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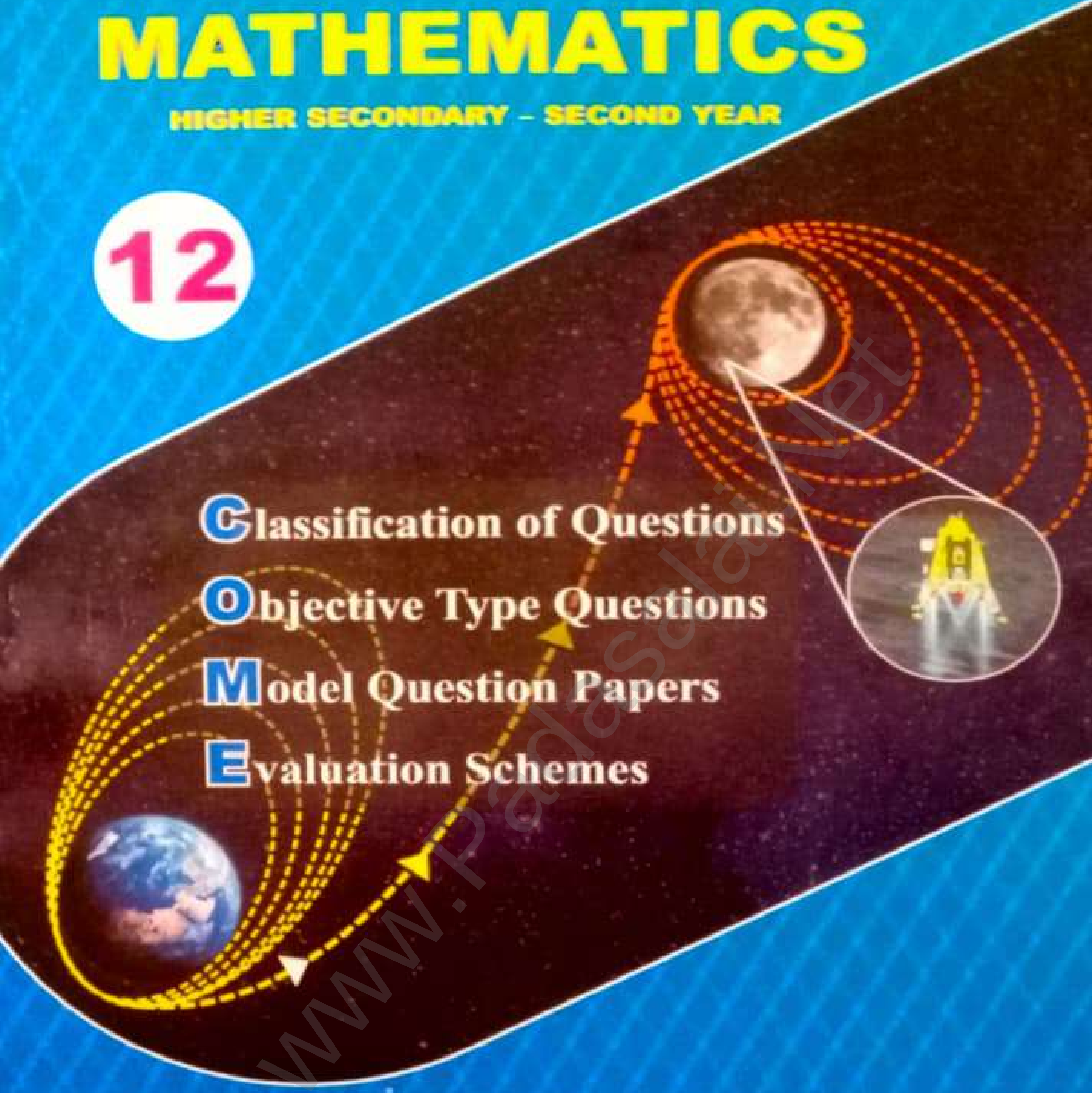

Classification of Questions
Objective Type Questions
Model Question Papers
Evaluation Schemes

This book has been brought out with the Guidance of

THE DIRECTORATE OF SCHOOL EDUCATION

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Chennai 114

This book has been printed on 60 GSM paper
Through Tamil Nadu Text Book and Educational Services Corporatio
Printed by Web offset at
Chandra Publishers Chennai - 600 095.

Price ₹ 160.00

The name of the book COME is created by Dr. K. SRINIVASAN
for
Parent Teacher Association

PREFACE

The members of the preparation team feel honoured in bringing out 'COME' Book for Higher Secondary Second Year Mathematics Text Book, Tamilnadu, and is a new venture in the annals of the Directorate of School Education, Government of Tamilnadu. All the questions included in the Text Book are classified into 2 marks, 3 marks and 5 marks and the question collection includes objective type questions with four possible alternatives. The book deals with six model question papers, their complete solutions, and the method of evaluation for clarity and objectivity. As a confidence building measure, public examination question papers are also appended at the end.

We earnestly acknowledge that the experience gained from the preparation of 2005 edition and its revised editions of 2012 COME Book of then prescribed Text Book, the constructive criticism and encouragement received from students, teachers, domain experts and, parents and the public kindled our interest in the making of the current COME Book to be published in a specialised and thoughtful manner.

I, on behalf of the preparation team, express our heartfelt gratitude to the administrators of the Department of School Education for aiding us to bring out this book in a speedy manner. Our special thanks to the Honourable Minister, Department of School Education, Principal Secretary, Department of School Education, Director of School Education and Secretary, Tamilnadu Parent Teacher Association. My thanks are also due to every member of the preparation team, and to Mr. S. Manoharan, who made our work appear in a book form.

(K. SRINIVASAN)

CONTENTS

CLASSIFICATION OF QUESTIONS

Part – II

(2 Mark Questions)

EXERCISE 1.1

1. Find the adjoint of the following:

(i) $\begin{bmatrix} -3 & 4 \\ 6 & 2 \end{bmatrix}$

2. Find the inverse (if it exists) of the following:

(i) $\begin{bmatrix} -2 & 4 \\ 1 & -3 \end{bmatrix}$

9. If $\operatorname{adj}(A) = \begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$, find A^{-1}.

EXERCISE 1.2

1. Find the rank of the following matrices by minor method: (each 2 marks)

(i) $\begin{bmatrix} 2 & -4 \\ -1 & 2 \end{bmatrix}$ (ii) $\begin{bmatrix} -1 & 3 \\ 4 & -7 \\ 3 & -4 \end{bmatrix}$ (iii) $\begin{bmatrix} 1 & -2 & -1 & 0 \\ 3 & -6 & -3 & 1 \end{bmatrix}$

EXERCISE 1.7

3. By using Gaussian elimination method, balance the chemical reaction equations:

$C_2H_6 + O_2 \rightarrow H_2O + CO_2$

Example 1.4

If A is a nonsingular matrix of odd order, prove that $|\operatorname{adj} A|$ is positive.

Example 1.7

If A is symmetric, prove that then $\operatorname{adj} A$ is also symmetric.

Example 1.11

Prove that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal.

Example 1.16

Find the rank of the following matrices which are in row-echelon form : (each 2 marks)

(i) $\begin{bmatrix} 2 & 0 & -7 \\ 0 & 3 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ (ii) $\begin{bmatrix} -2 & 2 & -1 \\ 0 & 5 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ (iii) $\begin{bmatrix} 6 & 0 & -9 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Example 1.39

By using Gaussian elimination method, balance the chemical reaction equation:

$C_3H_8 + O_2 \rightarrow CO_2 + H_2O$.

Chapter 2
Complex Numbers

EXERCISE 2.1

Simplify the following: (each 2 marks)

(1) $i^{1947} + i^{1950}$ (2) $i^{1948} - i^{-1869}$ (3) $\sum_{n=1}^{12} i^n$

(4) $i^{59} + \frac{1}{i^{59}}$ (5) $i.i^2.i^3 \ldots i^{2000}$ (6) $\sum_{n=1}^{10} i^{n+50}$

EXERCISE 2.2

1. Evaluate the following if $z = 5-2i$ and $w = -1+3i$ (each 2 marks)

(i) $z+w$ (ii) $z-iw$ (iii) $2z+3w$

(iv) zw (v) $z^2+2zw+w^2$ (vi) $(z+w)^2$

EXERCISE 2.4

1. Write the following in the rectangular form: (each 2 marks)

(i) $\overline{(5+9i)+(2-4i)}$ (ii) $\frac{10-5i}{6+2i}$ (iii) $\overline{3i} + \frac{1}{2-i}$

2. If $z = x+iy$, find the following in cartesian form. (each 2 marks)

(i) $\text{Re}\left(\frac{1}{z}\right)$ (ii) $\text{Re}(i\bar{z})$ (iii) $\text{Im}(3z+4\bar{z}-4i)$

3. If $z_1 = 2-i$ and $z_2 = -4+3i$, find the inverses of z_1z_2 and $\frac{z_1}{z_2}$. (each 2 marks)

5. Prove the following properties: (each 2 marks)

(i) z is real if and only if $z = \bar{z}$ (ii) $\text{Re}(z) = \frac{z+\bar{z}}{2}$ and $\text{Im}(z) = \frac{z-\bar{z}}{2i}$

7. Show that (i) $\left(2+i\sqrt{3}\right)^{10}-\left(2-i\sqrt{3}\right)^{10}$ is purely imaginary

EXERCISE 2.5

1. Find the modulus of the following complex numbers (each 2 marks)

(i) $\frac{2i}{3+4i}$ (ii) $\frac{2-i}{1+i}+\frac{1-2i}{1-i}$ (iii) $(1-i)^{10}$ (iv) $2i(3-4i)(4-3i)$

10. Find the square roots of (i) $4+3i$ (ii) $-6+8i$ (iii) $-5-12i$. (each 2 marks)

EXERCISE 2.6

4. Show that the following equations represent a circle, and find its centre and radius. (each 2 marks)

(i) $|z-2-i|=3$ (ii) $|2z+2-4i|=2$ (iii) $|3z-6+12i|=8$

EXERCISE 2.8

9. If $z=2-2i$, find the rotation of z by θ radians in the counter clockwise direction about the origin when (each 2 marks)

(i) $\theta=\frac{\pi}{3}$ (ii) $\theta=\frac{2\pi}{3}$ (iii) $\theta=\frac{3\pi}{2}$

Example 2.1

Simplify the following. (each 2 marks)

(i) i^7 (ii) i^{1729} (iii) $i^{-1924}+i^{2018}$ (iv) $\sum_{n=1}^{102} i^n$ (vi) $i\, i^2\, i^3 \cdots i^{40}$

Example 2.3

Write $\frac{3+4i}{5-12i}$ in the $x+iy$ form, hence find the real and imaginary parts.

Example 2.5

If $\frac{z+3}{z-5i}=\frac{1+4i}{2}$, find the complex number z in the rectangular form.

Example 2.6

If $z_1=3-2i$ and $z_2=6+4i$ find $\frac{z_1}{z_2}$ in the rectangular form.

Example 2.7

Find z^{-1}, if $z=(2+3i)(1-i)$.

Example 2.8

Show that (i) $\left(2+i\sqrt{3}\right)^{10}+\left(2-i\sqrt{3}\right)^{10}$ is real .

Example 2.9 (each 2 marks)

If $z_1=3+4i,\ z_2=5-12i,\ z_3=6+8i$, find $|z_1|, |z_2|, |z_3|, |z_1+z_2|, |z_2-z_3|$, and $|z_1+z_3|$.

Example 2.10 (each 2 marks)

Find the following (i) $\left|\frac{2+i}{-1+2i}\right|$ (ii) $\left|\overline{(1+i)}(2+3i)(4i-3)\right|$ (iii) $\left|\frac{i(2+i)^3}{(1+i)^2}\right|$.

Example 2.17

Find the square root of $6-8i$.

Example 2.22 (each 2 marks)

Find the modulus and principal argument of the following complex numbers.

(i) $\sqrt{3}+i$ (ii) $-\sqrt{3}+i$ (iii) $-\sqrt{3}-i$ (iv) $\sqrt{3}-i$

Example 2.24

Find the principal argument $Arg\, z$, when $z=\frac{-2}{1+i\sqrt{3}}$.

Example 2.25

Find the product $\frac{3}{2}\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right).6\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)$ in rectangular from.

Example 3.1

If α and β are the roots of the quadratic equation $17x^2+43x-73=0$, construct a quadratic equation whose roots are $\alpha+2$ and $\beta+2$.

Example 3.2

If α and β are the roots of the quadratic equation $2x^2-7x+13=0$, construct a quadratic equation whose roots are α^2 and β^2.

Example 3.3

If α, β, γ are the roots of the equation $x^3+px^2+qx+r=0$, find the value of $\sum\frac{1}{\beta\gamma}$ in terms of the coefficients.

Example 3.11

Show that the equation $2x^2-6x+7=0$ cannot be satisfied by any real values of x.

Example 3.12

If $x^2+2(k+2)x+9k=0$ has equal roots, find k.

