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# EXERCISE 1.8

Choose the Correct or the most suitable answer from the given four alternatives :-

1) If $|adj(adjA)| = |A|^9$, then the order of the square matrix A is

(a) 3 (b) 4 (c) 2 (d) 5

Sol:- $|adj(adjA)| = |A|^{(n-1)^2}$

$\therefore (n-1)^2 = 9$

$n-1 = 3$

$n = 3+1$

$n = 4$

$\therefore$ Ans: (b) 4

2) If A is a 3x3 non-Singular matrix such that $AA^T = A^TA$ and $B = A^{-1}A^T$, then $BB^T =$

(a) A (b) B (c) $I_3$ (d) $B^T$

Sol:-

$BB^T = (A^{-1}A^T)(A^{-1}A^T)^T$ $\quad (AB)^T = B^TA^T$

$= A^{-1}A^T(A^T)^T(A^{-1})^T$ $\quad (A^T)^T = A$

$= A^{-1}A^TA(A^{-1})^T$

$= A^{-1}AA^T(A^{-1})^T$

$= IA^T(A^{-1})^T$

$= I(A^{-1}A)^T$

$= I(I)^T$

$= I(I)$

$= I$

$\therefore$ Ans: (c) $I_3$

3) If $A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$, $B = adjA$ and $C = 3A$ then $\frac{|adjB|}{|C|} =$

(a) $\frac{1}{3}$ (b) $\frac{1}{9}$ (c) $\frac{1}{4}$ (d) 1

Sol:-

$B = adjA$

$= \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix}$

$adjB = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$

$|adjB| = \begin{vmatrix} 3 & 5 \\ 1 & 2 \end{vmatrix} = 6-5 = 1$

$C = 3A = 3\begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 3 & 6 \end{bmatrix}$

$|C| = \begin{vmatrix} 9 & 15 \\ 3 & 6 \end{vmatrix} = 54-15 = 9$

$\therefore \frac{|adjB|}{|C|} = \frac{1}{9}$

$\therefore$ Ans:- (b) $\frac{1}{9}$

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4) If $A\begin{bmatrix}1 & -2\\ 1 & 4\end{bmatrix} = \begin{bmatrix}6 & 0\\ 0 & 6\end{bmatrix}$, then $A =$

(a) $\begin{bmatrix}1 & -2\\ 1 & 4\end{bmatrix}$ (b) $\begin{bmatrix}1 & 2\\ -1 & 4\end{bmatrix}$ (c) $\begin{bmatrix}4 & 2\\ -1 & 1\end{bmatrix}$ (d) $\begin{bmatrix}4 & -1\\ 2 & 1\end{bmatrix}$

Sol:-

$AX = B$

$A = BX^{-1}$

$= \begin{bmatrix}6 & 0\\ 0 & 6\end{bmatrix}\begin{bmatrix}1 & -2\\ 1 & 4\end{bmatrix}^{-1}$

$= \begin{bmatrix}6 & 0\\ 0 & 6\end{bmatrix}\frac{1}{4+2}\begin{bmatrix}4 & 2\\ -1 & 1\end{bmatrix}$

$= \frac{1}{6}\begin{bmatrix}6 & 0\\ 0 & 6\end{bmatrix}\begin{bmatrix}4 & 2\\ -1 & 1\end{bmatrix}$

$= \frac{1}{6}\begin{bmatrix}24 & 12\\ -6 & 6\end{bmatrix}$

$= \begin{bmatrix}4 & 2\\ -1 & 1\end{bmatrix}$ ∴ Ans: (c) $\begin{bmatrix}4 & 2\\ -1 & 1\end{bmatrix}$

5) If $A = \begin{bmatrix}7 & 3\\ 4 & 2\end{bmatrix}$, then $9I_2 - A =$

(a) $A^{-1}$ (b) $\frac{A^{-1}}{2}$ (c) $3A^{-1}$ (d) $2A^{-1}$

Sol:-

$9I_2 - A = 9\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} - \begin{bmatrix}7 & 3\\ 4 & 2\end{bmatrix}$

$= \begin{bmatrix}9 & 0\\ 0 & 9\end{bmatrix} - \begin{bmatrix}7 & 3\\ 4 & 2\end{bmatrix}$

$= \begin{bmatrix}2 & -3\\ -4 & 7\end{bmatrix}$

$=$

$= 2A^{-1}$

$A^{-1} = \frac{1}{14-12}\begin{bmatrix}2 & -3\\ -4 & 7\end{bmatrix}$

$A^{-1} = \frac{1}{2}\begin{bmatrix}2 & -3\\ -4 & 7\end{bmatrix}$

$2A^{-1} = \begin{bmatrix}2 & -3\\ -4 & 7\end{bmatrix}$

∴ Ans: $2A^{-1}$

6) If $A = \begin{bmatrix}2 & 0\\ 1 & 5\end{bmatrix}$ and $B = \begin{bmatrix}1 & 4\\ 2 & 0\end{bmatrix}$ then $|adj(AB)| =$

(a) −40 (b) −80 (c) −60 (d) −20

Sol:-

$AB = \begin{bmatrix}2 & 0\\ 1 & 5\end{bmatrix}\begin{bmatrix}1 & 4\\ 2 & 0\end{bmatrix}$

$= \begin{bmatrix}2+0 & 8+0\\ 1+10 & 4+0\end{bmatrix}$

$AB = \begin{bmatrix}2 & 8\\ 11 & 4\end{bmatrix}$

$adj(AB) = \begin{bmatrix}4 & -8\\ -11 & 2\end{bmatrix}$

$|adj(AB)| = \begin{vmatrix}4 & -8\\ -11 & 2\end{vmatrix} = 8 - 88$

$= -80$

∴ Ans (b) −80

7) If $P = \begin{bmatrix}1 & x & 0\\ 1 & 3 & 0\\ 2 & 4 & -2\end{bmatrix}$ is the adjoint of 3×3 matrix A and $|A| = 4$, then $x$ is



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Sol:- (a) 15 (b) 12 (c) 14 (d) 11

$$|adj\,A| = |A|^{n-1}$$

$$|adj\,A| = |A|^{3-1}$$

$$|adj\,A| = |A|^2$$

$$\begin{vmatrix} 1 & x & 0 \\ 1 & 3 & 0 \\ 2 & 4 & -2 \end{vmatrix} = 16$$

$$1(-6-0) - x(-2-0) = 16$$

$$-6 + 2x = 16$$

$$2x = 16+6$$

$$2x = 22$$

$$x = 11$$

∴ Ans (d) 11

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8) If $A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & -1 \end{bmatrix}$ and $A^{-1}\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ then the value of $a_{23}$ is

(a) 0 (b) −2 (c) −3 (d) −1

Sol:-

$$|A| = \begin{vmatrix} 3 & 1 & -1 \\ 2 & -2 & 0 \\ 1 & 2 & -1 \end{vmatrix} = 3(2) - 1(-2) - 1(4+2)$$

$$= 6 + 2 - 6$$

$$= 2$$

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T$$

$$= \frac{1}{|A|}\begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

$a_{23} = \frac{1}{|A|}$ Cofactor of $a_{32}$

$$a_{23} = \frac{1}{2} \times -\begin{vmatrix} 3 & -1 \\ 2 & 0 \end{vmatrix}$$

$$= \frac{1}{2} \times -(0+2)$$

$$= \frac{1}{2} \times -2$$

$$= -1$$

∴ Ans (b) −1

9) If A, B and C are invertible matrices of some order, then which one of the following is not true?

(a) $adj\,A = |A|A^{-1}$ (b) $adj(AB) = (adj\,A)(adj\,B)$

(c) $\det A^{-1} = (\det A)^{-1}$ (d) $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

Sol:-

$$adj(AB) = (adj\,A)(adj\,B)$$

∴ Ans : (b) $adj(AB) = (adj\,A)(adj\,B)$

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10) If $(AB)^{-1} = \begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$, then $B^{-1} =$

(a) $\begin{bmatrix} 2 & -5 \\ -3 & 8 \end{bmatrix}$ (b) $\begin{bmatrix} 8 & 5 \\ 3 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 8 & -5 \\ -3 & 2 \end{bmatrix}$

Sol:-

$$(AB)^{-1} = B^{-1}A^{-1}$$

$$B^{-1}A^{-1} = \begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix}$$

$$B^{-1}\begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 12 & -17 \\ -19 & 27 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 36-34 & 12-17 \\ -57+54 & -19+27 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -5 \\ -3 & 8 \end{bmatrix}$$

∴ Ans: (a) $\begin{bmatrix} 2 & -5 \\ -3 & 8 \end{bmatrix}$

11) If $A^TA^{-1}$ is Symmetric, then $A^2 =$

(a) $A^{-1}$ (b) $(A^T)^2$ (c) $A^T$ (d) $(A^{-1})^2$

Sol:-

$$A^TA^{-1} = (A^TA)^T$$

$$= (A^{-1})^T(A^T)^T$$

$$A^TA^{-1} = (A^{-1})^T A$$

$$A = A^T \cdot A^TA^{-1}$$

$$A = (A^T)^2A^{-1}$$

$$A^2 = (A^T)^2A^{-1}A$$

$$= (A^T)^2 I$$

$$\therefore A^2 = (A^T)^2$$

∴ Ans: (b) $(A^T)^2$

12) If $A$ is a non-singular matrix such that $A^{-1} = \begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$, then $(A^T)^{-1} =$

(a) $\begin{bmatrix} -5 & 3 \\ 2 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$ (c) $\begin{bmatrix} -1 & -3 \\ 2 & 5 \end{bmatrix}$ (d) $\begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$

Sol:-

$$(A^T)^{-1} = (A^{-1})^T$$

$$= \begin{bmatrix} 5 & 3 \\ -2 & 1 \end{bmatrix}^T$$

$$= \begin{bmatrix} 5 & -2 \\ 3 & 1 \end{bmatrix}$$

∴ Ans: (d) $\begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$

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13) If $A=\begin{bmatrix} 3/5 & 4/5 \\ x & 3/5 \end{bmatrix}$ and $A^T = A^{-1}$, then the value of $x$ is

(a) $-4/5$ (b) $-3/5$ (c) $3/5$ (d) $4/5$

Sol:-

$$A^T = A^{-1}$$
$$A^T A = A^{-1} A$$
$$A^T A = I$$

$$\begin{bmatrix} 3/5 & x \\ 4/5 & 3/5 \end{bmatrix} \begin{bmatrix} 3/5 & 4/5 \\ x & 3/5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{9}{25}+x^2 & \frac{12}{25}+\frac{3x}{5} \\ \frac{12}{25}+\frac{3x}{5} & \frac{16}{25}+\frac{9}{25} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\frac{12}{25}+\frac{3x}{5} = 0$$
$$\frac{3x}{5} = -\frac{12}{25}$$
$$x = -\frac{4}{5}$$

∴ Ans: (a) $-4/5$

14) If $A=\begin{bmatrix} 1 & \tan\theta/2 \\ -\tan\theta/2 & 1 \end{bmatrix}$ and $AB = I_2$, then $B=$

(a) $(\cos^2\theta/2)A$ (b) $(\cos^2\theta/2)A^T$ (c) $(\cos^2\theta)I$ (d) $(\sin^2\theta/2)A$

Sol:

$$AB = I$$
$$B = A^{-1} I$$
$$B = A^{-1}$$
$$B = \frac{1}{1+\tan^2\theta/2}\begin{bmatrix} 1 & -\tan\theta/2 \\ \tan\theta/2 & 1 \end{bmatrix}$$
$$= \frac{1}{\sec^2\theta/2}\begin{bmatrix} 1 & -\tan\theta/2 \\ \tan\theta/2 & 1 \end{bmatrix}$$
$$= \cos^2\theta/2 \begin{bmatrix} 1 & \tan\theta/2 \\ -\tan\theta/2 & 1 \end{bmatrix}^T$$
$$= \left(\cos^2\frac{\theta}{2}\right) A^T$$

∴ Ans: (b) $\left(\cos^2\frac{\theta}{2}\right) A^T$

15) If $A=\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ and $A(\text{adj}A)=\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$, then $k=$

(a) 0 (b) $\sin\theta$ (c) $\cos\theta$ (d) 1

Sol:-

$$A(\text{adj}A) = (\text{adj}A)A = |A|I$$



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$$k = \begin{vmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{vmatrix} \qquad \therefore |A| = k$$

$$= \cos^2\theta + \sin^2\theta$$

$$k = 1 \qquad \therefore \text{Ans (d) } 1$$

16) If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $\lambda A^{-1} = A$, then $\lambda$ is

(a) 17 (b) 14 (c) 19 (d) 21

Sol:-

$$\lambda A^{-1} = A$$

$$\lambda A^{-1} \cdot A = A \cdot A$$

$$\lambda I = A^2$$

$$\lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$$

$$\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 4+15 & 6-6 \\ 10-10 & 15+4 \end{bmatrix}$$

$$\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 19 & 0 \\ 0 & 19 \end{bmatrix}$$

$$\therefore \lambda = 19 \qquad \therefore \text{Ans (c) } 19$$

17) If $\text{adj } A = \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$ and $\text{adj } B = \begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}$ then adj (AB) is

(a) $\begin{bmatrix} -7 & -1 \\ 7 & -9 \end{bmatrix}$ (b) $\begin{bmatrix} -6 & 5 \\ -2 & -10 \end{bmatrix}$ (c) $\begin{bmatrix} -7 & 7 \\ -1 & -9 \end{bmatrix}$ (d) $\begin{bmatrix} -6 & -2 \\ 5 & -10 \end{bmatrix}$

Sol:-

$$\text{adj } A = \begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix} \qquad \text{adj } B = \begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} -1 & -3 \\ -4 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} -1 & -3 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1-9 & -2-3 \\ -4+6 & -8+2 \end{bmatrix}$$

$$AB = \begin{bmatrix} -10 & -5 \\ 2 & -6 \end{bmatrix}$$

$$\text{adj}(AB) = \begin{bmatrix} -6 & 5 \\ -2 & -10 \end{bmatrix} \qquad \therefore \text{Ans: (b)} \begin{bmatrix} -6 & 5 \\ -2 & -10 \end{bmatrix}$$

18) The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \end{bmatrix}$ is

(a) 1 (b) 2 (c) 4 (d) 3



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**Sol:-**

Let $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \end{bmatrix}$ $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 + R_1$

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\therefore \rho(A) = 1$

$\therefore$ Ans: (a) 1

19) If $x^a y^b = e^m$, $x^c y^d = e^n$, $\Delta_1 = \begin{vmatrix} m & b \\ n & d \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} a & m \\ c & n \end{vmatrix}$, $\Delta_3 = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$, then the value of $x$ and $y$ are respectively

(a) $e^{(\Delta_2/\Delta_1)}, e^{(\Delta_3/\Delta_1)}$ (b) $\log(\Delta_1/\Delta_3), \log(\Delta_2/\Delta_3)$

(c) $\log(\Delta_2/\Delta_1), \log(\Delta_3/\Delta_1)$ (d) $e^{(\Delta_1/\Delta_3)}, e^{(\Delta_2/\Delta_3)}$

**Sol:-**

$$x^a y^b = e^m$$

Taking log on both sides

$$\log(x^a y^b) = \log e^m$$

$$\log x^a + \log y^b = m$$

$$a \log x + b \log y = m$$

$$x^c y^d = e^n$$

Taking log on both sides

$$\log(x^c y^d) = \log e^n$$

$$\log x^c + \log y^d = n$$

$$c \log x + d \log y = n$$

$\Delta_1 = \begin{vmatrix} m & b \\ n & d \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} a & m \\ c & n \end{vmatrix}$

Put $\Delta_3 = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$

$$\log x = \frac{\Delta_1}{\Delta_3}, \quad \log y = \frac{\Delta_2}{\Delta_3}$$

$$\log x = \frac{\Delta'}{\Delta_3}$$

$$x = e^{\Delta_1/\Delta_3} \text{ and } y = e^{\Delta_2/\Delta_3}$$

$\therefore$ Ans: (d) $e^{(\Delta_1/\Delta_3)}, e^{(\Delta_2/\Delta_3)}$

20) which of the following is/are Correct?

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(i) Adjoint of a symmetric matrix is also a symmetric matrix

(ii) Adjoint of a diagonal matrix is also a diagonal matrix

(iii) If $A$ is a square matrix of order $n$ and $\lambda$ is a scalar, then $adj(\lambda A) = \lambda^n adj(A)$.

(iv) $A(adj A) = (adj A)A = |A|I$.

(a) only (i) (b) (ii) and (iii)

(c) (iii) and (iv) (d) (i), (ii) and (iv)

∴ Ans : (d) (i), (ii) and (iv)

21) If $\rho(A) = \rho([A|B])$ then the system $AX = B$ of linear equation is

(a) Consistent and has a unique solution

(b) Consistent

(c) Consistent and has infinitely many solution

(d) inConsistent

∴ Ans : (b) Consistent

22) If $0 \le \theta \le \pi$ and the system of equations $x + (\sin\theta)y - (\cos\theta)z = 0$, $(\cos\theta)x - y + z = 0$, $(\sin\theta)x + y - z = 0$ has a non-trivial solution then $\theta$ is

(a) $\frac{2\pi}{3}$ (b) $\frac{3\pi}{4}$ (c) $\frac{5\pi}{6}$ (d) $\frac{\pi}{4}$

Sol:-

$$A = \begin{bmatrix} 1 & \sin\theta & -\cos\theta \\ \cos\theta & -1 & 1 \\ \sin\theta & 1 & -1 \end{bmatrix}$$

The system has non-trivial solution, $|A| = 0$

$$\begin{vmatrix} 1 & \sin\theta & -\cos\theta \\ \cos\theta & -1 & 1 \\ \sin\theta & 1 & -1 \end{vmatrix} = 0$$



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$$1\begin{vmatrix}-1 & 1\\ 1 & -1\end{vmatrix} - \sin\theta\begin{vmatrix}\cos\theta & 1\\ \sin\theta & -1\end{vmatrix} - \cos\theta\begin{vmatrix}\cos\theta & -1\\ \sin\theta & 1\end{vmatrix} = 0$$

$$1(1-1) - \sin\theta[-\cos\theta - \sin\theta] - \cos\theta[\cos\theta + \sin\theta] = 0$$

$$\sin\theta\cos\theta + \sin^2\theta - \cos^2\theta - \sin\theta\cos\theta = 0$$

$$\sin^2\theta - \cos^2\theta = 0$$

$$\therefore \cos^2\theta - \sin^2\theta = 0$$

$$\cos 2\theta = 0$$

$$\cos 2\theta = \cos\frac{\pi}{2}$$

$$2\theta = \frac{\pi}{2}$$

$$\theta = \frac{\pi}{4}$$

$\therefore$ Ans: (d) $\frac{\pi}{4}$

23) The augmented matrix of a system of linear equations is $\begin{bmatrix}1 & 2 & 7 & 3\\ 0 & 1 & 4 & 6\\ 0 & 0 & \lambda-7 & \mu+5\end{bmatrix}$.

The system has infinitely many solutions if

(a) $\lambda=7, \mu\neq-5$ (b) $\lambda=-7, \mu=5$ (c) $\lambda\neq7, \mu\neq-5$ (d) $\lambda=7, \mu=-5$

Sol:-

$\rho(A)=2$, and $\rho([A|B])=2$

number of unknows $=3$

$\rho(A) = \rho[A|B] = 2<3$

$\therefore$ The system is consistent and has infinitely many solutions.

When $\lambda-7=0$ and $\mu+5=0$

$\lambda=7$, $\mu=-5$

$\therefore$ Ans: (d) $\lambda=7, \mu=-5$

24) Let $A=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$ and $4B=\begin{bmatrix}3 & 1 & -1\\ 1 & 3 & x\\ -1 & 1 & 3\end{bmatrix}$,

If B is the inverse of A, then the value of x is

(a) 2 (b) 4 (c) 3 (d) 1

Sol:-

$B = A^{-1}$

$B.A = A^{-1}A$



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$$BA = I$$

$$4BA = 4I$$

$$\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & x \\ -1 & 1 & 3 \end{bmatrix} \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} = 4 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 6-1-1 & -3+2+1 & 3-1-2 \\ 2-3+x & -1+6-x & 1-3+2x \\ -2-1+3 & 1+2-3 & -1-1+6 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 0 & 0 \\ -1+x & 5-x & -2+2x \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$5-x = 4$$

$$x = 5-4$$

$$x = \underline{1}$$

$\therefore$ Ans: (d) 1

25) If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, then adj (adj A) is

(a) $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 6 & -6 & 8 \\ 4 & -6 & 8 \\ 0 & -2 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} -3 & 3 & -4 \\ -2 & 3 & -4 \\ 0 & 1 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & -3 & 4 \\ 0 & -1 & 1 \\ 2 & -3 & 4 \end{bmatrix}$

Sol:-

$$adj(adj A) = |A|^{n-2} A = |A| . A$$

$$|A| = \begin{vmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{vmatrix}$$

$$= 3(-3+4) + 3(2-0) + 4(-2-0)$$

$$= 3(1) + 3(2) + 4(-2)$$

$$= 3 + 6 - 8$$

$$|A| = 1$$

$$\therefore adj(adj A) = 1^{(3-2)} A$$

$$= 1 . \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

$\therefore$ Ans:- (a) $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$

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# EXERCISE 2.9

Choose the correct answer or the most suitable answer from the given four alternatives.

1) $i^n + i^{n+1} + i^{n+2} + i^{n+3}$ is

(a) 0 (b) 1 (c) $-1$ (d) $i$

Sol:-

$$i^n + i^{n+1} + i^{n+2} + i^{n+3} = i^n + i^n \cdot i + i^n \cdot i^2 + i^n \cdot i^3$$
$$= i^n [1 + i + i^2 + i^3]$$
$$= i^n [1 + i - 1 - i]$$
$$= i^n (0)$$
$$= 0$$

∴ Ans: (a) 0

2) The value of $\sum_{i=1}^{13} (i^n + i^{n-1})$ is

(a) $1+i$ (b) $i$ (c) 1 (d) 0

Sol:-

$$\sum_{i=1}^{13} i^n + i^{n-1} = \sum_{i=1}^{13} i^n + i^n \cdot i^{-1}$$
$$= \sum_{i=1}^{13} i^n + \frac{i^n}{i}$$
$$= \sum_{i=1}^{13} i^n \left(1 + \frac{1}{i}\right)$$
$$= \left(1 + \frac{1}{i}\right) \sum_{i=1}^{13} i^n$$
$$= \left(1 + \frac{1}{i}\right) [i^1 + i^2 + i^3 + i^4 + i^5 + i^6 + i^7 + i^8 + i^9 + i^{10} + i^{11} + i^{12} + i^{13}]$$
$$= \left(1 + \frac{1}{i}\right) [(i^1 - 1 - i^2 + 1) + (i^3 - 1 - i^4 + 1) + (i^5 - 1 - i^6 + 1) + i]$$
$$= \left(1 + \frac{1}{i}\right)(i)$$
$$= \left(\frac{i+1}{i}\right) i$$
$$= 1 + i$$

∴ Ans (a) $1+i$

3) The area of the triangle formed by the complex numbers $z$, $iz$ and $z+iz$ in the Argand's diagram is

(a) $\frac{1}{2}|z|^2$ (b) $|z|^2$ (c) $\frac{3}{2}|z|^2$ (d) $2|z|^2$

Sol:-

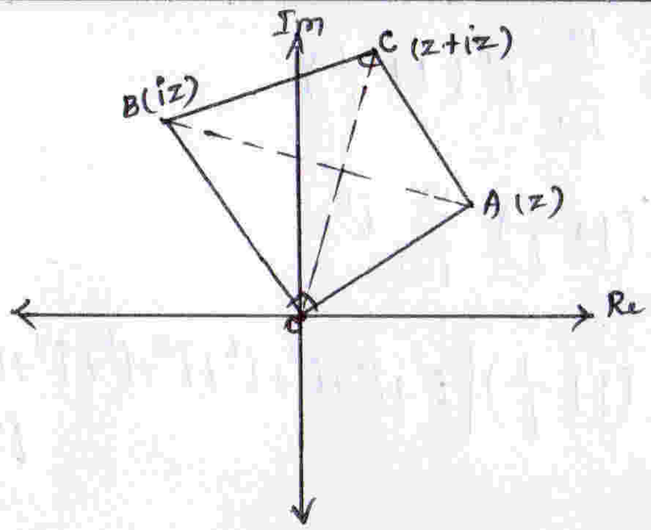
In $\triangle ABC$, $\angle ACB = 90^\circ$

$\triangle ABC$ is a right angled triangle.



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Area of $\triangle ABC = \frac{1}{2} AC \times BC$

$= \frac{1}{2} |(z+iz)-z| \times |z+iz-iz|$

$= \frac{1}{2} |iz|\,|z|$ $\quad \because |i| = 1$

$= \frac{1}{2} |i|\,|z|\,|z|$

$= \frac{1}{2} |z|^2$ $\quad \therefore$ Ans (a) $\frac{1}{2}|z|^2$

4) The Conjugate of a Complex numbers is $\frac{1}{i-2}$. Then, the Complex number is

(a) $\frac{1}{i+2}$ (b) $\frac{-1}{i+2}$ (c) $\frac{-1}{i-2}$ (d) $\frac{1}{i-2}$

Sol:- $z = \frac{1}{i-2}$

$\bar{z} = \frac{1}{-i-2}$

$= \frac{1}{-(i+2)} = -\frac{1}{i+2}$ $\quad \therefore$ Ans: (b) $\frac{-1}{i+2}$

5) If $z = \frac{(\sqrt{3}+i)^3 (3i+4)^2}{(8+6i)^2}$, then $|z|$ is equal to

(a) 0 (b) 1 (c) 2 (d) 3

Sol:-

$|z| = \frac{|\sqrt{3}+i|^3\,|3i+4|^2}{|8+6i|^2}$

$= \frac{(\sqrt{(\sqrt{3})^2+1^2})^3 (\sqrt{3^2+4^2})^2}{(\sqrt{8^2+6^2})^2}$

$= \frac{(\sqrt{3+1})^3 (\sqrt{9+16})^2}{(\sqrt{64+36})^2}$

$= \frac{(\sqrt{4})^3 (\sqrt{25})^2}{(\sqrt{100})^2}$

$= \frac{2^3\ 5^2}{10^2}$

$= \frac{{}^{2}\cancel{8} \times 25}{\cancel{100}_{4}}$ $\quad \therefore$ Ans: (c) 2

$= 2$

6) If $z$ is a non zero Complex number, such that $2iz^2 = \bar{z}$ then $|z|$ is

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 3

Sol:- $2iz^2 = \bar{z}$



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Taking modulus on both sides

$|2iz^2| = |\bar{z}|$ $\quad |\bar{z}| = |z|$

$|2i|\,|z^2| = |z|$ $\quad |i| = 1$

$2\,|z|^2 = |z|$

$2|z|^2 - |z| = 0$

$|z|\,[2|z| - 1] = 0$

$|z| = 0$ (not possible) $\quad 2|z| - 1 = 0$, $2|z| = 1$, $|z| = \frac{1}{2}$

$\therefore$ Ans : (a) $\frac{1}{2}$

7) If $|z - 2 + i| \le 2$, then the greatest value of $|z|$ is

(a) $\sqrt{3} - 2$ (b) $\sqrt{3} + 2$ (c) $\sqrt{5} - 2$ (d) $\sqrt{5} + 2$

Sol:-

$|z - 2 + i| \le 2$

$|z - (2 - i)| \le 2$

$|\,|z| - |2 - i|\,| \le 2$

$|\,|z| - \sqrt{2^2 + 1^2}\,| \le 2$

$|\,|z| - \sqrt{4+1}\,| \le 2$

$|\,|z| - \sqrt{5}\,| \le 2$

$|z| \le 2 + \sqrt{5}$

$\therefore$ The greatest value of $|z|$ is $2 + \sqrt{5}$

$\therefore$ Ans:- (d) $\sqrt{5} + 2$

8) If $|z - \frac{3}{z}| = 2$, then the least value of $|z|$ is

(a) 1 (b) 2 (c) 3 (d) 5

Sol:-

$|z - \frac{3}{z}| = 2$

Consider, $|z| = |z - \frac{3}{z} + \frac{3}{z}|$

$|z| \le |z - \frac{3}{z}| + |\frac{3}{z}|$

$\le |z - \frac{3}{z}| + \frac{3}{|z|}$

$\le 2 + \frac{3}{|z|}$

$|z| \le \frac{2|z| + 3}{|z|}$

$|z|^2 \le 2|z| + 3$

$|z|^2 - 2|z| \le 3$

Adding 1 on both sides

$|z|^2 - 2|z| + 1 \le 3 + 1$



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$$(|z|-1)^2 \leq 4$$

$$|z|-1 \leq \pm\sqrt{4}$$

$$|z|-1 \leq \pm 2$$

$$|z| \leq \pm 2+1$$

∴ The least values of $|z|$ is $|-2+1| = |-1|$

$= 1$

∴ Ans (a) 1

9) If $|z|=1$, then the value of $\frac{1+z}{1+\bar{z}}$ is

(a) $z$ (b) $\bar{z}$ (c) $\frac{1}{z}$ (d) 1

Sol:-

$$z\bar{z} = |z|^2$$

$$|z| = 1$$

$$z\bar{z} = 1$$

$$\bar{z} = \frac{1}{z}$$

$$\frac{1+z}{1+\bar{z}} = \frac{1+z}{1+\frac{1}{z}}$$

$$= \frac{1+z}{\left(\frac{z+1}{z}\right)}$$

$$= \frac{z(1+z)}{1+z}$$

$$= z$$

∴ Ans (a) $z$

10) The solution of the equation $|z|-z = 1+2i$ is

(a) $\frac{3}{2}-2i$ (b) $-\frac{3}{2}+2i$ (c) $2-\frac{3}{2}i$ (d) $2+\frac{3}{2}i$

