Sri Raghavendra Tuition Center

COMPLEX NUMBER : 2.1 to 2.6

12th Standard

Date : 28-Apr-24

Reg.No. :

Maths

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Time : 01:30:00 Hrs

Total Marks : 50

I. ANSWER ALL QUESTION

5 x 1 = 5

1) (a) 1+ i

2) (a) 0

3) (a) z

4) (a) $\frac{1}{2}|z|^2$

5) (c) x^2+y^2

II. ANSWER ANY 5 QUESTION

7 x 2 = 14

6) $\overline{(5+9i)+(2-4i)}$

= $\overline{(5+2)+(9i-4i)} = \overline{7+5i}$

= 7 - 5i [∵ Conjugate of 7 + 5i is 7 - 5i]

7) $i^{1+50} + i^{2+50} + \ldots + i^{10+50}$

= $i^{51} + i^{52} + \ldots + i^{60}$

Taking i^{50} common we get,

i^{50} [i + i^2+ i^3+ i^4) + (i^5+ i^6+ i^7+ i^8) + i^9+ i^{10}]

= i^{50}[0 + (i^{4+1}+ i^{4+2}+ i^{4+3}+ i^{4+4}) + (i^{8+1} + i^{8+2})]

= i^{50} [0 + 0 + i + i^2] [∵ i + i^2+ i^3+ i^4 = 0]

= i^{50} [i-1] = i^{48+2}(i-1)

= i^2(i-1) [∴ i^{48} = 1]

= -1(i-1) = -i+1 = 1-i

8) We compute $|6-8i| = \sqrt{6^2+(-8)^2} = 10$

and applying the formula for square root, we get

$\sqrt{6-8i} = \pm\left(\sqrt{\frac{10+6}{2}} - i\sqrt{\frac{10-6}{2}}\right)$ (∴ b is negative $\frac{b}{|b|} = -1$)

= $\pm\left(\sqrt{8}+i\sqrt{2}\right)$

= $\pm\left(2\sqrt{2}-i\sqrt{2}\right)$

9) 2i(3−4i)(4−3i)

Let z = 2i(3−4i)(4−3i).

∴ |z| = |2i(3-4i)(4-3i)|

= |2i| |3-4i| |4-3i|

= $\sqrt{2^2}\sqrt{3^2+(-4)^2}\sqrt{4^2+(-3)^2}$

= $2.\sqrt{9+16}\sqrt{16+9} = 2.\sqrt{25}.\sqrt{25}$

= 2(5)(5) = 50

10) $Re(z) = \frac{z+\bar{z}}{2}$ and $\text{Im}(z) = \frac{z-\bar{z}}{2i}$

Let z = x+iy where x is the Re(x) and y is the Im (z)

Then $\bar{z}$ = x-iy

z+$\bar{z}$ = x + iy + x - iy = 2x

∴ $\frac{z+\bar{x}}{2}$ = x

$\frac{z+\bar{x}}{2}$ = Re(z)

Also z-$\bar{z}$ = x+iy-(x-iy)

= x+iy-x+iy = 2iy

$\frac{z-\bar{z}}{2i}$ = y

∴ $\frac{z-\bar{z}}{2i}$ = Im(z)

Hence proved

11) |z+i| = |z-1|

⇒ |x + iy +i| = |x + iy-1|

⇒ |x + i(y + 1)| = |(x - 1) + iy|

$\Rightarrow \sqrt{x^2+(y+1)^2} = \sqrt{(x-1)^2+y^2}$

⇒ x^2 + $(y + 1)^2$ =$(x- 1)^2$ + y^2

[squaring both sides]

$\Rightarrow \cancel{x}^2 + \cancel{y}^2 + 2y + \cancel{1} = \cancel{x}^2 - 2x + \cancel{1} + \cancel{y}^2$

⇒ 2y + 2x = 0

⇒ x + y = 0

Hence, the Cartesian equation is x + y = 0

12) $i^{25} = (i^4)^6 \times i^1 = i^6 \times i = i$

∴ $|i^{25}| = |i| = 1$

III. ANSWER ANY 5 QUESTION

7 x 3 = 21

13) We have z = (2+3i)(1−i) = (2+3)+(3−2)i = 5+i

$\Rightarrow z^{-1} = \frac{1}{z} = \frac{1}{5+i}$

Multiplying the numerator and denominator by the conjugate of the denominator, we get

$z^{-1} = \frac{(5-i)}{(5+i)(5-i)} = \frac{5-i}{5^2+I^2} = \frac{5}{26} - i\frac{1}{26}$

$\Rightarrow z^{-1} = \frac{5}{26} - i\frac{1}{26}$

14) We consider $\frac{1+i}{1-i} = \frac{(1+i)(1+i)}{(1-i)(1+i)} = \frac{1+2i}{1+1} = \frac{2i}{2} = i$

and $\frac{1-i}{1+i} = \left(\frac{1+i}{1-i}\right)^{-1} = \frac{1}{i} = -i$

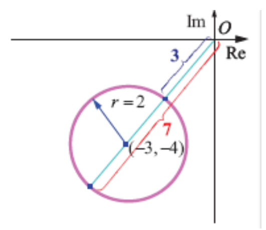
Therefore, $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1-i}\right)^2$ = i^3-$(-i)^3$ = - i - i = -2i