EXERCISE 4.1

2. Find the period and amplitude of

   (i) $y=\sin 7x$ (ii) $y=-\sin\left(\frac{1}{3}x\right)$ (iii) $y=4\sin(-2x)$. (each 2 marks)

4. Find the values of (i) $\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right)$ (ii) $\sin^{-1}\left(\sin\left(\frac{5\pi}{4}\right)\right)$. (each 2 marks)

5. For what value of x does $\sin x=\sin^{-1}x$?

EXERCISE 4.2

2. State the reason for $\cos^{-1}\left[\cos\left(-\frac{\pi}{6}\right)\right] \neq -\frac{\pi}{6}$.

3. Is $\cos^{-1}(-x) = \pi - \cos^{-1}(x)$ true? Justify your answer.

4. Find the principal value of $\cos^{-1}\left(\frac{1}{2}\right)$.

8. Find the value of

   (i) $\cos\left(\cos^{-1}\left(\frac{4}{5}\right) + \sin^{-1}\left(\frac{4}{5}\right)\right)$.

EXERCISE 4.3

1. Find the domain of the following functions : (each 2 marks)

   (i) $\tan^{-1}\left(\sqrt{9-x^2}\right)$ (ii) $\frac{1}{2}\tan^{-1}(1-x^2) - \frac{\pi}{4}$.

2. Find the value of (i) $\tan^{-1}\left(\tan\frac{5\pi}{4}\right)$ (ii) $\tan^{-1}\left(\tan\left(-\frac{\pi}{6}\right)\right)$. (each 2 marks)

3. Find the value of (i) $\tan\left(\tan^{-1}\left(\frac{7\pi}{4}\right)\right)$ (ii) $\tan\left(\tan^{-1}(1947)\right)$

   (iii) $\tan\left(\tan^{-1}(-0.2021)\right)$ (each 2 marks)

EXERCISE 4.4

1. Find the principal value of

   (i) $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (ii) $\cot^{-1}\left(\sqrt{3}\right)$ (iii) $\operatorname{cosec}^{-1}\left(-\sqrt{2}\right)$ (each 2 marks)

2. Find the value of

   (i) $\tan^{-1}\left(\sqrt{3}\right) - \sec^{-1}(-2)$ (ii) $\sin^{-1}(-1) + \cos^{-1}\left(\frac{1}{2}\right) + \cot^{-1}(2)$ (each 2 marks)

EXERCISE 4.5

1. Find the value, if it exits. If not, give the reason for non-existence. (each 2 marks)

   (i) $\sin^{-1}(\cos\pi)$ (ii) $\tan^{-1}\left(\sin\left(-\frac{5\pi}{2}\right)\right)$ (iii) $\sin^{-1}[\sin 5]$.

Example 4.1

Find the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$ (in radians and degrees).

Example 4.2

Find the principal value of $\sin^{-1}(2)$, if it exists.

Example 4.3

Find the principal value of

(i) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (ii) $\sin^{-1}\left(\sin\left(-\frac{\pi}{3}\right)\right)$ (iii) $\sin^{-1}\left(\sin\left(\frac{5\pi}{6}\right)\right)$. (each 2 marks)

Example 4.5

Find the principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$.

Example 4.6

Find (i) $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ (ii) $\cos^{-1}\left(\cos\left(-\frac{\pi}{3}\right)\right)$ (iii) $\cos^{-1}\left(\cos\left(\frac{7\pi}{6}\right)\right)$. (each 2 marks)

Example 4.8

Find the principal value of $\tan^{-1}\left(\sqrt{3}\right)$.

Example 4.9

Find (i) $\tan^{-1}\left(-\sqrt{3}\right)$ (ii) $\tan^{-1}\left(\tan\frac{3\pi}{5}\right)$ (iii) $\tan\left(\tan^{-1}(2019)\right)$. (each 2 marks)

Example 4.12

Find the principal value of

(i) $\operatorname{cosec}^{-1}(-1)$ (ii) $\sec^{-1}(-2)$. (each 2 marks)

Example 4.13

Find the value of $\sec^{-1}\left(-\frac{2\sqrt{3}}{3}\right)$.

Example 4.18

Find the value of (i) $\sin\left[\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right]$ (ii) $\cos\left[\frac{1}{2}\cos^{-1}\left(\frac{1}{8}\right)\right]$ (each 2 marks)

EXERCISE 5.1

5. Obtain the equation of the circle for which $(3,4)$ and $(2,-7)$ are the ends of a diameter.

10. Determine whether the points $(-2,1),(0,0)$, and $(-4,-3)$ lie outside, on or inside the circle $x^2+y^2-5x+2y-5=0$. (each 2 marks)

11. Find the centre and radius of the following circles: (each 2 marks)

(i) $x^2+(y+2)^2=0$ (ii) $x^2+y^2+6x-4y+4=0$

(iii) $x^2+y^2-x+2y-3=0$ (iv) $2x^2+2y^2-6x+4y+2=0$

EXERCISE 5.3

Identify the type of conic sections. (each 2 marks)

(1) $2x^2-y^2=7$ (2) $3x^2+3y^2-4x+3y+10=0$ (3) $3x^2+2y^2=14$

(4) $x^2+y^2+x-y=0$ (5) $11x^2-25y^2-44x+50y-256=0$ (6) $y^2+4x+3y+4=0$

Example 5.1

Find the general equation of a circle with centre $(-3,-4)$ and radius 3 units.

Example 5.3

Determine whether $x+y-1=0$ is the equation of a diameter of the circle $x^2+y^2-6x+4y+c=0$ for all possible values of c.

Example 5.4

Find the general equation of the circle whose diameter is the line segment joining the points $(-4,-2)$ and $(1,1)$.

Example 5.5

Examine the position of the point $(2,3)$ with respect to the circle $x^2+y^2-6x-8y+12=0$.

Example 5.25

Find the vertices, foci for the hyperbola $9x^2-16y^2=144$.

Example 5.27

The orbit of Halley's Comet is an ellipse 36.18 astronomical units long and by 9.12 astronomical units wide. Find its eccentricity.

Example 5.28

Identify the type of the conic for the following equations: (each 2 marks)

(i) $16y^2=-4x^2+64$ (ii) $x^2+y^2=-4x-y+4$

(iii) $x^2-2y=x+3$ (iv) $4x^2-9y^2-16x+18y-29=0$

EXERCISE 6.2

1. If $a=\hat{i}-2\hat{j}+3\hat{k}, \vec{b}=2\hat{i}+\hat{j}-2\hat{k}, \vec{c}=3\hat{i}+2\hat{j}+\hat{k}$, find $\vec{a}\cdot(\vec{b}\times\vec{c})$.
2. Find the volume of the parallelopiped whose coterminous edges are represented by the vectors $-6\hat{i}+14\hat{j}+10\hat{k}, 14\hat{i}-10\hat{j}-6\hat{k}$ and $2\hat{i}+4\hat{j}-2\hat{k}$.
3. The volume of the parallelopiped whose coterminus edges are $7\hat{i}+\lambda\hat{j}-3\hat{k}, \hat{i}+2\hat{j}-\hat{k}, -3\hat{i}+7\hat{j}+5\hat{k}$ is 90 cubic units. Find the value of λ.
6. Determine whether the three vectors $2\hat{i}+3\hat{j}+\hat{k}, \hat{i}-2\hat{j}+2\hat{k}$ and $3\hat{i}+\hat{j}+3\hat{k}$ are coplanar.
7. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=\vec{i}$ and $\vec{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$. If $c_1=1$ and $c_2=2$, find c_3 such that $\vec{a},\vec{b}$ and $\vec{c}$ are coplanar.
8. If $\vec{a}=\hat{i}-\hat{k}, \vec{b}=x\hat{i}+\hat{j}+(1-x)\hat{k}, \vec{c}=y\hat{i}+x\hat{j}+(1+x-y)\hat{k}$, show that $\left[\vec{a}, \hat{b}, \vec{c}\right]$ independent of x and y.
9. If the vectors $a\hat{i}+a\hat{j}+c\hat{k}, \hat{i}+\hat{k}$ and $c\hat{i}+c\hat{j}+b\hat{k}$ are coplanar, prove that c is the geometric mean of a and b.

EXERCISE 6.3

1. If $\vec{a}=\hat{i}-2\hat{j}+3\hat{k}$, $\vec{b}=2\hat{i}+\hat{j}-2\hat{k}$, $\vec{c}=3\hat{i}+2\hat{j}+\hat{k}$, find (i) $\left(\vec{a}\times\vec{b}\right)\times\vec{c}$ (ii) $\vec{a}\times\left(\vec{b}\times\vec{c}\right)$, (each 2 marks)

2. For any vector $\vec{a}$, prove that $\hat{i}\times\left(\hat{a}\times\hat{i}\right)+\hat{j}\times\left(\vec{a}\times\hat{j}\right)+\hat{k}\times\left(\vec{a}\times\hat{k}\right)=2\vec{a}$.

EXERCISE 6.4

5. Find the angle between the following lines: (each 2 marks)

   (i) $\vec{r}=\left(4\hat{i}-\hat{j}\right)+t\left(\hat{i}+2\hat{j}-2\hat{k}\right),\vec{r}=\left(\hat{i}-2\hat{j}+4\hat{k}\right)+s\left(-\hat{i}-2\hat{j}+2\hat{k}\right)$

   (ii) $\frac{x+4}{3}=\frac{y-7}{4}=\frac{z+5}{5}$ and $\vec{r}=4\hat{k}+t\left(2\hat{i}+\hat{j}+\hat{k}\right)$.

   (iii) $2x=3y=-z$ and $6x=-y=-4z$.

EXERCISE 6.6

5. Find the intercepts cut off by the plane $\vec{r}\cdot\left(6\hat{i}+4\hat{j}-3\hat{k}\right)=12$ on the coordinate axes.

EXERCISE 6.9

6. Find the distance (length of the perpendicular) from the point $(1,-2,3)$ to the plane $x-y+z=5$.

Example 6.12

If $\vec{a}=-3\hat{i}-\hat{j}+5\hat{k}$, $\vec{b}=\hat{i}-2\hat{j}+\hat{k}$, $\vec{c}=4\hat{j}-5\hat{k}$, find $\vec{a}\cdot(\vec{b}\times\vec{c})$.

Example 6.13

Find the volume of the parallelepiped whose coterminus edges (adjacent edges) are given by the vectors $2\hat{i}-3\hat{j}+4\hat{k},\hat{i}+2\hat{j}-\hat{k}$ and $3\hat{i}-\hat{j}+2\hat{k}$.

Example 6.14

Show that the vectors $\hat{i}+2\hat{j}-3\hat{k},2\hat{i}-\hat{j}+2\hat{k}$ and $3\hat{i}+\hat{j}-\hat{k}$ are coplanar.

Example 6.15

If $2\hat{i}-\hat{j}+3\hat{k}$, $3\hat{i}+2\hat{j}+\hat{k}$, $\hat{i}+m\hat{j}+4\hat{k}$ are coplanar, find the value of m.

Example 6.32

Show that the lines $\frac{x-1}{4}=\frac{2-y}{6}=\frac{z-4}{12}$ and $\frac{x-3}{-2}=\frac{y-3}{3}=\frac{5-z}{6}$ are parallel.

Example 6.39

If the Cartesian equation of a plane is $3x-4y+3z=-8$, find the vector equation of the plane in the standard form.

Example 6.45

Verify whether the line $\frac{x-3}{-4}=\frac{y-4}{-7}=\frac{z+3}{12}$ lies in the plane $5x-y+z=8$.

Example 6.47

Find the acute angle between the planes $\vec{r}\cdot\left(2\hat{i}+2\hat{j}+2\hat{k}\right)=11$ and $4x-2y+2z=15$.

Example 6.48

Find the angle between the straight line $\vec{r}=\left(2\hat{i}+3\hat{j}+\hat{k}\right)+t\left(\hat{i}-\hat{j}+\hat{k}\right)$ and the plane $2x-y+z=5$.

Example 6.49

Find the distance of a point $(2,5,-3)$ from the plane $\vec{r}\cdot\left(6\hat{i}-3\hat{j}+2\hat{k}\right)=5$.

Example 6.51

Find the distance between the parallel planes $x+2y-2z+1=0$ and $2x+4y-4z+5=0$.

EXERCISE 7.1

4. If the volume of a cube of side length x is $V=x^3$. Find the rate of change of the volume with respect to x when $x=5$ units.

6. A stone is dropped into a pond causing ripples in the form of concentric circles. The radius r of the outer ripple is increasing at a constant rate at 2 cm per second. When the radius is 5 cm find the rate of changing of the total area of the disturbed water?

EXERCISE 7.2

1. Find the slope of the tangent to the curves at the respective given points. (each 2 marks)

(i) $y=x^4+2x^2-x$ at $x=1$ (ii) $x=a\cos^3 t, y=b\sin^3 t$ at $t=\frac{\pi}{2}$.

