$ is

(a) $f\left(\frac{y}{x}\right) = kx$ (b) $x\,f\left(\frac{y}{x}\right) = k$ (c) $f\left(\frac{y}{x}\right) = ky$ (d) $y\,f\left(\frac{y}{x}\right) = k$

Exercise 5.3

Choose the correct Answer

1. $\Delta^2 y_0 =$

(a) $y_2 - 2y_1 + y_0$ (b) $y_2 + 2y_1 - y_0$ (c) $y_2 + 2y_1 + y_0$ (d) $y_2 + y_1 + 2y_0$

2. $\Delta f(x) =$

(a) $f(x+h)$ (b) $f(x) - f(x+h)$ (c) $f(x+h) - f(x)$ (d) $f(x) - f(x-h)$

3. $E \equiv$

(a) $1+\Delta$ (b) $1-\Delta$ (c) $1+\nabla$ (d) $1-\nabla$

4. If $h=1$, then $\Delta(x^2) =$

(a) $2x$ (b) $2x-1$ (c) $2x+1$ (d) 1

5. If c is a constant then $\Delta c =$

(a) c (b) Δ (c) Δ^2 (d) 0

6. If m and n are positive integers then $\Delta^m \Delta^n f(x) =$

(a) $\Delta^{m+n} f(x)$ (b) $\Delta^m f(x)$ (c) $\Delta^n f(x)$ (d) $\Delta^{m-n} f(x)$

7. If 'n' is a positive integer $\Delta^n \left[\Delta^{-n} f(x)\right]$

(a) $f(2x)$ (b) $f(x+h)$ (c) $f(x)$ (d) $\Delta f(x)$

8. $E f(x) =$

(a) $f(x-h)$ (b) $f(x)$ (c) $f(x+h)$ (d) $f(x+2h)$

9. $\nabla \equiv$

(a) 1+E (b) 1-E (c) $1-E^{-1}$ (d) $1+E^{-1}$

10. $\nabla f(a) =$

(a) $f(a) + f(a-h)$ (b) $f(a) - f(a+h)$

(c) $f(a) - f(a-h)$ (d) $f(a)$

11. For the given points (x_0, y_0) and (x_1, y_1) the Lagrange's formula is

(a) $y(x) = \frac{x - x_1}{x_0 - x_1} y_0 + \frac{x - x_0}{x_1 - x_0} y_1$ (b) $y(x) = \frac{x_1 - x}{x_0 - x_1} y_0 + \frac{x - x_0}{x_1 - x_0} y_1$

(c) $y(x) = \frac{x - x_1}{x_0 - x_1} y_1 + \frac{x - x_0}{x_1 - x_0} y_0$ (d) $y(x) = \frac{x_1 - x}{x_0 - x_1} y_1 + \frac{x - x_0}{x_1 - x_0} y_0$

12. Lagrange's interpolation formula can be used for

(a) equal intervals only (b) unequal intervals only

(c) both equal and unequal intervals (d) none of these.

13. If $f(x) = x^2 + 2x + 2$ and the interval of differencing is unity then $\Delta f(x)$

(a) $2x - 3$ (b) $2x + 3$ (c) $x + 3$ (d) $x - 3$

14. For the given data find the value of $\Delta^3 y_0$ is

x	5	6	9	11
y	12	13	15	18

(a) 1 (b) 0 (c) 2 (d) –1