15) Using the given value for z_1 and z_2 the value of $\frac{z_1}{z_2} = \frac{3-2i}{6+4} = \frac{3-2i}{6+4i} \times \frac{6-4i}{6-4i}$

= $\frac{(18-8)+i(12-12)}{6^2+4^2} = \frac{10-24i}{52} = \frac{10}{52} = \frac{24i}{52}$

= $\frac{5}{26} - \frac{6}{13}i$

16)



$|z+3+4i| \leq |z| + |3+4i| = 2+5 = 7$

$|z+3+4i| \leq 7$ .... (1)

$|z+3+4i| \geq ||z| - |3+4i|| = |2-5| = 3$

$|z+3+4i| \geq 3$ .... (2)

From (1) and (2) we get, $3 \leq |z+3+4i| \leq 7$

17) Given v = 3-4i, w = 4+3i and $\frac{1}{u} = \frac{1}{v} + \frac{1}{w}$

∴ $\frac{1}{u} = \frac{1}{3-4i} + \frac{1}{4+3i}$

= $\frac{3+4i}{(3-4i)(3+4i)} + \frac{4-3i}{(4+3i)(4-3i)}$

= $\frac{3+4i}{9-(4i)^2} + \frac{4-3i}{16-(3i)^2} = \frac{3+4i}{9+16} + \frac{4-3i}{16+9}$

= $\frac{3+4i}{25} + \frac{4-3i}{25} = \frac{3+4i+4-3i}{25}$

$\frac{1}{u} = \frac{7+i}{25}$

∴ u = $\frac{25}{7+i} \times \frac{7-i}{7-i} = \frac{25(7-i}{7^2-(i^2)}$

= $\frac{25(7-i)}{49+1} = \frac{25(7-i)}{50} = \frac{1}{2}$(7-i)

∴ u = $\frac{1}{2}$(7-i) or $\frac{7}{2}$ - $\frac{i}{2}$

18) Let the points be A (10 - 8i), B (11 + 6i) and C(1-i)

Distance between A and C is |(10-8i)-(1+i)|

= |10-8i-1-i| = |9-9i|

= $\sqrt{9^2+(-9)^2} = \sqrt{81+81} = \sqrt{2\times 81} = \sqrt{162}$

= $9\sqrt{2}$

Distance between Band C is s |(11+6i)-(1+i)|

= |11 + 6i-1-i| = |10 + 5i|

$=\sqrt{10^2+5^2}=\sqrt{100+25}=\sqrt{125}$

$=\sqrt{25\times 5}=5\sqrt{5}$

Since $5\sqrt{5}<9\sqrt{2}$, B is closest to C.

∴ 11 + 6i is closet to 1 + i.

19) $|2z+2-4i|=2$

2|z+1-2i| = 2

⇒ |z-(-1+2i)| = 1

It is of the form $|z - z_0|$ = r and so it represents a circle.

Its centre is (-1+2i) and radius is 1.

IV. ANSWER ALL QUESTION

4 x 5 = 25

20) Given (3 -i) x - (2 - i) y + 2i + 5

= 2x + (-1 + 2i) y + 3 + 2i

⇒ 3x - ix - 2y + iy + 2i + 5 = 2x - y + 2iy + 3 + 2i

choosing the real and imaginary parts

(3x-2y + 5) + i (-x + y + 2) = 2x - y + 3 + i (2y+ 2)

Equating the real and imaginary parts both sides, we get

3x- 2y+ 5 = 2x-y+3

⇒ 3x - 2y + 5 - 2x +y - 3 = 0

⇒ x-y = -2... (1)

-x+y+2 = 2y+2

⇒ -x+y+2-2y-2 = 0

⇒ -x-y = 0 ⇒ x+y = 0.. (2)

(1)-(2) we get,

x - y = -2

x + y = 0

2y = -2

y = 1

Substituting y = 1 in (2) we get.

x+1 = 0 ⇒ x = -1

∴ x = -1 and y = 1

21) Given $|z_1|$ = 1, $|z_2|$= 2, $|z_3|$ = 3, $|z_1 + z_2 + z_3|$ = 1

$|z_1|^2 = 1^2 \Rightarrow z_1\,\overline{z_1} = 1 \Rightarrow z_1 = \frac{1}{z_1}$

$|z_2|^2 = 4 \Rightarrow z_2\,\overline{z_2} = 1 \Rightarrow z_2 = \frac{4}{z_2}$

$|z_3|^2 = 9 \Rightarrow z_3\,\overline{z_3} = 1 \Rightarrow z_3 = \frac{9}{z_3}$