EXERCISE 7.3

1. Explain why Rolle's theorem is not applicable to the following functions in the respective intervals. (each 2 marks)

(i) $f(x)=\left|\frac{1}{x}\right|,\ x\in[-1,1]$ (ii) $f(x)=\tan x, x\in[0,\pi]$

(iii) $f(x)=x-2\log x,\ x\in[2,7]$

3. Explain why Lagrange's mean value theorem is not applicable to the following functions in the respective intervals : (each 2 marks)

(i) $f(x)=\frac{x+1}{x},\ x\in[-1,2]$ (ii) $f(x)=|3x+1|, x\in[-1,3]$

Example 7.2

The temperature in celsius in a long rod of length $10m$, insulated at both ends, is a function of length x given by $T=x(10-x)$. Prove that the rate of change of temperature at the midpoint of the rod is zero.

Example 7.5 (each 2 marks)

A particle is fired straight up from the ground to reach a height of s feet in t seconds, where $s=128t-16t^2$.

(i) Compute the maximum height of the particle reached?

(ii) What is the velocity when the particle hits the ground?

Example 7.33

Evaluate : $\lim_{x\to 1}\left(\frac{x^2-3x+2}{x^2-4x+3}\right)$.

Example 7.34

Compute the limit $\lim_{x\to a}\left(\frac{x^n-a^n}{x-a}\right)$.

Example 7.35

Evaluate : $\lim_{x\to 0}\left(\frac{\sin mx}{x}\right)$.

Example 7.36

Evaluate : $\lim_{x\to 0}\left(\frac{\sin x}{x^2}\right)$.

Example 7.47

Prove that the function $f(x)=x^2-2x-3$ is strictly increasing in $(2,\infty)$.

Example 7.50

Find the intervals of monotonicity and hence find the local extrema for the function $f(x)=x^2-4x+4$.

EXERCISE 8.2

1. Find the differentials dy for each of the following functions: (each 2 marks)

   (i) $y=\frac{(1-2x)^3}{3-4x}$ (ii) $y=3(3+\sin(2x))^{2/3}$ (iii) $y=e^{x^2-5x+7}\cos(x^2-1)$

2. Find df for $f(x)=x^2+3x$ and evaluate it for

   (i) $x=2$ and $dx=0.1$ (ii) $x=3$ and $dx=0.02$ (each 2 marks)

6. An egg of a particular bird is very nearly spherical. If the radius to the inside of the shell is 5mm and radius to the outside of the shell is 5.3mm, find the volume of the shell approximately.

7. Assume that the cross section of the artery of human is circular. A drug is given to a patient to dilate his arteries. If the radius of an artery is increased from 2mm to 2.1mm, how much is cross-sectional area increased approximately?

9. The relation between number of words y a person learns in x hours is given by $y=52\sqrt{x}, 0\le x\le 9$. What is the approximate number of words learned when x changes from

   (i) 1 to 1.1 hour? (ii) 4 to 4.1 hour? (each 2 marks)

EXERCISE 8.3

1. Evaluate $\lim_{(x,y)\to(1,2)} g(x,y)$, if the limit exists, where $g(x,y)=\frac{3x^2-xy}{x^2+y^2+3}$.

2. Evaluate $\lim_{(x,y)\to(0,0)} \cos\left(\frac{x^3+y^2}{x+y+2}\right)$, if the limit exists.

3. Let $f(x,y)=\dfrac{y^2-xy}{\sqrt{x}-\sqrt{y}}$ for $(x,y)\neq(0,0)$. Show that $\lim\limits_{(x,y)\to(0,0)} f(x,y)=0$.

4. Evaluate $\lim\limits_{(x,y)\to(0,0)} \cos\left(\dfrac{e^x \sin y}{y}\right)$, if the limit exists.

EXERCISE 8.4

10. A firm produces two types of calculators each week, x number of type A and y number of type B. The weekly revenue and cost functions (in rupees) are $R(x,y)=80x+90y+0.04xy-0.05x^2-0.05y^2$ and $C(x,y)=8x+6y+2000$ respectively.

(i) Find the profit function $P(x,y)$,

(ii) Find $\dfrac{\partial P}{\partial x}(1200,1800)$ and $\dfrac{\partial p}{\partial y}(1200,1800)$.

Remark : Revenue function, cost function and profit functions are not defined.

EXERCISE 8.7

1. In each of the following cases, determine whether the following function is homogeneous or not. If it is so, find the degree. (each 2 marks)

(i) $f(x,y)=x^2y+6x^3+7$

(ii) $h(x,y)=\dfrac{6x^2y^3-\pi y^5+9x^4y}{2020x^2+2019y^2}$

(iii) $g(x,y,z)=\dfrac{\sqrt{3x^2+5y^2+z^2}}{4x+7y}$

(iv) $U(x,y,z)=xy+\sin\left(\dfrac{y^2-2z^2}{xy}\right)$

Example 8.6

Let $g(x)=x^2+\sin x$. Calculate the differential dg.

Example 8.21

Show that $F(x,y)=\dfrac{x^2+5xy-10y^2}{3x+7y}$ is a homogeneous function of degree 1.

EXERCISE 9.3

1. Evaluate the following definite integrals :

(i) $\displaystyle\int_3^4 \frac{dx}{x^2-4}$

EXERCISE 9.6

1. Evaluate the following:

(i) $\displaystyle\int_0^{\frac{\pi}{2}} \sin^{10} x\, dx$ (ii) $\displaystyle\int_0^{\frac{\pi}{2}} \cos^7 x\, dx$

EXERCISE 9.7

1. Evaluate the following

(i) $\displaystyle\int_0^{\infty} x^5 e^{-3x}\, dx$

EXERCISE 9.8

1. Find the area of the region bounded by $3x-2y+6=0$, $x=-3$, $x=1$ and x-axis.

Example 9.7

Evaluate : $\int_0^{\frac{1}{2}} [2x]\,dx$ where $[\cdot]$ is the greatest integer function.

Example 9.20

Show that $\int_0^{\pi} g(\sin x)dx = 2\int_0^{\frac{\pi}{2}} g(\sin x)dx$, where $g(\sin x)$ is a function of $\sin x$.

Example 9.22

Show that $\int_0^{2\pi} g(\cos x)dx = 2\int_0^{\pi} g(\cos x)dx$, where $g(\cos x)$ is a function of $\cos x$.

Example 9.24

Evaluate : $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x\cos x\, dx$.

Example 9.25

Evaluate : $\int_{-\log 2}^{\log 2} e^{-|x|}\,dx$.

Example 9.37

Evaluate $\int_0^{\frac{\pi}{2}} \left(\sin^2 x + \cos^4 x\right)dx$

Example 9.47

Find the area of the region bounded by the line $6x+5y=30$, $x-$ axis and the lines $x=-1$ and $x=3$.

EXERCISE 10.3

8. Find the differential equation of the curve represented by $xy = ae^x + be^{-x} + x^2$. Here a and b are arbitrary constants.

EXERCISE 10.4

1. Show that each of the following expressions is a solution of the corresponding given differential equation. (each 2 marks)

   (i) $y = 2x^2$; $xy' = 2y$

   (ii) $y = ae^x + be^{-x}$; $y'' - y = 0$

2. Find value of m so that the function $y = e^{mx}$ is a solution of the given differential equation. (each 2 marks)

   (i) $y' + 2y = 0$ (ii) $y'' - 5y' + 6y = 0$

4. Show that $y = e^{-x} + mx + n$ is a solution of the differential equation $e^{x}\left(\frac{d^2y}{dx^2}\right) - 1 = 0$.

8. Show that $y = a\cos bx$ is a solution of the differential equation $\frac{d^2y}{dx^2} + b^2y = 0$.

Example 10.2

Find the differential equation for the family of all straight lines passing through the origin.

Example 10.3

Form the differential equation by eliminating the arbitrary constants A and B from $y = A\cos x + B\sin x$.

Example 10.5

Find the differential equation of the family of parabolas $y^2 = 4ax$, where a is an arbitrary constant.

Example 10.7

Show that $x^2 + y^2 = r^2$, where r is a constant, is a solution of the differential equation $\frac{dy}{dx} = -\frac{x}{y}$.

Example 10.8

Show that $y = mx + \frac{7}{m}, m \neq 0$ is a solution of the differential equation $xy' + 7\frac{1}{y'} - y = 0$.

Example 10.22

Solve $\frac{dy}{dx} + 2y = e^{-x}$.

EXERCISE 11.3

2. The probability density function of X is $f(x) = \begin{cases} x & 0 < x < 1 \\ 2 - x & 1 \leq x < 2 \\ 0 & \text{otherwise} \end{cases}$ (each 2 marks)

Find (i) $P(0.2 \leq X < 0.6)$ (ii) $P(1.2 \leq X < 1.8)$ (iii) $P(0.5 \leq X < 1.5)$

4. The probability density function of X is given by $f(x) = \begin{cases} ke^{-\frac{x}{3}} & \text{for } x > 0 \\ 0 & \text{for } x \leq 0 \end{cases}$ (each 2 marks)

Find (i) the value of k (ii) the distribution function (iii) $P(X < 3)$
(iv) $P(5 \leq X)$ (v) $P(X \leq 4)$

then find (i) the distribution function $F(x)$ (ii) $P(-0.5 \leq X \leq 0.5)$

6. If X is the random variable with distribution function $F(x)$ given by,

$$F(x) = \begin{cases} 0, & x < 0 \\ \frac{1}{2}(x^2 + x) & 0 \leq x < 1 \\ 1, & x \geq 1 \end{cases}$$

then find (i) the probability density function $f(x)$

EXERCISE 11.4

1. For the random variable X with the given probability mass function as below, find the mean and variance. (each 2 marks)

(i) $f(x)=\begin{cases}\frac{1}{10} & x=2,5\\ \frac{1}{5} & x=0,1,3,4\end{cases}$ (ii) $f(x)=\begin{cases}\frac{4-x}{6} & x=1,2,3\end{cases}$

(iii) $f(x)=\begin{cases}2(x-1) & 1<x<2\\ 0 & \text{otherwise}\end{cases}$ (iv) $f(x)=\begin{cases}\frac{1}{2}e^{-\frac{x}{2}} & \text{for } x>0\\ 0 & \text{otherwise}\end{cases}$

6. The time to failure in thousand hours of an electronic equipment used in a manufactured computer has the density function

$$f(x)=\begin{cases}3e^{-3x} & x>0\\ 0 & \text{elsewhere}\end{cases}$$

Find the expected life of this electronic equipment.

EXERCISE 11.5

1. Compute $P(X=k)$ for the binomial distribution, $B(n,p)$ where

(i) $n=6, p=\frac{1}{3}, k=3$ (ii) $n=10,\ p=\frac{1}{5},\ k=4$ (iii) $n=9,\ p=\frac{1}{2},\ k=7$ (each 2 marks)

3. Using binomial distribution find the mean and variance of X for the following experiments
   (i) A fair coin is tossed 100 times, and X denote the number of heads.
   (ii) A fair die is tossed 240 times, and X denote the number of times that four appeared.
   (each 2 marks)

4. The probability that a certain kind of component will survive a electrical test is $\frac{3}{4}$. Find the probability that exactly 3 of the 5 components tested survive.

Example 11.6

A pair of fair dice is rolled once. Find the probability mass function to get the number of fours.

Example 11.21 (each 2 marks or entire question 5 marks)

The mean and variance of a binomial variate X are respectively 2 and 1.5. Find
(i) $P(X=0)$ (ii) $P(X=1)$ (iii) $P(X\geq 1)$

EXERCISE 12.1

1. Determine whether $*$ is a binary operation on the sets given below:
   (i) $a*b=a\cdot|b|$ on $\mathbb{R}$ (ii) $a*b=\min(a,b)$ on $A=\{1,2,3,4,5\}$
   (iii) $(a*b)=a\sqrt{b}$ on $\mathbb{R}$. (each 2 marks)

2. On $\mathbb{Z}$, define $\otimes$ by $(m\otimes n)=m^n+n^m : \forall m,n\in\mathbb{Z}$. Is $\otimes$ binary on $\mathbb{Z}$?

3. Let $*$ be defined on $\mathbb{R}$ by $(a*b)=a+b+ab-7$. Is $*$ binary on $\mathbb{R}$? If so, find $3*\left(\frac{-7}{15}\right)$.

4. Let $A=\left\{a+\sqrt{5}b : a,b\in\mathbb{Z}\right\}$. Check whether the usual multiplication is a binary operation on A.

6. Fill in the following table so that the binary operation $*$ on $A=\{a,b,c\}$ is commutative.

*	a	b	c
a	b		
b	c	b	a
c	a		c

7. Consider the binary operation $*$ defined on the set $A=\{a,b,c,d\}$ by the following table :

*	a	b	c	d
a	a	c	b	d
b	d	a	b	c
c	c	d	a	a
d	d	b	a	c

Is it commutative and associative?