Sol:-

$$|z|-z = 1+2i$$

Put $z = x+iy$

$$|x+iy|-(x+iy) = 1+2i$$

$$\sqrt{x^2+y^2} - x - iy = 1+2i$$

$\sqrt{x^2+y^2} - x = 1$ | $-y = 2$

$\sqrt{x^2+(-2)^2} - x = 1$ | $y = -2$

$$\sqrt{x^2+4} - x = 1$$

$$\sqrt{x^2+4} = x+1$$

$$x^2+4 = (x+1)^2$$

$$x^2+4 = x^2+2x+1$$

$$2x = 4-1$$

$$2x = 3$$

$$x = \frac{3}{2}$$

∴ $z = \frac{3}{2}-2i$

∴ Ans : (a) $\frac{3}{2}-2i$

11) If $|z_1|=1$, $|z_2|=2$, $|z_3|=3$ and $|9z_1z_2 + 4z_1z_3 + z_2z_3| = 12$, then the value of $|z_1+z_2+z_3|$ is

(a) 1 (b) 2 (c) 3 (d) 4



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**Sol:-** $|z_1|=1$ $|z_2|=2$ $|z_3|=3$

$|z_1|^2=1$ $|z_2|^2=4$ $|z_3|^2=9$

$z_1\bar{z_1}=1$ $z_2\bar{z_2}=4$ $z_3\bar{z_3}=9$

$z_1=\frac{1}{\bar{z_1}}$ $z_2=\frac{4}{\bar{z_2}}$ $z_3=\frac{9}{\bar{z_3}}$

$$|z_1+z_2+z_3| = \left|\frac{1}{\bar{z_1}}+\frac{4}{\bar{z_2}}+\frac{9}{\bar{z_3}}\right|$$

$$= \left|\frac{\bar{z_2}\bar{z_3}+4\bar{z_1}\bar{z_3}+9\bar{z_1}\bar{z_2}}{\bar{z_1}\bar{z_2}\bar{z_3}}\right|$$

$$= \frac{|z_2z_3+4z_1z_3+9z_1z_2|}{|z_1||z_2||z_3|}$$

$$= \frac{12}{(1)(2)(3)}$$

$$= \frac{12}{6}$$

$$= 2$$

∴ **Ans: (b) 2**

12) If $z$ is a Complex number such that $z\in\mathbb{C}\setminus\mathbb{R}$ and $z+\frac{1}{z}\in\mathbb{R}$, the $|z|$ is

(a) 0 (b) 1 (c) 2 (d) 3

**Sol:-** Let $z=x+iy$, $y\neq 0$

$$z+\frac{1}{z} = x+iy+\frac{1}{x+iy}$$

$$= x+iy+\frac{1}{x+iy}\times\frac{x-iy}{x-iy}$$

$$= x+iy+\frac{x-iy}{x^2+y^2}$$

$$= x+iy+\frac{x}{x^2+y^2}-\frac{iy}{x^2+y^2}$$

$$= x+\frac{x}{x^2+y^2}+i\left(y-\frac{y}{x^2+y^2}\right)$$

$\therefore y-\frac{y}{x^2+y^2}=0$

$y\left[1-\frac{1}{x^2+y^2}\right]=0$

$y=0$ (not possible) | $1-\frac{1}{x^2+y^2}=0$

$1=\frac{1}{x^2+y^2}$

$x^2+y^2=1$

$\therefore |z| = |x+iy|$

$=\sqrt{x^2+y^2}$

$=\sqrt{1}=1$

∴ **Ans (b) 1**

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13) $Z_1, Z_2$ and $Z_3$ are Complex numbers such that $Z_1+Z_2+Z_3=0$ and $|Z_1|=|Z_2|=|Z_3|=1$ then $Z_1^2+Z_2^2+Z_3^2$ is

(a) 3 (b) 2 (c) 1 (d) 0

Sol:-

$|Z_1|=1$ $\quad |Z_2|=1$ $\quad |Z_3|=1$

$|Z_1|^2=1$ $\quad |Z_2|^2=1$ $\quad |Z_3|^2=1$

$Z_1\overline{Z_1}=1$ $\quad Z_2\overline{Z_2}=1$ $\quad Z_3\overline{Z_3}=1$

$Z_1=\frac{1}{\overline{Z_1}}$ $\quad Z_2=\frac{1}{\overline{Z_2}}$ $\quad Z_3=\frac{1}{\overline{Z_3}}$

$Z_1+Z_2+Z_3=0$

$\frac{1}{\overline{Z_1}}+\frac{1}{\overline{Z_2}}+\frac{1}{\overline{Z_3}}=0$

$\frac{\overline{Z_2}\overline{Z_3}+\overline{Z_1}\overline{Z_3}+\overline{Z_2}\overline{Z_1}}{\overline{Z_1}\overline{Z_2}\overline{Z_3}}=0$

$\overline{Z_2}\overline{Z_3}+\overline{Z_1}\overline{Z_3}+\overline{Z_2}\overline{Z_1}=0$

$\overline{Z_2Z_3+Z_1Z_3+Z_2Z_1}=0$

$Z_2Z_3+Z_1Z_3+Z_2Z_1=0$ ——— (1)

$Z_1+Z_2+Z_3=0$

$(Z_1+Z_2+Z_3)^2=0$

$Z_1^2+Z_2^2+Z_3^2+2(Z_1Z_2+Z_2Z_3+Z_3Z_1)=0$

$Z_1^2+Z_2^2+Z_3^2+2(0)=0$

$\therefore Z_1^2+Z_2^2+Z_3^2=0$ $\quad$ $\therefore$ Ans: (d) 0

14) If $\frac{Z-1}{Z+1}$ is purely imaginary, then $|Z|$ is

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 3

Sol:- $Z=x+iy$

$\frac{Z-1}{Z+1}=\frac{x+iy-1}{x+iy+1}$

$=\frac{x-1+iy}{x+1+iy}\times\frac{x+1-iy}{x+1-iy}$

$=\frac{(x-1)(x+1)-iy(x-1)+iy(x+1)+y^2}{(x+1)^2+y^2}$

Real part $=0$

$\frac{(x-1)(x+1)+y^2}{(x+1)^2+y^2}=0$

$(x-1)(x+1)+y^2=0$

$x^2-1+y^2=0$

$x^2+y^2=1$

$\therefore |Z|=|x+iy|$

$=\sqrt{x^2+y^2}$

$=\sqrt{1}$

$=1$

$\therefore$ Ans (b) 1

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15) If $z = x+iy$ is a Complex number such that $|z+2| = |z-2|$, then the locus of $z$ is

(a) real axis (b) imaginary axis (c) ellipse (d) circle

Sol:-

$$|z+2| = |z-2|$$

$$|x+iy+2| = |x+iy-2|$$

$$|x+2+iy| = |x-2+iy|$$

$$\sqrt{(x+2)^2+y^2} = \sqrt{(x-2)^2+y^2}$$

$$x^2+4x+4+y^2 = x^2-4x+4+y^2$$

$$4x = -4x$$

$$x+x = 0$$

$$2x = 0$$

$$x = 0$$

$z = 0+iy$

$z = iy$

∴ Ans: (b) imaginary axis

∴ The locus of imaginary axis

16) The principal argument of $\frac{3}{-1+i}$ is

(a) $-\frac{5\pi}{6}$ (b) $-\frac{2\pi}{3}$ (c) $-\frac{3\pi}{4}$ (d) $-\frac{\pi}{2}$

Sol:-

Let $z = \frac{3}{-1+i}$

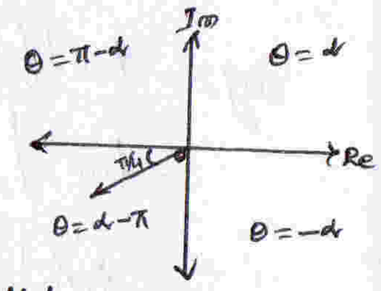
$$= \frac{3}{-1+i} \times \frac{-1-i}{-1-i}$$

$$= \frac{3(-1-i)}{(-1)^2-i^2}$$

$$= \frac{-3(1+i)}{1+1}$$

$$= -\frac{3}{2}(1+i)$$

$-\frac{3}{2}(1+i)$ lies in III rd quadrant




$|z| = r = \sqrt{\frac{9}{4}+\frac{9}{4}}$

$= \frac{3}{2}\sqrt{2}$

$\alpha = \tan^{-1}\left|\frac{y}{x}\right|$

$= \tan^{-1}1$

$\alpha = \frac{\pi}{4}$

$\therefore \theta = \alpha - \pi$

$= \frac{\pi}{4} - \pi$

$\therefore \theta = -\frac{3\pi}{4}$

∴ Ans: (c) $-\frac{3\pi}{4}$

17) The principal argument of $(\sin 40^\circ + i\cos 40^\circ)^5$ is

(a) $-110^\circ$ (b) $-70^\circ$ (c) $70^\circ$ (d) $110^\circ$

Sol:-

$$z = (\sin 40^\circ + i\cos 40^\circ)^5$$

$$= [\sin(90^\circ-50^\circ) + i\cos(90^\circ-50^\circ)]^5$$

$$= (\cos 50^\circ + i\sin 50^\circ)^5$$

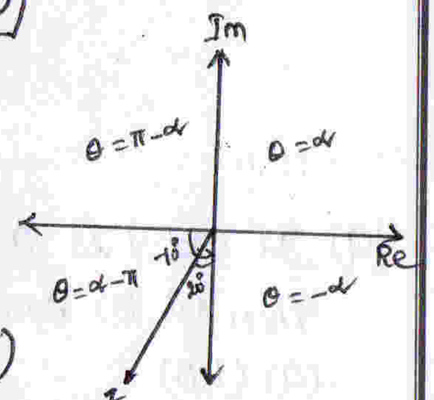
$$= \cos 5(50^\circ) + i\sin 5(50^\circ)$$

$$= \cos 250^\circ + i\sin 250^\circ$$

$$= \cos(180^\circ+70^\circ) + i\sin(180^\circ+70^\circ)$$

$$z = -\cos 70^\circ - i\sin 70^\circ$$

z lies in third quadrant






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$\arg z = \tan^{-1}\left|\frac{y}{x}\right|$

$= \tan^{-1}\left|\frac{-\sin 70^\circ}{-\cos 70^\circ}\right| - \pi$

$= \tan^{-1} \tan 70^\circ - \pi$

$= 70^\circ - 180^\circ$

$\arg z = -110^\circ$

$\alpha = 70^\circ$

$\theta = \alpha - \pi$

$= 70^\circ - 180^\circ$

$= -110^\circ$

∴ Ans: (a) $-110^\circ$

18) If $(1+i)(1+2i)(1+3i)\ldots(1+ni) = x+iy$, then $2.5.10\ldots(1+n^2)$ is

(a) 1 (b) $i$ (c) $x^2+y^2$ (d) $1+n^2$

Sol:-

$|(1+i)(1+2i)(1+3i)\ldots(1+ni)| = |x+iy|$

$|1+i|\,|1+2i|\,|1+3i|\ldots|1+ni| = |x+iy|$

$\sqrt{1^2+1^2}\cdot\sqrt{1^2+2^2}\cdot\sqrt{1^2+3^2}\ldots\sqrt{1^2+n^2} = \sqrt{x^2+y^2}$

$\sqrt{2}\ \sqrt{5}\ \sqrt{10}\ \ldots\sqrt{1+n^2} = \sqrt{x^2+y^2}$

$2.5.10\ldots(1+n^2) = x^2+y^2$

∴ Ans: (c) $x^2+y^2$

19) If $\omega \neq 1$ is a cubic root of unity and $(1+\omega)^7 = A+B\omega$, then $(A,B)$ equals

(a) (1,0) (b) (−1,1) (c) (0,1) (d) (1,1)

Sol:- $\omega \neq 1$ is a cube root of unity

∴ $1, \omega, \omega^2$ are the roots and $1+\omega+\omega^2 = 0$

$(1+\omega)^7 = A+B\omega$

$(-\omega^2)^7 = A+B\omega$

$-\omega^{14} = A+B\omega$

$-\omega^{12}\cdot\omega^2 = A+B\omega$

$-\omega^2 = A+B\omega$

$1+\omega = A+B\omega$

∴ $A=1$, $B=1 \Rightarrow (A,B)=(1,1)$

$\omega^{12} = (\omega^3)^4$

$= 1^4$

$= 1$

$1+\omega+\omega^2 = 0$

$1+\omega = -\omega^2$

∴ Ans: (d) (1,1)

20) The principal argument of the complex number $\frac{(1+i\sqrt{3})^2}{4i(1-i\sqrt{3})}$ is

(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{5\pi}{6}$ (d) $\frac{\pi}{2}$

Sol:- Let $z = 1+i\sqrt{3} = r(\cos\theta + i\sin\theta)$

$r = \sqrt{1+3} = \sqrt{4} = 2$

$\alpha = \tan^{-1}\left|\frac{\sqrt{3}}{1}\right|$

$\alpha = \frac{\pi}{3}$

$1+i\sqrt{3}$ lies in 1st quadrant

$\theta = \pi/3$

$1+i\sqrt{3} = 2(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})$

$(1-i\sqrt{3}) = 2\left[\cos\left(\frac{\pi}{3}\right) - i\sin(\pi/3)\right]$

$= 2\left[\cos(-\frac{\pi}{3}) + i\sin(-\frac{\pi}{3})\right]$

$i = 0+i$

$= \cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$

$\therefore \frac{(1+i\sqrt{3})^2}{4i(1-i\sqrt{3})} = \frac{\frac{2\pi}{3}}{\frac{\pi}{2}\cdot(-\frac{\pi}{3})}$

$= \frac{2\pi}{3} + \frac{\pi}{3} - \frac{\pi}{2}$

$= \pi - \frac{\pi}{2}$

$= \pi/2$

∴ Ans: (d) $\frac{\pi}{2}$



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21) If $\alpha$ and $\beta$ are the roots of $x^2+x+1=0$, then $\alpha^{2020}+\beta^{2020}$ is

(a) $-2$ (b) $-1$ (c) $1$ (d) $2$

Sol:- $\alpha$ and $\beta$ are the roots of $x^2+x+1=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-1\pm\sqrt{1-4(1)(1)}}{2(1)}$$

$$=\frac{-1\pm\sqrt{-3}}{2}$$

$$=\frac{-1\pm i\sqrt{3}}{2}$$

$$\therefore x=\frac{-1+i\sqrt{3}}{2},\ \frac{-1-i\sqrt{3}}{2}\Rightarrow \omega,\omega^2$$

$\alpha,\beta\rightarrow\omega,\omega^2$

$1+\omega+\omega^2=0$

$\omega+\omega^2=-1$

$\omega^{2020}=\omega^{3(673)+1}$

$=(\omega^3)^{673}(\omega^1)$

$=1^{673}(\omega)$

$=\omega$

$$\alpha^{2020}+\beta^{2020}=\omega^{2020}+(\omega^2)^{2020}$$

$$=\omega^{2020}+(\omega^{2020})^2$$

$$=\omega+\omega^2$$

$$=-1$$

$\therefore$ Ans: (b) $-1$

22) The product of all four values of $\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)^{\frac{3}{4}}$ is

(a) $-2$ (b) $-1$ (c) $1$ (d) $2$

Sol:- $z=\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)^{\frac{3}{4}}=\left(\cos\frac{3\pi}{3}+i\sin\frac{3\pi}{3}\right)^{\frac{1}{4}}$

$=(\cos\pi+i\sin\pi)^{1/4}$

Adding $2k\pi$

$=\left[\cos(\pi+2k\pi)+i\sin(\pi+2k\pi)\right]^{1/4}$

$=\cos\left(\frac{\pi+2k\pi}{4}\right)+i\sin\left(\frac{\pi+2k\pi}{4}\right)$ $k=0,1,2,3$

If $k=0$, $z=\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}=\frac{1}{\sqrt2}+i\frac{1}{\sqrt2}$

If $k=1$, $z=\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}$

If $k=2$, $z=\cos\frac{5\pi}{4}+i\sin\frac{5\pi}{4}$

If $k=3$, $z=\cos\frac{7\pi}{4}+i\sin\frac{7\pi}{4}$

product: $\cos\left(\frac{\pi}{4}+\frac{3\pi}{4}+\frac{5\pi}{4}+\frac{7\pi}{4}\right)+i\sin\left(\frac{\pi}{4}+\frac{3\pi}{4}+\frac{5\pi}{4}+\frac{7\pi}{4}\right)$

$=\cos 4\pi+i\sin 4\pi$

$=(\cos\pi+i\sin\pi)^4$

$=(-1+0)^4=(-1)^4=1$

$\therefore$ Ans: (c) 1

23) If $\omega\neq1$ is a cubic root of unity and $\begin{vmatrix}1&1&1\\1&-\omega^2-1&\omega^2\\1&\omega^2&\omega^7\end{vmatrix}=3k$, then $k$ is equal to

(a) $1$ (b) $-1$ (c) $\sqrt3 i$ (d) $-\sqrt3 i$

Sol:- $\begin{vmatrix}1&1&1\\1&-\omega^2-1&\omega^2\\1&\omega^2&\omega^7\end{vmatrix}=\begin{vmatrix}1&1&1\\1&\omega&\omega^2\\1&\omega^2&\omega\end{vmatrix}=3k$

$1+\omega+\omega^2=0$

$\omega=-1-\omega^2$

$\omega^7=\omega^3\omega^3.\omega=\omega$

$1(\omega^2-\omega^4)-1(\omega-\omega^2)+1(\omega^2-\omega)=3k$



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$\omega^2 - \omega^4 - \omega + \omega^2 + \omega^2 - \omega = 3k$

$\omega^2 - \omega - \omega + \omega^2 + \omega^2 - \omega = 3k$

$-3\omega + 3\omega^2 = 3k$

$-\omega + \omega^2 = k$

Cube root

$1, \omega, \omega^2 \Rightarrow 1, -\frac{1}{2} + i\frac{\sqrt{3}}{2}, -\frac{1}{2} - i\frac{\sqrt{3}}{2}$

$-\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) + \left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) = k$

$\frac{1}{2} - i\frac{\sqrt{3}}{2} - \frac{1}{2} - i\frac{\sqrt{3}}{2} = k$

$-2i\frac{\sqrt{3}}{2} = k$

$\therefore k = -\sqrt{3}i$

$\therefore$ Ans: (d) $-\sqrt{3}i$

24) The value of $\left(\frac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{10}$ is

(a) $cis\frac{2\pi}{3}$ (b) $cis\frac{4\pi}{3}$ (c) $-cis\frac{2\pi}{3}$ (d) $-cis\frac{4\pi}{3}$

Sol:-

$1+\sqrt{3}i = r(\cos\theta + i\sin\theta)$

$r = \sqrt{1^2+(\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$

$\alpha = \tan^{-1}\left|\frac{\sqrt{3}}{1}\right| = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$

$1+\sqrt{3}i$ lies in 1st quadrant

$\theta = \frac{\pi}{3}$

$1+\sqrt{3}i = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

$(1+\sqrt{3}i)^{10} = 2^{10}\left[\cos\frac{10\pi}{3} + i\sin\frac{10\pi}{3}\right]$

$(1-\sqrt{3}i)^{10} = 2^{10}\left[\cos\frac{10\pi}{3} - i\sin\frac{10\pi}{3}\right]$

$= 2^{10}\left[+\cos\left(-\frac{10\pi}{3}\right) + i\sin\left(-\frac{10\pi}{3}\right)\right]$

$\Rightarrow \frac{(1+\sqrt{3}i)^{10}}{(1-\sqrt{3}i)^{10}}$

$= \frac{2^{10}\left[\cos\frac{10\pi}{3} + i\sin\frac{10\pi}{3}\right]}{2^{10}\left[\cos\frac{10\pi}{3} + i\sin\frac{10\pi}{3}\right]}$

$= \cos\frac{20\pi}{3} + i\sin\frac{20\pi}{3}$

$= \cos\left(6\pi - \frac{2\pi}{3}\right) + i\sin\left(6\pi + \frac{2\pi}{3}\right)$

$= \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$

$= cis\frac{2\pi}{3}$

$\therefore$ Ans: (a) $cis\frac{2\pi}{3}$

25) If $\omega = cis\frac{2\pi}{3}$, then the number of distinct roots of

$$\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0$$

(a) 1 (b) 2 (c) 3 (d) 4

Sol:-

$\omega = \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$

$= \cos\left(\pi - \frac{\pi}{3}\right) + i\sin\left(\pi - \frac{\pi}{3}\right)$

$= -\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$

$\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$

$\therefore \omega$ is cuberoot of unity

$\omega^3 = 1, \quad 1+\omega+\omega^2 = 0$

$\omega^2 = \left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)^2$

$= \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}$

$= \cos\left(\pi + \frac{\pi}{3}\right) + i\sin\left(\pi + \frac{\pi}{3}\right)$

$= -\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}$

$\omega^2 = -\frac{1}{2} - i\frac{\sqrt{3}}{2}$.

$R_1 \to R_1 + R_2 + R_3$

$$\begin{vmatrix} z+1+\omega+\omega^2 & \omega+z+\omega^2+1 & \omega^2+1+z+\omega \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0$$

$$\begin{vmatrix} z & z & z \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0 \qquad C_1 \to C_1 - C_2, \; C_2 \to C_2 - C_3$$

$$\begin{vmatrix} 0 & 0 & z \\ \omega - z - \omega^2 & z+\omega^2-1 & 1 \\ \omega^2 - 1 & 1-z-\omega & z+\omega \end{vmatrix} = 0$$

$z\left[(\omega - z - \omega^2)(1-z-\omega) - (\omega^2-1)(z+\omega^2-1)\right] = 0$

$z\left[\omega - z\omega - \omega^2 - z + z^2 + z\omega - \omega^2 + z\omega^2 + 1 - (\omega^2 z + \omega - \omega^2 - z - \omega^2 + 1)\right] = 0$

$z\left[\omega - \omega^2 - z + z^2 - \omega^2 + z\omega^2 + 1 - \omega^2 z - \omega + \omega^2 + z + \omega^2 - 1\right] = 0$

$z(z^2) = 0$

$z^3 = 0$

$\therefore z = 0$

z has only one root

$\therefore$ Ans: (a) 1



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# EXERCISE 3.7

Choose the correct or the most suitable answer from the given four alternatives:

1) A zero of $x^3+64$ is

(a) 0 (b) 4 (c) $4i$ (d) $-4$

Sol:-

$x^3+64=0$

$x^3 = -64$

$x = (-64)^{1/3}$

$x = (-4^3)^{1/3}$

$x = -4$ is zero of $x^3+64$

∴ Ans: (d) $-4$

2) If $f$ and $g$ are polynomials of degree $m$ and $n$ respectively, if $h(x)=(fog)(x)$, then the degree of $h$ is

(a) $mn$ (b) $m+n$ (c) $m^n$ (d) $n^m$

Sol:-

$f(x) = x^m$ $\quad g(x) = x^n$

$(fog)(x) = f[g(x)]$

$= f(x^n)$

$= (x^n)^m$

$h(x) = x^{mn}$

∴ degree of $h$ is $mn$

∴ Ans: (a) $mn$

3) A polynomial equation in $x$ of degree $n$ always has

(a) $n$ distinct roots (b) $n$ real roots

(c) $n$ imaginary roots (d) at most one root.

Sol:-

A polynomial of degree $n$ always has $n$ roots

∴ Ans (c) $n$ imaginary roots

4) If $\alpha, \beta$ and $\gamma$ are the zeros of $x^3+px^2+qx+r$, then $\sum \frac{1}{\alpha}$ is

(a) $-\frac{q}{r}$ (b) $-\frac{p}{r}$ (c) $\frac{q}{r}$ (d) $-\frac{q}{p}$

Sol:-

Given $\alpha, \beta, \gamma$ are the roots of the equation



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$$x^3 + px^2 + qx + r = 0$$

$$\alpha+\beta+\gamma = \frac{-p}{1} = -p$$

$$\alpha\beta+\beta\gamma+\gamma\alpha = \frac{q}{1} = q$$

$$\alpha\beta\gamma = \frac{-r}{1} = -r$$

$$\therefore \sum \frac{1}{\alpha} = \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$$

$$= \frac{\beta\gamma + \gamma\alpha + \beta\alpha}{\alpha\beta\gamma}$$

$$= \frac{q}{-r}$$

$$\therefore \sum \frac{1}{\alpha} = -\frac{q}{r}$$

$\therefore$ Ans: (a) $-\frac{q}{r}$

5) According to the rational root theorem, which number is not possible rational zero of $4x^7 + 2x^4 - 10x^3 - 5$?

(a) $-1$ (b) $\frac{5}{4}$ (c) $\frac{4}{5}$ (d) 5

Sol:-

$$p(x) = 4x^7 + 2x^4 - 10x^3 - 5$$

$a_n = 4$, $a_0 = -5$

If $\frac{p}{q}$ is a zero of $p(x)$ where $(p, q) = 1$

$p$ divides 5 and $q$ devides 4

$p$ divides 5 $\Rightarrow$ The only numbers dividing 5 are $\pm 1$ and $\pm 5$

$q$ divides 4 $\Rightarrow$ The only numbers dividing 4 are $\pm 1, \pm 2$, and $\pm 4$

$\therefore$ The possible $\frac{p}{q}$ are $\pm 1, \pm\frac{5}{2}, \pm\frac{5}{4}$

$$p\left(\tfrac{5}{4}\right) = 4\left(\tfrac{5}{4}\right)^7 + 2\left(\tfrac{5}{4}\right)^4 - 10\left(\tfrac{5}{4}\right)^3 - 5$$

$p\left(\frac{5}{4}\right) \neq 0$

$\therefore$ The rational number $\frac{5}{4}$ cannot be a rational zero of $p(x)$.

$\therefore$ Ans: (b) $\frac{5}{4}$

6) The polynomial $x^3 - kx^2 + 9x$ has three real zeros if and only if, $k$ satisfies

(a) $|k| \leq 6$ (b) $k = 0$ (c) $|k| > 6$ (d) $|k| \geq 6$



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Sol:-

$$x^3 - kx^2 + 9x = 0$$

$$x(x^2 - kx + 9) = 0$$

$x = 0$ (or) $x^2 - kx + 9 = 0$

$x = 0$ is one root which is real

$x^2 - kx + 9 = 0$ are real roots

$a = 1, \ b = -k, \ c = 9$

$$\Delta \geq 0$$

$$b^2 - 4ac \geq 0$$

$$(-k)^2 - 4(1)(9) \geq 0$$

$$k^2 - 36 \geq 0$$

$$k^2 \geq 36$$

$$|k| \geq 6$$

$\therefore x^3 - kx^2 + 9x = 0$ has all the three roots real if $|k| \geq 6$

$\therefore$ Ans: (d) $k \geq 6$

7) The number of real numbers in $[0, 2\pi]$ satisfying $\sin^4 x - 2\sin^2 x + 1$ is

(a) 2 (b) 4 (c) 1 (d) $\infty$

Sol:- $\sin^4 x - 2\sin^2 x + 1 = 0$

Let $y = \sin^2 x$

$$y^2 - 2y + 1 = 0$$

$$(y-1)^2 = 0$$

$$y - 1 = 0$$

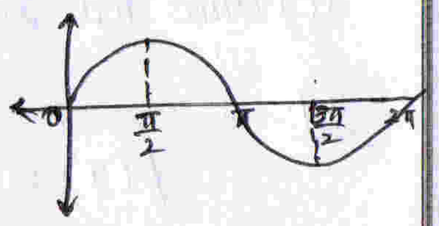
$$y = 1$$

$$\sin^2 x = 1$$

$$\sin x = \pm 1$$

$\sin x = 1, -1$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$




$\therefore$ Ans: (a) 2

$\therefore$ No. of real numbers is 2

8) If $x^3 + 12x^2 + 10ax + 1999$ definitely has a positive zero, if and only if

(a) $a \geq 0$ (b) $a > 0$ (c) $a < 0$ (d) $a \leq 0$

Sol:- $p(x) = x^3 + 12x^2 + 10ax + 1999$

if $a < 0$, No. of sign changes of $p(x) = 2$

$\therefore p(x)$ has atmost 2 positive roots

$\therefore$ Ans: (c) $a < 0$



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9) The polynomial $x^3 + 2x + 3$ has

(a) One negative and two imaginary zeros

(b) One positive and two imaginary zeros

(c) three real zeros (d) no zeros

Sol:-

$$p(x) = x^3 + 2x + 3$$

There are no sign changes for $p(x)$

$\therefore p(x)$ has no positive root

$$p(-x) = (-x)^3 + 2(-x) + 3$$

$$= -x^3 - 2x + 3$$

There are 1 sign changes for $p(-x)$

$\therefore p(x)$ has atmost 1 negative root

$p(x)$ has no positive root and atmost one negative root

$p(x)$ of degree 3 it has 3 roots

$\therefore$ Imaginary root = Total no. of roots − real root

$= 3 - 1 = 2$

$\therefore p(x)$ has one negative and two imaginary root

$\therefore$ Ans: (a)

10) The number of positive zeros of the polynomial $\sum_{j=0}^{n} {}^nC_r (-1)^r x^r$ is

(a) 0 (b) n (c) $< n$ (d) r

Sol:-

Let $p(x) = \sum_{j=0}^{n} {}^nC_r (-1)^r x^r$

$p(x) = {}^nC_0(-1)^0x^0 + {}^nC_1(-1)^1x^1 + {}^nC_2(-1)^2x^2 + \cdots$

$= {}^nC_0 x^0 - {}^nC_1 x^1 + {}^nC_2 x^2 - {}^nC_3 x^3 \cdots$

No. of sign changes $= n$

$\therefore$ maximum No. of positive roots $= n$

$\therefore$ Ans: (b) n

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# EXERCISE 4.6

Choose the correct or the most suitable answer from the given four alternatives.

