COMMON SECOND MID-TERM TEST - 2019

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Standard XII

MATHEMATICS

Part - I

Time: 1 30 hours

Reg No

Marks: 45

I. Choose the correct answer: 10 x 1 = 10

1. What is the value of the limit $\lim_{x \to 0}\left(\cot x - \frac{1}{x}\right)$?
   a) 0 b) 1 c) 2 d) ∞
2. The maximum product of two positive numbers, when their sum of the squares is 200, is
   a) 100 b) $25\sqrt{7}$ c) 28 d) $24\sqrt{14}$
3. The value of 'a' so that the curves $y = 2e^x$ and $y = ae^{-x}$ intersect orthogonally is
   a) $\frac{1}{2}$ b) $-\frac{1}{2}$ c) 2 d) $2e^2$
4. Which of the following is not correct?
   a) All stationary numbers are critical numbers
   b) At stationary point, the first derivative is zero
   c) At critical point, the first derivative does not exist
   d) All critical numbers are stationary numbers
5. If $u = (x-y)^4 + (y-z)^4 + (z-x)^4$, then the value of ΣU_x = ?
   a) 4 b) 1 c) 0 d) -4
6. If $u \log\left(\frac{x^2+y^2}{x+y}\right)$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ = ?
   a) 1 b) 2 c) e^2 d) log x
7. Linear approximation for g(x) = cos x at $x = \frac{\pi}{2}$ is
   a) $x + \frac{\pi}{2}$ b) $-x + \frac{\pi}{2}$ c) $x - \frac{\pi}{2}$ d) $-x - \frac{\pi}{2}$
8. Which one is the inverse of the statement $(p \vee q) \to p \wedge q$?
   a) $p \wedge q \to p \vee q$ b) $\neg(p \vee q) \to p \wedge q$
   c) $(\neg p \vee \neg q) \to (\neg p \wedge \neg q)$ d) $(\neg p \wedge \neg q) \to (\neg p \vee \neg q)$
9. Subtraction is not a binary operation in
   a) R b) Z c) N d) Q
10. If * is defined by $a * b = a^2 + b^2 + ab + 1$, then the value of $(2 * 3) * 2$ is
    a) 20 b) 40 c) 400 d) 445

Part - II

II. Answer any 3 questions: (Ques.No.15 is compulsory) 3 x 2 = 6

11. Prove that the function $f(x) = x^2 + 2$ is strictly increasing in (2,7) and strictly decreasing in (−2,0).
12. If the volume of the cube of side length x is $V = x^3$. Find the rate of change of the volume with respect to x when x = 5 units.
13. Let $F(x,y) = x^3y + y^2x + 7$ for all $(x,y) \in R^2$. Calculate $\frac{\partial F}{\partial x}(1,3)$ and $\frac{\partial F}{\partial y}(-2,1)$.
14. Show that the percentage error in the n^{th} roots of a number is approximately $\frac{1}{n}$ times the percentage error in the number.

15. In the set of integers under the operation * defined by $a * b = a + b - 1$, find the identity element.

Part - III

III. Answer any 3 questions: (Ques.No.20 is compulsory) $3 \times 3 = 9$

16. Suppose f(x) is differentiable function for all x with $f'(x) \leq 29$ and $f(2) = 17$. What is the maximum value of f(7)?

17. Construct the truth table for the statement $(p \vee q) \wedge (p \vee \sim q)$

18. Find f(3.02) using the linear approximation for the function $f(x) = 3x^2 + 5x + 3$

19. If $f(x,y) = \tan^{-1}\left(\frac{x}{y}\right)$, show that $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$

20. Find two positive numbers whose sum is 12 and their product is maximum.

Part - IV

IV. Answer all the questions: $4 \times 5 = 20$

21. a) If the curves $ax^2 + by^2 = 1$ and $cx^2 + dy^2 = 1$ intersect each other orthogonally, then prove that $\frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$

(or)

b) For the function $f(x) = 4x^3 + 3x^2 - 6x + 1$, find the intervals of monotonicity, local extrema, intervals of concavity and the points of inflection.

22. a) If $u = \tan^{-1}\left(\frac{x^3 - y^3}{x - y}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$.

(or)

b) If $w(x,y,z) = xy + yz + zx$, $x = u - v$, $y = uv$, $z = u + v$, $u, v \in R$, find $\frac{\partial w}{\partial u}, \frac{\partial w}{\partial v}$ and evlauate them at $(\frac{1}{2}, 1)$.

23. a) Evaluate $\lim_{x \to 0} (\cot x)^{\sin x}$

(or)

b) Let $M = \left\{ \begin{pmatrix} x & x \\ x & x \end{pmatrix} : x \in R - \{0\} \right\}$ and let * be the matrix multiplication. Determine whether m is closed under *. If so examine the commutative and associative properties satisfied by * on M.

24. a) Show that $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

(or)

b) If $v = \log r$ and $r^2 + x^2 + y^2 + z^2$, then prove that $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} = \frac{1}{r^2}$

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