8. Let $A=\begin{pmatrix}1&0&1&0\\0&1&0&1\\1&0&0&1\end{pmatrix}$, $B=\begin{pmatrix}0&1&0&1\\1&0&1&0\\1&0&0&1\end{pmatrix}$, $C=\begin{pmatrix}1&1&0&1\\0&1&1&0\\1&1&1&1\end{pmatrix}$ be any three Boolean matrices of the same type. Find (i) $A\vee B$ (ii) $A\wedge B$ (iii) $(A\vee B)\wedge C$ (iv) $(A\wedge B)\vee C$. (each 2 marks)

EXERCISE 12.2

1. Let p : Jupiter is a planet and
   q : India is an island be any two simple statements. Give verbal sentence describing each of the following statements. (each 2 marks)
   (i) $\neg p$ (ii) $p\wedge\neg q$ (iii) $\neg p\vee q$ (iv) $p\rightarrow\neg q$ (v) $p\leftrightarrow q$
2. Write each of the following sentences in symbolic form using statement variables p and q.
   (i) 19 is not a prime number and all the angles of a triangle are equal.
   (ii) 19 is a prime number or all the angles of a triangle are not equal
   (iii) 19 is a prime number and all the angles of a triangle are equal
   (iv) 19 is not a prime number (any two, 2 marks)
3. Determine the truth value of each of the following statements
   (i) If $6+2=5$, then the milk is white.
   (ii) China is in Europe or $\sqrt{3}$ is an integer
   (iii) It is not true that $5+5=9$ or Earth is a planet
   (iv) 11 is a prime number and all the sides of a rectangle are equal (any two, 2 marks)
4. Which one of the following sentences is a proposition? (any two, 2 marks)
   (i) $4+7=12$ (ii) What are you doing? (iii) $3^n\leq 81, n\in\mathbb{N}$
   (iv) Peacock is our national bird (v) How tall this mountain is!

5. Write the converse, inverse, and contrapositive of each of the following implication.

(i) If x and y are numbers such that $x = y$, then $x^2 = y^2$.

(ii) If a quadrilateral is a square then it is a rectangle (each 2 marks)

Example12.1

Examine the closure property for $a*b = a+3ab-5b^2; \forall a,b \in \mathbb{Z}$

Example 12.8

Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ be any two boolean matrices of the same type. Find $A \vee B$ and $A \wedge B$.

Example 12.11

Identify the valid statements from the following sentences (for two sentences – 2 marks).

Example 12.12 (each 2 marks)

Write the statements in words corresponding to $\neg p, p \wedge q, p \vee q$ and $q \vee \neg p$, where p is 'It is cold' and q is 'It is raining.'

Example 12.13 (each 2 marks)

How many rows are needed to form truth tables for the following compound statements ?

(i) $p \vee \neg t \wedge (p \vee \neg s)$ (ii) $((p \wedge q) \vee (\neg r \vee \neg s)) \wedge (\neg t \wedge v)$

Example 12.15 (each 2 marks)

Write down the (i) conditional statement (ii) converse statement (iii) inverse statement, and (iv) contrapositive statement for the two statements p and q given below.

p : The number of primes is infinite. q : Ooty is in Kerala.

Example 12.17

Establish the equivalence property: $p \to q \equiv \neg p \vee q$.

OTHER QUESTIONS TAKEN FROM THE TEXT BOOK

Theorem 1.4 (each 2 marks)

If A is nonsingular, then

(i) $|A^{-1}|=\frac{1}{|A|}$ (ii) $(A^T)^{-1}=(A^{-1})^T$ (iii) $(\lambda A)^{-1}=\frac{1}{\lambda}A^{-1}$, where λ is a non-zero number.

Theorem 1.5 (Left Cancellation Law)

Let $A, B,$ and C be square matrices of order n. If A is non-singular and $AB = AC$, then $B = C$.

Theorem 1.6 (Right Cancellation Law)

Let $A, B,$ and C be square matrices of order n. If A is non-singular and $BA = CA$, then $B = C$.

Theorem 1.7 (Reversal Law for Inverses)

If A and B are nonsingular matrices of the same order, then the product AB is also nonsingular and $(AB)^{-1} = B^{-1}A^{-1}$.

Theorem 1.8 (Law of Double Inverse)

If A is nonsingular, then A^{-1} is also nonsingular and $(A^{-1})^{-1} = A$.

Theorem 1.10

If A and B are any two non-singular square matrices of order n, then

$$\text{adj}(AB) = (\text{adj } B)(\text{adj } A).$$

2.5.1 Properties of Modulus of complex number (each 2 marks)

(1) $|z| = |\bar{z}|$ (3) $|z_1 z_2| = |z_1||z_2|$

(5) $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$, $z_2 \neq 0$ (6) $|z^n| = |z|^n$ where n is an integer

(7) $\text{Re}(z) \leq |z|$ (8) $\text{Im}(z) \leq |z|$

Theorem 6.9 (Jacobi's identity)

$$\vec{a}\times(\vec{b}\times\vec{c})+\vec{b}\times(\vec{c}\times\vec{a})+\vec{c}\times(\vec{a}\times\vec{b})=\vec{0}.$$

Theorem 6.10 (Lagrange's identity)

For any four vectors $\vec{a},\vec{b},\vec{c},\vec{d}$, we have $(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d}) = \begin{vmatrix} \vec{a}\cdot\vec{c} & \vec{a}\cdot\vec{d} \\ \vec{b}\cdot\vec{c} & \vec{b}\cdot\vec{d} \end{vmatrix}$.

Theorem 7.1 Intermediate value theorem

If f is continuous in the closed interval $[a,b]$, and $f(a) \leq k \leq f(b)$, then there exists at least one $c \in [a,b]$ such that $f(c) = k$.

Theorem 7.2 (Rolle's Theorem)

If a function $f(x)$ is (i) continuous in the closed interval $[a,b]$, (ii) differentiable in the open interval (a,b) and (iii) $f(a) = f(b)$ then there exists at least one $c \in (a,b)$ such that $f'(c) = 0$.

Theorem 7.3 (Lagrange's Mean Value Theorem)

If a function $f(x)$ is (i) continuous in the closed interval $[a,b]$, (ii) differentiable in the open interval (a,b) then there exists at least one $c \in (a,b)$ such that $f'(c) = \frac{f(b)-f(a)}{b-a}$.

Theorem 7.8 Extreme Value Theorem

If $f(x)$ is continuous on a closed interval $[a,b]$, then f has both an absolute maximum and an absolute minimum (extreme values) in the interval $[a,b]$.

Theorem 7.9 (Fermat)

If $f(x)$ has a relative extrema at $x=c$ then c is a critical number.

Chapter 11 (each 2 marks)

1. $E(aX+b) = a\ E(X)+b$
2. $Var(aX+b) = a^2 Var(X)$

Theorem 12.1: (Uniqueness of Identity)

In an algebraic structure the identity element (if exists) must be unique.

Theorem 12.2(Uniqueness of Inverse)

In an algebraic structure the inverse of an element must be unique.

Part – III
(3 Mark Questions)

EXERCISE 1.1

1. Find the adjoint of the following: (each 3 marks)

(ii) $\begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$ (iii) $\frac{1}{3}\begin{bmatrix} 2 & 2 & 1 \\ -2 & 1 & 2 \\ 1 & -2 & 2 \end{bmatrix}$

2. Find the inverse (if it exists) of the following: (each 3 marks)

(ii) $\begin{bmatrix} 5 & 1 & 1 \\ 1 & 5 & 1 \\ 1 & 1 & 5 \end{bmatrix}$ (iii) $\begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$

4. If $A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$, show that $A^2 - 3A - 7I_2 = O$. Hence find A^{-1}.

5. If $A = \frac{1}{9}\begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$, prove that $A^{-1} = A^T$.

6. If $A = \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$, verify that $A(adjA) = (adjA)A = |A|I$.

7. If $A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & -3 \\ 5 & 2 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1}A^{-1}$.

8. If $\text{adj}(A) = \begin{bmatrix} 2 & -4 & 2 \\ -3 & 12 & -7 \\ -2 & 0 & 2 \end{bmatrix}$, find A.

10. Find $adj(adj(A))$ if $adjA = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}$.

11. $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, show that $A^T A^{-1} = \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$.

12. Find the matrix A for which $A\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix}$.

13. Given $A = \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -2 \\ 1 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$, find a matrix X such that $AXB = C$.

EXERCISE 1.2

1. Find the rank of the following matrices by minor method: (each 3 marks)

(iv) $\begin{bmatrix} 1 & -2 & 3 \\ 2 & 4 & -6 \\ 5 & 1 & -1 \end{bmatrix}$ (v) $\begin{bmatrix} 0 & 1 & 2 & 1 \\ 0 & 2 & 4 & 3 \\ 8 & 1 & 0 & 2 \end{bmatrix}$

2. Find the rank of the following matrices by row reduction method: (each 3 marks)

(i) $\begin{bmatrix} 1 & 1 & 1 & 3 \\ 2 & -1 & 3 & 4 \\ 5 & -1 & 7 & 11 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 1 & -2 & 3 \\ 1 & -1 & 1 \end{bmatrix}$ (iii) $\begin{bmatrix} 3 & -8 & 5 & 2 \\ 2 & -5 & 1 & 4 \\ -1 & 2 & 3 & -2 \end{bmatrix}$

EXERCISE 1.3

1. Solve the following system of linear equations by matrix inversion method:

(i) $2x+5y=-2,\ x+2y=-3$ (ii) $2x-y=8,\ 3x+2y=-2$ (each 3 marks)

EXERCISE 1.4

1. Solve the following systems of linear equations by Cramer's rule: (each 3 marks)

(i) $5x-2y+16=0, x+3y-7=0$

(ii) $\frac{3}{x}+2y=12, \frac{2}{x}+3y=13$

2. In a competitive examination, one mark is awarded for every correct answer while $\frac{1}{4}$ mark is deducted for every wrong answer. A student answered 100 questions and got 80 marks. How many questions did he answer correctly ?

EXERCISE 1.6

1. Test for consistency and if possible, solve the following systems of equations by rank method.

(iii) $2x+2y+z=5,\quad x-y+z=1,\quad 3x+y+2z=4$

Example 1.3

Find the inverse of the matrix $\begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix}$.

Example 1.5

Find a matrix A if $\text{adj}(A)=\begin{bmatrix} 7 & 7 & -7 \\ -1 & 11 & 7 \\ 11 & 5 & 7 \end{bmatrix}$.

Example 1.6

If $\text{adj } A = \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, find A^{-1}

Example 1.8

Verify the property $(A^T)^{-1} = (A^{-1})^T$ with $A = \begin{bmatrix} 2 & 9 \\ 1 & 7 \end{bmatrix}$.

Example 1.9

Verify $(AB)^{-1} = B^{-1}A^{-1}$ with $A = \begin{bmatrix} 0 & -3 \\ 1 & 4 \end{bmatrix}, B = \begin{bmatrix} -2 & -3 \\ 0 & -1 \end{bmatrix}$.

Example 1.10

If $A = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$, find x and y such that $A^2 + xA + yI_2 = O$ Hence, find A^{-1}.

Prove that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal.

Example 1.13

Reduce the matrix $\begin{bmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{bmatrix}$ to a row-echelon form.

Example 1.14

Reduce the matrix $\begin{bmatrix} 0 & 3 & 1 & 6 \\ -1 & 0 & 2 & 5 \\ 4 & 2 & 0 & 0 \end{bmatrix}$ to row-echelon form.

Example 1.15 (each 3 marks)

Find the rank of each of the following matrices:

(i) $\begin{bmatrix} 3 & 2 & 5 \\ 1 & 1 & 2 \\ 3 & 3 & 6 \end{bmatrix}$ (ii) $\begin{bmatrix} 4 & 3 & 1 & -2 \\ -3 & -1 & -2 & 4 \\ 6 & 7 & -1 & 2 \end{bmatrix}$.

Example 1.17

Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \\ 3 & 0 & 5 \end{bmatrix}$ by reducing it to a row-echelon form.

Example 1.18

Find the rank of the matrix $\begin{bmatrix} 2 & -2 & 4 & 3 \\ -3 & 4 & -2 & -1 \\ 6 & 2 & -1 & 7 \end{bmatrix}$ by reducing it to an echelon form.

Example 1.19

Show that the matrix $\begin{bmatrix} 3 & 1 & 4 \\ 2 & 0 & -1 \\ 5 & 2 & 1 \end{bmatrix}$ is non-singular and reduce it to the identity matrix by elementary row transformations.

Example 1.22

Solve the following system of linear equations, using matrix inversion method:
$5x+2y=3,\ 3x+2y=5$.