1) The value of $\sin^{-1}(\cos x)$, $0 \le x \le \pi$ is

(a) $\pi - x$ (b) $x - \frac{\pi}{2}$ (c) $\frac{\pi}{2} - x$ (d) $x - \pi$

Sol:-

$$\sin^{-1}(\cos x) = \sin^{-1}\left(\sin\left(\frac{\pi}{2} - x\right)\right)$$

$$= \frac{\pi}{2} - x$$

∴ Ans:- (c) $\frac{\pi}{2} - x$

2) If $\sin^{-1}x + \sin^{-1}y = \frac{2\pi}{3}$, then $\cos^{-1}x + \cos^{-1}y$ is equal to

(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $\pi$

Sol:-

$$\sin^{-1}x + \sin^{-1}y = \frac{2\pi}{3}$$

$$\frac{\pi}{2} - \cos^{-1}x + \frac{\pi}{2} - \cos^{-1}y = \frac{2\pi}{3}$$

$$\frac{\pi}{2} + \frac{\pi}{2} - \frac{2\pi}{3} = \cos^{-1}x + \cos^{-1}y$$

$$\pi - \frac{2\pi}{3} = \cos^{-1}x + \cos^{-1}y$$

$$\therefore \cos^{-1}x + \cos^{-1}y = \frac{\pi}{3}$$

∴ Ans: (b) $\frac{\pi}{3}$

$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$
$\sin^{-1}x = \frac{\pi}{2} - \cos^{-1}x$
$\sin^{-1}y = \frac{\pi}{2} - \cos^{-1}y$



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3) $\sin^{-1}\frac{3}{5} - \cos^{-1}\frac{12}{13} + \sec^{-1}\frac{5}{3} - \operatorname{cosec}^{-1}\frac{13}{12}$ is equal to

(a) $2\pi$ (b) $\pi$ (c) 0 (d) $\tan^{-1}\frac{12}{65}$

Sol:-

$$\sin^{-1}\frac{3}{5} - \cos^{-1}\frac{12}{13} + \sec^{-1}\frac{5}{3} - \operatorname{cosec}^{-1}\frac{13}{12}$$

$$= \sin^{-1}\frac{3}{5} - \cos^{-1}\frac{12}{13} + \cos^{-1}\frac{3}{5} - \sin^{-1}\frac{12}{13}$$

$$= \sin^{-1}\frac{3}{5} + \cos^{-1}\frac{3}{5} - \left(\sin^{-1}\frac{12}{13} + \cos^{-1}\frac{12}{13}\right)$$

$$= 1 - 1$$

$$= 0$$

∴ Ans: (c) 0

4) If $\sin^{-1}x = 2\sin^{-1}\alpha$ has a solution, then

(a) $|\alpha| \le \frac{1}{\sqrt{2}}$ (b) $|\alpha| \ge \frac{1}{\sqrt{2}}$ (c) $|\alpha| < \frac{1}{\sqrt{2}}$ (d) $|\alpha| > \frac{1}{\sqrt{2}}$

Sol:-

$$-\frac{\pi}{2} \le \sin^{-1}x \le \frac{\pi}{2}$$

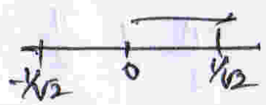
$$-\frac{\pi}{2} \le 2\sin^{-1}\alpha \le \frac{\pi}{2} \quad \text{(divide by 2)}$$

$$-\frac{\pi}{4} \le \sin^{-1}\alpha \le \frac{\pi}{4}$$

$$\sin\left(-\frac{\pi}{4}\right) \le \alpha \le \sin\left(\frac{\pi}{4}\right)$$

$$-\sin\frac{\pi}{4} \le \alpha \le \sin\frac{\pi}{4}$$

$$-\frac{1}{\sqrt{2}} \le \alpha \le \frac{1}{\sqrt{2}}$$




∴ Ans: (a) $|\alpha| \le \frac{1}{\sqrt{2}}$

5) $\sin^{-1}(\cos x) = \frac{\pi}{2} - x$ is valid for

(a) $-\pi \le x \le 0$ (b) $0 \le x \le \pi$ (c) $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ (d) $-\frac{\pi}{4} \le x \le \frac{3\pi}{4}$

Sol:-

$$\sin^{-1}(\cos x) = \begin{cases} \frac{\pi}{2} - x, & \text{if } x \in [0,\pi] \\ \frac{\pi}{2} - y, & \text{if } x \notin [0,\pi] \text{ and } \cos x = \cos y, \\ & y \in [0,\pi] \end{cases}$$

∴ Ans: (b) $0 \le x \le \pi$

6) If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$, the value of $x^{2017} + y^{2018} + z^{2019} - \frac{9}{x^{101}+y^{101}+z^{101}}$ is

(a) 0 (b) 1 (c) 2 (d) 3

Sol:-

$$\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$$

$$\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{\pi}{2} + \frac{\pi}{2} + \frac{\pi}{2}$$

| $\sin^{-1}x = \frac{\pi}{2}$ | $\sin^{-1}y = \frac{\pi}{2}$ | $\sin^{-1}z = \frac{\pi}{2}$ |
|---|---|---|
| $x = \sin\frac{\pi}{2}$ | $y = \sin\frac{\pi}{2}$ | $z = \sin\frac{\pi}{2}$ |
| $x = 1$ | $y = 1$ | $z = 1$ |

$$x^{2017} + y^{2018} + z^{2019} - \frac{9}{x^{101}+y^{101}+z^{101}} = 1+1+1 - \frac{9}{1+1+1}$$

$$= 3 - \frac{9}{3}$$

$$= 0$$

∴ Ans: (a) 0



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7) If $\cot^{-1}x = \frac{2\pi}{5}$ for some $x \in R$, the value of $\tan^{-1}x$ is

(a) $-\frac{\pi}{10}$ (b) $\frac{\pi}{5}$ (c) $\frac{\pi}{10}$ (d) $-\frac{\pi}{5}$

Sol:-

$$\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$$

$$\tan^{-1}x = \frac{\pi}{2} - \cot^{-1}x$$

$$= \frac{\pi}{2} - \frac{2\pi}{5}$$

$$= \frac{5\pi - 4\pi}{10}$$

$$= \frac{\pi}{10}$$

∴ Ans: (c) $\frac{\pi}{10}$

8) The domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$ is

(a) $[1,2]$ (b) $[-1,1]$ (c) $[0,1]$ (d) $[-1,0]$

Sol:- the domain of $\sin^{-1}x$ is $[-1,1]$

$$-1 \le \sqrt{x-1} \le 1$$

$$0 \le x-1 \le 1$$

$$0+1 \le x \le 1+1$$

$$1 \le x \le 2$$

The domain of the function is $[1,2]$

∴ Ans: (a) $[1,2]$

9) If $x = \frac{1}{5}$, the value of $\cos(\cos^{-1}x + 2\sin^{-1}x)$ is

(a) $-\sqrt{\frac{24}{25}}$ (b) $\sqrt{\frac{24}{25}}$ (c) $\frac{1}{5}$ (d) $-\frac{1}{5}$

Sol:-

$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

$$\cos(\cos^{-1}x + 2\sin^{-1}x)$$

$$= \cos(\cos^{-1}x + \sin^{-1}x + \sin^{-1}x)$$

$$= \cos\left(\frac{\pi}{2} + \sin^{-1}x\right)$$

$\cos\left(\frac{\pi}{2}+\theta\right) = -\sin\theta$

$$= -\sin(\sin^{-1}x)$$

$$= -x$$

$$= -\frac{1}{5}$$

∴ Ans: (d) $-\frac{1}{5}$

10) $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right)$ is equal to

(a) $\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$ (b) $\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)$ (c) $\frac{1}{2}\tan^{-1}\left(\frac{3}{5}\right)$ (d) $\tan^{-1}\left(\frac{1}{2}\right)$

Sol:- $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$

$$\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right) = \tan^{-1}\left(\frac{\frac{1}{4}+\frac{2}{9}}{1-\frac{1}{4}\left(\frac{2}{9}\right)}\right)$$

$$= \tan^{-1}\left(\frac{\frac{9+8}{36}}{\frac{36-2}{36}}\right)$$

$$= \tan^{-1}\frac{17}{34}$$

$$= \tan^{-1}\left(\frac{1}{2}\right)$$

∴ Ans: (d) $\tan^{-1}\left(\frac{1}{2}\right)$



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11) If the function $f(x) = \sin^{-1}(x^2-3)$, then $x$ belongs to

(a) $[-1,1]$ (b) $[\sqrt{2},2]$ (c) $[-2,-\sqrt{2}] \cup [\sqrt{2},2]$ (d) $[-2,-\sqrt{2}]$

Sol:-

The range of $\sin^{-1} x : [-1,1] \to [-\frac{\pi}{2}, \frac{\pi}{2}]$

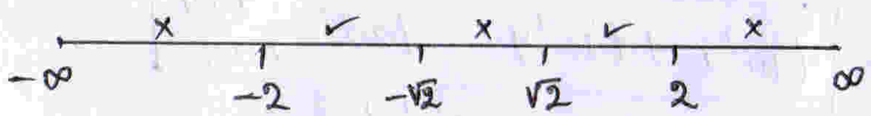
$$-1 \le x^2-3 \le 1$$

$$-1+3 \le x^2 \le 1+3$$

$$2 \le x^2 \le 4$$

$x^2 = 2$ | $x^2 = 4$

$x = \pm\sqrt{2}$ | $x = \pm 2$




$x$ is $[-2,-\sqrt{2}] \cup [\sqrt{2},2]$

∴ Ans: (c) $[-2,-\sqrt{2}] \cup [\sqrt{2},2]$

12) If $\cot^{-1}2$ and $\cot^{-1}3$ are two angles of a triangle, then the third angle is

(a) $\frac{\pi}{4}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{3}$

Sol:-

Let the third angle is $x$

Sum of three angle is $180^\circ$

$$\cot^{-1}2 + \cot^{-1}3 + x = \pi$$

$$x = \pi - \cot^{-1}2 - \cot^{-1}3$$

$$= \pi - [\cot^{-1}2 + \cot^{-1}3]$$

$$= \pi - \left[\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3}\right]$$

$$= \pi - \left[\tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\left(\frac{1}{3}\right)}\right)\right]$$

$$= \pi - \left[\tan^{-1}\left(\frac{\frac{3+2}{6}}{\frac{6-1}{6}}\right)\right]$$

$$= \pi - \tan^{-1}\left(\frac{5}{5}\right)$$

$$= \pi - \tan^{-1}(1)$$

$$= \pi - \frac{\pi}{4}$$

$$= \frac{4\pi-\pi}{4}$$

$$= \frac{3\pi}{4}$$

∴ Ans: (b) $\frac{3\pi}{4}$

$\cot^{-1}x = \tan^{-1}\frac{1}{x}$

$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$

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13) $\sin^{-1}\left(\tan\frac{\pi}{4}\right) - \sin^{-1}\left(\sqrt{\frac{3}{x}}\right) = \frac{\pi}{6}$. Then $x$ is a root of the equation.

(a) $x^2-x-6=0$ (b) $x^2-x-12=0$ (c) $x^2+x-12=0$ (d) $x^2+x-6=0$

Sol:-

$$\sin^{-1}\left(\tan\tfrac{\pi}{4}\right) - \sin^{-1}\left(\sqrt{\tfrac{3}{x}}\right) = \frac{\pi}{6}$$

$$\sin^{-1}(1) - \sin^{-1}\left(\sqrt{\tfrac{3}{x}}\right) = \frac{\pi}{6}$$

$$\frac{\pi}{2} - \sin^{-1}\left(\sqrt{\tfrac{3}{x}}\right) = \frac{\pi}{6}$$

$$\cos^{-1}\sqrt{\frac{3}{x}} = \frac{\pi}{2}$$

$$\sqrt{\frac{3}{x}} = \cos\frac{\pi}{6}$$

$$\frac{\sqrt{3}}{\sqrt{x}} = \frac{\sqrt{3}}{2}$$

$$\sqrt{x} = 2$$

$$x = 4$$

∴ Ans: (b) $x^2-x-12=0$

$\sin\frac{\pi}{2} = 1$

$\frac{\pi}{2} = \sin^{-1}(1)$

$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

$\frac{\pi}{2} - \sin^{-1}x = \cos^{-1}x$

When $x=4$

$x^2-x-12=0$

$4^2-4-12=0$

$16-16=0$

$0=0$

14) $\sin^{-1}(2\cos^2x-1) + \cos^{-1}(1-2\sin^2x) =$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

Sol:-

$$\sin^{-1}(2\cos^2x-1) + \cos^{-1}(1-2\sin^2x)$$

$$= \sin^{-1}(\cos 2x) + \cos^{-1}(\cos 2x)$$

$$= \frac{\pi}{2}$$

$\cos 2x = 2\cos^2x - 1$

$\cos 2x = 1-2\sin^2x$

$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

∴ Ans: (a) $\frac{\pi}{2}$

15) If $\cos^{-1}(\sqrt{\sin\alpha}) + \tan^{-1}(\sqrt{\sin\alpha}) = u$, then $\cos 2u$ is equal to

(a) $\tan^2\alpha$ (b) 0 (c) $-1$ (d) $\tan 2\alpha$

Sol:- $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$

$$\cos^{-1}(\sqrt{\sin\alpha}) + \tan^{-1}(\sqrt{\sin\alpha}) = u$$

$$\frac{\pi}{2} = u$$

$$\therefore \cos 2u = \cos 2\left(\tfrac{\pi}{2}\right)$$

$$= \cos\pi$$

$$= -1$$

∴ Ans: (c) $-1$

16) If $|x| \le 1$, then $2\tan^{-1}x - \sin^{-1}\frac{2x}{1+x^2}$ is equal to

(a) $\tan^{-1}x$ (b) $\sin^{-1}x$ (c) 0 (d) $\pi$

Sol:- $2\tan^{-1}x - \sin^{-1}\frac{2x}{1+x^2}$

$$= 2\tan^{-1}x - 2\tan^{-1}x$$

$$= 0$$

$\sin^{-1}\frac{2x}{1+x^2} = 2\tan^{-1}x$

∴ Ans: (c) 0



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17) The equation $\tan^{-1}x - \cot^{-1}x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ has

(a) no solution (b) unique solution

(c) two solutions (d) infinite number of solutions

Sol:- $\tan^{-1}x - \cot^{-1}x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$

$\cot^{-1}x = \frac{\pi}{2} - \tan^{-1}x$

$\tan^{-1}x - \left[\frac{\pi}{2} - \tan^{-1}x\right] = \frac{\pi}{6}$

$\tan^{-1}x - \frac{\pi}{2} + \tan^{-1}x = \frac{\pi}{6}$

$2\tan^{-1}x = \frac{\pi}{6} + \frac{\pi}{2}$

$2\tan^{-1}x = \frac{\pi + 3\pi}{6}$

$2\tan^{-1}x = \frac{4\pi}{6}$

$\tan^{-1}x = \frac{\pi}{3}$

$x = \tan\frac{\pi}{3}$

$x = \sqrt{3}$

∴ Ans: (b) Unique solution

18) If $\sin^{-1}x + \cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$, then $x$ is equal to

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{5}}$ (c) $\frac{2}{\sqrt{5}}$ (d) $\frac{\sqrt{3}}{2}$

Sol:- $\sin^{-1}x + \cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$

$\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$

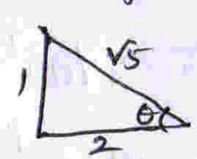
$\cot^{-1}x = \frac{\pi}{2} - \tan^{-1}x$

$\sin^{-1}x = \frac{\pi}{2} - \cot^{-1}\left(\frac{1}{2}\right)$

$\tan^{-1}x = \frac{\pi}{2} - \cot^{-1}x$

$\sin^{-1}x = \tan^{-1}\left(\frac{1}{2}\right)$

$\tan = \frac{opp}{adj} = \frac{1}{2}$




$hyp = \sqrt{1+2^2}$

$= \sqrt{1+4} = \sqrt{5}$

$\sin^{-1}x = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$

$x = \frac{1}{\sqrt{5}}$

∴ Ans: (b) $\frac{1}{\sqrt{5}}$

19) If $\sin^{-1}\frac{x}{5} + \operatorname{Cosec}^{-1}\frac{5}{4} = \frac{\pi}{2}$, then the value of $x$ is

(a) 4 (b) 5 (c) 2 (d) 3

Sol:- $\sin^{-1}\frac{x}{5} + \operatorname{Cosec}^{-1}\frac{5}{4} = \frac{\pi}{2}$

$\operatorname{Cosec}^{-1}x = \sin^{-1}\frac{1}{x}$

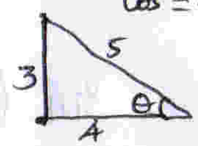
$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

$\cos^{-1}x = \frac{\pi}{2} - \sin^{-1}x$

$\sin^{-1}\frac{x}{5} + \sin^{-1}\frac{4}{5} = \frac{\pi}{2}$

$\sin^{-1}\frac{x}{5} = \frac{\pi}{2} - \sin^{-1}\left(\frac{4}{5}\right)$

$\cos = \frac{adj}{hyp} = \frac{4}{5}$




$opp = \sqrt{25-16}$

$= \sqrt{9}$

$= 3$

$\sin^{-1}\frac{x}{5} = \cos^{-1}\frac{4}{5}$

$\sin^{-1}\frac{x}{5} = \sin^{-1}\frac{3}{5}$

$x = \frac{5}{5}^{3} = 3$

∴ Ans: (d) 3

20) $\sin(\tan^{-1}x)$, $|x| < 1$ is equal to

(a) $\frac{x}{\sqrt{1-x^2}}$ (b) $\frac{1}{\sqrt{1-x^2}}$ (c) $\frac{1}{\sqrt{1+x^2}}$ (d) $\frac{x}{\sqrt{1+x^2}}$

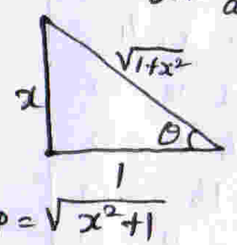
Sol:-

$\sin(\tan^{-1}x)$

$= \sin\left(\sin^{-1}\frac{x}{\sqrt{1+x^2}}\right)$

$= \frac{x}{\sqrt{1+x^2}}$

$\tan = \frac{opp}{adj} = \frac{x}{1}$




$hyp = \sqrt{x^2+1}$

∴ Ans: (d) $\frac{x}{\sqrt{1+x^2}}$

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# EXERCISE 5.6

Choose the correct or the most suitable answer from the given four alternatives:

1) The equation of the circle passing through (1,5) and (4,1) and touching y-axis is $x^2+y^2-5x-6y+9+\lambda(4x+3y-19)=0$ where $\lambda$ is equal to

(a) $0, -\frac{40}{9}$ (b) $0$ (c) $\frac{40}{9}$ (d) $-\frac{40}{9}$

Sol:-

Given: $x^2+y^2-5x-6y+9+\lambda(4x+3y-19)=0$

$$x^2+y^2-5x-6y+9+4\lambda x+3\lambda y-19\lambda=0$$

$$x^2+y^2+x(-5+4\lambda)+y(-6+3\lambda)+9-19\lambda=0$$

$$x^2+y^2+2gx+2fy+c=0$$

$$g=\frac{4\lambda-5}{2},\quad f=\frac{3\lambda-6}{2},\quad c=9-19\lambda$$

Centre $(-g,-f)=\left(\frac{5-4\lambda}{2},\frac{6-3\lambda}{2}\right)$

$$\text{Radius}=\sqrt{g^2+c^2-c}$$

$$=\sqrt{\left(\frac{4\lambda-5}{2}\right)^2+\left(\frac{3\lambda-6}{2}\right)^2-(9-19\lambda)}$$

$$\therefore \frac{5-4\lambda}{2}=\sqrt{\left(\frac{4\lambda-5}{2}\right)^2+\left(\frac{3\lambda-6}{2}\right)^2-(9-19\lambda)}$$

$$\left(\frac{5-4\lambda}{2}\right)^2=\left(\frac{4\lambda-5}{2}\right)^2+\left(\frac{3\lambda-6}{2}\right)^2-(9-19\lambda)$$

$$9-19\lambda=\frac{(3\lambda-6)^2}{4}$$

$$4(9-19\lambda)=9\lambda^2-36\lambda+36$$

$$36-76\lambda=9\lambda^2-36\lambda+36$$

$$9\lambda^2-36\lambda+76\lambda=0$$

$$9\lambda^2+40\lambda=0$$

$$\lambda(9\lambda+40)=0$$

$\lambda=0$, $9\lambda+40=0$

$$9\lambda=-40$$

$$\lambda=\frac{-40}{9}$$

∴ Ans: (a) $0, \frac{-40}{9}$

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2) The eccentricity of the hyperbola whose latus rectum is 8 and conjugate axis is equal to half the distance between the foci is

(a) $\frac{4}{3}$ (b) $\frac{4}{\sqrt{3}}$ (c) $\frac{2}{\sqrt{3}}$ (d) $\frac{3}{2}$

Sol:-

latus rectum is 8

$$\frac{2b^2}{a} = 8$$

$$b^2 = 4a$$

Given: $2b = \frac{1}{2}(2ae)$

$$\therefore 2b = ae \Rightarrow b = \frac{ae}{2}$$

$$b^2 = a^2(e^2-1)$$

$$b^2 = a^2e^2 - a^2$$

$$b^2 = (2b)^2 - a^2$$

$$b^2 = 4b^2 - a^2$$

$$a^2 = 4b^2 - b^2$$

$$a^2 = 3b^2$$

$$= 3\left(\frac{ae}{2}\right)^2 \qquad b = \frac{ae}{2}$$

$$a^2 = 3\frac{a^2e^2}{4}$$

$$4 = 3e^2$$

$$e^2 = \frac{4}{3}$$

$$e = \frac{2}{\sqrt{3}}$$

∴ Ans: (c) $\frac{2}{\sqrt{3}}$

3) The circle $x^2+y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct point if

(a) $15 < m < 65$ (b) $35 < m < 85$ (c) $-85 < m < -35$ (d) $-35 < m < 15$

Sol:-

The circle $x^2+y^2-4x-8y-5=0$ intersects the line $3x-4y-m=0$,

The distance between the centre and the line is always less than the radius.

$$x^2+y^2+2gx+2fy+c=0$$



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$g = \frac{-4}{2} = -2$, $f = \frac{-8}{2} = -4$, $c = -5$

Centre $(-g, -f) = (2, 4)$

Radius $= \sqrt{g^2 + f^2 - c}$

$= \sqrt{(-2)^2 + (-4)^2 + 5}$

$= \sqrt{4 + 16 + 5}$

$= \sqrt{25}$

$= 5$

$d \Rightarrow \left| \frac{6 - 16 - m}{\sqrt{9+16}} \right| < 5$

$\frac{10 + m}{\sqrt{25}} < 5$

$10 + m < 25$

$-25 < 10 + m < 25$

$-25 - 10 < m < 25 - 10$

$-35 < m < 15$

$\therefore$ Ans : (d) $-35 < m < 15$

$\frac{lx + my + n}{\sqrt{l^2 + m^2}}$

$lx + my + n \Rightarrow 3x - 4y - m$

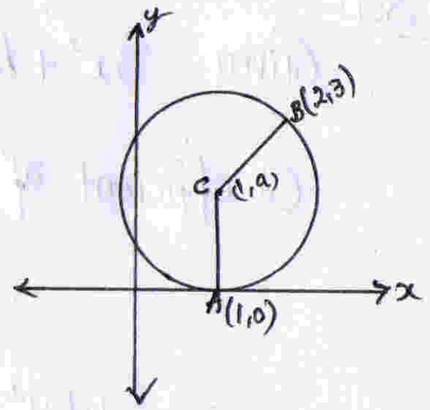
$l = 3, m = -4, \left(\frac{x}{2}, \frac{y}{4}\right)$

$n = -m$

4) The length of the diameter of the circle which touches the x-axis at the point (1,0) and passes through the point (2,3).

(a) $\frac{6}{5}$ (b) $\frac{5}{3}$ (c) $\frac{10}{3}$ (d) $\frac{3}{5}$

Sol:-




$CA = CB$

$\sqrt{(1-1)^2 + (0-a)^2} = \sqrt{(1-2)^2 + (a-3)^2}$

$a^2 = 1 + (a-3)^2$

$a^2 = 1 + a^2 - 6a + 9$

$6a = 9$

$a = \frac{5}{3}$

radius $= \sqrt{(1-1)^2 + \left(\frac{5}{3}\right)^2}$ (distance $\Rightarrow$ AT)

$= \sqrt{0 + \left(\frac{5}{3}\right)^2}$

$= \frac{5}{3}$

$\therefore$ diameter $= 2\left(\frac{5}{3}\right)$

$= \frac{10}{3}$

$\therefore r = 1, \frac{5}{3}$

$\therefore$ Ans : (c) $\frac{10}{3}$



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5) The radius of the circle $3x^2+by^2+4bx-6by+b^2=0$ is

(a) 1 (b) 3 (c) $\sqrt{10}$ (d) $\sqrt{11}$

Sol:-

Given: $3x^2+by^2+4bx-6by+b^2=0$

Co-efficient of $x^2$ = Co-efficient of $y^2$

$\therefore 3 = b$

$3x^2+by^2+4bx-6by+b^2=0$

$3x^2+3y^2+12x-18y+9=0$

$\div 3$ $x^2+y^2+4x-6y+3=0$

$\boxed{x^2+y^2+2gx+2fy+c=0}$

$g=\frac{4}{2}=2, \quad f=\frac{-6}{2}=-3, \quad c=3$

radius $=\sqrt{g^2+f^2-c}$

$=\sqrt{4+9-3}$

$=\sqrt{10}$

$\therefore$ Ans: (c) $\sqrt{10}$

6) The centre of the circle inscribed in a square formed by the lines $x^2-8x+12=0$ and $y^2-14y+45=0$ is

(a) (4,7) (b) (7,4) (c) (9,4) (d) (4,9)

Sol:-

Given:

$x^2-8x-12=0$

$(x-6)(x-2)=0$ $\quad -6-2=+12$

$x-6=0 \mid x-2=0$

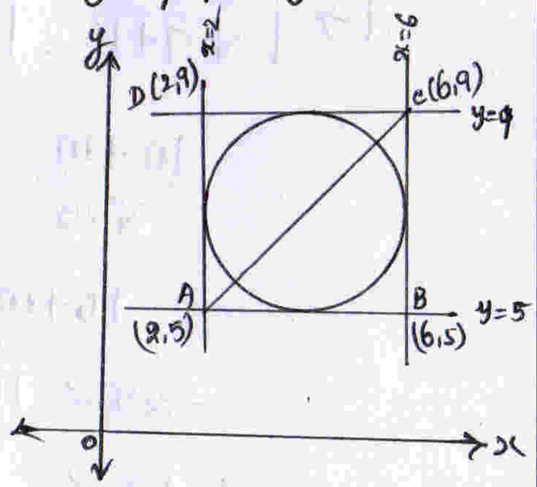
$x=6 \mid x=2$

$y^2-14y+45=0$

$(y-9)(y-5)=0$ $\quad -9-5=45$

$y-9=0 \mid y-5=0$

$y=9 \mid y=5$




Mid point of AC

$=\left(\frac{2+6}{2}, \frac{5+9}{2}\right)$

$=\left(\frac{8}{2}, \frac{14}{2}\right)$

$=(4,7)$

Centre is the mid point of x values and y values.

