CO

COMMON SECOND MID-TERM TEST - 2019

Reg.No.: | | | | | | |

Standard XII

MATHEMATICS

Part - I

Time: 1.30 hours. Marks: 50

10 x 1 = 10

I. Choose the correct answer:

1. The tangent to the curve $y^2 - xy + 9 = 0$ is vertical when
   a) $y = 0$ b) $y = \pm\sqrt{3}$ c) $y = \frac{1}{2}$ d) $y = \pm 3$
2. The maximum value of the function $x^2 e^{-2x}$, $x > 0$ is
   a) $\frac{1}{e}$ b) $\frac{1}{2e}$ c) $\frac{1}{e^2}$ d) $\frac{4}{e^4}$
3. If $u(x,y) = e^{x^2+y^2}$, then $\frac{\partial u}{\partial x}$ is equal to
   a) $e^{x^2+y^2}$ b) $2xu$ c) x^2u d) y^2u
4. If $f(x,y,z) = xy + yz + zx$, then $f_x - f_z$ is equal to
   a) $z - x$ b) $y - z$ c) $x - z$ d) $y - x$
5. The value of $\int_{-1}^{2} |x|\,dx$ is
   a) $\frac{1}{2}$ b) $\frac{3}{2}$ c) $\frac{5}{2}$ d) $\frac{7}{2}$
6. The area between $y^2 = 4x$ and its latus rectum is
   a) $\frac{2}{3}$ b) $\frac{4}{3}$ c) $\frac{8}{3}$ d) $\frac{5}{3}$
7. The order and degree of the differential equation $\sqrt{\sin x}\,(dx + dy) = \sqrt{\cos x}\,(dx - dy)$ is
   a) 1,2 b) 2,2 c) 1,1 d) 2,1
8. If sin x is the integrating factor of the linear differential equation $\frac{dy}{dx} + Py = Q$, then P is
   a) log sin x b) cos x c) tan x d) cot x
9. $\int_0^a x f(x)\,dx = \frac{a}{2}\int_0^a f(x)\,dx$ if
   a) $f(x-a) = f(x)$ b) $f(a-x) = f(x)$ c) $f(a-x) = -f(x)$ d) $f(x-a) = -f(x)$
10. If the differential equation is of the form $\frac{dx}{dy} + Px = Q$, where P and Q are functions of y only, then the integrating factor is
    a) $e^{\int Q dx}$ b) $e^{\int P dy}$ c) $e^{\int P dx}$ d) $e^{\int x dy}$

Part - II

II. Answer any 5 questions: (Ques.No.17 is compulsory) 5 x 2 = 10

11. Find the points on the curve $y = x^3 - 3x^2 + x - 2$ at which at the tangent is parallel to the line $y = x$.
12. Evaluate : $\lim_{x \to \pi/2^-} \frac{\sec x}{\tan x}$
13. If the radius of a sphere, with radius 10 cm, has to decrease by 0.1 cm, approximate how much will its volume decrease?

14. Evaluate: $\int_0^1 x^3(1-x)^4\,dx$

15. Evaluate $\int_{-5}^{5} x\cos\left(\frac{e^x-1}{e^x+1}\right)dx$ using properties of integration.

16. Form the differential equation by eliminating the arbitrary constants A and B from $y = A\cos x + B\sin x$

17. Find the value of m so tht the function $y = e^{mx}$ is a solution of the given differential equation $y'' - 5y' + 6y = 0$

Part - III

III. Answer any 5 questions: (Ques.No.24 is compulsory) $5 \times 3 = 15$

18. Using the Lagrange's mean value theorem, determine the values of x at which the tangent is parallel to the Secant line at the end points of the given interval: $f(x) = (x-2)(x-7)$, $x \in [3,11]$

19. A conical water tank with vertex down of 12 m height has a radius of 5 m at the top. If the water flows into the tank at a rate 10 cubic $^{m}/_{min}$, how fast is the depth of the water increases when the water is 8 metres deep?

20. Let $g(x,y) = \frac{e^y \sin x}{x}$ for $x \neq 0$ and $g(0,0) = 1$. Show that g is continuous at (0,0)

21. Evaluate : $\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}\,dx$

22. Evaluate : $\int_0^1 x^3 e^{-2x}\,dx$

23. Solve: $\frac{dy}{dx} + 2y = e^{-x}$

24. If $u(x,y,z) = xy^2z^3$, $x = \sin t$, $y = \cos t$, $z = 1 + e^{2t}$, find $\frac{du}{dt}$

Part - IV

IV. Answer all the questions: $3 \times 5 = 15$

25. a) If the curves $ax^2 + by^2 = 1$ and $cx^2 + dy^2 = 1$ intersect each other orthogonally then show that $\frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$ ***(or)***

b) Evaluate $\int_1^4 (2x^2+3)\,dx$, as the limit of a sum.

26. a) Find the dimensions of the largest rectangle that can be inscribed in a semi circle of a radius r cm. ***(or)***

b) If $v(x,y) = \log\left(\frac{x^2+y^2}{x+y}\right)$, prove that $x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} = 1$

27. a) Find the area of the region in the first quadrant bounded by the parabol $y^2 = 4x$, the line $x + y = 3$ and y-axis. ***(or)***

b) Solve: $2xy\,dx + (x^2 + 2y^2)\,dy = 0$

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