EXERCISE 2.2

2. Given the complex number $z=2+3i$, represent the complex numbers
   (i) z, iz, and $z+iz$ (ii) z, $-iz$, and $z-iz$ in Argand diagram. (each 3 marks)

3. Find the values of the real numbers x and y, if the complex numbers
   $(3-i)x-(2-i)y+2i+5$ and $2x+(-1+2i)y+3+2i$ are equal

EXERCISE 2.3

1. If $z_1=1-3i$, $z_2=-4i$, and $z_3=5$, show that (each 3 marks)
   (i) $(z_1+z_2)+z_3=z_1+(z_2+z_3)$ (ii) $(z_1z_2)z_3=z_1(z_2z_3)$
2. If $z_1=3$, $z_2=-7i$, and $z_3=5+4i$, show that (each 3 marks)
   (i) $z_1(z_2+z_3)=z_1z_2+z_1z_3$ (ii) $(z_1+z_2)z_3=z_1z_3+z_2z_3$
3. If $z_1=2+5i$, $z_2=-3-4i$, $z_3=1+i$, find the additive and multiplicative inverse of z_1, z_2, and z_3 (each 3 marks)

EXERCISE 2.4

4. The complex numbers u, v, and w are related by $\frac{1}{u}=\frac{1}{v}+\frac{1}{w}$.
   If $v=3-4i$ and $w=4+3i$, find u in rectangular form.
6. Find the least value of the positive integer n for which $\left(\sqrt{3}+i\right)^n$
   (i) real (ii)purely imaginary
7. Show that
   (ii) $\left(\frac{19-7i}{9+i}\right)^{12}+\left(\frac{20-5i}{7-6i}\right)^{12}$ is real

EXERCISE 2.5

2. For any two complex numbers z_1 and z_2, such that $|z_1| = |z_2| = 1$ and $z_1 z_2 \neq -1$, then show that $\frac{z_1 + z_2}{1 + z_1 z_2}$ is real number.
3. Which one of the points $10 - 8i, 11 + 6i$ is closed to $1 + i$.
4. If $|z| = 3$, show that $7 \leq |z + 6 - 8i| \leq 13$.
5. If $|z| = 1$, show that $2 \leq |z^2 - 3| \leq 4$.
6. If $|z| = 2$, show that $8 \leq |z + 6 + 8i| \leq 12$.
8. If the area of the triangle formed by the vertices z, iz, and $z + iz$ is 50 square units, find the value of $|z|$.
9. Show that the equation $z^3 + 2\bar{z} = 0$ has five solutions.

EXERCISE 2.6

1. If $z = x + iy$ is a complex number such that $\left|\frac{z - 4i}{z + 4i}\right| = 1$ show that the locus of z is real axis.
3. Obtain the Cartesian equation for the locus of $z = x + iy$ in each of the following cases:

   (i) $[\text{Re}(iz)]^2 = 3$ (ii) $\text{Im}[(1 - i)z + 1] = 0$ (iii) $|z + i| = |z - 1|$ (iv) $\bar{z} = z^{-1}$

   (each 3 mar

5. Obtain the Cartesian equation for the locus of $z = x + iy$ in each of the following cases:

   (i) $|z - 4| = 16$ (ii) $|z - 4|^2 + |z - 1|^2 = 16$

EXERCISE 2.7

1. Write in polar form of the following complex numbers (each 3 marks)

   (i) $2 + i2\sqrt{3}$ (ii) $3 - i\sqrt{3}$ (iii) $-2 - i2$ (iv) $\frac{i - 1}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}$

2. Find the Cartesian form of the complex numbers (each 3 marks)

   (i) $\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$ (ii) $\frac{\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}}{2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)}$

4. If $\frac{1 + z}{1 - z} = \cos 2\theta + i\sin 2\theta$, show that $z = i\tan\theta$.

EXERCISE 2.8

1. If $\omega \neq 1$ is a cube root of unity, show that $\frac{a + b\omega + c\omega^2}{b + c\omega + a\omega^2} + \frac{a + b\omega + cw^2}{c + a\omega + b\omega^2} = -1$.
2. Show that $\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5 = -\sqrt{3}$.
7. Find the value of $\sum_{k=1}^{8}\left(\cos\frac{2k\pi}{9} + i\sin\frac{2k\pi}{9}\right)$.

8. If $\omega \neq 1$ is a cube root of unity, show that (each 3 marks)

(i) $\left(1-\omega+\omega^2\right)^6+\left(1+\omega-\omega^2\right)^6=128$

(ii) $(1+\omega)\left(1+\omega^2\right)\left(1+\omega^4\right)\left(1+\omega^8\right)\cdots\left(1+\omega^{2^{11}}\right)=1$

Example 2.2

Find the value of the real numbers x and y, if the complex numbers $(2+i)x+(1-i)y+2i-3$ and $x+(-1+2i)y+1+i$ are equal

Example 2.4

Simplify $\left(\frac{1+i}{1-i}\right)^3-\left(\frac{1-i}{1+i}\right)^3$ into rectangular form.

Example 2.8

Show that (ii) $\left(\frac{19+9i}{5-3i}\right)^{15}-\left(\frac{8+i}{1+2i}\right)^{15}$ is purely imaginary.

Example 2.11

Which one of the points $i, -2+i$ and 3 is farthest from the origin?

Example 2.13

If $|z|=2$ show that $3\leq|z+3+4i|\leq 7$.

Example 2.14

Show that the points $1, \frac{-1}{2}+i\frac{\sqrt{3}}{2}$, and $\frac{-1}{2}-i\frac{\sqrt{3}}{2}$ are the vertices of an equilateral triangle.

Example 2.16

Show that the equation $z^2=\bar{z}$ has four solutions.

Example 2.19

Show that $|3z-5+i|=4$, represents a circle, and find its centre and radius.

Example 2.20

Show that $|z+2-i|<2$ represents interior points of a circle. Find the centre and radius.

Example 2.21 (each 3 marks)

Obtain the Cartesian equation for the locus of z in each of the following cases.

(i) $|z|=|z-i|$ (ii) $|2z-3-i|=3$.

Example 2.23

Represent the complex number (i) $-1-i$ (ii) $1+i\sqrt{3}$ in polar form.

Example 2.26

Find the quotient $\dfrac{2\left(\cos\frac{9\pi}{4}+i\sin\frac{9\pi}{4}\right)}{4\left(\cos\left(\frac{-3\pi}{2}\right)+i\sin\left(\frac{-3\pi}{2}\right)\right)}$ in rectangular form.

Example 2.28 (each 3 marks)

If $z=(\cos\theta+i\sin\theta)$, show that $z^n+\frac{1}{z^n}=2\cos n\theta$ and $z^n-\frac{1}{z^n}=2i\sin n\theta$.

Example 2.29

Simplify $\left(\sin\frac{\pi}{6}+i\cos\frac{\pi}{6}\right)^{18}$.

Example 2.31

Simplify (i) $(1+i)^{18}$

EXERCISE 3.1

1. If the sides of a cubic box are increased by 1,2,3 units respectively to form a rectangul cuboid, then the volume is increased by 52 cubic units. Find the volume of the cuboid.
2. Construct a cubic equation with roots (each 3 marks)

   (i) 1,2, and 3 (ii) 1,1, and -2 (iii) $2,\frac{1}{2}$ and 1.

3. If α,β and γ are the roots of the cubic equation $x^3+2x^2+3x+4=0$, form a cubic equatic whose roots are

   (i) $2\alpha,2\beta,2\gamma$ (ii) $\frac{1}{\alpha},\frac{1}{\beta},\frac{1}{\gamma}$ (iii) $-\alpha,-\beta,-\gamma$ (each 3 marks)

7. If $\alpha,\beta,$ and γ are the roots of the polynomial equation $ax^3+bx^2+cx+d=0$, find the valu of $\sum\frac{\alpha}{\beta\gamma}$ in terms of the coefficients.
8. If α,β,γ and δ are the roots of the polynomial equation $2x^4+5x^3-7x^2+8=0$, find quadratic equation with integer coefficients whose roots are $\alpha+\beta+\gamma+\delta$ and $\alpha\beta\gamma\delta$.
11. A 12 metre tall tree was broken into two parts. It was found that the height of the part whic was left standing was the cube root of the length of the part that was cut away. Formulate th into a mathematical problem to find the height of the part which was cut away.

EXERCISE 3.2

2. Find a polynomial equation of minimum degree with rational coefficients, having $2+\sqrt{3}i$ a root.
3. Find a polynomial equation of minimum degree with rational coefficients, having $2i+3$ as root.
4. Find a polynomial equation of minimum degree with rational coefficients, having $\sqrt{5}-\sqrt{}$ as a root.

EXERCISE 3.5

1. Solve the following equations:

   (i) $\sin^2 x-5\sin x+4=0$

EXERCISE 3.6

3. Show that the equation $x^9-5x^5+4x^4+2x^2+1=0$ has atleast 6 imaginary solutions.
5. Find the exact number of real zeros and imaginary zeros of the polynomia $x^9+9x^7+7x^5+5x^3+3x$.

Example 3.4

Find the sum of the squares of the roots of $ax^4 + bx^3 + cx^2 + dx + e = 0,\ a \neq 0$.

Example 3.8

Find the monic polynomial equation of minimum degree with real coefficients having $2-\sqrt{3}i$ as a root.

Example 3.9

Find a polynomial equation of minimum degree with rational coefficients, having $2-\sqrt{3}$ as a root.

Example 3.10

Form a polynomial equation with integer coefficients with $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ as a root.

Example 3.13

Show that, if p,q,r are rational, the roots of the equation $x^2 - 2px + p^2 - q^2 + 2qr - r^2 = 0$ are rational.

Example 3.25

Solve the equation $x^3 - 5x^2 - 4x + 20 = 0$.

Example 3.29

Find solution, if any, of the equation $2\cos^2 x - 9\cos x + 4 = 0$.

Example 3.30

Show that the polynomial $9x^9 + 2x^5 - x^4 - 7x^2 + 2$ has at least six imaginary zeros.

EXERCISE 4.1

1. Find all the values of x such that

   (i) $-10\pi \le x \le 10\pi$ and $\sin x = 0$ (ii) $-3\pi \le x \le 3\pi$ and $\sin x = -1$. (each 3 marks)

3. Sketch the graph of $y = \sin\left(\frac{1}{3}x\right)$ for $0 \le x < 6\pi$.

6. Find the domain of the following

   (i) $f(x) = \sin^{-1}\left(\frac{x^2+1}{2x}\right)$ (ii) $g(x) = 2\sin^{-1}(2x-1) - \frac{\pi}{4}$. (each 3 marks)

7. Find the value of $\sin^{-1}\left(\sin\frac{5\pi}{9}\cos\frac{\pi}{9} + \cos\frac{5\pi}{9}\sin\frac{\pi}{9}\right)$.

EXERCISE 4.2

1. Find all values of x such that

   (i) $-6\pi \le x \le 6\pi$ and $\cos x = 0$ (ii) $-5\pi \le x \le 5\pi$ and $\cos x = -1$. (each 3 marks)

5. Find the value of

(i) $2\cos^{-1}\left(\frac{1}{2}\right)+\sin^{-1}\left(\frac{1}{2}\right)$　　(ii) $\cos^{-1}\left(\frac{1}{2}\right)+\sin^{-1}(-1)$

(iii) $\cos^{-1}\left(\cos\frac{\pi}{7}\cos\frac{\pi}{17}-\sin\frac{\pi}{7}\sin\frac{\pi}{17}\right)$. (each 3 marks)

6. Find the domain of

(ii) $g(x)=\sin^{-1}x+\cos^{-1}x$.

7. For what values of x, the inequality $\frac{\pi}{2}<\cos^{-1}(3x-1)<\pi$ holds?

8. Find the value of

(ii) $\cos^{-1}\left(\cos\left(\frac{4\pi}{3}\right)\right)+\cos^{-1}\left(\cos\left(\frac{5\pi}{4}\right)\right)$.

EXERCISE 4.3

4. Find the value of

(i) $\tan\left(\cos^{-1}\left(\frac{1}{2}\right)-\sin^{-1}\left(-\frac{1}{2}\right)\right)$　　(ii) $\sin\left(\tan^{-1}\left(\frac{1}{2}\right)-\cos^{-1}\left(\frac{4}{5}\right)\right)$

(iii) $\cos\left(\sin^{-1}\left(\frac{4}{5}\right)-\tan^{-1}\left(\frac{3}{4}\right)\right)$. (each 3 marks)

EXERCISE 4.4

2. Find the value of

(iii) $\cot^{-1}(1)+\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)-\sec^{-1}\left(-\sqrt{2}\right)$.

EXERCISE 4.5

2. Find the value of the expression in terms of x, with the help of a reference triangle.

(i) $\sin\left(\cos^{-1}(1-x)\right)$　(ii) $\cos\left(\tan^{-1}(3x-1)\right)$　(iii) $\tan\left(\sin^{-1}\left(x+\frac{1}{2}\right)\right)$. (each 3 marks

3. Find the value of

(i) $\sin^{-1}\left(\cos\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right)$　　(ii) $\cot\left(\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{4}{5}\right)$

(iii) $\tan\left(\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right)$. (each 3 marks)

4. Prove that

(i) $\tan^{-1}\frac{2}{11}+\tan^{-1}\frac{7}{24}=\tan^{-1}\frac{1}{2}$.　(ii) $\sin^{-1}\frac{3}{5}-\cos^{-1}\frac{12}{13}=\sin^{-1}\frac{16}{65}$　(each 3 marks)

8. Simplify : $\tan^{-1}\frac{x}{y}-\tan^{-1}\frac{x-y}{x+y}$.

Example 4.4

Find the domain of $\sin^{-1}(2-3x^2)$.