$\therefore$ Ans: (a) (4,7)



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7) The equation of the normal to the circle $x^2+y^2-2x-2y+1=0$ which is parallel to the line $2x+4y=3$ is

(a) $x+2y=3$ (b) $x+2y+3=0$ (c) $2x+4y+3=0$ (d) $x-2y+3=0$

Sol:-

Given: $x^2+y^2-2x-2y+1=0$ is in the form of

$$x^2+y^2+2gx+2fy+c=0$$

$$g=\frac{-2}{2}=-1, \quad f=\frac{-2}{2}=-1$$

Centre $(-g,-f)=(1,1)$

the normal passes through the centre

$$2x+4y+k=0$$
$$2(1)+4(1)+k=0$$
$$k=-6$$
$$\therefore 2x+4y-6=0$$
$$\div 2 \quad x+2y-3=0$$
$$x+2y=3$$

$\therefore$ Ans: (a) $x+2y=3$

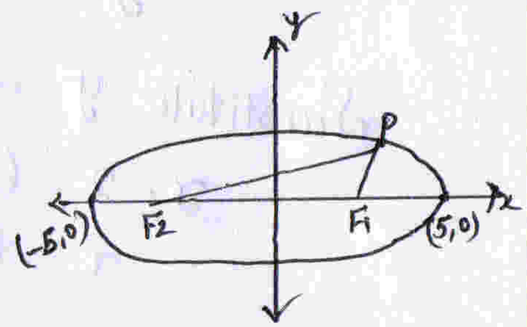
8) If $P(x,y)$ be any point on $16x^2+25y^2=400$ with foci $F_1(3,0)$ and $F_2(-3,0)$ then $PF_1+PF_2$ is

(a) 8 (b) 6 (c) 10 (d) 12

Sol:-

Given: $16x^2+25y^2=400$

$$\div 400 \quad \frac{16x^2}{400}+\frac{25y^2}{400}=1$$
$$\frac{x^2}{25}+\frac{y^2}{16}=1$$




$a^2=25, \quad b^2=16$

$a=5 \quad b=4$

$$PF_1+PF_2=2a$$
$$=2(5)$$
$$=10$$

$\therefore$ Ans: (c) 10

9) The radius of the circle passing through the point (6, 2) two of whose diameter are $x+y=6$ and $x+2y=4$ is

(a) 10 (b) $2\sqrt{5}$ (c) 6 (d) 4



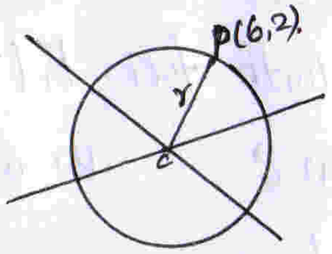
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**Sol:-**

Given: $x+y=6$ ——— (1)

$x+2y=4$ ——— (2)




(1) − (2) ⇒ $x+y=6$

$x+2y=4$

(−) (−) (−)

$-y=2$

$y=-2$

Substitute $y=-2$ in (1)

$x-2=6$

$x=8$

∴ Centre is $(8,-2)$

The distance between $(6,2)$ and the centre is the radius.

radius $= \sqrt{(8-6)^2+(-2-2)^2}$

$= \sqrt{2^2+(-4)^2}$

$= \sqrt{4+16}$

$= \sqrt{20}$

$\therefore r = 2\sqrt{5}$

∴ **Ans: (b) $2\sqrt{5}$**

10) Area of quadrilateral with foci of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and $\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$ is

(a) $4(a^2+b^2)$ (b) $2(a^2+b^2)$ (c) $a^2+b^2$ (d) $\frac{1}{2}(a^2+b^2)$

**Sol:-**

First hyperbola

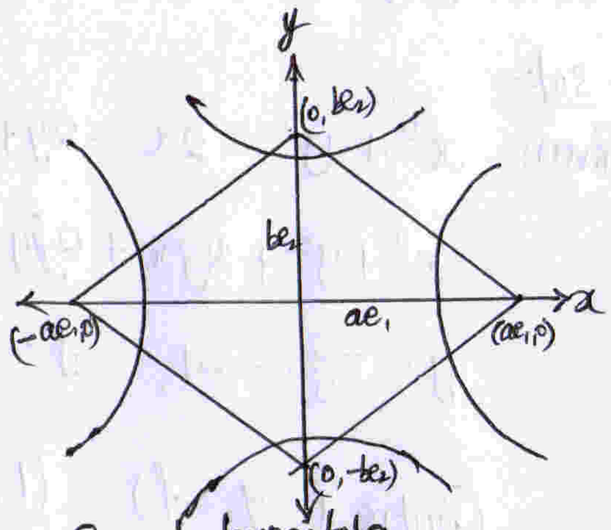
$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$b^2=a^2(e_1^2-1)$

$\frac{b^2}{a^2}=e_1^2-1$

$e_1^2=1+\frac{b^2}{a^2}$

$e_1=\sqrt{\frac{a^2+b^2}{a^2}}$




Second hyperbola

$-\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$\frac{y^2}{b^2}-\frac{x^2}{a^2}=1$

$e_2=\sqrt{\frac{b^2+a^2}{b^2}}$

Required Area

$= \frac{A}{2} \times \frac{1}{2} ae_1 \times be_2$

$= 2abe_1e_2$

$= 2ab\sqrt{\frac{a^2+b^2}{a^2}}\sqrt{\frac{b^2+a^2}{b^2}}$

$= 2ab\frac{\sqrt{a^2+b^2}}{a}\frac{\sqrt{a^2+b^2}}{b}$

$= 2(a^2+b^2)$

∴ **Ans: (b) $2(a^2+b^2)$**



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11) If the normals of the parabola $y^2 = 4x$ drawn at the end points of its latus rectum are tangents to the circle $(x-3)^2 + (y+2)^2 = r^2$, then the value of $r^2$ is

(a) 2 (b) 3 (c) 1 (d) 4

Sol:-

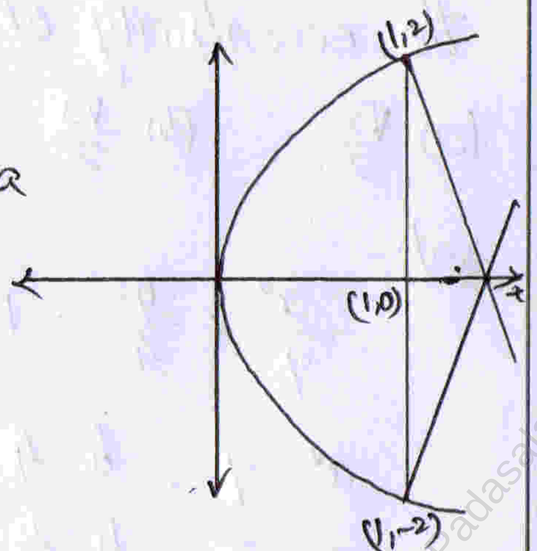
Equation of the parabola

$y^2 = 4ax$

Given: $y^2 = 4x$

$\therefore 4a = 4$

$a = 1$




Equation of the tangent: $y^2 = 4x$

at $(1,2)$ $(x_1, y_1)$

$yy_1 = 4 \cdot \frac{1}{2}(x + x_1)$

$y(2) = 2(x+1)$

$2y = 2x + 2$

$2x - 2y + 2 = 0$

$x - y + 1 = 0$

Equation of the Normal: $x + y + k = 0$

at $(1,2)$

$1 + 2 + k = 0$

$k = -3$

$\therefore x + y - 3 = 0$

Given: $(x-3)^2 + (y+2)^2 = r^2$

Centre: $(3, -2)$

The distance between $(3,-2)$ and $x + y - 3 = 0$

$= \left| \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}} \right|$

$= \left| \frac{1(3) + 1(-2) - 3}{\sqrt{1+1}} \right|$

$= \left| \frac{3 - 2 - 3}{\sqrt{2}} \right|$

$= \left| -\frac{2}{\sqrt{2}} \right|$

$= \frac{2}{\sqrt{2}}$

$r = \sqrt{2}$

$\therefore r^2 = 2$

$\therefore$ Ans: (a) 2



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12) If $x+y=k$ is a normal to the parabola $y^2=12x$, then the value of $k$ is

(a) 3 (b) $-1$ (c) 1 (d) 9

Sol:-

$y^2=12x$

Equation of tangent

$yy_1 = 6(x+x_1)$

$yy_1 = 6x+6x_1$

$6x-yy_1+6x_1=0$

Equation of Normal

$y_1x+6y+k=0$

at $(x_1,y_1)$

$y_1x_1+6y_1+k=0$

$k=-6y_1-x_1y_1$

$\therefore y_1x+6y-6y_1-x_1y_1=0$

Comparing, $x+y-k=0$

$y_1=6$

$6^2=12(x_1)$

$36=12x_1$

$x_1=3$

$x_1=3,\ y_1=6$

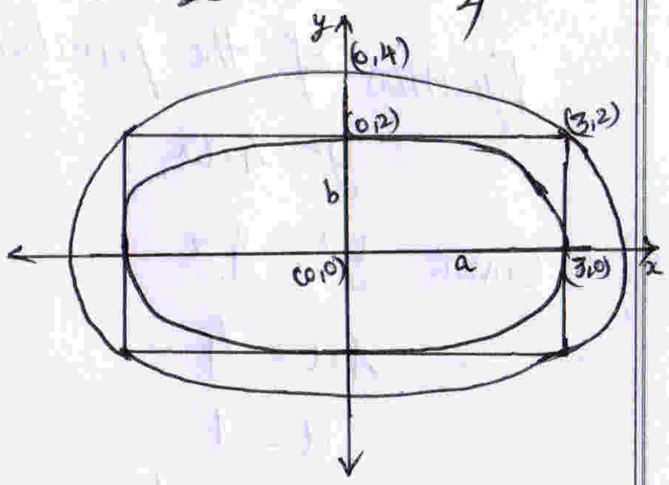
$3+6-k=0$

$k=9$

$\therefore$ Ans : (d) 9

13) The ellipse $E_1: \frac{x^2}{9}+\frac{y^2}{4}=1$ is inscribed in a rectangle R whose sides are parallel to the coordinate axes. Another ellipse $E_2$ passing through the point (0,4) circumscribes the rectangle R. The eccentricity of the ellipse is

(a) $\frac{\sqrt{2}}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$




Sol:-

$\frac{x^2}{9}+\frac{y^2}{4}=1$

$a^2=9,\ b^2=4$

$a=3 \qquad b=2$

ellipse, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

passing through (3,2), $b=4$

$\frac{3^2}{a^2}+\frac{2^2}{4^2}=1$

$\frac{9}{a^2}+\frac{4}{16}=1$

$\frac{9}{a^2}=1-\frac{1}{4}$

$\frac{9}{a^2}=\frac{3}{4}$

$3a^2=36$

$a^2=12$

$\therefore b^2=16$



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$a^2 = b^2(1-e^2)$

$12 = 16(1-e^2)$

$\frac{12}{16} = 1-e^2$

$e^2 = 1 - \frac{3}{4}$

$e^2 = \frac{1}{4}$

$\therefore e = \frac{1}{2}$

$\therefore$ Ans : (c) $\frac{1}{2}$

14) Tangents are drawn to the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ parallel to the straight line $2x - y = 1$. One of the points of Contact of tangents on the hyperbola is

(a) $\left(\frac{9}{2\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$ (b) $\left(\frac{-9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ (c) $\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ (d) $(3\sqrt{3}, -2\sqrt{2})$

Sol:-

Given: $\frac{x^2}{9} - \frac{y^2}{4} = 1$

$a^2 = 9 \quad b^2 = 4$

$a = 3 \quad b = 2$

Slope of $2x - y = 1$

$y = 2x - 1$

Comparing, $y = mx + c$

$\therefore m = 2$

$\therefore c^2 = a^2m^2 - b^2$

$= 9(4) - 4$

$= 36 - 4$

$c^2 = 32$

$c = \pm\sqrt{32} = \pm 4\sqrt{2}$

$\therefore$ The point of contact is $\left(\frac{-a^2m}{c}, \frac{-b^2}{c}\right)$

$= \frac{-9(2)}{\pm 4\sqrt{2}}, \frac{-4}{\pm 4\sqrt{2}} \quad \left|\frac{-9(2)}{-4\sqrt{2}}, \frac{-4}{-4\sqrt{2}}\right.$

$= \left(\frac{-9}{2\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$ (or) $\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$\therefore$ Ans : (c) $\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

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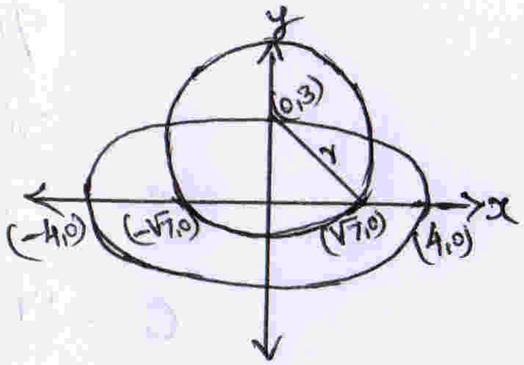
15) The equation of the circle passing through the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ having Centre at (0,3) is

(a) $x^2+y^2-6y-7=0$ (b) $x^2+y^2-6y+7=0$

(c) $x^2+y^2-6y-5=0$ (d) $x^2+y^2-6y+5=0$

Sol:-

Given: $\frac{x^2}{16}+\frac{y^2}{9}=1$




$a^2=16, \quad b^2=9$

$a=4, \quad b=3$

$(ae)^2 = a^2-b^2$

$= 16-9$

$(ae)^2 = 7$

$ae = \sqrt{7}$

$\therefore e = \frac{\sqrt{7}}{a}$

$e = \frac{\sqrt{7}}{4}$

Foci $(\pm ae, 0) = (\pm\sqrt{7}, 0)$

$r = \sqrt{(\sqrt{7}-0)^2+(0-3)^2}$

$= \sqrt{7+9}$

$= \sqrt{16}$

$\therefore r = 4$

Given: Centre $(h,k) = (0,3)$

$(x-h)^2+(y-k)^2 = r^2$

$(x-0)^2+(y-3)^2 = 4^2$

$x^2+y^2-6y+9 = 16$

$x^2+y^2-6y+9-16 = 0$

$x^2+y^2-6y-7 = 0$

$\therefore$ Ans : (a) $x^2+y^2-6y-7=0$

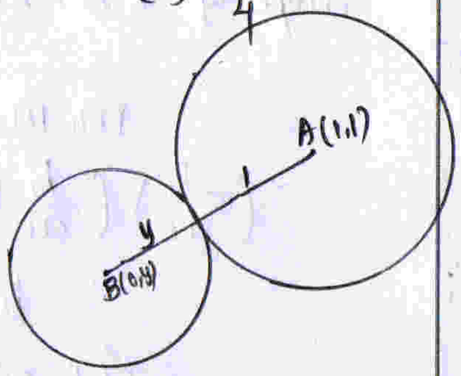
16) Let C be the circle with centre at (1,1) and radius = 1. If T is the circle centered at (0,y) passing through the origin and touching the circle C externally, then the radius of T is equal to



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(a) $\frac{\sqrt{3}}{\sqrt{2}}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$




Sol:-

$AB = y+1$

distance of $AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$y+1 = \sqrt{(0-1)^2+(y-1)^2}$

$y+1 = \sqrt{(-1)^2+(y-1)^2}$

$(y+1)^2 = 1 + (y-1)^2$

$y^2+2y+1 = 1+y^2-2y+1$

$4y = 1$

$y = \frac{1}{4}$

∴ radius, $y = \frac{1}{4}$

∴ Ans: (d) $\frac{1}{4}$

17) Consider an ellipse whose centre is of the origin and its major axis is along x-axis. If its eccentricity is $\frac{3}{5}$ and the distance between its foci is 6, then the area of the quadrilateral inscribed in the ellipse with diagonals as major and minor axis of the ellipse is

(a) 8 (b) 32 (c) 80 (d) 40

Sol:-

Given: $e = \frac{3}{5}$

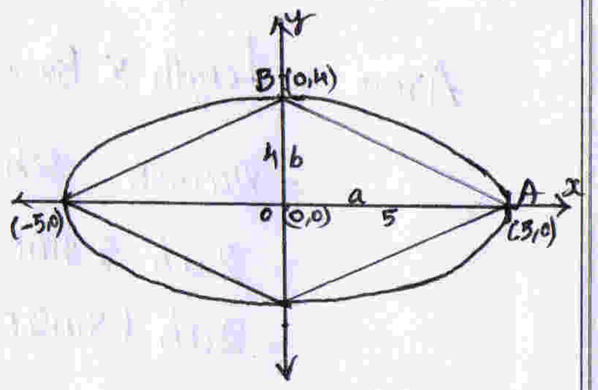
$2ae = 6$

$\therefore ae = 3$

$a\left(\frac{3}{5}\right) = 3$

$a = 3\left(\frac{5}{3}\right)$

$\therefore a = 5$




$b^2 = a^2(1-e^2)$

$= a^2 - a^2e^2$

$= 25 - 25\left(\frac{9}{25}\right)$

$= 25-9$

$b^2 = 16$

$\therefore b = 4$

Area of quadrilateral,

$A = 4\left(\frac{1}{2}ab\right)$

$= 4 \times \frac{1}{2} \times 5 \times 4$

$\therefore$ Area = 40 sq. units.

∴ Ans: (d) 40

18) Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is



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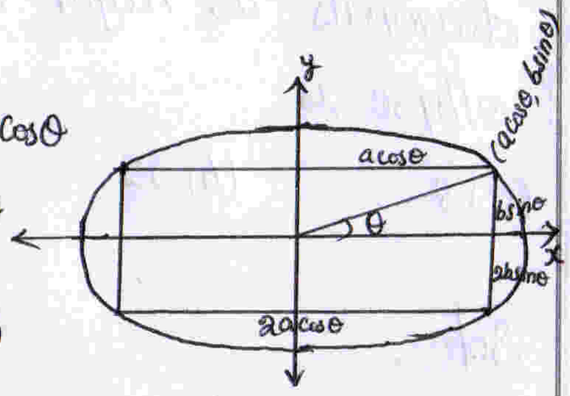
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(a) $2ab$ (b) $ab$ (c) $\sqrt{ab}$ (d) $\frac{a}{b}$

Sol:-

Length of the rectangle = $2a\cos\theta$

Breath of the rectangle = $2b\sin\theta$




Area = Length × Breadth

$= 2a\cos\theta \cdot 2b\sin\theta$

$= 2ab \cdot 2\sin\theta\cos\theta$

$= 2ab \cdot (\sin 2\theta)$

Maximum Value of $\sin 2\theta = 1$

$\therefore$ Max. Area $= 2ab(1)$

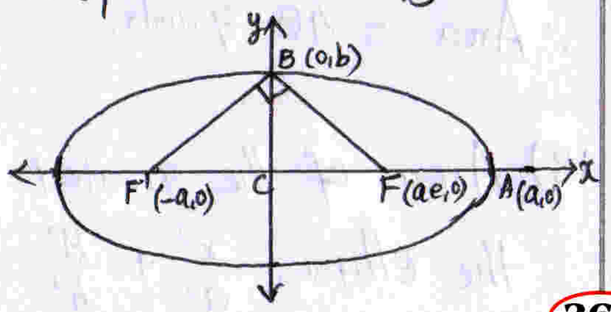
$= 2ab$

$\therefore$ Ans : (a) $2ab$

19) An ellipse has OB as semi minor axes, F and F' its foci and the angle FBF' is a right angle. Then the eccentricity of the ellipse is

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{\sqrt{3}}$

Sol:-




slope of $FB = \frac{b-0}{0-ae}$

$\therefore m_1 = \frac{-b}{ae}$

Slope of $F'B = m_2 = \frac{b-0}{0+ae} = \frac{b}{ae}$

$m_1 m_2 = -1$

$\left(\frac{-b}{ae}\right)\left(\frac{b}{ae}\right) = -1$

$b^2 = a^2e^2$

$a^2(1-e^2) = a^2e^2$

$1-e^2 = e^2$

$2e^2 = 1$

$e^2 = \frac{1}{2}$

eccentricity, $e = \frac{1}{\sqrt{2}}$

$\therefore$ Ans : (a) $\frac{1}{\sqrt{2}}$

20) The eccentricity of the ellipse $(x-3)^2 + (y-4)^2 = \frac{y^2}{9}$ is

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{3\sqrt{2}}$ (d) $\frac{1}{\sqrt{3}}$

Sol:-

For ellipse, $e = \sqrt{1-\frac{b^2}{a^2}}$

$\therefore b^2$ is smaller (co-eff $x^2, y^2$)

$\therefore a^2$ is greater (co-eff $x^2, y^2$)

$\therefore e = \sqrt{1-\frac{S}{G}}$ S → Smaller, G → Greater

$= \sqrt{1-\frac{1-\frac{1}{9}}{1}}$

$e = \sqrt{1-\frac{8}{9}} = \sqrt{\frac{1}{9}} = \frac{1}{3}$

$\therefore$ Ans : (b) $\frac{1}{3}$



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21) If the two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles then the locus of P is

(a) $2x+1=0$ (b) $x=-1$ (c) $2x-1=0$ (d) $x=1$

Sol:-

If the two tangents are at right angles then the locus of point of intersection is the directrix.

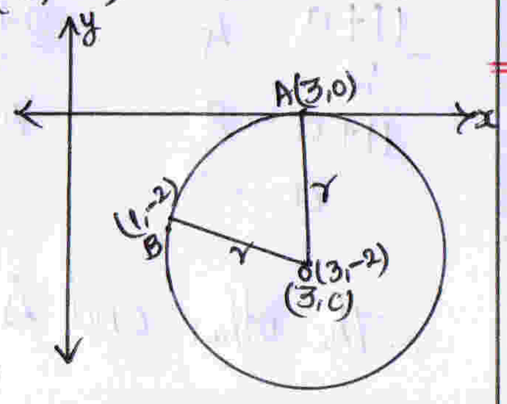
$\therefore x = -1$

$\therefore$ Ans: (b) $x=-1$

22) The circle passing through (1,-2) and touching the axis of $x$ at (3,0) passing through the point

(a) (-5,2) (b) (2,-5) (c) (5,-2) (d) (-2,5)

Sol:-




$OA = OB$

$\sqrt{(3-3)^2+(c-0)^2} = \sqrt{(3-1)^2+(c+2)^2}$

$0 + c^2 = 4 + c^2 + 4c + 4$

$4c = -8$

$\therefore$ radius $c = -2$

the equation is: $(x-h)^2 + (y-k)^2 = r^2$

$(x-3)^2 + (y+2)^2 = 4$

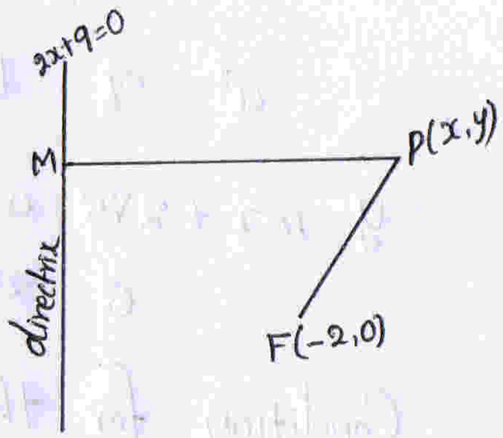
Verify the points satisfies.

$\therefore$ Ans: (c) (5,-2)

23) The locus of a point whose distance from (-2,0) is $\frac{2}{3}$ times its distance from the line $x = \frac{-9}{2}$ is

(a) a parabola (b) a hyperbola (c) an ellipse (d) a circle

Sol:-




$PF = \frac{2}{3} PM$

$\frac{PF}{PM} = \frac{2}{3} = e$

$= e < 1$

$\therefore$ ellipse

$\therefore$ Ans: (c) an ellipse

24) The values of m for which the line $y = mx + 2\sqrt{5}$ touches the hyperbola $16x^2 - 9y^2 = 144$ are the roots of $x^2 - (a+b)x - 4 = 0$, then the value of $(a+b)$ is

(a) 2 (b) 4 (c) 0 (d) -2



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Sol:- Given: $16x^2 - 9y^2 = 144$

$\div 144$ $\quad \frac{16x^2}{144} - \frac{9y^2}{144} = 1$

$\frac{x^2}{9} - \frac{y^2}{16} = 1$.

$a^2 = 9, \quad b^2 = 16$

$y = mx + 2\sqrt{5}$ is in the form of $y = mx + c$

$c = 2\sqrt{5}$

Condition for the line $y = mx + c$ to touches the hyperbola is

$c^2 = a^2m^2 - b^2$

$(2\sqrt{5})^2 = 9(m^2) - 16$

$20 + 16 = 9m^2$

$36 = 9m^2$

$m^2 = 4$

$m = \pm 2$ (Sum $= a+b = 0$)

Sum of roots : $2-2=0$

Product of roots : $2(-2) = -4$

$m = 2, -2$

Equation is : $x^2 - (\text{sum of roots})x + (\text{product of roots}) = 0$

$x^2 - 4 = 0$, $\quad \therefore$ Ans: (c) 0

$\therefore a+b=0$

25) If the coordinates at one end of a diameter of the circle $x^2+y^2-8x-4y+c=0$ are $(11,2)$, the coordinates of the other end are

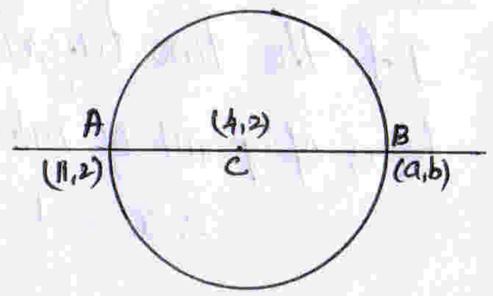
(a) $(3,5)$ (b) $(-3,2)$ (c) $(11,-8)$ (d) $(5,3)$

Sol:- $x^2+y^2-8x-4y+c=0$

$2g = -8 \quad 2f = -4$

$g = -4 \quad f = -2$

Centre $(-g,-f) = (4,2)$




mid point of AB $= \left(\frac{11+a}{2}, \frac{2+b}{2}\right)$

$(4,2) = \left(\frac{11+a}{2}, \frac{2+b}{2}\right)$

$\frac{11+a}{2} = 4 \quad \frac{2+b}{2} = 2$

$11+a = 8 \quad 2+b = 4$

$a = -3 \quad b = 2$

$\therefore$ the other end is $(-3,2)$

$\therefore$ Ans: (b) $(-3,2)$

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# EXERCISE 6.10

Choose the correct or the most suitable answer from the given four alternatives:-

1) If $\vec{a}$ and $\vec{b}$ are parallel vectors, then $[\vec{a}, \vec{c}, \vec{b}]$ is equal to

(a) 2 (b) −1 (c) 1 (d) 0

Sol:-

$\vec{a}$ and $\vec{b}$ are parallel

$\vec{a} = s\vec{b}$

$[\vec{a}, \vec{c}, \vec{b}] = [s\vec{b}, \vec{c}, \vec{b}]$

$= 0$

∴ Ans:- (d) 0

2) If a vector $\vec{\alpha}$ lies in the plane of $\vec{\beta}$ and $\vec{\gamma}$, then

(a) $[\vec{\alpha}, \vec{\beta}, \vec{\gamma}] = 1$ (b) $[\vec{\alpha}, \vec{\beta}, \vec{\gamma}] = -1$ (c) $[\vec{\alpha}, \vec{\beta}, \vec{\gamma}] = 0$ (d) $[\vec{\alpha}, \vec{\beta}, \vec{\gamma}] = 2$

Sol:-

$\vec{a}$ lies in the plane of $\vec{\beta}$ and $\vec{\gamma}$

$\vec{\alpha}, \vec{\beta}, \vec{\gamma}$ are coplanar vectors

$\therefore [\vec{\alpha}, \vec{\beta}, \vec{\gamma}] = 0$ ∴ Ans:- $[\vec{\alpha}, \vec{\beta}, \vec{\gamma}] = 0$

3) If $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$, then the value of $[\vec{a}, \vec{b}, \vec{c}]$ is

(a) $|\vec{a}||\vec{b}||\vec{c}|$ (b) $\frac{1}{3}|\vec{a}||\vec{b}||\vec{c}|$ (c) 1 (d) −1

sol:-

$\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$

$\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors

$\vec{a} \perp \vec{b}$ and $\vec{a} \perp \vec{c}$

∴ $\vec{a}$ is parallel to $(\vec{b} \times \vec{c})$

∴ $[\vec{a}, \vec{b}, \vec{c}] = \vec{a}.(\vec{b} \times \vec{c})$

$= |\vec{a}|\,|\vec{b} \times \vec{c}| \cos 0$

$= |\vec{a}|\,|\vec{b}|\,|\vec{c}| \sin 90°$

$= |\vec{a}||\vec{b}||\vec{c}|$ ∴ Ans:- (a) $|\vec{a}||\vec{b}||\vec{c}|$



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4) If $\vec{a}, \vec{b}, \vec{c}$ are three unit vectors such that $\vec{a}$ is perpendicular to $\vec{b}$, and is parallel to $\vec{c}$ then $\vec{a} \times (\vec{b} \times \vec{c})$ is equal to

(a) $\vec{a}$ (b) $\vec{b}$ (c) $\vec{c}$ (d) $\vec{0}$

Sol:-

$\vec{a}, \vec{b}, \vec{c}$ are unit vectors $\quad \vec{a} \perp \vec{b}, \ \vec{a} \perp \vec{c}$

$|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

$\therefore \vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$

$= (|\vec{a}||\vec{c}| \cos\theta)\vec{b} - 0$

$= |\vec{a}||\vec{c}|\vec{b}$

$= (1)(1)\vec{b}$

$= \vec{b}$

$\therefore$ Ans:- (b) $\vec{b}$

5) If $[\vec{a}, \vec{b}, \vec{c}] = 1$, then the value of

$$\frac{\vec{a} \cdot (\vec{b} \times \vec{c})}{(\vec{c} \times \vec{a}) \cdot \vec{b}} + \frac{\vec{b} \cdot (\vec{c} \times \vec{a})}{(\vec{a} \times \vec{b}) \cdot \vec{c}} + \frac{\vec{c} \cdot (\vec{a} \times \vec{b})}{(\vec{c} \times \vec{b}) \cdot \vec{a}} \text{ is}$$

(a) 1 (b) $-1$ (c) 2 (d) 3

Sol:-

$[\vec{a}, \vec{b}, \vec{c}] = \vec{a} \cdot (\vec{b} \times \vec{c}) = \vec{b} \cdot (\vec{c} \times \vec{a}) = \vec{c} \cdot (\vec{a} \times \vec{b}) = 1$

$\vec{a} \cdot (\vec{c} \times \vec{b}) = \vec{b} \cdot (\vec{a} \times \vec{c}) = \vec{c} \cdot (\vec{b} \times \vec{a}) = -1$

$$\therefore \frac{\vec{a} \cdot (\vec{b} \times \vec{c})}{(\vec{c} \times \vec{a}) \cdot \vec{b}} + \frac{\vec{b} \cdot (\vec{c} \times \vec{a})}{(\vec{a} \times \vec{b}) \cdot \vec{c}} + \frac{\vec{c} \cdot (\vec{a} \times \vec{b})}{(\vec{c} \times \vec{b}) \cdot \vec{a}} = \frac{1}{1} + \frac{1}{1} + \frac{1}{-1}$$

$= 1 + 1 - 1$

$= 1$

$\therefore$ Ans:- (a) 1

6) The volume of the parallelepiped with its edges represented by the vectors $\hat{i}+\hat{j}$, $\hat{i}+2\hat{j}$, $\hat{i}+\hat{j}+\pi\hat{k}$ is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\pi$ (d) $\frac{\pi}{4}$

Sol:-

$\vec{a} = \hat{i}+\hat{j}, \quad \vec{b} = \hat{i}+2\hat{j}, \quad \vec{c} = \hat{i}+\hat{j}+\pi\hat{k}$

Volume of parallelepiped $= [\vec{a}, \vec{b}, \vec{c}] = \vec{a} \cdot (\vec{b} \times \vec{c})$

$$= \begin{vmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 1 & \pi \end{vmatrix}$$



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$= 1(0) - 1(0) + \pi(2-1)$

$= \pi(1)$

$= \pi$ ∴ Ans :- (c) $\pi$

7) If $\vec{a}$ and $\vec{b}$ are unit vectors such that $[\vec{a}, \vec{b}, \vec{a}\times\vec{b}] = \frac{1}{4}$, then the angle between $\vec{a}$ and $\vec{b}$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

Sol:-

$\vec{a}$ and $\vec{b}$ are unit vectors

$|\vec{a}| = |\vec{b}| = 1$

$[a, b, a\times b] = (a\times b)\cdot(a\times b) = \frac{1}{4}$

$|a\times b|^2 = \frac{1}{4}$

$|\vec{a}|^2|\vec{b}|^2 \sin^2\theta = 1/4$

$(1)(1) \quad \sin^2\theta = 1/4$

$\sin\theta = \frac{1}{2}$

$\theta = \sin^{-1}\left(\frac{1}{2}\right)$

$\theta = \frac{\pi}{6}$ ∴ Ans:- (a) $\frac{\pi}{6}$

8) If $\vec{a} = \hat{i}+\hat{j}+\hat{k}$, $\vec{b} = \hat{i}+\hat{j}$, $\vec{c} = \hat{i}$ and $(\vec{a}\times\vec{b})\times\vec{c} = \lambda\vec{a} + \mu\vec{b}$, then the value of $\lambda+\mu$ is

(a) 0 (b) 1 (c) 6 (d) 3

Sol:-

$\vec{a} = \hat{i}+\hat{j}+\hat{k}$, $\vec{b} = \hat{i}+\hat{j}$, $\vec{c} = \hat{i}$

$\vec{a}\cdot\vec{c} = 1$

$\vec{b}\cdot\vec{c} = 1$

$(\vec{a}\times\vec{b})\times\vec{c} = \lambda\vec{a} + \mu\vec{b}$

$(\vec{a}\cdot\vec{c})\vec{b} - (\vec{b}\cdot\vec{c})\vec{a} = \lambda\vec{a} + \mu\vec{b}$