$\therefore \left|9, \frac{1}{\overline{z_1}}.\frac{4}{\overline{z_2}} + 4.\frac{1}{\overline{z_1}}.\frac{9}{\overline{z_3}} + \frac{4}{\overline{z_2}}.\frac{9}{\overline{z_3}}\right|$

$\left|\frac{36}{\overline{z_1 z_2}} + \frac{36}{\overline{z_1 z_3}} + \frac{36}{\overline{z_2 z_3}}\right| = \left|36\left(\frac{\overline{z_3}+\overline{z_2}+\overline{z_1}}{\overline{z_1 z_2 z_3}}\right)\right|$

$\left[\because |\overline{z_1}+\overline{z_2}+\overline{z_3}| = |\overline{z_1+z_2+z_3}|\right]$

$= \frac{36|\overline{z_1+z_2+z_3}|}{|\overline{z_1}||\overline{z_2}||\overline{z_3}|} = 36\frac{\overline{|\overline{z_1}+\overline{z_2}+\overline{z_3}|}}{|\overline{z_1}||\overline{z_2}||\overline{z_3}|}$

$\left[\because |\overline{z_1}| = |z_1|, |\overline{z_2} = |z_{21}|, |\overline{z_3}| = |\overline{z_3}|\right]$

$= \frac{36(1)}{1(2)(3)} = \frac{36}{6} = 6$

∴ $|9z_1 + z_2 + 4z_1z_3 + z_2z_3|$ = 6

22) cosider $|z_1+z_2|^2 = (z_1+z_2)\left(\overline{z_1+z_2}\right)$ (since $z\bar{z} = |z|^2$

$= (z_1+z_2)(\overline{z_1}+\overline{z_2})$

$= (z_1\overline{z_1} + z_1\overline{z_2} + z_2\overline{z_1} + z_2\overline{z_2})$

$= |z_1|^2 + z_1\overline{z_2} + \overline{z_1\overline{z_2}}|z_2|^2$

$= |z_1|^2 + |z_2|^2 + 2\cdot \mathrm{Re}(z_1\overline{z_2})$ since $z_1 + z_2 = 2\,\mathrm{Re}(z)$

$\le |z_1|^2 + |z_2|^2 + 2\,|z_1\overline{z_2}|$

$\le |z_1|^2 + |z_2|^2 + 2\,|z_1|\,|z_2|$

$\le (|z_1+z_2|)^2$

Taking positive square root both sides

$|z_1 + z_2| \leq |z_1| + |z_2|$

23) a) $\overline{(2-i)^{12} + (2+i)^{12}} = \overline{(2-i)^{12}} + \overline{(2+i)^{12}}$

$\left(2+i\sqrt{3}\right)^{10} - \left(2-i\sqrt{3}\right)^{10}$

Now $\bar{z}$ = $\overline{(2+\sqrt{3})^{10} - (2-i\sqrt{3})^{10}}$

$\bar{z}$ = $\overline{(2+i\sqrt{3})^{10}} - (2-i\sqrt{3})^{10}$

$[\because \overline{z_1 - z_2} = \overline{z_1} - \overline{z_2}]$

= $(2-i\sqrt{3})^{10} - (2+i\sqrt{3})^{10}$

= -$\left[(2-i\sqrt{3})^{10} - (2+i\sqrt{3})^{10}\right]$

$\therefore$ $\bar{z}$ = - z ⇒ z is purely imaginary

Hence $(2-i\sqrt{3})^{10} - (2+i\sqrt{3})^{10}$ is purely imaginary

(OR)

b) Let z = $\left(\frac{19+9i}{5-3i}\right)^{15} - \left(\frac{8+i}{I+2i}\right)^{15}$

Here, $\frac{19+9i}{5-3i} = \frac{(19+9i)(5+3i)}{(5-3i)(5+3i)}$

= $\frac{(95-27)+i(45+57)}{5^2+3^2} = \frac{68+102i}{34}$

= 2 + 3i ....(1)

and $\frac{8+i}{1+2i} = \frac{(8+i)(1-2i)}{(1+2i)(1-2i)}$

= $\frac{(8+2)+i(1-16)}{1^2+2^2} = \frac{10-15i}{5}$

= 2 - 3i .... (2)

Now z = $\left(\frac{19+9i}{5-3i}\right)^{15} - \left(\frac{8+i}{1+2i}\right)^{15}$

⇒ z = $(2 + 3i)^{15} - (2 - 3i)^{15}$ (by (1) and (2))

Then by definition, $\bar{z} = \left(\overline{(2+3i)^{15} - (2-3i)^{15}}\right)$

= $\left(\overline{2+3i}\right)^{15} - \left(\overline{2-3i}\right)^{15}$ (using properties of conjugates)

= $(2 - 3i)^{15} - (2 + 3i)^{15} = -((2 + 3i)^{15} - (2-3i)^{15})$

⇒ $\bar{z}$ = -z

Therefore, $\left(\frac{19+9i}{5-3i}\right)^{15} - \left(\frac{8+i}{1+2i}\right)^{15}$ is purely imaginary.