Example 4.7

Find the domain of $\cos^{-1}\left(\dfrac{2+\sin x}{3}\right)$.

Example 4.10

Find the value of $\tan^{-1}(-1)+\cos^{-1}\left(\dfrac{1}{2}\right)+\sin^{-1}\left(-\dfrac{1}{2}\right)$.

Example 4.14

If $\cot^{-1}\left(\dfrac{1}{7}\right)=\theta$, find the value of $\cos\theta$.

Example 4.15

Show that $\cot^{-1}\left(\dfrac{1}{\sqrt{x^2-1}}\right)=\sec^{-1}x$, $|x|>1$.

Example 4.16

Prove that $\dfrac{\pi}{2}\le\sin^{-1}x+2\cos^{-1}x\le\dfrac{3\pi}{2}$.

Example 4.17 (each 3 marks)

Simplify (i) $\cos^{-1}\left(\cos\left(\dfrac{13\pi}{3}\right)\right)$ (ii) $\tan^{-1}\left(\tan\left(\dfrac{3\pi}{4}\right)\right)$

(iii) $\sec^{-1}\left(\sec\left(\dfrac{5\pi}{3}\right)\right)$ (iv) $\sin^{-1}[\sin 10]$

Example 4.19

Prove that $\tan(\sin^{-1}x)=\dfrac{x}{\sqrt{1-x^2}}$ for $|x|<1$.

Example 4.21

Prove that (i) $\tan^{-1}\dfrac{1}{2}+\tan^{-1}\dfrac{1}{3}=\dfrac{\pi}{4}$ (ii) $2\tan^{-1}\dfrac{1}{2}+\tan^{-1}\dfrac{1}{7}=\tan^{-1}\dfrac{31}{17}$.

Example 4.25

Solve $\sin^{-1}x>\cos^{-1}x$.

Example 4.26

Show that $\cot(\sin^{-1}x)=\dfrac{\sqrt{1-x^2}}{x}, -1\le x\le 1$ and $x\neq 0$.

Example 4.27

Solve : $\tan^{-1}2x+\tan^{-1}3x=\dfrac{\pi}{4}$, if $6x^2<1$.

EXERCISE 5.1

1. Obtain the equation of the circles with radius 5 cm and touching x-axis at the origin in general form.

2. Find the equation of the circle with centre $(2,-1)$ and passing through the point $(3,6)$ in standard form.
3. Find the equations of circles that touch both the axes and pass through $(-4,-2)$ in general form.
4. Find the equation of the circles with centre $(2,3)$ and passing through the intersection of the lines $3x-2y-1=0$ and $4x+y-27=0$.
7. A circle of area 9π square units has two of its diameters along the lines $x+y=5$ and $x-y=1$. Find the equation of the circle.
8. If $y=2\sqrt{2}x+c$ is a tangent to the circle $x^2+y^2=16$, find the value of c.
12. If the equation $3x^2+(3-p)xy+qy^2-2px-8pq=0$ represents a circle, find p and q. Also determine the centre and radius of the circle.

EXERCISE 5.2

1. Find the equation of the parabola in each of the cases given below : (each 3 marks)
   (i) focus $(4,0)$ and directrix $x=-4$.
   (ii) passes through $(2,-3)$, symmetric about y axis, open downwards and the vertex $(0,0)$.
   (iii) Vertex $(1-2)$ and focus $(4,-2)$.
   (iv) end points of latus rectum are $(4,-8),(4,8)$, open rightwords and the vertex $(0,0)$.
2. Find the equation of the ellipse in each of the cases given below : (each 3 marks)
   (i) Foci $(\pm3,0)$, $e=\frac{1}{2}$
   (ii) foci $(0,\pm4)$ and end points of major axis are $(0,\pm5)$.
   (iii) length of latus rectum 8, eccentricity $=\frac{3}{5}$, major axis on x-axis and the centre $(0,0)$.
   (iv) length of latus rectum 4, distance between foci $4\sqrt{2}$, major axis as y-axis and the centre $(0,0)$.
3. Find the equation of the hyperbola in each of the cases given below: (each 3 marks)
   (i) Foci $(\pm2,0)$, $e=\frac{3}{2}$
   (ii) Centre $(2,1)$, one of the foci $(8,1)$ and corresponding directrix $x=4$.
   (iii) Passing through $(5,-2)$, length of the transverse axis along x-axis and of length 8 units and the centre is $(0,0)$.
4. Find the vertex, focus, equation of directrix and length of the latus rectum of the following:
   (i) $y^2=16x$ (ii) $x^2=24y$ (iii) $y^2=-8x$ (each 3 marks)
6. Prove that the length of the latus rectum of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $\frac{2b^2}{a}$.
7. Show that the absolute value of difference of the focal distances of any point P on the hyperbola is the length of its transverse axis.

Example 5.2

Find the equation of the circle described on the chord $3x+y+5=0$ of the circle $x^2+y^2=16$ as diameter.

Example 5.6

The line $3x+4y-12=0$ meets the coordinate axes at A and B. Find the equation of the circle drawn on AB as diameter.

Example 5.7

A line $3x+4y+10=0$ cuts a chord of length 6 units on a circle with centre of the circle $(2,1)$. Find the equation of the circle in general form.

Example 5.8

A circle of radius 3 units touches both the axes. Find the equations of all possible circles formed in the general form.

Example 5.9

Find the centre and radius of the circle $3x^2+(a+1)y^2+6x-9y+a+4=0$.

Example 5.11

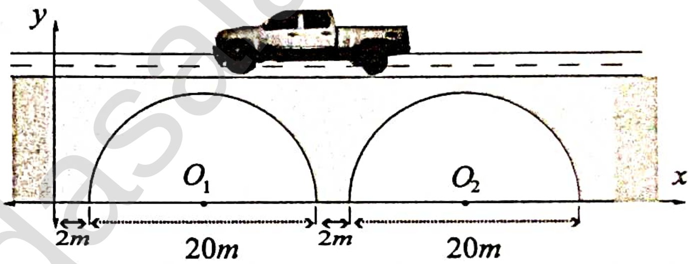
Find the equations of the tangent and normal to the circle $x^2+y^2=25$ at $P(-3,4)$.

Example 5.12

If $y=4x+c$ is a tangent to the circle $x^2+y^2=9$ find c,

Example 5.13

A road bridge over an irrigation canal have two semi circular vents each with a span of $20m$ and the supporting pillars of width $2m$ use the figure to write the equations that model the arches.



Example 5.14

Find the Latus rectum of the parabola $y^2=4ax$.

Example 5.15

Find the Latus rectum of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

Example 5.16

Find the equation of the parabola with focus $\left(-\sqrt{2},0\right)$ and directrix $x=\sqrt{2}$.

Example 5.17

Find the equation of the parabola whose vertex is $(5,-2)$ and focus $(2,-2)$.

Example 5.18

Find the equation of the parabola with vertex $(-1,-2)$, axis parallel to y-axis and passing through $(3,6)$.

Example 5.20

Find the equation of the ellipse with foci $(\pm 2,0)$, vertices $(\pm 3,0)$.

directrix is $x=7$. Find the length of the major and minor axes of the ellipse.

Example 5.24

Find the equation of the hyperbola with vertices $(0,\pm 4)$ and foci $(0,\pm 6)$.

Example 5.32

The maximum and minimum distances of the Earth from the Sun respectively are 152×10^6 km and 94.5×10^6 km. The Sun is at one focus of the elliptical orbit. Find the distance from the Sun to the other focus.

Example 5.33

A concrete bridge is designed as a parabolic arch. The road over bridge is $40m$ long and the maximum height of the arch is $15m$. Write an equation of the parabolic arch.

Example 5.34

The parabolic communications antenna has a focus at $2m$ distance from the vertex of the antenna. Find the width of the antenna $3m$ from the vertex.

Example 5.35

The cross section of a parabolic mirror is the equation $y=\frac{1}{32}x^2$ that are used for solar energy. There is a heating tube located at the focus of each parabola; how high is this tube located above the vertex of the parabola?

Example 5.36

A search light has a parabolic reflector (has a cross section that forms a 'bowl'). The parabolic bowl is $40cm$ wide from rim to rim and $30cm$ deep. The bulb is located at the focus

(1)What is the equation of the parabola used for reflector?

(2) How far from the vertex is the bulb to be placed so that the maximum distance covered?

Example 5.37

An equation of the elliptical part of an optical lens system is $\frac{x^2}{16}+\frac{y^2}{9}=1$. The parabolic part of the system has a focus in common with the right focus of the ellipse .The vertex of the parabola is at the origin and the parabola opens to the right. Determine the equation of the parabola.

Example 5.38

A room $34m$ long is constructed to be a whispering gallery. The room has an elliptical ceiling, as shown. If the maximum height of the ceiling is $8m$, determine where the foci are located.

Example 5.39

If the equation of the ellipse is $\frac{(x-11)^2}{484}+\frac{y^2}{64}=1$ (x and y are measured in centimeters) where to the nearest centimeter, should the patient's kidney stone be placed so that the reflected sound hits the kidney stone?

EXERCISE 6.1

1. Prove by vector method that if a line is drawn from the centre of a circle to the mid point of a chord, then the line is perpendicular to the chord.
2. Prove by vector method that the median to the base of an isosceles triangle is perpendicular to the base.
3. Prove by vector method that an angle in a semi-circle is a right angle.

4. Prove by vector method that the diagonals of a rhombus intersect each other at right angles.
5. Using vector method, prove that if the diagonals of a parallelogram are equal, then it is a rectangle.
6. Prove by vector method that the area of quadrilateral $ABCD$ having diagonals AC and BD is $\frac{1}{2}\left|\overrightarrow{AC}\times\overrightarrow{BD}\right|$.
7. Prove by vector method that the parallelograms on the same base and between the same parallels are equal in area.
11. A particle acted on by constant forces $8\hat{i}+2\hat{j}-6\hat{k}$ and $6\hat{i}+2\hat{j}-2\hat{k}$ is displaced from the point $(1,2,3)$ to the point $(5,4,1)$. Find the total work done by the forces.
12. Forces of magnitudes $5\sqrt{2}$ and $10\sqrt{2}$ units acting in the directions $3\hat{i}+4\hat{j}+5\hat{k}$ and $10\hat{i}+6\hat{j}-8\hat{k}$, respectively, act on a particle which is displaced from the point with position vector $4\hat{i}-3\hat{j}-2\hat{k}$ to the point with position vector $6\hat{i}+\hat{j}-3\hat{k}$. Find the work done by the forces.
13. Find the magnitude and direction cosines of the torque (moment) of a force represented by $3\hat{i}+4\hat{j}-5\hat{k}$ about the point with position vector $2\hat{i}-3\hat{j}+4\hat{k}$ acting through a point whose position vector is $4\hat{i}+2\hat{j}-3\hat{k}$.
14. Find the torque (moment) of the resultant of the three forces represented by $-3i+6\hat{j}-3\hat{k}, 4\hat{i}-10\hat{j}+12\hat{k}$ and $4\hat{i}+7\hat{j}$ acting at the point with position vector $8\hat{i}-6\hat{j}-4\hat{k}$, about the point with position vector $18\hat{i}+3\hat{j}-9\hat{k}$.

EXERCISE 6.2

4. If $\vec{a},\vec{b},\vec{c}$ are three non-coplanar vectors represnted by concurrent edges of a parallelopiped of volume 4 cubic units, find the value of $(\vec{a}+\vec{b})\cdot(\vec{b}\times\vec{c})+(\vec{b}+\vec{c})\cdot(\vec{c}\times\vec{a})+(\vec{c}+\vec{a})\cdot(\vec{a}\times\vec{b})$.
5. Find the altitude of a parallelopiped determined by the vectors $\vec{a}=-2\hat{i}+5\hat{j}+3\hat{k}, \vec{b}=\hat{i}+3\hat{j}-2\hat{k}$ and $\vec{c}=-3\hat{i}+\hat{j}+4\hat{k}$ if the base is taken as the parallelogram determined by $\vec{b}$ and $\vec{c}$.
10. Let $\vec{a},\vec{b},\vec{c}$ be three non-zero vectors such that $\vec{c}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$. If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$, show that $\left[\vec{a}, \vec{b}, \vec{c}\right]^2=\frac{1}{4}|\vec{a}|^2|\vec{b}|^2$.