$\therefore \lambda = -(\vec{b}\cdot\vec{c}) = -1$

$\mu = (\vec{a}\cdot\vec{c}) = 1$

$\therefore \lambda+\mu = -1+1$

$= 0$ ∴ Ans:- (a) 0



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9) If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar, non-zero vectors such that $[\vec{a}, \vec{b}, \vec{c}] = 3$, then $\{[\vec{a}\times\vec{b}, \vec{b}\times\vec{c}, \vec{c}\times\vec{a}]\}^2$

(a) 81 (b) 9 (c) 27 (d) 18

Sol:-

$\vec{a}, \vec{b}, \vec{c}$ are non-coplanar, non-zero vectors

$$\{[a\times b, b\times c, c\times a]\}^2 = [[\vec{a}, \vec{b}, \vec{c}]^2]^2$$

$$= (3^2)^2$$

$$= 9^2$$

$$= 81$$

∴ Ans: (a) 81

---

10) If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors, unit vectors such that $\vec{a}\times(\vec{b}\times\vec{c}) = \frac{\vec{b}+\vec{c}}{\sqrt{2}}$, then the angle between $\vec{a}$ and $\vec{b}$ is

(a) $\frac{\pi}{2}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{4}$ (d) $\pi$

Sol:-

$$\vec{a}\times(\vec{b}\times\vec{c}) = \frac{\vec{b}+\vec{c}}{\sqrt{2}}$$

$$(\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c} = \frac{1}{\sqrt{2}}\vec{b} + \frac{1}{\sqrt{2}}\vec{c}$$

$$\therefore -(\vec{a}\cdot\vec{b}) = \frac{1}{\sqrt{2}}$$

$$\vec{a}\cdot\vec{b} = -\frac{1}{\sqrt{2}}$$

$$|\vec{a}||\vec{b}|\cos\theta = -\frac{1}{\sqrt{2}}$$

$$(1)(1)\cos\theta = -\frac{1}{\sqrt{2}}$$

$$\theta = \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$$

$$= \pi - \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$= \pi - \frac{\pi}{4}$$

$$\theta = \frac{3\pi}{4}$$

∴ Ans: (b) $\frac{3\pi}{4}$

---

11) If the volume of the parallelepiped with $\vec{a}\times\vec{b}, \vec{b}\times\vec{c}, \vec{c}\times\vec{a}$ as coterminous edges is 8 cubic units, then the volume of the parallelepiped



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with $(\vec{a}\times\vec{b})\times(\vec{b}\times\vec{c})$, $(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a})$ and $(\vec{c}\times\vec{a})\times(\vec{a}\times\vec{b})$ as coterminous edges is,

(a) 8 cubic units (b) 5/2 cubic units (c) 64 cubic units (d) 24 cubic units

Sol:-

Let $\vec{A}=\vec{a}\times\vec{b}$, $\vec{B}=\vec{b}\times\vec{c}$, $\vec{C}=\vec{c}\times\vec{a}$

$$[(\vec{a}\times\vec{b})\times(\vec{b}\times\vec{c}),(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a}),(\vec{c}\times\vec{a})\times(\vec{a}\times\vec{b})]$$

$$=[\vec{A}\times\vec{B},\vec{B}\times\vec{C},\vec{C}\times\vec{A}]$$

$$=[\vec{A},\vec{B},\vec{C}]^2$$

$$=8^2$$

$$=64$$

∴ Ans:-(c) 64 cubic units

12) Consider the vectors $\vec{a},\vec{b},\vec{c},\vec{d}$ such that $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=\vec{0}$. Let $P_1$ and $P_2$ be the planes determined by the pairs of vectors $\vec{a},\vec{b}$ and $\vec{c},\vec{d}$ respectively. Then the angle between $P_1$ and $P_2$ is

(a) 0° (b) 45° (c) 60° (d) 90°

Sol:-

$$(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=\vec{0}$$

$\vec{a}\times\vec{b}$ and $\vec{c}\times\vec{d}$ are parallel

∴ $P_1$ and $P_2$ are parallel

∴ the angle between the planes is 0°

∴ Ans:-(a) 0°

13) If $\vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}\times\vec{b})\times\vec{c}$, where $\vec{a},\vec{b},\vec{c}$ are any three vectors such that $\vec{b}\cdot\vec{c}\neq0$ and $\vec{a}\cdot\vec{b}\neq0$, then $\vec{a}$ and $\vec{c}$ are

(a) perpendicular (b) parallel

(c) inclined at an angle $\frac{\pi}{3}$ (d) inclined at an angle $\frac{\pi}{6}$

Sol:-

$$\vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}\times\vec{b})\times\vec{c}$$

$$(\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c}=(\vec{a}\cdot\vec{c})\vec{b}-(\vec{b}\cdot\vec{c})\vec{a}$$

$$(\vec{a}\cdot\vec{b})\vec{c}=(\vec{b}\cdot\vec{c})\vec{a}$$

$\vec{c}$ and $\vec{a}$ are parallel ∴ Ans:-(b) parallel



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14) If $\vec{a} = 2\hat{i}+3\hat{j}-\hat{k}$, $\vec{b} = \hat{i}+2\hat{j}-5\hat{k}$, $\vec{c} = 3\hat{i}+5\hat{j}-\hat{k}$, then a vector perpendicular to $\vec{a}$ and lies in the plane containing $\vec{b}$ and $\vec{c}$ is

(a) $-17\hat{i}+21\hat{j}-97\hat{k}$ (b) $17\hat{i}+21\hat{j}-123\hat{k}$

(c) $-17\hat{i}-21\hat{j}+97\hat{k}$ (d) $-17\hat{i}-21\hat{j}-97\hat{k}$

Sol:-

$\vec{a} = 2\hat{i}+3\hat{j}-\hat{k}$, $\vec{b} = \hat{i}+2\hat{j}-5\hat{k}$, $\vec{c} = 3\hat{i}+5\hat{j}-\hat{k}$

$$\vec{b}\times\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -5 \\ 3 & 5 & -1 \end{vmatrix}$$

$= \hat{i}(-2+25) - \hat{j}(-1+15) + \hat{k}(5-6)$

$= 23\hat{i} - 14\hat{j} - \hat{k}$

$$\vec{a}\times(\vec{b}\times\vec{c}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -1 \\ 23 & -14 & -1 \end{vmatrix}$$

$= \hat{i}(-3-14) - \hat{j}(-2+23) + \hat{k}(-28-69)$

$= -17\hat{i} - 21\hat{j} - 97\hat{k}$

∴ Ans:- (d) $-17\hat{i}-21\hat{j}-97\hat{k}$

15) The angle between the lines $\frac{x-2}{3} = \frac{y+1}{-2}$, $z=2$ and $\frac{x-1}{1} = \frac{2y+3}{3} = \frac{z+5}{2}$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

Sol:-

Given lines

$L_1$: $\frac{x-2}{3} = \frac{y+1}{-2} = \frac{z-2}{0}$

$\vec{b_1} = 3\hat{i}-2\hat{j}+0\hat{k}$

$L_2$: $\frac{x-1}{1} = \frac{y+3/2}{3/2} = \frac{z+5}{2} \Rightarrow \frac{x-1}{2} = \frac{y+3/2}{3} = \frac{z+5}{4}$

$\therefore \vec{b_2} = 2\hat{i}+3\hat{j}+4\hat{k}$

the angle between the lines

$$\cos\theta = \frac{\vec{b_1}\cdot\vec{b_2}}{|\vec{b_1}||\vec{b_2}|}$$

$$\cos\theta = \frac{6-6+0}{\sqrt{9+4}\ \sqrt{4+9+16}} = 0$$

$\theta = \cos^{-1}(0)$

$\therefore \theta = 90^\circ$ (or) $\frac{\pi}{2}$

∴ Ans:- (d) $\frac{\pi}{2}$



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16) If the line $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ lies in the plane $x+3y-\alpha z+\beta=0$, then $(\alpha, \beta)$ is

(a) $(-5,5)$ (b) $(-6,7)$ (c) $(5,-5)$ (d) $(6,-7)$

Sol:-

Given line $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$

point on line $(2,1,-2)$

$\vec{b} = 3\hat{i}-5\hat{j}+2\hat{k}$
$\vec{n} = \hat{i}+3\hat{j}-\alpha\hat{k}$
$\vec{b}\cdot\vec{n} = 0$
$3-15-2\alpha=0$
$-2\alpha = 12$
$\alpha = -6$

$2+3(1)-\alpha(-2)+\beta=0$

$5+2\alpha+\beta=0$

$2\alpha+\beta=-5$ ——(1)

$2(-6)+\beta=-5$

$\beta = -5+12 = 7$

$\therefore (\alpha,\beta) = (-6,7)$ $\therefore$ Ans:- (b) $(-6,7)$

17) The angle between the line $\vec{r} = (\hat{i}+2\hat{j}-3\hat{k}) + t(2\hat{i}+\hat{j}-2\hat{k})$ and the plane $\vec{r}\cdot(\hat{i}+\hat{j})+4=0$ is

(a) $0^\circ$ (b) $30^\circ$ (c) $45^\circ$ (d) $90^\circ$

Sol:-

$\vec{b} = 2\hat{i}+\hat{j}-2\hat{k}$, $\vec{n} = \hat{i}+\hat{j}$

The angle between the lines

$$\sin\theta = \frac{\vec{b}\cdot\vec{n}}{|\vec{b}||\vec{n}|}$$

$$= \frac{(2\hat{i}+\hat{j}-2\hat{k})\cdot(\hat{i}+\hat{j})}{\sqrt{4+1+4}\ \sqrt{1+1}}$$

$$= \frac{2+1+0}{\sqrt{9}\ \sqrt{2}}$$

$$= \frac{3}{3\sqrt{2}}$$

$$\sin\theta = \frac{1}{\sqrt{2}}$$

$$\theta = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$\therefore \theta = 45^\circ$ (or) $\frac{\pi}{4}$

$\therefore$ Ans:- (c) $45^\circ$

18) The coordinates of the point where the line $\vec{r} = (6\hat{i}-\hat{j}-3\hat{k}) + t(-\hat{i}+4\hat{k})$ meets the plane $\vec{r}\cdot(\hat{i}+\hat{j}-\hat{k}) = 3$ are

(a) $(2,1,0)$ (b) $(7,-1,-7)$ (c) $(1,2,-6)$ (d) $(5,-1,1)$

Sol:-

Let $\frac{x-6}{-1} = \frac{y+1}{0} = \frac{z+3}{4} = t$



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$\frac{x-6}{-1} = t$ $\quad \frac{y+1}{0} = t$ $\quad \frac{z+3}{4} = t$

$x = -t+6$, $\quad y = -1$, $\quad z = 4t-3$

the point $(-t+6, -1, 4t-3)$ lies on $x+y-z=3$

$$-t+6-1-4t+3 = 3$$
$$-5t = -5$$
$$t = 1$$

$\therefore$ The point is $(5,-1,1)$ $\quad \therefore$ Ans: (d) $(5,-1,1)$

19) Distance from the origin to the plane $3x-6y+2z+7=0$ is

(a) 0 $\quad$ (b) 1 $\quad$ (c) 2 $\quad$ (d) 3

Sol:-

$$\delta = \frac{|d|}{\sqrt{a^2+b^2+c^2}}$$
$$= \frac{7}{\sqrt{9+36+4}}$$
$$= \frac{7}{\sqrt{49}}$$
$$\delta = \frac{7}{7} = 1$$

$\therefore$ Ans:- (b) 1

20) The distance between the planes $x+2y+3z+7=0$ and $2x+4y+6z+7=0$ is

(a) $\frac{\sqrt{7}}{2\sqrt{2}}$ $\quad$ (b) $\frac{7}{2}$ $\quad$ (c) $\frac{\sqrt{7}}{2}$ $\quad$ (d) $\frac{7}{2\sqrt{2}}$

Sol:- Given planes

$P_1$ : $x+2y+3z+7=0$

$P_2$ : $2x+4y+6z+7=0$

$\div 2$, $\quad x+2y+3z+\frac{7}{2}=0$

$a=1, b=2, c=3$
$d_1=7, d_2=7/2$

$$d = \frac{|d_1-d_2|}{\sqrt{a^2+b^2+c^2}}$$
$$= \frac{|7-\frac{7}{2}|}{\sqrt{1+4+9}}$$
$$= \frac{7}{2\sqrt{14}} = \frac{7^{\sqrt{7}}}{2\sqrt{2}\sqrt{7}}$$
$$= \frac{\sqrt{7}}{2\sqrt{2}}$$

$\therefore$ Ans:- (a) $\frac{\sqrt{7}}{2\sqrt{2}}$

21) If the direction cosines of a line are $\frac{1}{c}, \frac{1}{c}, \frac{1}{c}$, then

(a) $c = \pm 3$ $\quad$ (b) $c = \pm\sqrt{3}$ $\quad$ (c) $c > 0$ $\quad$ (d) $0 < c < 1$



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Sol:-

$$\left(\frac{1}{c}\right)^2 + \left(\frac{1}{c}\right)^2 + \left(\frac{1}{c}\right)^2 = 1$$

$$\frac{1}{c^2} + \frac{1}{c^2} + \frac{1}{c^2} = 1$$

$$\frac{3}{c^2} = 1$$

$$c^2 = 3$$

$$\therefore c = \pm\sqrt{3}$$

∴ Ans:-(b) $c = \pm\sqrt{3}$

22) The vector equation $\vec{r} = (\hat{i} - 2\hat{j} - \hat{k}) + t(6\hat{j} - \hat{k})$ represents a straight line passing through the points

(a) (0, 6, -1) and (1, -2, -1) (b) (0, 6, -1) and (-1, -4, -2)

(c) (1, -2, -1) and (1, 4, -2) (d) (1, -2, -1) and (0, -6, 1)

Sol:- $\vec{r} = (\hat{i} - 2\hat{j} - \hat{k}) + t(6\hat{j} - \hat{k})$

$$\frac{x-1}{0} = \frac{y+2}{6} = \frac{z+1}{-1}$$

(a) dr : 1-0, -2-6, -1+1 = (1, -8, 0)

(b) dr : -1-0, -4-6, -2+1 = (-1, -10, -1)

(c) dr : 0, 4+2, -2+1 = (0, 6, -1) ✓

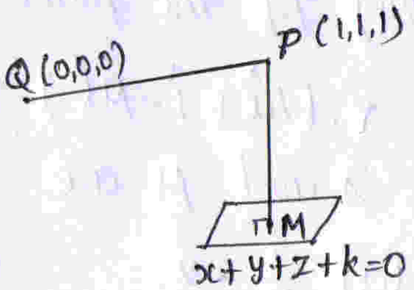
(d) dr : 0-1, -6+2, 1+1 = (-1, -4, 2)

∴ Ans:-(c) (1, -2, -1) and (1, 4, -2)

23) If the distance of the point (1, 1, 1) from the origin is half of its distance from the plane $x + y + z + k = 0$, then the values of k are

(a) ±3 (b) ±6 (c) -3, 9 (d) 3, -9

Sol:-




$$PQ = \frac{1}{2} PM$$

$$PQ = \sqrt{(1-0)^2 + (1-0)^2 + (1-0)^2}$$

$$= \sqrt{1+1+1}$$

$$= \sqrt{3}$$

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$$PM = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2+B^2+C^2}}$$

$$= \frac{|1+1+1+k|}{\sqrt{3}}$$

$$\therefore PM = \left|\frac{3+k}{\sqrt{3}}\right|$$

$$\therefore \sqrt{3} = \frac{1}{2}\left|\left(\frac{3+k}{\sqrt{3}}\right)\right|$$

$$\pm 6 = 3 + k$$

$$3 + k = \pm 6$$

| | |
|---|---|
| $3+k=6$ | $3+k=-6$ |
| $k=3$ | $k=-9$ |

$\therefore$ Ans:- (d) $(3,-9)$

24) If the planes $\vec{r}.(2\hat{i}-\lambda\hat{j}+\hat{k}) = 3$ and $\vec{r}.(4\hat{i}+\hat{j}-\mu\hat{k}) = 5$ are parallel. then the value of $\lambda$ and $\mu$ are

(a) $\frac{1}{2}, -2$ (b) $-\frac{1}{2}, 2$ (c) $-\frac{1}{2}, -2$ (d) $\frac{1}{2}, 2$

Sol:-

Given planes are parallel

$$\therefore \frac{2}{4} = \frac{-\lambda}{1} = \frac{1}{-\mu}$$

$$\frac{2}{4} = -\lambda \qquad \frac{1}{4_2} = -\frac{1}{\mu}$$

$$\therefore \lambda = -\frac{1}{2} \qquad \frac{1}{2} = -\frac{1}{\mu}$$

$$\therefore \mu = -2$$

$\therefore$ Ans:- (c) $-\frac{1}{2}, -2$

25) If the length of the perpendicular from the origin to the plane $2x+3y+\lambda z=1$, $\lambda > 0$ is $\frac{1}{5}$, then the value of $\lambda$ is

(a) $2\sqrt{3}$ (b) $3\sqrt{2}$ (c) 0 (d) 1

Sol:-

$$\frac{|-1|}{\sqrt{4+9+\lambda^2}} = \frac{1}{5}$$

$$\frac{1}{\sqrt{\lambda^2+13}} = \frac{1}{5}$$

$$\lambda^2 + 13 = 25$$

$$\lambda^2 = 12$$

$$\lambda = \pm\sqrt{12} = \pm 2\sqrt{3}$$

$\lambda > 0$, $\therefore \lambda = 2\sqrt{3}$

$\therefore$ Ans:- (a) $2\sqrt{3}$



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# EXERCISE 7.10

1) The volume of a sphere is increasing in volume at the rate of $3\pi\ cm^3/sec$.

(a) 3 cm/s (b) 2 cm/s (c) 1 cm/s (d) $\frac{1}{2}$ cm/s

Sol:-

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \frac{dr}{dt}$$

$$3\pi = \frac{4}{3}\pi 3\left(\frac{1}{2}\right)^2 \frac{dr}{dt}$$

$$3 = 4\left(\frac{1}{4}\right)\frac{dr}{dt}$$

$$\therefore \frac{dr}{dt} = 3$$

$\frac{dV}{dt} = 3\pi\ cm^3/s$

$r = \frac{1}{2}$ cm

$\frac{dr}{dt} = ?$

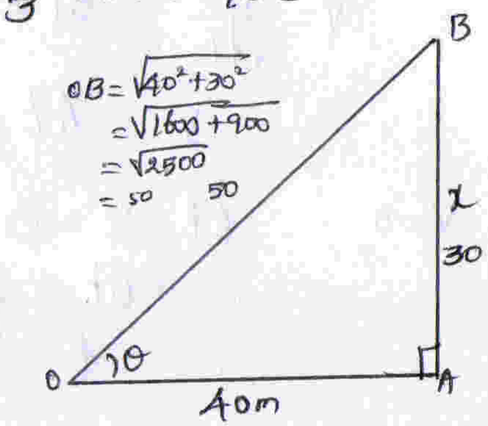
∴ Ans: (a) 3 cm/s

2) A balloon rises straight up at 10 m/s. An observer is 40m away from the spot where the balloon left the ground. Find the rate of change of the balloon's angle of elevation. in radian Per second when the balloon is 30 metres above the ground.

(a) $\frac{3}{25}$ radians/sec (b) $\frac{4}{25}$ radians/sec

(c) $\frac{1}{5}$ radians/sec (d) $\frac{1}{3}$ radians/sec

Sol:-




$$\tan\theta = \frac{x}{40}$$

$$x = 40\tan\theta$$

$$\frac{dx}{dt} = 40\sec^2\theta \frac{d\theta}{dt}$$

$$10 = 40\left(\frac{50}{40}\right)^2 \frac{d\theta}{dt}$$

$$\frac{d\theta}{dt} = \frac{10 \times 16}{25 \times 40}$$

$$= \frac{4}{25} \text{ radians/sec}$$

$\sec\theta = \frac{hyp}{adj}$

$\frac{dx}{dt} = 10 m/s$

$\frac{d\theta}{dt} = ?$ when $x = 30m$

∴ Ans: (b) $\frac{4}{25}$ radians/sec

3) The position of a particle moving along a horizontal line of any time t is given by $s(t) = 3t^2 - 2t - 8$. The time at which the particle is at rest is

(a) $t = 0$ (b) $t = \frac{1}{3}$ (c) $t = 1$ (d) $t = 3$



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Sol:-

$$s(t) = 3t^2 - 2t - 8$$

$$v = \frac{ds}{dt} = 3(2t) - 2$$

$$v = 6t - 2$$

$v = 0$, $\quad 6t - 2 = 0$

$$6t = 2$$

$$t = \frac{1}{3}$$

∴ Ans: (b) $t = \frac{1}{3}$

4) A stone is thrown up vertically. The height it reaches at time t seconds is given by $x = 80t - 16t^2$. The stone reaches the maximum height in time t seconds is given by

(a) 2 (b) 2.5 (c) 3 (d) 3.5

Sol:-

$$x = 80t - 16t^2$$

$$\frac{dx}{dt} = 80 - 16(2t)$$

$$= 80 - 32t$$

$\frac{dx}{dt} = 0$, $\quad 80 - 32t = 0$

$$32t = 80$$

$$t = \frac{5}{2} = 2.5$$

∴ Ans: (b) 2.5

5) Find the point on the curve $6y = x^3 + 2$ at which y-Coordinate changes 8 times as fast as x-Coordinate is

(a) (4, 11) (b) (4, -11) (c) (-4, 11) (d) (-4, -11)

Sol:-

$$6y = x^3 + 2$$

Given: $\frac{dy}{dt} = 8\frac{dx}{dt}$ ——(1)

$$6\frac{dy}{dt} = 3x^2\frac{dx}{dt}$$

$$6\left(8\frac{dx}{dt}\right) = 3x^2\frac{dx}{dt}$$

$$x^2 = 16$$

$$x = \pm 4$$

when $x = 4$, $\quad 6y = 64 + 2$

$$6y = 66$$

$$y = 11$$

when $x = -4$, $\quad 6y = -64 + 2$

$$6y = -62$$

$$y = -\frac{31}{3}$$

∴ Ans: (a) (4, 11)

∴ the points are (4, 11) and $\left(-4, \frac{-31}{3}\right)$



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6) the abscissa of the point on the curve $f(x) = \sqrt{8-2x}$ at which the slope of the tangent is $-0.25$?

(a) $-8$ (b) $-4$ (c) $-2$ (d) $0$

Sol:-

$$f(x) = \sqrt{8-2x}$$

$$f'(x) = \frac{1}{2\sqrt{8-2x}}(-2)$$

$$f'(x) = \frac{-1}{\sqrt{8-2x}}$$

$f'(x) = -0.25 = -\frac{1}{4}$

$f'(x) = -\frac{1}{4}$, $\frac{-1}{\sqrt{8-2x}} = \frac{-1}{4}$

$\sqrt{8-2x} = 4$

$8-2x = 16$

$2x = 8-16$

$2x = -8^{4}$

$x = -4$

∴ Ans: (b) $-4$

7) The slope of the line normal to the curve $f(x) = 2\cos 4x$ at $x = \frac{\pi}{2}$ is

(a) $-4\sqrt{3}$ (b) $-4$ (c) $\frac{\sqrt{3}}{12}$ (d) $4\sqrt{3}$

Sol:-

$$f(x) = 2\cos 4x$$

$$f'(x) = 2(-\sin 4x)4$$

$$= -8\sin 4x$$

at $x = \frac{\pi}{12}$, $= -8\sin 4\left(\frac{\pi}{12}\right)$

$= -8\sin\frac{\pi}{3}$

$= -4\left(\frac{\sqrt{3}}{2}\right)$

Slope of tangent, $m = -4\sqrt{3}$

∴ Slope of Normal $= \frac{1}{4\sqrt{3}}$

$= \frac{1}{4\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{\sqrt{3}}{12}$

∴ Ans: (c) $\frac{\sqrt{3}}{12}$

8) The tangent to the curve $y^2 - xy + 9 = 0$ is vertical when

(a) $y = 0$ (b) $y = \pm\sqrt{3}$ (c) $y = \frac{1}{2}$ (d) $y = \pm 3$

Sol:-

$$y^2 - xy + 9 = 0 \quad \text{——— (1)}$$

$$2y\frac{dy}{dx} - \left[(1)y + x\frac{dy}{dx}\right] = 0$$

$$2y\frac{dy}{dx} - y - x\frac{dy}{dx} = 0$$



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$$\frac{dy}{dx}(2y-x) = y$$

$$\frac{dy}{dx} = \frac{y}{2y-x}$$

tangent is vertical, $m = \tan 90^\circ = \infty = \frac{1}{0}$

$$\frac{y}{2y-x} = \frac{1}{0}$$

$$2y - x = 0$$

$$2y = x$$

Substitute $x = 2y$ in (1)

$$y^2 - (2y)y + 9 = 0$$

$$y^2 - 2y^2 + 9 = 0$$

$$-y^2 + 9 = 0$$

$$y^2 = 9$$

$$y = \pm 3$$

∴ Ans: (d) $\pm 3$

9) Angle between $y^2 = x$ and $x^2 = y$ at the origin is

(a) $\tan^{-1}(\frac{3}{4})$ (b) $\tan^{-1}(\frac{4}{3})$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

Sol:-

$y^2 = x$

$2y\frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{2y}$

$\therefore m_1 = \frac{1}{2y}$

$x^2 = y$

$2x = \frac{dy}{dx}$

$\frac{dy}{dx} = 2x$

$\therefore m_2 = 2x$

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left|\frac{\frac{1}{2y} - 2x}{1 + \frac{1}{2y}(2x)}\right|$$

$$= \left|\frac{\frac{1-2x}{2y}}{\frac{2y+2x}{2y}}\right|$$

$$= \left|\frac{1-2x}{2y+2x}\right|$$ at (0,0)

$$= \left|\frac{1-0}{0+0}\right|$$

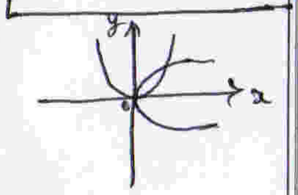
$$= \frac{1}{0}$$

$\tan\theta = \infty$

$\therefore \tan\theta = \frac{\pi}{2}$

∴ Ans: (c) $\frac{\pi}{2}$

* The angle between the parabola is the angle between the axes $= \frac{\pi}{2}$






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10) The value of the Limit $\lim_{x\to 0}\left(\cot x - \frac{1}{x}\right)$ is

(a) 0 (b) 1 (c) 2 (d) $\infty$

Sol:-

$$\lim_{x\to 0}\left(\cot x - \frac{1}{x}\right) = \lim_{x\to 0}\left(\frac{\cos x}{\sin x} - \frac{1}{x}\right)$$

$$= \lim_{x\to 0}\left(\frac{x\cos x - \sin x}{x\sin x}\right)$$

$$= \frac{0}{0} \quad \text{(Indeterminate form)}$$

Applying l'Hopital Rule

$$= \lim_{x\to 0}\frac{x(-\sin x) + \cos x(1) - \cos x}{x\cos x + \sin x}$$

$$= \lim_{x\to 0}\frac{-x\sin x}{x\cos x + \sin x}$$

$$= \frac{0}{0} \quad \text{(Indeterminate form)}$$

Applying L'Hopital Rule

$$= \lim_{x\to 0}\frac{x(-\cos x) + (-\sin x)(1)}{x(-\sin x) + \cos x(1) + \cos x}$$

$$= \lim_{x\to 0}\left(\frac{-x\cos x - \sin x}{-x\sin x + 2\cos x}\right)$$

$$= \frac{0-0}{0+2}$$

$$= \frac{0}{2}$$

$$= 0$$

$\therefore$ Ans: (a) 0

11) the function $\sin^4 x + \cos^4 x$ is increasing in the interval

(a) $\left[\frac{5\pi}{8}, \frac{3\pi}{4}\right]$ (b) $\left[\frac{\pi}{2}, \frac{5\pi}{8}\right]$ (c) $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$ (d) $\left[0, \frac{\pi}{4}\right]$

Sol:-

$$f(x) = \sin^4 x + \cos^4 x$$

$$f'(x) = 4\sin^3 x(\cos x) + 4\cos^3 x(-\sin x)$$

$$= 4\sin x\cos x\left[\sin^2 x - \cos^2 x\right]$$

$$= -2\cdot 2\sin x\cos x\left(\cos^2 x - \sin^2 x\right)$$

$$= -2\sin 2x\cos 2x$$

$$f'(x) = -\sin 4x$$

$f'(x) = 0$, $\quad -\sin 4x = 0$

$\sin 2A = 2\sin A\cos A$
$\cos 2A = \cos^2 A - \sin^2 A$



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$\sin 4x = 0$

$4x = n\pi$

$x = \frac{n\pi}{4}$ $(n = 0, 1, 2, 3, \ldots)$

$\therefore x = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \ldots$

Intervals are $[0, \frac{\pi}{4}], [\frac{\pi}{4}, \frac{\pi}{2}], [\frac{\pi}{2}, \frac{3\pi}{4}], \ldots$

In $[0, \frac{\pi}{4}]$, $f'(x) = -ve$, $f(x)$ is decreasing

In $[\frac{\pi}{4}, \frac{\pi}{2}]$, $f'(x) = +ve$, $f(x)$ is increasing

$\therefore$ Ans: (c) $[\frac{\pi}{4}, \frac{\pi}{2}]$

12) The number given by the Rolle's theorem for the function $x^3 - 3x^2$, $x \in [0, 3]$ is

(a) 1 (b) $\sqrt{2}$ (c) $\frac{3}{2}$ (d) 2

Sol:-

$f(x) = x^3 - 3x^2$

$f'(x) = 3x^2 - 6x$

$f'(c) = 3c^2 - 6c$

$f'(c) = 0$, $3c^2 - 6c = 0$

$3c(c-2) = 0$

$3c = 0$ $\quad c - 2 = 0$

$c = 0 \notin (0,3)$ $\quad c = 2 \in (0,3)$

$\therefore$ Ans: (d) 2

13) The number given by the Mean value theorem for the function $\frac{1}{x}, x \in [1, 9]$ is