EXERCISE 6.3

3. Prove that $\left[\vec{a}-\vec{b}, \vec{b}-\vec{c}, \vec{c}-\vec{a}\right]=0$.
5. $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}, \vec{b}=-\hat{i}+2\hat{j}-4\hat{k}, \vec{c}=\hat{i}+\hat{j}+\hat{k}$ then find the value of $(\vec{a}\times\vec{b})\cdot(\vec{a}\times\vec{c})$.
6. If $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are coplanar vectors, show that $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=\vec{0}$.
7. If $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}, \vec{b}=2\hat{i}-\hat{j}+\hat{k}, \vec{c}=3\hat{i}+2\hat{j}+\hat{k}$ and $\vec{a}\times(\vec{b}\times\vec{c})=l\vec{a}+m\vec{b}+n\vec{c}$, find the values of l,m,n.

8. If $\hat{a}, \hat{b}, \hat{c}$ are three unit vectors such that $\hat{b}$ and $\hat{c}$ are non-parallel and $\hat{a}\times(\hat{b}\times\hat{c})=\frac{1}{2}\hat{b}$, find the angle between $\hat{a}$ and $\hat{c}$.

EXERCISE 6.4

1. Find the non-parametric form of vector equation and Cartesian equations of the straight line passing through the point with position vector $4\hat{i}+3\hat{j}-7\hat{k}$ and parallel to the vector $2\hat{i}-6\hat{j}+7\hat{k}$.
2. Find the parametric form of vector equation and Cartesian equations of the straight line passing through the point $(-2,3,4)$ and parallel to the straight line $\frac{x-1}{-4}=\frac{y+3}{5}=\frac{8-z}{6}$.
3. Find the points where the straight line passes through $(6,7,4)$ and $(8,4,9)$ cuts the xz and yz planes.
4. Find the direction cosines of the straight line passing through the points $(5,6,7)$ and $(7,9,13)$. Also find the parametric form of vector equation and Cartesian equations of the straight line passing through two given points.
6. The vertices of ΔABC are $A(7,2,1), B(6,0,3)$, and $C(4,2,4)$. Find $\angle ABC$.
7. If the straight line joining the points $(2,1,4)$ and $(a-1,4,-1)$ is parallel to the line joining the points $(0,2,b-1)$ and $(5,3,-2)$, find the values of a and b.
8. If the straight lines $\frac{x-5}{5m+2}=\frac{2-y}{5}=\frac{1-z}{-1}$ and $x=\frac{2y+1}{4m}=\frac{1-z}{-3}$ are perpendicular to each other, find the value of m.
9. Show that the points $(2,3,4),(-1,4,5)$ and $(8,1,2)$ are collinear.

EXERCISE 6.5

1. Find the parametric form of vector equation and Cartesian equations of a straight line passing through $(5,2,8)$ and is perpendicular to the straight lines $\vec{r}=(\hat{i}+\hat{j}-\hat{k})+s(2\hat{i}-2\hat{j}+\hat{k})$ and $\vec{r}=(2\hat{i}-\hat{j}-3\hat{k})+t(\hat{i}+2\hat{j}+2\hat{k})$.
3. If the two lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-m}{2}=z$ intersect at a point, find the value of m.

EXERCISE 6.6

1. Find the vector equation of a plane which is at a distance of 7 units from the orgin having $3,-4,5$ as direction ratios of a normal to it.
2. Find direction cosines of the normal to the plane $12x+3y-4z=65$. Also, find the non-parametric form of vector equation of a plane and the length of the perpendicular to the plane from the origin.
3. Find the vector and Cartesian equations of the plane passing through the point with position vector $2\hat{i}+6\hat{j}+3\hat{k}$ and normal to the vector $\hat{i}+3\hat{j}+5\hat{k}$.
4. A plane passes through the point $(-1,1,2)$ and the normal to the plane of magnitude $3\sqrt{3}$ makes equal acute angles with the coordinate axes. Find the equation of the plane.
6. If a plane meets the coordinate axes at A,B,C such that the centroid of the triangle ABC is the point (u,v,w), find the equation of the plane.

EXERCISE 6.9

1. Find the Cartesian equation of the plane passing through the line of intersection of the planes $\vec{r}\cdot(2\hat{i}-7\hat{j}+4\hat{k})=3$ and $3x-5y+4z+11=0$, and the point $(-2,1,3)$.

3. Find the angle between the line $\vec{r}=(2\hat{i}-\hat{j}+\hat{k})+t(\hat{i}+2\hat{j}-2\hat{k})$ and the plane $\vec{r}\cdot(6\hat{i}+3\hat{j}+2\hat{k})=8$.

4. Find the angle between the planes $\vec{r}\cdot(\hat{i}+\hat{j}-2\hat{k})=3$ and $2x-2y+z=2$.

5. Find the Cartesian equation of the plane which passes through the point $(3,4,-1)$ and is parallel to the plane $2x-3y+5z+7=0$. Also, find the distance between the two planes.

Example 6.4

With usual notations, in any triangle ABC, prove by vector method that

$$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$$

Example 6.9

A particle acted upon by constant forces $2\hat{i}+5\hat{j}+6\hat{k}$ and $-\hat{i}-2\hat{j}-\hat{k}$ is displaced from the point $(4,-3,-2)$ to the point $(6,1,-3)$. Find the total work done by the forces.

Example 6.10

A particle is acted upon by the forces $3\hat{i}-2\hat{j}+2\hat{k}$ and $2\hat{i}+\hat{j}-\hat{k}$ is displaced from the point $(1,3,-1)$ to the point $(4,-1,\lambda)$. If the work done by the forces is 16 units, find the value of λ.

Example 6.11

Find the magnitude and the direction cosines of the torque about the point $(2,0,-1)$ of a force $2\hat{i}+\hat{j}-\hat{k}$ whose line of action passes through the origin.

Example 6.17

If the vectors $\vec{a},\vec{b},\vec{c}$ are coplanar, then prove that the vectors $\vec{a}+\vec{b},\vec{b}+\vec{c},\vec{c}+\vec{a}$ are also coplanar.

Example 6.18

For any three vector $\vec{a},\vec{b},\vec{c}$ prove that $[\vec{a}+\vec{c},\vec{a}+b,\vec{a}+\vec{b}+\vec{c}]=[\vec{a},\vec{b},\vec{c}]$.

Example 6.19

Prove that $[\vec{a}\times\vec{b},\ \vec{b}\times\vec{c},\ \vec{c}\times\vec{a}]=[\vec{a},\vec{b},\vec{c}]^2$.

Example 6.20

Prove that $(\vec{a}\cdot(\vec{b}\times\vec{c}))\vec{a}=(\vec{a}\times\vec{b})\times(\vec{a}\times\vec{c})$.

Example 6.26

Find the vector (parametric) and Cartesian equations of the line passing through $(-4,2,-3)$ and is parallel to the line $\frac{-x-2}{4}=\frac{y+3}{-2}=\frac{2z-6}{3}$.

Example 6.28

Find the angle between the straight line $\frac{x+3}{2}=\frac{y-1}{2}=\frac{z}{-1}$ with coordinate axes.

Example 6.29

Find the acute angle between the lines $\vec{r} = (\hat{i} + 2\hat{j} + 4\hat{k}) + t(2\hat{i} + 2\hat{j} + \hat{k})$ and the straight line passing through the points $(5,1,4)$ and $(9,2,12)$.

Example 6.30

Find the angle between the straight lines $\frac{x-4}{2} = \frac{y}{1} = \frac{z+1}{-2}$ and $\frac{x-1}{4} = \frac{y+1}{-4} = \frac{z-2}{2}$ and state whether they are parallel or perpendicular.

Example 6.31

Show that the straight line passing through the points $A(6,7,5)$ and $B(8,10,6)$ is perpendicular to the straight line passing through the points $C(10,2,-5)$ and $D(8,3,-4)$.

Example 6.33

Find the point of intersection of the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$.

Example 6.34

Find the vectpr equation (parametric) of a straight line passing through the point of intersection of the straight lines $\vec{r} = \hat{i} + 3\hat{j} - \hat{k} + t(2\hat{i} + 3\hat{j} + 2\hat{k})$ and $\frac{x-2}{1} = \frac{y-4}{2} = \frac{z+3}{4}$, and perpendicular to both straight lines.

Example 6.35

Determine whether the pair of straight lines $\vec{r} = (2\hat{i} + 6\hat{j} + 3\hat{k}) + t(2\hat{i} + 3\hat{j} + 4\hat{k})$,

$\vec{r} = (2\hat{j} - 3\hat{k}) + s(\hat{i} + 2\hat{j} + 3\hat{k})$ are parallel.Find the shortest distance between them.

Example 6.36

Find the shortest distance between the given straight lines $\vec{r} = (2\hat{i} + 3\hat{j} + 4\hat{k}) + t(-2\hat{i} + \hat{j} - 2\hat{k})$ and $\frac{x-3}{2} = \frac{y}{-1} = \frac{z+2}{2}$.

Example 6.38

Find the vector and Cartesian equations of a plane which is at a distance of 12 units from the origin and perpendicular to $6\hat{i} + 2\hat{j} - 3\hat{k}$.

Example 6.40

Find the direction cosines of the normal and the length of the normal from the origin to the plane $\vec{r} \cdot (3\hat{i} - 4\hat{j} + 12\hat{k}) = 5$.

Example 6.41

Find the vector and Cartesian equations of the plane passing through the point with position vector $4\hat{i} + 2\hat{j} - 3\hat{k}$ and normal to vector $2\hat{i} - \hat{j} + \hat{k}$.

Example 6.42

A variable plane moves in such a way that the sum of the reciprocals of its intercepts on the coordinate axes is a constant. Show that the plane passes through a fixed point

Example 6.52

Find the distance between the planes $\vec{r} \cdot \left(2\hat{i} - \hat{j} - 2\hat{k}\right) = 6$ and $\vec{r} \cdot \left(6\hat{i} - 3\hat{j} - 6\hat{k}\right) = 27$.

Example 6.56

Find the point where the straight line $\vec{r}=(2\hat{i}-\hat{j}+2\hat{k})+t(3\hat{i}+4\hat{j}+2\hat{k})$ meets the plane $x-y+z-5=0$.

EXERCISE 7.1

1. A point moves along a line in such a way that after t seconds its distance from the origin is $s=2t^2+3t$ metres.
   (i) Find the average velocity of the points between $t=3$ and $t=6$ seconds.
   (ii) Find the instantaneous velocities at $t=3$ and $t=6$ seconds.
5. If the mass $m(x)$ (in kilograms) of a thin rod of length x (in metres) is given by, $m(x)=\sqrt{3x}$ then what is the rate of change of mass with respect to the length when it is $x=3$ and $x=27$ metres.

EXERCISE 7.2

2. Find the point on the curve $y=x^2-5x+4$ at which the tangent is parallel to the line $3x+y=7$.
3. Find the points on the curve $y=x^3-6x^2+x+3$ where the normal is parallel to the line $x+y=1729$.
4. Find the points on the curve $y^2-4xy=x^2+5$ for which the tangent is horizontal.
5. Find the tangent and normal to the following curves at the given points on the curve.
   (i) $y=x^2-x^4$ at $(1,0)$ (ii) $y=x^4+2e^x$ at $(0,2)$
   (iii) $y=x\sin x$ at $\left(\frac{\pi}{2},\frac{\pi}{2}\right)$ (iv) $x=\cos t, y=2\sin^2 t$ at $t=\frac{\pi}{3}$ (each 3 marks)
6. Find the equations of the tangents to the curve $y=1+x^3$ for which the tangent is orthogonal with the line $x+12y=12$.
7. Find the equations of the tangents to the curve $y=\frac{x+1}{x-1}$ which are parallel to the line $x+2y=6$.
8. Find the equation of tangent and normal to the curve given by $x=7\cos t$ and $y=2\sin t, t\in\mathbb{R}$ at any point on the curve.

EXERCISE 7.3

2. Using the Rolle's theorem, determine the values of x at which the tangent is parallel to the x-axis for the following functions : (each 3 marks)
   (i) $f(x)=x^2-x, x\in[0,1]$ (ii) $f(x)=\frac{x^2-2x}{x+2}, x\in[-1,6]$
   (iii) $f(x)=\sqrt{x}-\frac{x}{3}, x\in[0,9]$
4. Using the Lagrange's mean value theorem determine the values of x at which the tangent is parallel to the secant line at the end points of the given interval : (each 3 marks)
   (i) $f(x)=x^3-3x+2, x\in[-2,2]$ (ii) $f(x)=(x-2)(x-7), x\in[3,11]$

5. Show that the value in the conclusion of the mean value theorem for

(i) $f(x)=\frac{1}{x}$ on a closed interval of positive numbers $[a,b]$ is $\sqrt{ab}$.

(ii) $f(x)=Ax^2+Bx+C$ on any interval $[a,b]$ is $\frac{a+b}{2}$. (each 3 marks)

6. A race car is racing at 20[th] km. If its speed never exceeds 150 km/hr, what is the maximum distance he can cover in the next two hours?

7. Suppose that the function $f(x), f'(x)\le 1$ for all $1\le x\le 4$. Show that $f(4)-f(1)\le 3$.

8. Does there exist a differentiable function $f(x)$ such that $f(0)=-1, f(2)=4$ and $f'(x)\le 2$ for all x. Justify your answer.