(a) 2 (b) 2.5 (c) 3 (d) 3.5

Sol:-

$f(x) = \frac{1}{x}$, $x \in [1, 9]$

$f(1) = \frac{1}{1} = 1$

$f(9) = \frac{1}{9}$

$f'(x) = \frac{-1}{x^2}$

$f'(c) = \frac{-1}{c^2}$

Mean Value theorem

$$f'(c) = \frac{f(b) - f(a)}{b - a} = \frac{f(9) - f(1)}{9 - 1}$$

$$\frac{-1}{c^2} = \frac{\frac{1}{9} - 1}{9 - 1}$$



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$$\frac{-1}{c^2} = \frac{1-9}{9(8)}$$

$$\frac{-1}{c^2} = \frac{-8}{9(8)}$$

$c^2 = 9$ $\quad\therefore$ Ans : (c) 3

$c = \pm 3$

$c = -3 \notin (1,9)$, $\quad c = 3 \in (1,9)$

14) The minimum value of the function $|3-x|+9$ is

(a) 0 (b) 3 (c) 6 (d) 9

Sol:-

$$f(x) = |3-x| + 9$$

When $x=3$, $|3-3|+9 = 0+9 = 9$

When $x=4$, $|3-4|+9 = |-1|+9 = 1+9 = 10$

$\therefore$ Minimum Value is 9 at $x=3$

$\therefore$ Ans: (d) 9

15) The maximum slope of the tangent to the curve $y = e^x \sin x$, $x \in [0, 2\pi]$ is at

(a) $x = \frac{\pi}{4}$ (b) $x = \frac{\pi}{2}$ (c) $x = \pi$ (d) $x = \frac{3\pi}{2}$

Sol:-

$$y = e^x \sin x \quad , \quad x \in [0, 2\pi]$$

$$\frac{dy}{dx} = e^x \sin x + e^x \cos x$$

$uv \to u'v + uv'$

slope: $m = e^x (\sin x + \cos x)$

$$f(x) = e^x (\sin x + \cos x)$$

$$f'(x) = e^x (\sin x + \cos x) + e^x (\cos x - \sin x)$$

$$= e^x [\sin x + \cos x + \cos x - \sin x]$$

$$f'(x) = e^x \, 2\cos x$$

$f'(x) = 0$, $\quad e^x 2\cos x = 0$

$\cos x = 0 \quad \therefore x = \pi/2, 3\pi/2$

$$f''(x) = 2[e^x \cos x + e^x(-\sin x)]$$

$$= 2e^x [\cos x - \sin x]$$

$$f''(\pi/2) = 2e^{\pi/2} [\cos \pi/2 - \sin \pi/2]$$

$$= 2e^{\pi/2} (0-1)$$

$$= -2e^{\pi/2} < 0$$

$f(x)$ is a maximum at $x = \pi/2$



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$f''\left(\frac{3\pi}{2}\right) = 2e^{3\pi/2}\left[\cos\frac{3\pi}{2} - \sin\frac{3\pi}{2}\right]$

$= 2e^{3\pi/2}(1)$

$= 2e^{3\pi/2} > 0$

$\therefore f(x)$ has minimum at $x = 3\pi/2$

$\therefore$ Ans: (b) $x = \frac{\pi}{2}$

16) The maximum value of the function $x^2e^{-2x}$, $x>0$ is

(a) $\frac{1}{e}$ (b) $\frac{1}{2e}$ (c) $\frac{1}{e^2}$ (d) $\frac{4}{e^4}$

Sol:-

$y = x^2e^{-2x}$

$y' = 2xe^{-2x} + x^2 - 2e^{-2x}$

$y' = 2xe^{-2x}(1-x)$

$y'=0, \quad 2xe^{-2x}(1-x) = 0$

$2xe^{-2x} = 0 \qquad 1-x = 0$

$x = 0 \qquad x = 1$

$\therefore x>0$

$y' = 2xe^{-2x}(1-x)$

$= 2e^{-2x}(x-x^2)$

$y'' = 2e^{-2x}(-2)(x-x^2) + 2e^{-2x}(1-2x)$

$y''(1) = 2e^{-2}(-1)$

$= -2e^{-2} < 0$

$\therefore$ It is maximum at $x = 1$

$\therefore y(1) = 1^2e^{-2(1)}$

$= e^{-2}$

$= \frac{1}{e^2}$ $\therefore$ Ans:- (c) $\frac{1}{e^2}$

17) One of the closest points on the curve $x^2 - y^2 = 4$ to the point $(6,0)$ is

(a) $(2,0)$ (b) $(\sqrt{5},1)$ (c) $(3,\sqrt{5})$ (d) $(\sqrt{13},-\sqrt{3})$

Sol:-

$(x,y) = (6,0)$

$d = \sqrt{(x-6)^2 + (y-0)^2}$

$= \sqrt{(x-6)^2 + y^2}$

$d = \sqrt{(x-6)^2 + x^2 - 4}$

$x^2 - y^2 = 4$

$x^2 - 4 = y^2$



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$d^2 = (x-6)^2 + x^2 - 4$

$f(x) = (x-6)^2 + x^2 - 4$

$f'(x) = 2(x-6)(1) + 2x$

$f'(x)=0$, $2(x-6) + 2x = 0$

$2x - 12 + 2x = 0$

$4x = +12^3$

$x = 3$

$f''(x) = 2(1) + 2$

$= 4 > 0$ It's minimum

when $x = 3$, $x^2 - y^2 = 4$

$3^2 - y^2 = 4$

$y^2 = 9 - 4$

$y^2 = 5$

$y = \pm\sqrt{5}$

$\therefore (3, \sqrt{5})$ and $(3, -\sqrt{5})$

$\therefore$ Ans : (c) $(3, \sqrt{5})$

18) The maximum value of the product of two Positive numbers, when their Sum of the squares is 200, is

(a) 100 (b) $25\sqrt{7}$ (c) 28 (d) $24\sqrt{14}$

Sol:- Let the positive numbers be $x, y$

$x^2 + y^2 = 200$

$y^2 = 200 - x^2$

$y = \sqrt{200 - x^2}$

$P = xy$

$= x\sqrt{200 - x^2}$

$f(x) = P^2 = x^2(200 - x^2)$

$= 200x^2 - x^4$

$f'(x) = 200(2x) - 4x^3$

$= 400x - 4x^3$

$f'(x) = 4x(100 - x^2)$

$f'(x)=0$, $4x(100 - x^2) = 0$

$4x = 0$ $\quad 100 - x^2 = 0$

$x = 0$ $\quad x^2 = 100$

$x = \pm 10$

$\therefore x = 10$

$f''(x) = 400 - 12x^2$

$f''(10) = 400 - 12(100) = 400 - 1200 = -800 < 0$

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$\therefore$ It's maximum at $x=10$

$P = x\sqrt{200-x^2}$ when $x=3$,

$= 10\sqrt{200-100}$

$= 10\sqrt{100}$

$P = 10(10) = 100$ $\therefore$ Ans: (a) 100

---

19) the curve $y = ax^4 + bx^2$ with $ab>0$

(a) has no horizontal tangent (b) is concave up

(c) is Concave down (d) has no points of inflection

Sol:- $y = f(x) = ax^4 + bx^2$

$f'(x) = a.4x^3 + b.2x$

$= 4ax^3 + 2bx = 2x[2ax^2+b]$

$f'(x)=0$, $2x[2ax^2+b]=0$

$2x=0$ $\quad 2ax^2+b=0$

$x=0$ $\quad 2ax^2=-b$

$x^2 = -b/2a$ (not possible)

$f''(x) = 4a.3x^2 + 2b$

$f''(x)=0$, $12ax^2+2b=0$

$12ax^2 = -2b \Rightarrow x^2 = \frac{-2b}{12a}$

(not possible)

$f''(x)$ does not changes its sign

$\therefore$ No point of inflection

$\therefore$ Ans: (d) has no points of inflection

---

20) The point of inflection of the curve $y=(x-1)^3$ is

(a) (0,0) (b) (0,1) (c) (1,0) (d) (1,1)

Sol:- $f(x) = (x-1)^3$

$f'(x) = 3(x-1)^2$

$f''(x) = 6(x-1)$

$f''(x)=0$, $6(x-1)=0$

$x-1=0$

$x=1$




| Intervals | $(-\infty, 1)$ | $(1, \infty)$ |
|---|---|---|
| sign of $f''(x)$ | $f''(0)$ = -ve | $f''(2)$ = +ve |
| Concavity | Concave down | Concave up |

at $x=1$, $y=(1-1)^3=0$

$\therefore$ The point of inflection is (1,0) $\therefore$ Ans: (c) (1,0)



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# EXERCISE 8.8

1) A circular template has a radius of 10cm. The measurement of radius has an approximate error of 0.02 cm. Then the percentage error in calculating area of this template is

(a) 0.2% (b) 0.4% (c) 0.04% (d) 0.08%

Sol:-

$r = 10\,cm, \quad dr = 0.02\,cm$

Area, $A = \pi r^2$

$dA = 2\pi r\,dr$

$\frac{dA}{A} \times 100 = \frac{2\pi r\,dr}{\pi r^2} \times 100$

$= \frac{2}{10}(0.02) \times 100$

$= 0.4\%$

Ans: (b) 0.4%

2) The percentage error of fifth root of 31 is approximately how many times the percentage error in 31?

(a) $\frac{1}{31}$ (b) $\frac{1}{5}$ (c) 5 (d) 31

Sol:-

The percentage error in the $n^{th}$ root of a number is approximately $\frac{1}{n}$ times the percentage error in the number

$n = 5, \quad = \frac{1}{5}$

∴ Ans: (b) $\frac{1}{5}$

3) If $u(x,y) = e^{x^2+y^2}$, then $\frac{\partial u}{\partial x}$ is equal to

(a) $e^{x^2+y^2}$ (b) $2xu$ (c) $x^2u$ (d) $y^2u$

Sol:-

$u(x,y) = e^{x^2+y^2}$

$\frac{\partial u}{\partial x} = e^{x^2+y^2} 2x$

$= 2xu$

∴ Ans (b) $2xu$

4) If $v(x,y) = \log(e^x + e^y)$, then $\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y}$ is equal to

(a) $e^x + e^y$ (b) $\frac{1}{e^x+e^y}$ (c) 2 (d) 1

Sol:-

$v(x,y) = \log(e^x + e^y)$

$\frac{\partial v}{\partial x} = \frac{1}{e^x+e^y} \cdot e^x = \frac{e^x}{e^x+e^y}$



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$$\frac{\partial v}{\partial y} = \frac{e^y}{e^x+e^y}$$

$$\therefore \frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} = \frac{e^x}{e^x+e^y} + \frac{e^y}{e^x+e^y}$$

$$= \frac{e^x+e^y}{e^x+e^y}$$

$$= 1$$

∴ Ans: (d) 1

5) If $w(x,y) = x^y$, $x>0$, then $\frac{\partial w}{\partial x}$ is equal to

(a) $x^y \log x$ (b) $y \log x$ (c) $yx^{y-1}$ (d) $x \log y$

Sol:-

$w(x,y) = x^y$ $\quad x^n \to nx^{n-1}$

$\frac{\partial w}{\partial x} = yx^{y-1}$

∴ Ans: (c) $yx^{y-1}$

6) If $f(x,y) = e^{xy}$, then $\frac{\partial^2 f}{\partial x \partial y}$ is equal to

(a) $xye^{xy}$ (b) $(1+xy)e^{xy}$ (c) $(1+y)e^{xy}$ (d) $(1+x)e^{xy}$

Sol:-

$f(x,y) = e^{xy}$

$\frac{\partial f}{\partial y} = e^{xy} \cdot x = xe^{xy}$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)$$

$$= \frac{\partial}{\partial x}\left(xe^{xy}\right)$$

$$= x \cdot e^{xy} y + e^{xy}(1)$$

$$= e^{xy}(xy+1)$$

∴ Ans: (b) $(1+xy)e^{xy}$

7) If we measure the side of a cube to be 4 cm with an error of 0.1 cm, then the error in our calculation of the volume is

(a) 0.4 cu.cm (b) 0.45 cu.cm (c) 2 cu.cm (d) 4.8 cu.cm

Sol:- $a = 4\text{cm}$, $da = 0.1\text{ cm}$

$V = a^3$

$dv = 3a^2 da$

$= 3(4)^2(0.1)$

$= 4.8$ cu.cm

Ans: (d) 4.8 cu.cm

8) The change in the surface area $S = 6x^2$ of a cube when the edge length varies from $x_0$ to $x_0 + dx$ is

(a) $12x_0 + dx$ (b) $12x_0 dx$ (c) $6x_0 dx$ (d) $6x_0 + dx$



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Sol:- $S = 6x^2$

$ds = 12x\,dx$ $(x = x_0)$

$\therefore ds = 12x_0\,dx$ $\therefore$ Ans: (b) $12x_0\,dx$

9) The approximate change in the volume V of a cube of side x metres caused by increasing the side by 1% is

(a) $0.3x\,dx\,m^3$ (b) $0.03x\,m^3$ (c) $0.03x^2m^3$ (d) $0.03x^3m^3$

Sol:- $V = x^3$

$dV = 3x^2\,dx$ $dx = \frac{1}{100}x = 0.01x$

$= 3x^2(0.01x)$

$= 0.03x^3m^3$ $\therefore$ Ans: (d) $0.03x^3m^3$

10) If $g(x,y) = 3x^2 - 5y + 2y^2$, $x(t) = e^t$ and $y(t) = \cos t$, then $\frac{dg}{dt}$ is equal to

(a) $6e^{2t} + 5\sin t - 4\cos t \sin t$ (b) $6e^{2t} - 5\sin t + 4\cos t \sin t$

(c) $3e^{2t} + 5\sin t + 4\cos t \sin t$ (d) $3e^{2t} - 5\sin t + 4\cos t \sin t$

Sol:- $\frac{dg}{dt} = \frac{\partial g}{\partial x}\frac{dx}{dt} + \frac{\partial g}{\partial y}\frac{dy}{dt}$ $x = e^t$, $y = \cos t$

$= 6x \cdot e^t + (-5 + 4y)(-\sin t)$

$= 6e^{2t} + (-5 + 4\cos t)(-\sin t)$

$= 6e^{2t} + 5\sin t - 4\cos t \sin t$ $\therefore$ Ans (a) $6e^{2t} + 5\sin t - 4\cos t \sin t$

11) If $f(x) = \frac{x}{x+1}$, then its differential is given by

(a) $\frac{-1}{(x+1)^2}dx$ (b) $\frac{1}{(x+1)^2}dx$ (c) $\frac{1}{x+1}dx$ (d) $\frac{-1}{x+1}dx$

Sol:- $df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy$

$= \frac{(1)(x+1) - x(1)}{(x+1)^2}dx + 0$

$= \frac{x+1-x}{(x+1)^2}$

$= \frac{1}{(x+1)^2}$ $\therefore$ Ans: (b) $\frac{1}{(x+1)^2}dx$

12) If $u(x,y) = x^2 + 3xy + y - 2019$, then $\frac{\partial u}{\partial x}\Big|_{(4,-5)}$ is equal to

(a) $-4$ (b) $-3$ (c) $-7$ (d) 13

Sol:- $\frac{\partial u}{\partial x} = 2x + 3y$

$\frac{\partial u}{\partial x}(4,-5) = 2(4) + 3(-5)$

$= 8 - 15$

$= -7$ $\therefore$ Ans: (c) $-7$

13) Linear approximation for $g(x) = \cos x$ at $x = \frac{\pi}{2}$ is

(a) $x + \frac{\pi}{2}$ (b) $-x + \frac{\pi}{2}$ (c) $x - \frac{\pi}{2}$ (d) $-x - \frac{\pi}{2}$



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Sol:- $L(x) = g(x_0) + g'(x_0)(x - x_0)$ $\quad g(x) = \cos x$, $g'(x) = -\sin x$

$= \cos\frac{\pi}{2} + (-\sin\frac{\pi}{2})(x - \frac{\pi}{2})$

$= 0 - 1(x - \frac{\pi}{2})$

$= -x + \frac{\pi}{2}$

∴ Ans: (b) $-x + \frac{\pi}{2}$

14) If $w(x, y, z) = x^2(y - z) + y^2(z - x) + z^2(x - y)$, then $\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z}$ is

(a) $xy + yz + zx$ (b) $x(y + z)$ (c) $y(z + x)$ (d) 0

Sol:- $w(x, y, z) = x^2(y - z) + y^2(z - x) + z^2(x - y)$

$\frac{\partial w}{\partial x} = 2x(y - z) + y^2(-1) + z^2(1)$

$= 2xy - 2xz - y^2 + z^2$

$\frac{\partial w}{\partial y} = x^2(1) + 2y(z - x) + z^2(-1)$

$= x^2 + 2yz - 2xy - z^2$

$\frac{\partial w}{\partial z} = x^2(-1) + y^2(1) + 2z(x - y)$

$= -x^2 + y^2 + 2xz - 2yz$

$\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} = 2xy - 2xz - y^2 + z^2 + x^2 + 2yz - 2xy - z^2 - x^2 + y^2 + 2xz - 2yz$

$= 0$

∴ Ans: (d) 0

15) If $f(x, y, z) = xy + yz + zx$, then $f_x - f_z$ is equal to

(a) $z - x$ (b) $y - z$ (c) $x - z$ (d) $y - x$

Sol:- $f(x, y, z) = xy + yz + zx$

$f_x - f_z = \frac{\partial f}{\partial x} - \frac{\partial f}{\partial z}$

$= [(1)y + 0 + z(1)] - [0 + y + x]$

$= y + z - y - x$

$= z - x$

∴ Ans: (a) $z - x$



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# EXERCISE 9.10

1) The value of $\int_0^{2/3} \frac{dx}{\sqrt{4-9x^2}}$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) $\pi$

**Sol:-**

$$\int_0^{2/3} \frac{dx}{\sqrt{4-9x^2}} = \int_0^{2/3} \frac{dx}{\sqrt{2^2-(3x)^2}}$$

$\boxed{\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + c}$

$$= \frac{1}{3}\left[\sin^{-1}\frac{3x}{2}\right]_0^{2/3}$$

$$= \frac{1}{3}\left[\sin^{-1}\frac{3}{2}\left(\frac{2}{3}\right) - \sin^{-1}0\right]$$

$$= \frac{1}{3}\left[\sin^{-1}(1)\right]$$

$$= \frac{1}{3}\cdot\frac{\pi}{2}$$

$$= \frac{\pi}{6}$$

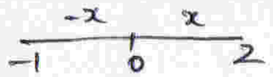
∴ **Ans**: (a) $\frac{\pi}{6}$

2) The value of $\int_{-1}^{2} |x|\,dx$ is

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) $\frac{5}{2}$ (d) $\frac{7}{2}$

**Sol:-**

$$\int_{-1}^{2} |x|\,dx = \int_{-1}^{0} -x\,dx + \int_{0}^{2} x\,dx$$




$$= -\left[\frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2}\right]_{0}^{2}$$

$$= -\left[0 - \frac{1}{2}\right] + \left[\frac{4}{2} - 0\right]$$

$$= \frac{1}{2} + 2 = \frac{1+4}{2}$$

$$= \frac{5}{2}$$

∴ **Ans**: (c) $\frac{5}{2}$

3) For any value of $n \in Z$, $\int_0^{\pi} e^{\cos^2 x}\cos^3[(2n+1)x]\,dx$ is

(a) $\frac{\pi}{2}$ (b) $\pi$ (c) 0 (d) 2

**Sol:-**

$$I = \int_0^{\pi} e^{\cos^2 x}\cos^3[(2n+1)x]\,dx \quad \text{——— (1)}$$

$$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$

$$I = \int_0^{\pi} e^{\cos^2(\pi-x)}\cos^3[(2n+1)(\pi-x)]\,dx$$



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$$= \int_0^{\pi} e^{\cos^2 x} \cdot -\cos^3[(2n+1)x]\,dx$$

$$I = -\int_0^{\pi} e^{\cos^2 x} \cos^3[(2n+1)x]\,dx \quad \text{——— (2)}$$

From (1) and (2)

$I + I = 0$

$\therefore 2I = 0$

$\therefore I = 0$

$\therefore$ Ans: (c) 0

---

4) The value of $\int_{-\pi/2}^{\pi/2} \sin^2 x \cos x\,dx$ is

(a) $\frac{3}{2}$ (b) $\frac{1}{2}$ (c) 0 (d) $\frac{2}{3}$

Sol:-

$$\int_{-\pi/2}^{\pi/2} \sin^2 x \cos x\,dx$$

Let $f(x) = \sin^2 x \cos x$

$f(-x) = \sin^2(-x)\cos(-x)$

$= (-\sin x)^2 \cos x$

$= \sin^2 x \cos x = f(x)$

$f(x)$ is even function.

$$\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$$

$$\therefore \int_{-\pi/2}^{\pi/2} \sin^2 x \cos x\,dx = 2\int_0^{\pi/2} \sin^2 x \cos x\,dx$$

$t = \sin x$

$dt = \cos x\,dx$

| $x$ | 0 | $\pi/2$ |
|---|---|---|
| $t$ | 0 | 1 |

$$= 2\int_0^1 t^2\,dt$$

$$= 2\left[\frac{t^3}{3}\right]_0^1$$

$$= \frac{2}{3}(1-0)$$

$$= \frac{2}{3}$$

$\therefore$ Ans: (b) $\frac{2}{3}$

---

5) The value of $\int_{-4}^{4} \left[\tan^{-1}\left(\frac{x^2}{x^4+1}\right) + \tan^{-1}\left(\frac{x^4+1}{x^2}\right)\right]dx$ is

(a) $\pi$ (b) $2\pi$ (c) $3\pi$ (d) $4\pi$

Sol:-

$$\int_{-4}^{4} \tan^{-1}\left(\frac{x^2}{x^4+1}\right) + \tan^{-1}\left(\frac{x^4+1}{x^2}\right)$$

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$$\tan^{-1}x = \cot^{-1}\frac{1}{x}$$

$$= \int_{-4}^{4} \tan^{-1}\left(\frac{x^2}{x^4+1}\right) + \cot^{-1}\left(\frac{x^2}{x^4+1}\right) dx$$

$\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$

$$= \int_{-4}^{4} \frac{\pi}{2}\, dx$$

$$= \frac{\pi}{2}\left[x\right]_{-4}^{4}$$

$$= \frac{\pi}{2}\left[4+4\right] = \frac{8\pi}{2}$$

$$= 4\pi$$

∴ Ans: (d) $4\pi$

6) The value of $\int_{-\pi/4}^{\pi/4} \left(\frac{2x^7 - 3x^5 + 7x^3 - x + 1}{\cos^2 x}\right) dx$ is

(a) 4 (b) 3 (c) 2 (d) 0

Sol:-

$$\int_{-\pi/4}^{\pi/4} \left(\frac{2x^7 - 3x^5 + 7x^3 - x + 1}{\cos^2 x}\right) dx$$

$$= \int_{-\pi/4}^{\pi/4} \left[\frac{2x^7 - 3x^5 + 7x^3 - x}{\cos^2 x} + \frac{1}{\cos^2 x}\right] dx$$

$$= \int_{-\pi/4}^{\pi/4} \frac{2x^7 - 3x^5 + 7x^3 - x}{\cos^2 x}\, dx + \int_{-\pi/4}^{\pi/4} \sec^2 x\, dx$$

$$f(x) = \frac{2x^7 - 3x^5 + 7x^3 - x}{\cos^2 x}$$

$$f(-x) = \frac{-2x^7 + 3x^5 - 7x^3 + x}{\cos^2 x}$$

$f(-x) = -f(x)$ $f(x)$ is odd function

$\int_{-a}^{a} f(x)\, dx = 0$

$$= 0 + \left[\tan x\right]_{-\pi/4}^{\pi/4}$$

$$= \tan\frac{\pi}{4} + \tan\frac{\pi}{4}$$

$$= 1 + 1$$

$$= 2$$

∴ Ans: (c) 2

7) If $f(x) = \int_0^2 t\cos t\, dt$, then $\frac{df}{dx} =$

(a) $\cos x - x\sin x$ (b) $\sin x + x\cos x$ (c) $x\cos x$ (d) $x\sin x$

Sol:-

$$f(x) = \int_0^x t\cos t\, dt$$

I L A T E



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$u = t \qquad dv = \cos t\, dt$

$du = dt \qquad v = \sin t$

$uv - \int v\,du = t\sin t - \int \sin t\, dt$

$= [t\sin t + \cos t]_0^x$

$\therefore f(x) = x\sin x + \cos x - 1$

$\frac{\partial f}{\partial x} = x\cos x + \sin x(1) - \sin x$

$= x\cos x$ $\qquad \therefore$ Ans : (C) $x\cos x$

8) The area between $y^2 = 4x$ and its latus rectum is

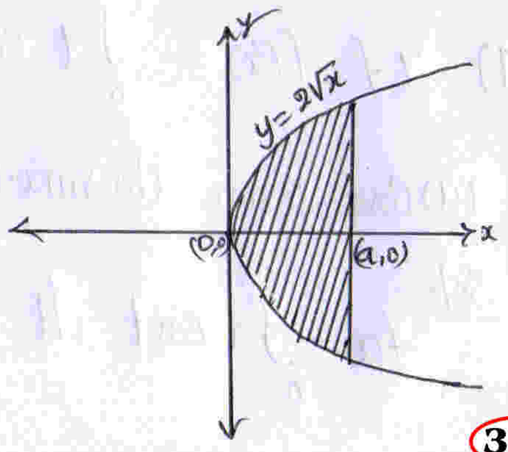
(a) $\frac{2}{3}$ (b) $\frac{4}{3}$ (c) $\frac{8}{3}$ (d) $\frac{5}{3}$

Sol:-

$y^2 = 4x$

$y = 2\sqrt{x}$

$\text{Area} = 2\int_0^1 y\, dx$




$= 2\int_0^1 2\sqrt{x}\, dx$

$= 4\int_0^1 x^{1/2}\, dx$

$= 4\left[\frac{x^{3/2}}{3/2}\right]_0^1$

$= 4\cdot\frac{2}{3}[1-0]$

$= \frac{8}{3}$ $\qquad \therefore$ Ans : (C) $\frac{8}{3}$

9) The value of $\int_0^1 x(1-x)^{99}\, dx$ is

(a) $\frac{1}{11000}$ (b) $\frac{1}{10100}$ (c) $\frac{1}{10010}$ (d) $\frac{1}{10001}$

Sol:-

$\int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx$

$f(x) = \int_0^1 x(1-x)^{99}\, dx$

$f(1-x) = \int_0^1 (1-x)(1-(1-x))^{99}\, dx$

$= \int_0^1 (1-x)(1-1+x)^{99}\, dx$



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$$= \int_0^1 (1-x)\, x^{99}\, dx$$

$$= \int_0^1 (x^{99} - x^{100})\, dx$$

$$= \left[\frac{x^{100}}{100} - \frac{x^{101}}{101}\right]_0^1$$

$$= \left(\frac{1}{100} - \frac{1}{101}\right) - 0$$

$$= \frac{101-100}{10100}$$

$$= \frac{1}{10100}$$

$\therefore$ Ans : (b) $\frac{1}{10100}$

10) The value of $\int_0^{\pi} \frac{dx}{1+5^{\cos x}}$ is

(a) $\frac{\pi}{2}$ (b) $\pi$ (c) $\frac{3\pi}{2}$ (d) $2\pi$

Sol:-

$$I = \int_0^{\pi} \frac{dx}{1+5^{\cos x}} \quad \text{——— (1)}$$

$$\int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx$$

$$I = \int_0^{\pi} \frac{dx}{1+5^{\cos(\pi-x)}}$$

$$= \int_0^{\pi} \frac{dx}{1+5^{-\cos x}}$$

$$= \int_0^{\pi} \frac{dx}{1+\frac{1}{5^{\cos x}}}$$

$$= \int_0^{\pi} \frac{dx}{\frac{5^{\cos x}+1}{5^{\cos x}}}$$

$$I = \int_0^{\pi} \frac{5^{\cos x}\, dx}{5^{\cos x}+1} \quad \text{——— (2)}$$

(1) + (2) $\Rightarrow$

$$2I = \int_0^{\pi} \frac{1+5^{\cos x}}{5^{\cos x}+1}\, dx$$

$$= [x]_0^{\pi}$$

$$2I = \pi$$

$\therefore I = \frac{\pi}{2}$ $\therefore$ Ans : (a) $\frac{\pi}{2}$

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11) If $\frac{\Gamma(n+2)}{\Gamma(n)} = 90$ then $n$ is

(a) 10 (b) 5 (c) 8 (d) 9

Sol:-

$$\frac{\Gamma(n+2)}{\Gamma(n)} = 90 \qquad \Gamma n = (n-1)!$$

$$\frac{(n+1)!}{(n-1)!} = 90$$

$$\frac{(n+1)\,n\,(n-1)!}{(n-1)!} = 90$$

$$(n+1)\,n = 10 \times 9$$

$$\therefore \boxed{n = 9}$$

$n+1 = 10$
$n = 9$

$\therefore$ Ans : (d) 9

12) The value of $\int_0^{\pi/6} \cos^3 3x\,dx$ is

(a) $\frac{2}{3}$ (b) $\frac{2}{9}$ (c) $\frac{1}{9}$ (d) $\frac{1}{3}$

Sol:-

$$\int_0^{\pi/6} \cos^3 3x\,dx$$

$t = 3x$
$dt = 3\,dx$
$dt/3 = dx$

$$= \frac{1}{3}\int_0^{\pi/2} \cos^3 t\,dt$$

$$= \frac{1}{3}\cdot\frac{2}{3}$$

$$= \frac{2}{9}$$

| $x$ | 0 | $\pi/6$ |
|---|---|---|
| $t$ | 0 | $\pi/2$ |

$n$ is odd

$$\int_0^{\pi/2} \cos^n x\,dx = \frac{n-1}{n} \times \frac{n-3}{n-2} \times \cdots \times \frac{2}{3}$$

$\therefore$ Ans : (b) $\frac{2}{9}$

13) The value of $\int_0^{\pi} \sin^4 x\,dx$ is

(a) $\frac{3\pi}{10}$ (b) $\frac{3\pi}{8}$ (c) $\frac{3\pi}{4}$ (d) $\frac{3\pi}{2}$

Sol:-

$$\int_0^{\pi} \sin^4 x\,dx$$

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

$$= \int_0^{\pi} \left(\frac{1-\cos 2x}{2}\right)^2 dx$$

$$= \frac{1}{4}\int_0^{\pi} (1-\cos 2x)^2\,dx$$

$$= \frac{1}{4}\int_0^{\pi} (1 - 2\cos 2x + \cos^2 2x)\,dx$$



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$$= \frac{1}{4}\int_0^{\pi}\left(1 - 2\cos 2x + \frac{1+\cos 4x}{2}\right) dx$$

$$= \frac{1}{4}\left[x - \frac{2\sin 2x}{2} + \frac{x}{2} + \frac{\sin 4x}{8}\right]_0^{\pi}$$

$$= \frac{1}{4}\left[\pi + \frac{\pi}{2}\right]$$

$$= \frac{1}{4}\left[\frac{3\pi}{2}\right]$$

$$= \frac{3\pi}{8}$$

∴ Ans: (b) $\frac{3\pi}{8}$

Alternate method:

$$\int_0^{2a} f(x)\,dx = 2\int_0^{a} f(2a-x)\,dx$$

$$\int_0^{\pi} \sin^4 x\,dx = 2\int_0^{\pi/2} \sin^4(\pi - x)\,dx$$

$$= 2\int_0^{\pi/2} \sin^4 x\,dx$$

$$= 2\left(\frac{3}{4} \times \frac{1}{2} \times \frac{\pi}{2}\right)$$

$$= \frac{3\pi}{8}$$

n is even

$$\int_0^{\pi/2} \sin^n x\,dx = \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdots \frac{1}{2} \cdot \frac{\pi}{2}$$

14) The value of $\int_0^{\infty} e^{-3x} x^2\,dx$ is

(a) $\frac{1}{27}$ (b) $\frac{5}{27}$ (c) $\frac{4}{27}$ (d) $\frac{2}{27}$

Sol:-

$$\int_0^{\infty} e^{-ax} x^n\,dx = \frac{n!}{a^{n+1}} \qquad a=3,\ n=2$$

$$\therefore \int_0^{\infty} e^{-3x} x^2\,dx = \frac{2!}{3^{2+1}}$$

$$= \frac{2\times 1}{3^3}$$

$$= \frac{2}{27}$$

∴ Ans: (d) $\frac{2}{27}$

15) If $\int_0^{a} \frac{1}{4+x^2}\,dx = \frac{\pi}{8}$ then a is

(a) 4 (b) 1 (c) 3 (d) 2

Sol:-

$$\int_0^{a} \frac{1}{4+x^2}\,dx = \frac{\pi}{8} \qquad \int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}$$

$$\int_0^{a} \frac{1}{x^2+2^2}\,dx = \frac{\pi}{8}$$

$$\left[\frac{1}{2}\tan^{-1}\frac{x}{2}\right]_0^{a} = \frac{\pi}{8}$$

$$\frac{1}{2}\tan^{-1}\frac{a}{2} - 0 = \frac{\pi}{8}$$

$$\tan^{-1}\frac{a}{2} = \frac{\pi}{8} \times 2$$



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$$\frac{a}{2} = \tan\frac{\pi}{4}$$

$$\frac{a}{2} = 1$$

$$\therefore a = 2$$

$\therefore$ Ans : (d) 2

16) The volume of solid of revolution of the region bounded by $y^2 = x(a-x)$ about $x$-axis is

(a) $\pi a^3$ (b) $\frac{\pi a^3}{4}$ (c) $\frac{\pi a^3}{5}$ (d) $\frac{\pi a^3}{6}$

Sol:-

$$y^2 = x(a-x)$$

$y = 0$, $\quad x(a-x) = 0$

$\therefore x = 0, \quad x = a$

$$\text{Volume} = \int_0^a \pi y^2 \, dx$$

$$= \pi \int_0^a x(a-x) \, dx$$

$$= \pi \int_0^a (ax - x^2) \, dx$$

$$= \pi \left[ \frac{ax^2}{2} - \frac{x^3}{3} \right]_0^a$$

$$= \pi \left[ \left( \frac{a^3}{2} - \frac{a^3}{3} \right) - 0 \right]$$

$$= \pi \left( \frac{3a^3 - 2a^3}{6} \right)$$

$$= \pi \left( \frac{a^3}{6} \right)$$

$$= \frac{\pi a^3}{6} \text{ cu. units.}$$

$\therefore$ Ans : (d) $\frac{\pi a^3}{6}$

17) If $f(x) = \int_1^x \frac{e^{\sin u}}{u} \, du$, $x > 1$ and $\int_1^3 \frac{e^{\sin x^2}}{x} \, dx = \frac{1}{2}[f(a) - f(1)]$, then one of the possible value of $a$ is

(a) 3 (b) 6 (c) 9 (d) 5



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Sol:- $\int_1^3 \frac{e^{\sin x^2}}{x}\,dx = \frac{1}{2}[f(a)-f(1)]$

Let $u = x^2$

$x = \sqrt{u}$

$dx = \frac{1}{2\sqrt{u}}\,du$

| $x$ | 1 | 3 |
|---|---|---|
| $u$ | 1 | 9 |

$\int_1^9 \frac{e^{\sin u}}{\sqrt{u}} \cdot \frac{1}{2\sqrt{u}}\,du = \frac{1}{2}[f(a)-f(1)]$

$\frac{1}{2}\int_1^9 \frac{e^{\sin u}}{u}\,du = \frac{1}{2}[f(a)-f(1)]$

$\frac{1}{2}\int_1^9 f(x) = \frac{1}{2}[f(a)-f(1)]$

$\frac{1}{2}[f(9)-f(1)] = \frac{1}{2}[f(a)-f(1)]$

$\therefore a = 9$

$\therefore$ Ans : (c) 9

18) The value of $\int_0^1 (\sin^{-1}x)^2\,dx$ is

(a) $\frac{\pi^2}{4}-1$ (b) $\frac{\pi^2}{4}+2$ (c) $\frac{\pi^2}{4}+1$ (d) $\frac{\pi^2}{4}-2$

Sol:- $\int_0^1 (\sin^{-1}x)^2\,dx$

$t = \sin^{-1}x$

$\sin t = x$

$dx = \cos t\,dt$

| $x$ | 0 | 1 |
|---|---|---|
| $t$ | 0 | $\pi/2$ |

$\int_0^{\pi/2} t^2 \cos t\,dt$ ILATE

$\int uv\,dx = uv_1 - u'v_2 + u^2v_3 - \cdots$

$u = t^2$ $v = \cos t$

$u' = 2t$ $v_1 = \sin t$

$u^2 = 2$ $v_2 = -\cos t$

$= \left[t^2 \sin t + 2t\cos t + 2(-\sin t)\right]_0^{\pi/2}$

$= \left(\frac{\pi}{2}\right)^2 + 2\left(\frac{\pi}{2}\right)(0) - 2$

$= \frac{\pi^2}{4} - 2$

$\therefore$ Ans : (d) $\frac{\pi^2}{4}-2$

19) The value of $\int_0^a (\sqrt{a^2-x^2})^3\,dx$ is

(a) $\frac{\pi a^3}{16}$ (b) $\frac{3\pi a^4}{16}$ (c) $\frac{3\pi a^2}{8}$ (d) $\frac{3\pi a^4}{8}$

Sol:- $\int_0^a (\sqrt{a^2-x^2})^3\,dx$

Let $x = a\sin\theta$

$dx = a\cos\theta\,d\theta$

| $x$ | 0 | $a$ |
|---|---|---|
| $\theta$ | 0 | $\pi/2$ |

$= \int_0^{\pi/2} (\sqrt{a^2 - a^2\sin^2\theta})^3 a\cos\theta\,d\theta$



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$$= \int_0^{\pi/2} \left(\sqrt{a^2(1-\sin^2\theta)}\right)^3 a\cos\theta\, d\theta$$

$$= \int_0^{\pi/2} \left(\sqrt{a^2\cos^2\theta}\right)^3 a\cos\theta\, d\theta$$

$$= \int_0^{\pi/2} (a\cos\theta)^3 a\cos\theta\, d\theta$$

$$= a^4 \int_0^{\pi/2} \cos^4\theta\, d\theta$$

$$= a^4\left[\frac{3}{4}\times\frac{1}{2}\times\frac{\pi}{2}\right]$$

$$= \frac{3\pi a^4}{16}$$

∴ Ans : (b) $\frac{3\pi a^4}{16}$

n is even

$\int_0^{\pi/2} \cos^n x\, dx =$

$\frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdots\frac{1}{2}\cdot\frac{\pi}{2}$

20) If $\int_0^x f(t)\,dt = x + \int_x^1 t f(t)\,dt$, then the value of $f(1)$ is

(a) $\frac{1}{2}$ (b) 2 (c) 1 (d) $\frac{3}{4}$

Sol:-

$$\int_0^x f(t)\,dt = x + \int_x^1 t f(t)\,dt$$

differentiate on both derivatives

$$f(x) = 1 + \left[-\int_1^x t f(t)\,dt\right]$$

by theorem 9.1

$$f(x) = 1 - x f(x)$$

Substitute $x = 1$

$$f(1) = 1 - f(1)$$

$$f(1) + f(1) = 1$$

$$2f(1) = 1$$

$$\therefore f(1) = \frac{1}{2}$$

∴ Ans : (a) $\frac{1}{2}$

$F(x) = \int_a^x f(u)\,du$,

$\frac{d}{dx} F(x) = f(x)$

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# EXERCISE 10.9

Choose the Correct or the most suitable answer from the given four alternatives:

1) The order and degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{1/3} + x^{1/4} = 0$ are respectively.

(a) 2, 3 (b) 3, 3 (c) 2, 6 (d) 2, 4

Sol:-

$$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{1/3} + x^{1/4} = 0$$

$$\frac{d^2y}{dx^2} + x^{1/4} = -\left(\frac{dy}{dx}\right)^{1/3}$$

$$\left[\frac{d^2y}{dx^2} + x^{1/4}\right]^3 = -\frac{dy}{dx}$$

$$\left(\frac{d^2y}{dx^2}\right)^3 + \left(x^{1/4}\right)^3 = -\frac{dy}{dx}$$

∴ order = 2
degree = 3

∴ Ans: (a) 2, 3

2) The differential equation representing the family of curves $y = A\cos(x+B)$, where A and B are parameters, is

(a) $\frac{d^2y}{dx^2} - y = 0$ (b) $\frac{d^2y}{dx^2} + y = 0$ (c) $\frac{d^2y}{dx^2} = 0$ (d) $\frac{d^2x}{dy^2} = 0$

Sol:-

$y = A\cos(x+B)$ A and B are parameters

$\frac{dy}{dx} = -A\sin(x+B)$

$\frac{d^2y}{dx^2} = -A\cos(x+B)$

$\frac{d^2y}{dx^2} = -y$

∴ $\frac{d^2y}{dx^2} + y = 0$

∴ Ans: (b) $\frac{d^2y}{dx^2} + y = 0$

3) The order and degree of the differential equation $\sqrt{\sin x}\,(dx+dy) = \sqrt{\cos x}\,(dx-dy)$ is

(a) 1, 2 (b) 2, 2 (c) 1, 1 (d) 2, 1

Sol:- $\sqrt{\sin x}\,(dx+dy) = \sqrt{\cos x}\,(dx-dy)$

$\sqrt{\sin x}\,dx + \sqrt{\sin x}\,dy = \sqrt{\cos x}\,dx - \sqrt{\cos x}\,dy$



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$$\sqrt{\sin x}\,dy + \sqrt{\cos x}\,dy = \sqrt{\cos x}\,dx - \sqrt{\sin x}\,dx$$

$$dy\left(\sqrt{\sin x} + \sqrt{\cos x}\right) = dx\left(\sqrt{\cos x} - \sqrt{\sin x}\right)$$

$$\frac{dy}{dx} = \frac{\sqrt{\cos x} - \sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}$$

$\therefore$ Order = 1,
degree = 1

$\therefore$ Ans: (c) 1, 1

4) The order of the differential equation of all circles with centre at $(h,k)$ and radius 'a' is

(a) 2 (b) 3 (c) 4 (d) 1.

Sol:- Equation of circle is

$$(x-h)^2 + (y-k)^2 = a^2$$

h, k, a are arbitrary Constants

differentiate at 3 times

order = 3

$\therefore$ Ans: (b) 3

5) The differential equation of the family of curves $y = Ae^x + Be^{-x}$, where A and B are arbitrary Constants is

(a) $\frac{d^2y}{dx^2} + y = 0$ (b) $\frac{d^2y}{dx^2} - y = 0$ (c) $\frac{dy}{dx} + y = 0$ (d) $\frac{dy}{dx} - y = 0$

Sol:-

$$y = Ae^x + Be^{-x}$$

A and B are arbitrary constants

$$\frac{dy}{dx} = Ae^x + Be^{-x}(-1)$$

$$= Ae^x - Be^{-x}$$

$$\frac{d^2y}{dx^2} = Ae^x - Be^{-x}(-1)$$

$$= Ae^x + Be^{-x}$$

$$\frac{d^2y}{dx^2} = y$$

$$\frac{d^2y}{dx^2} - y = 0$$

$\therefore$ Ans: (b) $\frac{d^2y}{dx^2} - y = 0$

6) The general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$ is

(a) $xy = k$ (b) $y = k\log x$ (c) $y = kx$ (d) $\log y = kx$



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**Sol:-**

$$\frac{dy}{dx} = \frac{y}{x}$$

$$\int \frac{dy}{y} = \int \frac{dx}{x} \qquad (c=k)$$

$$\log y = \log x + \log k$$

$$\log y = \log xk$$

$$y = xk$$

∴ **Ans**: (c) $y=kx$

7) The solution of the differential equation $2x\frac{dy}{dx} - y = 3$ represents

(a) Straight lines (b) circles

(c) Parabola (d) ellipse

**Sol:-**

$$2x\frac{dy}{dx} - y = 3$$

$$2x\frac{dy}{dx} = y+3$$

$$\int \frac{2\,dy}{y+3} = \int \frac{dx}{x}$$

$$2\log|y+3| = \log x + \log c$$

$$\log|y+3|^2 = \log cx$$

$(y+3)^2 = cx$ is the equation of parabola

∴ **Ans**: (c) Parabola

8) The solution of $\frac{dy}{dx} + P(x)y = 0$ is

(a) $y = ce^{\int P dx}$ (b) $y = ce^{-\int P dx}$ (c) $x = ce^{-\int P dy}$ (d) $x = ce^{\int P dy}$

**Sol:-**

$$\frac{dy}{dx} + P(x)y = 0$$

$\frac{dy}{dx} + Py = Q$

$P = P(x)$, $Q = 0$

$$IF = e^{\int P dx} = e^{\int P(x)dx}$$

**Solution**: $y(IF) = \int Q(IF)\,dx + C$

$$y e^{\int P(x)dx} = 0 + C$$

$$y = \frac{c}{e^{\int P dx}} \Rightarrow y = ce^{-\int P dx}$$

∴ **Ans**: (b) $y = ce^{-\int P dx}$

9) The integrating factor of the differential equation $\frac{dy}{dx} + y = \frac{1+y}{x}$ is

(a) $\frac{x}{e^x}$ (b) $\frac{e^x}{x}$ (c) $\lambda e^x$ (d) $e^x$



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Sol:-

$$\frac{dy}{dx} + y = \frac{1+y}{x}$$

$$\frac{dy}{dx} + y = \frac{1}{x} + \frac{y}{x}$$

$$\frac{dy}{dx} + y - \frac{y}{x} = \frac{1}{x}$$

$$\frac{dy}{dx} + \left(1 - \frac{1}{x}\right) y = \frac{1}{x}$$

$\boxed{\frac{dy}{dx} + Py = Q}$

$P = 1 - \frac{1}{x}$ , $Q = \frac{1}{x}$

$$IF = e^{\int P dx} = e^{\int 1 - \frac{1}{x}\, dx}$$

$$= e^{x - \log x}$$

$$= e^{x} \cdot e^{-\log x}$$

$$= e^{x} \cdot e^{\log x^{-1}}$$

$$= e^{x} \cdot x^{-1}$$

$$= \frac{e^x}{x}$$

$\therefore$ Ans: (b) $\frac{e^x}{x}$

10) The integrating factor of the differential equation $\frac{dy}{dx} + P(x)\, y = Q(x)$ is $x$, then $P(x)$

(a) $x$ (b) $\frac{x^2}{2}$ (c) $\frac{1}{x}$ (d) $\frac{1}{x^2}$

Sol:-

$$\frac{dy}{dx} + P(x)\, y = Q(x)$$

Given: $IF \Rightarrow e^{\int P dx} = x$

$$\log e^{\int P dx} = \log x$$

$$\int P dx = \log x$$

$\Rightarrow \int \frac{1}{x} dx = \log x$ $\quad \therefore P = \frac{1}{x}$ $\quad \therefore$ Ans: (c) $\frac{1}{x}$

11) The degree of the differential equation $y(x) = 1 + \frac{dy}{dx} + \frac{1}{1.2}\left(\frac{dy}{dx}\right)^2 + \frac{1}{1.2.3}\left(\frac{dy}{dx}\right)^3 + \cdots$ is

(a) 2 (b) 3 (c) 1 (d) 4

Sol:- $y(x) = 1 + \frac{dy}{dx} + \frac{1}{1.2}\left(\frac{dy}{dx}\right)^2 + \frac{1}{1.2.3}\left(\frac{dy}{dx}\right)^3 + \cdots$

$$y = 1 + \frac{y'}{1!} + \frac{(y')^2}{2!} + \frac{(y')^3}{3!} + \cdots$$

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$y = e^{y'}$ (ie) $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

$\log y = \log e^{y'}$

$\log y = y'$

$\frac{dy}{dx} = \log y$

order = 1
degree = 1

$\therefore$ Ans: (c) 1

12) If P and Q are the order and degree of the differential equation $y\frac{dy}{dx} + x^3\left(\frac{d^2y}{dx^2}\right) + xy = \cos x$, when

(a) $P<Q$ (b) $P=Q$ (c) $P>Q$ (d) P exists and Q does not exists.

Sol:-

$$y\frac{dy}{dx} + x^3\left(\frac{d^2y}{dx^2}\right) + xy = \cos x$$

order, P = 2
degree, Q = 1
$\therefore P>Q$ as $2>1$

$\therefore$ Ans:- (c) $P>Q$

13) The solution of the differential equation $\frac{dy}{dx} + \frac{1}{\sqrt{1-x^2}} = 0$ is

(a) $y + \sin^{-1}x = C$ (b) $x + \sin^{-1}y = 0$

(c) $y^2 + 2\sin^{-1}x = C$ (d) $x^2 + 2\sin^{-1}y = 0$

Sol:-

$$\frac{dy}{dx} + \frac{1}{\sqrt{1-x^2}} = 0$$

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}$$

$$\int dy = -\int \frac{1}{\sqrt{1-x^2}}\,dx$$

$$y = -\sin^{-1}x + C$$

$$y + \sin^{-1}x = C$$

$\therefore$ Ans: (a) $y + \sin^{-1}x = C$

14) The solution of the differential equation $\frac{dy}{dx} = 2xy$ is

(a) $y = Ce^{x^2}$ (b) $y = 2x^2 + C$ (c) $y = Ce^{-x^2} + C$ (d) $y = x^2 + C$



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Sol:-

$$\frac{dy}{dx} = 2xy$$

$$\int \frac{dy}{y} = \int 2x\,dx$$

$$\log y = \cancel{2}\cdot\frac{x^2}{\cancel{2}} + \log c$$

$$\log y - \log c = x^2$$

$$\log \frac{y}{c} = x^2$$

$$\frac{y}{c} = e^{x^2}$$

$$y = ce^{x^2}$$

∴ Ans: (a) $y = ce^{x^2}$

$$\frac{dy}{dx} = e^{x+y}$$

$$\frac{dy}{dx} = e^x \cdot e^y$$

$$\frac{dy}{e^y} = e^x\,dx$$

$$\int e^{-y}\,dy = \int e^x\,dx$$

$$\frac{e^{-y}}{-1} = e^x + c \Rightarrow e^{-y} = -e^x - c$$

$$e^x + e^{-y} + c = 0$$

$$e^x + e^{-y} = -c$$

$$\therefore e^x + e^{-y} = c$$

∴ Ans: (b) $e^x + e^{-y} = c$

15) The general solution of the differential equation $\log\left(\frac{dy}{dx}\right) = x + y$ is

(a) $e^x + e^y = c$ (b) $e^x + e^{-y} = c$

(c) $e^{-x} + e^y = c$ (d) $e^{-x} + e^{-y} = c$

Sol:- $\log\left(\frac{dy}{dx}\right) = x + y$

16) The solution of $\frac{dy}{dx} = 2^{y-x}$ is

(a) $2^x + 2^y = c$ (b) $2^x - 2^y = c$ (c) $\frac{1}{2^x} - \frac{1}{2^y} = c$ (d) $x + y = c$

Sol:-

$$\frac{dy}{dx} = 2^{y-x}$$

$$\frac{dy}{dx} = 2^y \cdot 2^{-x}$$



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$$\frac{dy}{2^y} = 2^{-x}\,dx$$

$$\int 2^{-y}\,dy = \int 2^{-x}\,dx \qquad \int a^x dx = \frac{a^x}{\log a}$$

$$\frac{2^{-y}}{\log 2} = \frac{2^{-x}}{\log 2} + C$$

$$\frac{2^{-x}}{\log 2} - \frac{2^{-y}}{\log 2} = C \Rightarrow \frac{2^{-x} - 2^{-y}}{\log 2} = C$$

$$2^{-x} - 2^{-y} = (\log 2)\,C$$

$$\frac{1}{2^x} - \frac{1}{2^y} = C$$

∴ Ans: (c) $\frac{1}{2^x} - \frac{1}{2^y} = C$

17) The solution of the differential equation $\frac{dy}{dx} = \frac{y}{x} + \frac{\phi(y/x)}{\phi'(y/x)}$ is

(a) $x\phi\left(\frac{y}{x}\right) = k$ (b) $\phi\left(\frac{y}{x}\right) = kx$ (c) $y\phi\left(\frac{y}{x}\right) = k$ (d) $\phi\left(\frac{y}{x}\right) = ky$

Sol:-

$$\frac{dy}{dx} = \frac{y}{x} + \frac{\phi\left(\frac{y}{x}\right)}{\phi'\left(\frac{y}{x}\right)} \qquad \text{---------- (1)}$$

It is homogeneous equation

$y = vx \Rightarrow \frac{y}{x} = v$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$ substitute in (1)

$$v + x\frac{dv}{dx} = v + \frac{\phi(v)}{\phi'(v)}$$

$$\int \frac{\phi'(v)\,dv}{\phi(v)} = \int \frac{dx}{x}$$

$t = \phi(v)$

$dt = \phi'(v)\,dv$

$$\int \frac{dt}{t} = \int \frac{dx}{x}$$

$$\log t = \log x + \log C$$

$$\log t = \log kx$$

$$t = kx$$

Replacing $v = y/x$

$$\phi(v) = kx$$

$$\phi\left(\frac{y}{x}\right) = kx$$

∴ Ans: (b) $\phi\left(\frac{y}{x}\right) = kx$

18) If $\sin x$ is the integrating factor of the linear differential equation $\frac{dy}{dx} + Py = Q$, then P is



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(a) $\log \sin x$ (b) $\cos x$ (c) $\tan x$ (d) $\cot x$

Sol:-

$$IF \Rightarrow e^{\int P dx} = \sin x$$

$$\log e^{\int P dx} = \log \sin x$$

$$\int P dx = \log \sin x$$

$$\int \cot dx = \log \sin x$$

$$\therefore P = \cot x$$

$\therefore$ Ans: (d) $\cot x$

19) The number of arbitrary constants in the general solutions of order $n$ and $n+1$ are respectively.

(a) $n-1, n$ (b) $n, n+1$ (c) $n+1, n+2$ (d) $n+1, n$

Sol:-

order $n \rightarrow$ arbitrary constants $= n$

order $n+1 \rightarrow$ arbitrary constants $= n+1$

$\therefore$ Ans: (b) $n, n+1$

20) The number of arbitrary constants in the particular solutions of a differential equation of third order is

(a) 3 (b) 2 (c) 1 (d) 0

Sol:- In General solution

if order 3 $\rightarrow$ arbitrary constants is 3

In particular solution

there is no arbitrary constants $\therefore$ Ans: (d) 0

21) Integrating factor of the differential equation $\frac{dy}{dx} = \frac{x+y+1}{x+1}$ is

(a) $\frac{1}{x+1}$ (b) $x+1$ (c) $\frac{1}{\sqrt{x+1}}$ (d) $\sqrt{x+1}$

Sol:-

$$\frac{dy}{dx} = \frac{x+y+1}{x+1}$$

$$\frac{dy}{dx} = \frac{x+1}{x+1} + \frac{y}{x+1}$$

$$\frac{dy}{dx} - \frac{y}{x+1} = 1$$



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$\boxed{\frac{dy}{dx} + Py = Q}$

$P = \frac{-1}{x+1}$ , $Q = 1$

$IF = e^{\int P dx} = e^{\int \frac{-1}{x+1} dx}$

$= e^{-\log(x+1)}$

$= e^{\log(x+1)^{-1}}$

$= (x+1)^{-1}$

$IF = \frac{1}{x+1}$

∴ Ans: (a) $\frac{1}{x+1}$

22) The population P in any year t is such that the rate of increase in the population is proportional to the population. Then

(a) $P = Ce^{kt}$ (b) $P = Ce^{-kt}$ (c) $P = Ckt$ (d) $P = C$

Sol:- Given $\frac{dP}{dt} \propto P$

$\frac{dP}{dt} = kP$

$\int \frac{dP}{P} = \int k\, dt$

$\log P = kt + \log C$

$\log P - \log C = kt$

$\log\left(\frac{P}{C}\right) = kt$

$\frac{P}{C} = e^{kt}$

$\therefore P = Ce^{kt}$

∴ Ans: (a) $P = Ce^{kt}$

23) P is the amount of certain substance left in after time t. If the rate of evaporation of the substance is proportional to the amount remaining, then

(a) $P = Ce^{kt}$ (b) $P = Ce^{-kt}$ (c) $P = Ckt$ (d) $Pt = C$

Sol:- Given $\frac{dP}{dt} \propto P$

$\frac{dP}{dt} = -kP$

$\int \frac{dP}{P} = -\int k\, dt$

$\log P = -kt + \log C$

$\log P - \log C = -kt$



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$$\log\left(\frac{P}{C}\right) = -kt$$

$$\frac{P}{C} = e^{-kt}$$

$$\therefore P = Ce^{-kt}$$

$\therefore$ Ans : (b) $P = Ce^{-kt}$

24) If the solution of the differential equation $\frac{dy}{dx} = \frac{ax+3}{2y+f}$ represents a circle, then the value of a is

(a) 2 (b) −2 (c) 1 (d) −1

Sol:- $\frac{dy}{dx} = \frac{ax+3}{2y+f}$

$$\int (2y+f)\,dy = \int (ax+3)\,dx$$

$$\frac{2y^2}{2} + fy = \frac{ax^2}{2} + 3x + C$$

$\frac{ax^2}{2} - y^2 + 3x - fy + C = 0$ is equation of circle.

Co-efficient of $x^2$ = Co-efficient of $y^2$

$$\frac{a}{2} = -1$$

$$\therefore a = -2$$

$\therefore$ Ans : (b) −2

25) the slope at any point of a curve $y = f(x)$ is given by $\frac{dy}{dx} = 3x^2$ and it passes through (−1, 1). Then the equation of the curve is

(a) $y = x^3 + 2$ (b) $y = 3x^2 + 4$ (c) $y = 3x^3 + 4$ (d) $y = x^3 + 5$

Sol:-

$$\frac{dy}{dx} = 3x^2$$

$$\int dy = \int 3x^2\,dx$$

$$y = \frac{3x^3}{3} + C$$

$$y = x^3 + C$$

it Passes through (−1, 1)

$$1 = (-1)^3 + C$$

$$1 = -1 + C$$

$$C = 1 + 1 = 2$$

$\therefore y = x^3 + 2$

$\therefore$ Ans : (a) $y = x^3 + 2$

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# EXERCISE 11.6

Choose the correct or the most suitable answer from the given four alternatives:-

1) Let X be random variable with probability density function

$$f(x) = \begin{cases} \frac{2}{x^3} & x \geq 1 \\ 0 & x < 1 \end{cases}$$

Which of the following statement is correct?

(a) both mean and Variance exist

(b) mean exists but Variance does not exist

(c) both mean and Variance do not exist

(d) Variance exists but Mean does not exist.

**Sol:-** Given Probability density function

$$f(x) = \begin{cases} \frac{2}{x^3} & x \geq 1 \\ 0 & x < 1 \end{cases}$$

$$\text{Mean} = E(X) = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$= \int_{-\infty}^{1} x f(x)\,dx + \int_{1}^{\infty} x f(x)\,dx$$

$$= 0 + \int_{1}^{\infty} x \frac{2}{x^3}\,dx$$

$$= 2\int_{1}^{\infty} \frac{1}{x^2}\,dx$$

$$= 2\left[-\frac{1}{x}\right]_1^{\infty}$$

$$= 2\left[\left(-\frac{1}{\infty}\right) - \left(-\frac{1}{1}\right)\right]$$

$$= 2(0+1)$$

$$\therefore E(X) = 2$$ Mean exist

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx$$

$$= \int_{1}^{\infty} x^2 \cdot \frac{2}{x^3}\,dx$$

$$= 2\int_{1}^{\infty} \frac{1}{x}\,dx$$



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$= 2[\log x]_1^\infty$

Variance does not exist

$\therefore$ Ans : (b) Mean exists but Variance doesnot exist

2) A rod of length $2l$ is broken into two pieces at random. The probability density function of the shorter of the two pieces is

$$f(x) = \begin{cases} \frac{1}{l}, & 0 < x < l \\ 0, & l \le x < 2l \end{cases}$$

The mean and variance of the shorter of the two pieces are respectively.

(a) $\frac{l}{2}, \frac{l^2}{3}$ (b) $\frac{l}{2}, \frac{l^2}{6}$ (c) $l, \frac{l^2}{12}$ (d) $\frac{l}{2}, \frac{l^2}{12}$

Sol:-

$$f(x) = \begin{cases} 1/l, & 0 < x < l \\ 0, & l \le x < 2l \end{cases}$$

$$\text{Mean} = E(x) = \int_{-\infty}^{\infty} x f(x)\, dx$$

$$= \int_0^l x f(x) dx + \int_l^{2l} x f(x) dx$$

$$= \int_0^l x \cdot \frac{1}{l}\, dx + 0$$

$$= \frac{1}{l}\left[\frac{x^2}{2}\right]_0^l$$

$$= \frac{1}{l}\left[\frac{l^2}{2} - 0\right]$$

$$= \frac{1}{l}\left(\frac{l^2}{2}\right)$$

$$\therefore \text{Mean} = \frac{l}{2}$$

$$E(x^2) = \int_0^l x^2 f(x)\, dx$$

$$= \int_0^l x^2 \cdot \frac{1}{l}\, dx$$

$$= \frac{1}{l}\left[\frac{x^3}{3}\right]_0^l$$

$$= \frac{1}{l}\left[\frac{l^3}{3} - 0\right]$$



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$$= \frac{1}{l}\left(\frac{l^3}{3}\right)$$

$$= \frac{l^2}{3}$$

$\therefore$ Variance $= E(x^2) - [E(x)]^2$

$$= \frac{l^2}{3} - \left(\frac{l}{2}\right)^2$$

$$= \frac{l^2}{3} - \frac{l^2}{4}$$

$$= \frac{4l^2 - 3l^2}{12}$$

$\therefore$ Variance $= \frac{l^2}{12}$

$\therefore$ Ans:- (d) $\frac{l}{2}, \frac{l^2}{12}$

3) Consider a game where the player tosses a six-sided fair die. If the face that comes up is 6, the player wins ₹ 36, otherwise he loses ₹ $k^2$, where k is the face that comes up $k = \{1,2,3,4,5\}$. The expected amount to win at this game in ₹ is

(a) $\frac{19}{6}$ (b) $-\frac{19}{6}$ (c) $\frac{3}{2}$ (d) $-\frac{3}{2}$

Sol:-

Let X denote the winning amount

$$X = -1, -4, -9, -16, -25, 36$$

probability mass function is

| x | -1 | -4 | -9 | -16 | -25 | 36 |
|---|---|---|---|---|---|---|
| f(x) | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ |

Mean $= E(X) = \sum x f(x)$

$$= -1\left(\frac{1}{6}\right) - 4\left(\frac{1}{6}\right) - 9\left(\frac{1}{6}\right) - 16\left(\frac{1}{6}\right) - 25\left(\frac{1}{6}\right) + 36\left(\frac{1}{6}\right)$$

$$= \frac{-1-4-9-16-25+36}{6}$$

$$= \frac{-19}{6}$$

$\therefore$ Ans: (b) $-\frac{19}{6}$



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4) A pair of dice numbered 1,2,3,4,5,6 of a six-sided die and 1,2,3,4 of a four-sided die is rolled and the sum is determined. Let the random variable X denote this sum. Then the number of elements in the inverse image of 7 is

(a) 1 (b) 2 (c) 3 (d) 4

Sol:-

Sample points to get sum '7' are

$\{(3,4),(4,3),(5,2),(6,1)\}$

∴ Number of elements in inverse image of 7 is 4

∴ Ans: (d) 4

5) A random variable X has binomial distribution with $n=25$ and $p=0.8$ then standard deviation of X is

(a) 6 (b) 4 (c) 3 (d) 2

Sol:-

Given: $n = 25$

$p = 0.8$

$q = 1-p = 1-0.8 = 0.2$

Standard deviation $= \sqrt{npq}$

$= \sqrt{25(0.8)(0.2)}$

$= \sqrt{25(0.16)}$

$= \sqrt{4}$

$= 2$

∴ Ans: (d) 2

6) Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed n times. Then the possible values of X are

(a) $i+2n$, $i=0,1,2,\ldots n$ (b) $2i-n$, $i=0,1,2,\ldots n$

(c) $n-i$, $i=0,1,2,\ldots n$ (d) $2i+2n$, $i=0,1,2,\ldots n$



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**Sol:-**

| Head | Tail | Difference |
|---|---|---|
| $n$ | $0$ | $n-0 = n-0$ (or) $0-n$ |
| $n-1$ | $1$ | $n-1-1 = n-2$ (or) $2-n$ |
| $n-2$ | $2$ | $n-2-2 = n-4$ (or) $4-n$ |

$X = 2i-n$

when $i = 0,2,3,\ldots$

∴ Ans: (b) $2i-n$, $i=0,1,2,\ldots n$

8) Four buses carrying 160 students from the same school arrive at a football stadium. the buses carry, respectively, 42, 36, 34 and 48 students. One of the students is randomly selected. Let X denotes the number of students that were on the bus carrying the randomly selected student. One of the 4 bus drivers is also randomly selected. Let Y denote the number of students on that bus.
then E(X) and E(Y) respectively are

(a) 50, 40 (b) 40, 50 (c) 40.75, 40 (d) 41, 41

**Sol:-**

| | $B_1$ | $B_2$ | $B_3$ | $B_4$ |
|---|---|---|---|---|
| $X$ | 42 | 36 | 34 | 48 |
| $f(x)$ | $\frac{42}{160}$ | $\frac{36}{160}$ | $\frac{34}{160}$ | $\frac{48}{160}$ |

$$E(X) = \sum x f(x)$$

$$= 42\left(\frac{42}{160}\right) + 36\left(\frac{36}{160}\right) + 34\left(\frac{34}{160}\right) + 48\left(\frac{48}{160}\right)$$

$$= \frac{1764 + 1296 + 1156 + 2304}{160}$$

$$= \frac{6520}{160}$$

$$E(X) = 40.75$$

| Drivers ⇒ | | $B_1$ | $B_2$ | $B_3$ | $B_4$ |
|---|---|---|---|---|---|
| | $Y$ | 1 | 1 | 1 | 1 |
| | $f(x)$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ |

$$E(Y) = \sum y f(y)$$

$$= 42\left(\frac{1}{4}\right) + 36\left(\frac{1}{4}\right) + 34\left(\frac{1}{4}\right) + 48\left(\frac{1}{4}\right)$$



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$$= \frac{42 + 36 + 34 + 48}{4}$$

$$= \frac{160}{4}$$

$E(Y) = 40$ ∴ Ans: (c) 40.75, 40

7) If the function $f(x) = \frac{1}{12}$ for $a < x < b$, represents a probability density function of a continuous random variable X, then which of the following cannot be the value of a and b?

(a) 0 and 12 (b) 5 and 17 (c) 7 and 19 (d) 16 and 24

Sol:-

$f(x)$ is a probability density function

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{a}^{b} \frac{1}{12}\,dx = 1$$

$$\frac{1}{12}[x]_a^b = 1$$

$$\frac{1}{12}(b-a) = 1$$

$$b - a = 12$$

(a) 0 and 12 ⇒ $12 - 0 = 12$

(b) 5 and 17 ⇒ $17 - 5 = 12$

(c) 7 and 19 ⇒ $19 - 7 = 12$

(d) 16 and 24 ⇒ $24 - 16 \neq 12$ ∴ Ans: (d) 16 and 24

9) Two coins are to be flipped. The first coin will land on heads with probability 0.6, the second with probability 0.5. Assume that the results of the flips are independent, and let X equal the total number of heads that result. The value of $E(X)$ is

(a) 0.11 (b) 1.1 (c) 11 (d) 1

Sol:-

Given: $P(H_1) = 0.6$ $P(H_2) = 0.5$

$P(T_1) = 0.4$ $P(T_2) = 0.5$



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$X = 0, 1, 2$

$P(X=0) = P(T_1) \cdot P(T_2) = 0.4\,(0.5) = 0.20$

$P(X=1) = P(H_1)\,P(T_2) + P(T_1)\,P(H_2)$

$= 0.6\,(0.5) + 0.4\,(0.5)$

$= 0.30 + 0.20$

$= 0.50$

$P(X=2) = P(H_1)\,P(H_2) = 0.6(0.5) = 0.30$

| $X$ | 0 | 1 | 2 |
|---|---|---|---|
| $f(x)$ | 0.20 | 0.50 | 0.30 |

$E(X) = \sum x\, f(x)$

$= 0(0.20) + 1(0.50) + 2(0.30)$

$= 0 + 0.50 + 0.60$

$= 1.1$

$\therefore$ Ans: (b) 1.1

10) On a multiple-choice exam with 3 possible destructives for each of the 5 questions, the probability that a student will get 4 or more Correct answer just by guessing is

(a) $\frac{11}{243}$ (b) $\frac{3}{8}$ (c) $\frac{1}{243}$ (d) $\frac{5}{243}$

Sol:-

Given: 3 possible answers

$p = \frac{1}{3}$, $n = 5$

$q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$

Let $x$ be the number of correct answers

$P(X=x) = {}^nC_x\, p^x (1-p)^{n-x}$, $x = 0, 1, 2, \ldots, n$

$P(X \geq 4) = P(X=4) + P(X=5)$

$= {}^5C_4 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)^{5-4} + {}^5C_5 \left(\frac{1}{3}\right)^5 \left(\frac{2}{3}\right)^{5-5}$

$= 5\left(\frac{1}{3^4}\right)\left(\frac{2}{3}\right) + (1)\left(\frac{1}{3^5}\right)(1)$

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$$= \frac{10}{3^5} + \frac{1}{3^5}$$

$$= \frac{11}{3^5}$$

$$= \frac{11}{243}$$

$\therefore$ Ans : (a) $\frac{11}{243}$

11) If $P(X=0) = 1 - P(X=1)$. If $E(X) = 3\,Var(X)$ then $P(X=0)$ is

(a) $\frac{2}{3}$ (b) $\frac{2}{5}$ (c) $\frac{1}{5}$ (d) $\frac{1}{3}$

Sol:-

$E(X) = np$ $\quad Var(X) = npq$

Given: $E(X) = 3\,Var(X)$

$np = 3npq$

$1 = 3q$

$\therefore q = \frac{1}{3}$ $\quad \because 1-p = q$

$1 - p = \frac{1}{3}$

$\therefore p = 1 - \frac{1}{3} = \frac{2}{3}$

Given: $P(X=0) = 1 - P(X=1)$

$P(X=0) + P(X=1) = 1$ $\quad \therefore {}^nC_x\, p^x q^{n-x}$

$nC_0\, p^0 q^n + nC_1\, p^1 q^{n-1} = 1$

$q^n + n\,pq^{n-1} = 1$

$\left(\frac{1}{3}\right)^n + n\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^{n-1} = 1$

$\frac{1}{3^n} + \frac{2n}{3}\,\frac{1}{3^{n-1}} = 1$

$\frac{1}{3^n} + \frac{2n}{3^n} = 1$

$\frac{1+2n}{3^n} = 1$

$1 + 2n = 3^n$

If $n=1 \Rightarrow 1 + 2n = 3^n$ $\quad \therefore \boxed{n=1}$

$P(X=0) = {}^1C_0 \left(\frac{2}{3}\right)^0 \left(\frac{1}{3}\right)^{1-0}$

$= (1)(1)\left(\frac{1}{3}\right)$

$= \frac{1}{3}$

$\therefore$ Ans: (b) $\frac{1}{3}$



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12) If X is a binomial random variable with expected value 6 and variance 2.4, then $P(X=5)$ is

(a) $\binom{10}{5}\left(\frac{3}{5}\right)^6\left(\frac{2}{5}\right)^4$ (b) $\binom{10}{5}\left(\frac{3}{5}\right)^{10}$

(c) $\binom{10}{5}\left(\frac{3}{5}\right)^4\left(\frac{2}{5}\right)^6$ (d) $\binom{10}{5}\left(\frac{3}{5}\right)^5\left(\frac{2}{5}\right)^5$

Sol:-

Given: Mean, $np = 6$

Variance $npq = 2.4$

$6q = 2.4$

$q = 0.4$ $(q = 1-p)$

$\therefore p = 1-0.4 = 0.6$

$np = 6$

$n(0.6) = 6$

$\therefore n = 10$

$P(X=x) = {}^nC_x\, p^x (1-p)^{n-x}$, $x = 0,1,2,\ldots,n$

$P(X=5) = {}^{10}C_5\,(0.6)^5\,(0.4)^{10-5}$

$= {}^{10}C_5\left(\frac{6}{10}\right)^5\left(\frac{4}{10}\right)^5$

$= {}^{10}C_5\left(\frac{3}{5}\right)^5\left(\frac{2}{5}\right)^5$

$\therefore$ Ans: (d) $\binom{10}{5}\left(\frac{3}{5}\right)^5\left(\frac{2}{5}\right)^5$

13) The random variable X has the probability density function

$$f(x) = \begin{cases} ax+b & 0<x<1 \\ 0 & \text{otherwise} \end{cases}$$

and $E(X) = \frac{7}{12}$, then a and b are respectively.

(a) 1 and $\frac{1}{2}$ (b) $\frac{1}{2}$ and 1 (c) 2 and 1 (d) 1 and 2

Sol:-

$f(x)$ is a probability density function

$$\int_0^1 (ax+b)\,dx = 1$$

$$\left[\frac{ax^2}{2} + bx\right]_0^1 = 1$$



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$$\frac{a}{2}+b=1 \Rightarrow a+2b=2 \quad \text{——(1)}$$

Given: $E(X)=\frac{7}{12}$

$$\int_0^1 x(ax+b)\,dx=\frac{7}{12}$$

$$\int_0^1 (ax^2+bx)\,dx=\frac{7}{12}$$

$$\left[\frac{ax^3}{3}+\frac{bx^2}{2}\right]_0^1=\frac{7}{12}$$

$$\frac{a}{3}+\frac{b}{2}=\frac{7}{12}$$

$$\frac{2a+3b}{6}=\frac{7}{12_2}$$

$$2a+3b=\frac{7}{2}$$

$$4a+6b=7 \quad \text{——(2)}$$

① $\Rightarrow$ $a+2b=2$

② $\Rightarrow$ $4a+6b=7$

① $\times 3 \Rightarrow 3a+6b=6$

$\hookrightarrow 4a+6b=7$

$-a=-1$

$a=1$

Substitute $a=1$ in (1)

$1+2b=2$

$2b=1$

$b=\frac{1}{2}$

$\therefore$ Ans: (a) 1 and $\frac{1}{2}$

14) Suppose that X takes on one of the values 0, 1 and 2. If for some constant k, $P(X=i)=kP(X=i-1)$ for $i=1,2$ and $P(X=0)=\frac{1}{7}$, then the value of k is

(a) 1 (b) 2 (c) 3 (d) 4

Sol:-

Given: $P(X=i)=kP(X=i-1)$



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Substitute $i=1$

$P(X=1) = kP(X=0)$ $\quad P(X=0)=\frac{1}{7}$

$P(X=1) = k\left(\frac{1}{7}\right)$

$\therefore P(X=1) = \frac{k}{7}$

when $i=2$,

$P(X=2) = kP(X=1)$

$= k\left(\frac{k}{7}\right)$

$\therefore P(X=2) = \frac{k^2}{7}$

| X | 0 | 1 | 2 |
|---|---|---|---|
| P(X) | $\frac{1}{7}$ | $\frac{k}{7}$ | $\frac{k^2}{7}$ |

$\therefore \frac{1}{7} + \frac{k}{7} + \frac{k^2}{7} = 1$

$\frac{k^2+k+1}{7} = 1$

$k^2+k+1 = 7$

$k^2+k+1-7 = 0$

$k^2+k-6 = 0$

$-6$ : $3 \times -2$

$(k+3)(k-2) = 0$

$k+3=0$ $\quad k-2=0$

$k=-3$ $\quad \therefore k=2$

(not possible)

$\therefore$ Ans :- (b) 2

15) Which of the following is a discrete random Variable?

I. The number of cars Crossing a particular Signal in a day

II. The number of customers in a queue to buy train tickets at a moment.

III. The time taken to Complete a telephone call.

(a) I and II (b) II only (c) III only (d) II and III

$\therefore$ Ans : (a) I and II



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16) If $f(x) = \begin{cases} 2x & 0 \le x \le a \\ 0 & \text{otherwise} \end{cases}$ is a probability density function of a random variable, then the value of a is

(a) 1 (b) 2 (c) 3 (d) 4

Sol:- $f(x)$ is a probability density function

$$\int_0^a f(x)\,dx = 1$$

$$\int_0^a 2x\,dx = 1$$

$$2\int_0^a x\,dx = 1$$

$$2\left[\frac{x^2}{2}\right]_0^a = 1$$

$$a^2 - 0 = 1$$

$$a^2 = 1$$

$$a = 1$$

∴ Ans:- (a) 1

17) The probability mass function of a random variable is defined as:

| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
|---|---|---|---|---|---|
| $f(x)$ | $k$ | $2k$ | $3k$ | $4k$ | $5k$ |

Then $E(X)$ is equal to.

(a) $\frac{1}{15}$ (b) $\frac{1}{10}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$

Sol:-

$$\sum f(x) = 1$$

$$k + 2k + 3k + 4k + 5k = 1$$

$$15k = 1$$

$$\therefore k = \frac{1}{15}$$

$$E(X) = \sum x f(x)$$

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$= -2(k) - 1(2k) + 0(3k) + 1(4k) + 2(5k)$

$= -2k - 2k + 0 + 4k + 10k$

$= -4k + 14k$

$= 10k \qquad (\because k = \frac{1}{15})$

$= 10\left(\frac{1}{15}\right)$

$= \frac{2}{3}$

$\therefore$ Ans : (d) $\frac{2}{3}$

18) Let X have a Bernoulli distribution with mean 0.4, then the varience of (2X − 3) is

(a) 0.24 (b) 0.48 (c) 0.6 (d) 0.96

Sol:-

X is a Bernoulli's random variable following with Parameter P, the Mean $\mu$ and variance $\sigma^2$ of Bernoulli's distribution are $\mu = P$ and $\sigma^2 = pq$

Mean $= 0.4$

$\mu = p = 0.4$

$q = 1 - 0.4 = 0.6$

$\sigma^2 = pq$

$= (0.4)(0.6)$

$Var(X) = \sigma^2 = 0.24$

$\therefore Var(2X - 3) = Var(2X) - Var(3)$

$= 2^2 Var(X) - 0$

$= 4(0.24)$

$= 0.96$

$\therefore$ Ans : (d) 0.96

19) If in 6 trials, X is a binomial variable which follows the relation $9P(X=4) = P(X=2)$, then the probability of success is

(a) 0.125 (b) 0.25 (c) 0.375 (d) 0.75



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**Sol:-** trials, $n=6$

$$P(X=x) = {}^nC_x\, p^x q^{n-x}, \quad x=0,1,2,\ldots,n$$

Given: $9P(X=4) = P(X=2)$

$$9\,{}^6C_4\, p^4 q^2 = {}^6C_2\, p^2 q^4 \qquad (\because {}^6C_4 = {}^6C_2)$$

$$9p^2 = q^2$$

Square root on both sides

$$3p = q$$

$$3p = 1-p$$

$$4p = 1$$

$$p = \frac{1}{4}$$

$$p = 0.25$$

$\therefore$ **Ans:** (b) 0.25

20) A Computer Salesperson knows from his past experience that he sells Computers to one in every twenty customers who enter the showroom. What is the probability that he will sell a Computer to exactly two of the next three customers?

(a) $\frac{57}{20^3}$ (b) $\frac{57}{20^2}$ (c) $\frac{19^3}{20^3}$ (d) $\frac{57}{20}$

**Sol:-**

Given: $n=3$,

$$p = \frac{1}{20}$$

$$q = 1-\frac{1}{20} = \frac{19}{20} \qquad (\because q=1-p)$$

$$P(X=x) = {}^nC_x\, p^x (1-p)^{n-x}, \quad x=0,1,2,\ldots,n$$

Exactly two

$$P(X=2) = {}^3C_2 \left(\frac{1}{20}\right)^2 \left(\frac{19}{20}\right)^{3-2}$$

$$= \frac{3\times 2}{1\times 2}\left(\frac{1}{20^2}\right)\left(\frac{19}{20}\right)$$

$$= 3\left(\frac{19}{20^3}\right)$$

$$= \frac{57}{20^3}$$

$\therefore$ **Ans:** (a) $\frac{57}{20^3}$

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# EXERCISE 12.3

Choose the correct or the most suitable answer from the given four alternatives.

1) A binary operation on a set $S$ is a function from

(a) $S \to S$ (b) $(S \times S) \to S$ (c) $S \to (S \times S)$ (d) $(S \times S) \to (S \times S)$

Sol:-

$* : (S \times S) \to S$ ; $*(a,b) = a * b \in S$

∴ Ans:- (b) $(S \times S) \to S$

2) Subtraction is not a binary operation in

(a) R (b) Z (c) N (d) Q

Sol:-

$2, 5 \in N$

but $2 - 5 = -3 \notin N$

∴ Ans: (c) N

3) Which on of the following is a binary operation on N?

(a) Subtraction (b) Multiplication

(c) Division (d) All the above

Sol:- $a, b \in N \Rightarrow a * b \in N$, $\forall a, b \in N$

$2, 3 \in N$

$2 \times 3 = 6 \in N$

∴ Ans: (b) Multiplication

4) In the set R of real numbers '*' is defined as follows. Which one of the following is not a binary operation on R?

(a) $a * b = \min(a, b)$ (b) $a * b = \max(a, b)$

(c) $a * b = a$ (d) $a * b = a^b$

Sol:- $a * b = a^b$ is not binary operation on R.

$-2, \frac{1}{2} \in R$

but $(-2)^{\frac{1}{2}} \notin R$

∴ Ans:- (d) $a * b = a^b$

5) The operation * defined by $a * b = \frac{ab}{7}$ is not a binary operation on

(a) $Q^+$ (b) Z (c) R (d) C

Sol:- $\frac{ab}{7}$ need not be an integer. It's not binary on Z



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ex:

$4, 3 \in Z \Rightarrow 4 * 3 = \frac{4*3}{7}$

$= \frac{12}{7} \notin Z$

∴ Ans: (b) Z

---

6) In the set Q define $a \odot b = a + b + ab$.

For what value of y, $3 \odot (y \odot 5) = 7$?

(a) $y = \frac{2}{3}$ (b) $y = \frac{-2}{3}$ (c) $y = \frac{-3}{2}$ (d) $y = 4$

Sol:-

Given: $3 \odot (y \odot 5) = 7$

$3 \odot (y + 5 + 5y) = 7$

$3 + y + 5 + 5y + 3(y + 5 + 5y) = 7$

$6y + 8 + 3y + 15 + 15y = 7$

$24y + 23 = 7$

$24y = 7 - 23$

$24y = -16$

$y = \frac{-2}{3}$

∴ Ans: (b) $y = \frac{-2}{3}$

---

7) If $a * b = \sqrt{a^2 + b^2}$ on the real numbers then * is

(a) Commutative but not associative

(b) associative but not Commutative

(c) both Commutative and associative

(d) neither Commutative nor associative.

Sol:-

$a * b = \sqrt{a^2 + b^2} = \sqrt{b^2 + a^2} = b * a$

∴ * is Commutative

$(a * b) * c = (\sqrt{a^2 + b^2}) * c = \sqrt{a^2 + b^2 + c^2}$

$a * (b * c) = a * (\sqrt{b^2 + c^2}) = \sqrt{a^2 + b^2 + c^2}$

∴ * is associative

∴ Ans: (c) both Commutative and associative

---

8) Which one of the following statements has the truth value T?

(a) Sin x is an even function

(b) Every square matrix is non-singular



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(c) The product of Complex number and its Conjugate is purely imaginary

(d) $\sqrt{5}$ is an irrational number.

Sol:-

Ans : (d) $\sqrt{5}$ is an irrational number

9) Which one of the following statements has truth value F?

(a) Chennai is in India or $\sqrt{2}$ is an integer

(b) chennai is in India or $\sqrt{2}$ is an irrational number

(c) chennai is in China or $\sqrt{2}$ is an integer

(d) chennai is in China or $\sqrt{2}$ is an irrational number

Sol:-

(a) $T \vee F \to T$ (c) $F \vee F \to F$

(b) $T \vee T \to T$ (d) $F \vee T \to T$

$\therefore$ Ans : (c) chennai is in China or $\sqrt{2}$ is an integer

10) If a compound statement involve 3 simple statements, then the number of rows in the truth table is

(a) 9 (b) 8 (c) 6 (d) 3

Sol:- No. of rows are

$2^3 = 8$

$\therefore$ Ans : (b) 8

11) Which one is the inverse of the statement $(P \vee q) \to (P \wedge q)$?

(a) $(P \wedge q) \to (P \vee q)$ (b) $\neg(P \vee q) \to (P \wedge q)$

(c) $(\neg P \vee \neg q) \to (\neg P \wedge \neg q)$ (d) $(\neg P \wedge \neg q) \to (\neg P \vee \neg q)$

Sol:-

Inverse of $(P \vee q) \to (P \wedge q)$ is

$\neg(P \vee q) \to \neg(P \wedge q)$

$(\neg P \wedge \neg q) \to (\neg P \vee \neg q)$

$\therefore$ Ans : (d) $(\neg P \wedge \neg q) \to (\neg P \vee \neg q)$



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12) Which one is the Contrapositive of the Statement $(p \vee q) \rightarrow r$?

(a) $\neg r \rightarrow (\neg p \wedge \neg q)$ (b) $\neg r \rightarrow (p \vee q)$

(c) $r \rightarrow (p \wedge q)$ (d) $p \rightarrow (q \vee r)$

Sol:-

Contrapositive of $(p \vee q) \rightarrow r$ is

$\neg r \rightarrow \neg (p \vee q)$

$\neg r \rightarrow \neg p \wedge \neg q$ ∴ Ans: (a) $\neg r \rightarrow (\neg p \wedge \neg q)$

13) The truth table for $(p \wedge q) \vee \neg q$ is given below

| p | q | $(p \wedge q) \vee \neg q$ |
|---|---|---|
| T | T | (1) |
| T | F | (2) |
| F | T | (3) |
| F | F | (4) |

Which one of the following is true?

| | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| (a) | T | T | T | T |
| (b) | T | F | T | T |
| (c) | T | T | F | T |
| (d) | T | F | F | F |

Sol:-

| p | q | $p \wedge q$ | $\neg q$ | $(p \wedge q) \vee \neg q$ |
|---|---|---|---|---|
| T | T | T | F | T |
| T | F | F | T | T |
| F | T | F | F | F |
| F | F | F | T | T |

| | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| ∴ Ans: (c) | T | T | F | T |

14) In the last column of the truth table for $\neg(p \vee \neg q)$ the number of final outcomes of the truth value 'F' are

(a) 1 (b) 2 (c) 3 (d) 4

Sol:- Truth table for $\neg(p \vee \neg q)$

| p | q | $\neg q$ | $p \vee \neg q$ | $\neg(p \vee \neg q)$ |
|---|---|---|---|---|
| T | T | F | T | F |
| T | F | T | T | F |
| F | T | F | F | T |
| F | F | T | T | F |



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The last column $\neg(P \vee \neg q)$

The number of final outcomes 'F' are 3 times

$\therefore$ Ans: (c) 3

15) Which one of the following is incorrect? For any two propositions p and q, we have

(a) $\neg(P \vee q) \equiv \neg P \wedge \neg q$ (b) $\neg(P \wedge q) \equiv \neg P \vee \neg q$

(c) $\neg(P \vee q) \equiv \neg P \vee \neg q$ (d) $\neg(\neg P) \equiv P$

Sol:-

Ans:- (c) $\neg(P \vee q) \equiv \neg P \vee \neg q$

16)

| P | q | $(P \wedge q) \rightarrow \neg P$ |
|---|---|---|
| T | T | (1) |
| T | F | (2) |
| F | T | (3) |
| F | F | (4) |

Which one of the following is Correct for the truth Value of $(P \wedge q) \rightarrow \neg P$?

| | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| (a) | T | T | T | T |
| (b) | F | T | T | T |
| (c) | F | F | T | T |
| (d) | T | T | T | F |

Sol:-

| P | q | $P \wedge q$ | $\neg P$ | $P \wedge P \rightarrow \neg P$ |
|---|---|---|---|---|
| T | T | T | F | F |
| T | F | F | F | T |
| F | T | F | T | T |
| F | F | T | T | T |

| | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| Ans:- (b) | F | T | T | T |

17) The dual of $\neg(P \vee q) \vee [P \vee (P \wedge \neg r)]$ is

(a) $\neg(P \wedge q) \wedge [P \vee (P \wedge \neg r)]$ (b) $(P \wedge q) \wedge [P \wedge (P \vee \neg r)]$

(c) $\neg(P \wedge q) \wedge [P \wedge (P \wedge r)]$ (d) $\neg(P \wedge q) \wedge [P \wedge (P \vee \neg r)]$

Sol:- The dual of $\neg(P \vee q) \vee [P \vee (P \wedge \neg r)]$ is

$\neg(P \wedge q) \wedge [P \wedge (P \vee \neg r)]$

$\therefore$ Ans: (d) $\neg(P \wedge q) \wedge [P \wedge (P \vee \neg r)]$

dual
$\vee, \wedge \rightarrow$ Replace to $\wedge, \vee$



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18) The proposition $p \wedge (\neg p \vee q)$ is

(a) a tautology (b) a Contradiction

(c) logically equivalent to $p \wedge q$ (d) logically equivalent to $p \vee q$

Sol:-

The proposition

$$p \wedge (\neg p \vee q) \equiv (p \wedge \neg p) \vee (p \wedge q)$$
$$\equiv F \vee (p \wedge q)$$
$$\equiv p \wedge q$$

∴ Ans:- (c) Logically equivalent to $p \wedge q$

19) Determine the truth value of each of the following statements

(i) $4+2=5$ and $6+3=9$ (ii) $3+2=5$ and $6+1=7$

(iii) $4+5=9$ and $1+2=4$ (iv) $3+2=5$ and $4+7=11$

| | (i) | (ii) | (iii) | (iv) |
|---|---|---|---|---|
| (a) | F | T | F | T |
| (b) | T | F | T | F |
| (c) | T | T | F | F |
| (d) | F | F | T | T |

Sol:-

(i) $F \wedge T = F$ (ii) $T \wedge T = T$

(iii) $T \wedge F = F$ (iv) $T \wedge T = T$

| | (i) | (ii) | (iii) | (iv) |
|---|---|---|---|---|
| ∴ Ans:- (a) | F | T | F | T |

20) Which of the following is not true?

(a) Negation of a negation of a statement is the statement itself.

(b) If the last column of the truth table contains only T then it is a tautology.

(c) If the last column of its truth table contains only F then it is a contradiction

(d) If p and q are any two statements then $p \leftrightarrow q$ is a tautology.

Sol:-

The last Column of $p \leftrightarrow q$ is not all T.

Ans: (d) If p and q are any two statements then $p \leftrightarrow q$ is a tautology.

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