9. Show that there lies a point on the curve $f(x)=x(x+3)e^{-\frac{\pi}{2}}, -3\le x\le 0$ where tangent drawn is parallel to the x-axis.

10. Using mean value theorem prove that for, $a>0,\ b>0, |e^{-a}-e^{-b}|<|a-b|$.

EXERCISE 7.4

1. Write the Maclaurin series expansion of the following functions: (each 3 marks)
   (i) e^x (ii) $\sin x$ (iii) $\cos x$
   (iv) $\log(1-x); -1\le x<1$

2. Write the Taylor series expansion of the function $\log x$ about $x=1$ upto three non-zero terms for $x>0$.

3. Expand $\sin x$ in ascending powers $x-\frac{\pi}{4}$ upto three non-zero terms.

4. Expand the polynomial $f(x)=x^2-3x+2$ in powers of $x-1$.

EXERCISE 7.5

Evaluate the following limits, if necessary use I'Hôpital Rule : (each 3 marks)

1. $\lim_{x\to 0}\frac{1-\cos x}{x^2}$ 2. $\lim_{x\to\infty}\frac{2x^2-3}{x^2-5x+3}$ 3. $\lim_{x\to\infty}\frac{x}{\log x}$

4. $\lim_{x\to\frac{\pi}{2}}\frac{\sec x}{\tan x}$ 5. $\lim_{x\to\infty}e^{-x}\sqrt{x}$ 6. $\lim_{x\to 0}\left(\frac{1}{\sin x}-\frac{1}{x}\right)$

7. $\lim_{x\to 1^+}\left(\frac{2}{x^2-1}-\frac{x}{x-1}\right)$

EXERCISE 7.6

1. Find the absolute extrema of the following functions on the given closed interval.

   (i) $f(x)=x^2-12x+10$; $[1,2]$ (iii) $f(x)=6x^{\frac{4}{3}}-3x^{\frac{1}{3}}$; $[-1,1]$

   (iv) $f(x)=2\cos x+\sin 2x$; $\left[0,\frac{\pi}{2}\right]$ (each 3 marks)

EXERCISE 7.7

2. Find the local extrema for the following functions using second derivative test:
   (iii) $f(x)=x^2e^{-2x}$

EXERCISE 7.8

2. Find two positive numbers whose product is 20 and their sum is minimum.

Example 7.3

A person learnt 100 words for an English test. The number of words the person remembers in t days after learning is given by $W(t)=100\times(1-0.1t)^2$, $0\le t\le 10$. What is the rate at which the person forgets the words '2' days after learning?

Example 7.4

A particle moves so that the distance moved is according to the law $s(t)=\frac{t^3}{3}-t^2+3$. At what time the velocity and acceleration are zero respectively?

Example 7.11

Find the equations of tangent and normal to the curve $y=x^2+3x-2$ at the point $(1,2)$.

Example 7.12

For what values of x the tangent of the curve $y=x^3-3x^2+x-2$ is parallel to the line $y=x$.

Example 7.13

Find the equation of the tangent and normal to the curve $x=2\cos 3t$ and $y=3\sin 2t, t\in R$.

Example 7.16

Find the angle of intersection of the curve $y=\sin x$ with the positive x-axis.

Example 7.19

Compute the value of 'c' satisfied by the Rolle's theorem for the function $f(x)=x^2(1-x)^2, x\in[0,1]$.

Example 7.20

Find the values in the interval $\left(\frac{1}{2},2\right)$ satisfied by Rolle's theorem for the function $f(x)=x+\frac{1}{x}, x\in\left[\frac{1}{2},2\right]$.

Example 7.21

Compute the value of 'c' satisfied by Rolle's theorem for the function $f(x)=\log\left(\frac{x^2+6}{5x}\right)$ in the interval $[2,3]$.

Example 7.22

Without actually solving show that the equation $x^4+2x^3-2=0$ has only one real root in the interval $(0,1)$.

Example 7.23

Prove using the Rolle's theorem that between any two distinct real zeros of the polynomial

$$a_nx^n+a_{n-1}x^{n-1}+\cdots+a_1x+a_0$$

there is a zero of the polynomial

$$na_nx^{n-1}+(n-1)a_{n-1}x^{n-2}+\cdots+a_1.$$

Example 7.24

Prove that there is a zero of the polynomial, $2x^3-9x^2-11x+12$ in the interval $(2,7)$ given that 2 and 7 are the zeros of the polynomial $x^4-6x^3-11x^2+24x+28$.

Example 7.25

Find the values in the interval $(1,2)$ of the mean value theorem satisfied by the function $f(x)=x-x^2$ for $1\le x\le 2$.

Example 7.26

A truck travels on a toll road with a speed limit of 80 km/hr. The truck completes a 164 km journey in 2 hours. At the end of the toll road the trucker is issued with a speed violation ticket. Justify this using the Mean Value Theorem.

Example 7.27

Suppose $f(x)$ is a differentiable function for all x with $f'(x)\le 29$ and $f(2)=17$ then what is the maximum value of $f(7)$?

Example 7.28

Prove using mean value theorem that, $|\sin\alpha-\sin\beta|\le|\alpha-\beta|$, $\alpha,\beta\in\mathbb{R}$.

Example 7.29

A thermometer was taken from a freezer and placed in a boiling water. It took 22 seconds for the thermometer to raise from $-10°C$ to $100°C$. Show that the rate of change of temperature at some time t is $5°C$ per second.

Example 7.37

If $\lim\limits_{\theta\to 0}\left(\dfrac{1-\cos m\theta}{1-\cos n\theta}\right)=1$, then prove that $m=\pm n$.

Example 7.38

Evaluate : $\lim\limits_{x\to 1^-}\left(\dfrac{\log(1-x)}{\cot(\pi x)}\right)$.

Example 7.39

Evaluate : $\lim\limits_{x\to 0^+}\left(\dfrac{1}{x}-\dfrac{1}{e^x-1}\right)$.

Example 7.40

Evaluate : $\lim\limits_{x\to 0^+} x\log x$.

Example 7.41

Evaluate : $\lim\limits_{x\to\infty}\left(\dfrac{x^2+17x+29}{x^4}\right)$.

Example 7.42

Evaluate : $\lim\limits_{x\to\infty}\left(\dfrac{e^x}{x^m}\right), m\in N$.

Example 7.46

Prove that the function $f(x)=x^2+2$ is strictly increasing in the interval $(2,7)$ and strictly decreasing in the interval $(-2,0)$.

Example 7.48

Find the absolute maximum and absolute minimum values of the function $f(x)=2x^3+3x^2-12x$ on $[-3,2]$.

Example 7.49

Find the absolute extrema of the function $f(x)=3\cos x$ on the closed interval $[0,2\pi]$.

Example 7.52

Prove that the function $f(x)=x-\sin x$ is increasing on the real line. Also discuss for the existence of local extrema.

EXERCISE 8.1

2. Use the linear approximation to find approximate values of (each 3 marks)

   (i) $(123)^{\frac{2}{3}}$ (ii) $\sqrt[4]{15}$ (iii) $\sqrt[3]{26}$.

3. Find a linear approximation for the following functions at the indicated points.

   (i) $f(x)=x^3-5x+12,\ x_0=2$ (ii) $g(x)=\sqrt{x^2+9},\ x_0=-4$

   (iii) $h(x)=\dfrac{x}{x+1},\ x_0=1$. (each 3 marks)

4. The radius of a circular plate is measured as 12.65 cm instead of the actual length 12.5 cm.

   (i) Absolute error (ii) Relative error

   (iii) Percentage error, in calculating the area (each 3 marks)

5. A sphere is made of ice having radius 10 cm. Its radius decreases from 10 cm to 9.8 cm. Find approximations for the following: (each 3 marks)

   (i) decrease in the volume (ii) change in the surface area

6. The time T, taken for a complete oscillation of a single pendulum with length l, is given by the equation $T=2\pi\sqrt{\dfrac{l}{g}}$, where g is a constant. Find the approximate percentage error in the calculated value of T corresponding to an error of 2 percent in the value of t.

7. Show that the percentage error in the n^{th} root of a number is approximately $\dfrac{1}{n}$ times the percentage error in the number.

EXERCISE 8.2

3. Find Δf and df for the function f for the indicated values of $x, \Delta x$ and compare

   (i) $f(x)=x^3-2x^2; x=2,\ \Delta x=dx=0.5$

   (ii) $f(x)=x^2+2x+3;\ x=-0.5,\ \Delta x=dx=0.1$ (each 3 marks)

4. Assuming $\log_{10}e=0.4343$, find an approximate value of $\log_{10}1003$.

8. In a newly developed city, it is estimated that the voting population (in thousands) will increase according to $V(t)=30+12t^2-t^3$, $0\le t\le 8$ where t is the time in years. Find the approximate change in voters for the time change from 4 to $4\frac{1}{6}$ year.

10. A circular plate expands uniformly under the influence of heat. If it's radius increases from 10.5cm to 10.75cm, then find an approximate change in the area and the approximate percentage changes in the area.

11. A coat of paint of thickness 0.2 cm is applied to the faces of a cube whose edge is 10 cm. Use the differentials to find approximately how many cubic centimeters of paint is used to paint this cube. Also calculate the exact amount of paint used to paint this cube.

EXERCISE 8.3

5. Let $g(x,y)=\frac{x^2y}{x^4+y^2}$ for $(x,y)\neq(0,0)$ and $g(0,0)=0$.

   (i) Show that $\lim_{(x,y)\to(0,0)} g(x,y)=0$ along every line $y=mx, m\in\mathbb{R}$.

   (ii) Show that $\lim_{(x,y)\to(0,0)} g(x,y)=\frac{k}{1+k^2}$ along every parabola $y=kx^2, k\in\mathbb{R}\setminus\{0\}$.

6. Show that $f(x,y)=\frac{x^2-y^2}{y^2+1}$ is continuous at every $(x,y)\in\mathbb{R}^2$.

7. Let $g(x,y)=\frac{e^y\sin x}{x}$, for $x\neq 0$ and $g(0,0)=1$. Show that g is continuous at $(0,0)$.

EXERCISE 8.4

1. Find the partial derivatives of the following functions at the indicated points.

   (i) $f(x,y)=3x^2-2xy+y^2+5x+2, (2,-5)$

   (ii) $g(x,y)=3x^2+y^2+5x+2, (1,-2)$

   (iii) $h(x,y,z)=x\sin(xy)+z^2x, \left(2,\frac{\pi}{4},1\right)$

   (iv) $G(x,y)=e^{x+3y}\ln(x^2+y^2), (-1,1)$ (each 3 marks)

4. If $u=\log(x^3+y^3+z^3)$ find $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}$.

8. If $w(x,y)=xy+\sin(xy)$, then prove that $\frac{\partial^2 w}{\partial y\partial x}=\frac{\partial^2 w}{\partial x\partial y}$.

EXERCISE 8.5

1. If $w(x,y)=x^3-3xy+2y^2, x,y\in\mathbb{R}$, find the linear approximation for w at $(1,-1)$.
2. Let $z(x,y)=x^2y+3xy^4, x,y\in\mathbb{R}$. Find the linear approximation for z at $(2,-1)$.
3. If $v(x,y)=x^2-xy+\frac{1}{4}y^2+7, \quad x, y\in\mathbb{R}$, find the differential dv.
4. Let $V(x,y,z)=xy+yz+zx, \quad x,y,z\in\mathbb{R}$. Find the differential dV.

EXERCISE 8.7

4. If $u(x,y)=\frac{x^2+y^2}{\sqrt{x+y}}$, prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{3}{2}u$.

Example 8.1

Find the linear approximation for $f(x)=\sqrt{1+x}, x\geq -1$, at $x_0=3$. Use the linear approximation to estimate $f(3.2)$.

Example 8.2

Use linear approximation to find an approximate value of $\sqrt{9.2}$ without using a calculator.

Example 8.3

Let us assume that the shape of a soap bubble is a sphere. Use linear approximation to approximate the increase in the surface area of a soap bubble as its radius increases from 5 cm to 5.2 cm. Also, calculate the percentage error.

Example 8.5

Let $f,g:(a,b)\to\mathbb{R}$ be differentiable functions. Show that $d(fg)=fdg+gdf$.

Example 8.7

If the radius of a sphere, with radius 10 cm, were to decrease by 0.1 cm, approximately how much would its volume decrease?

Example 8.8

Let $f(x,y)=\frac{3x-5y+8}{x^2+y^2+1}$ for all $(x,y)\in\mathbb{R}^2$. Show that f is continuous on $\mathbb{R}^2$.

Example 8.9

Consider $f(x,y)=\frac{xy}{x^2+y^2}$ if $(x,y)\neq(0,0)$ and $f(0,0)=0$. Show that f is not continuous at $(0,0)$ and continuous at all other points of $\mathbb{R}^2$.

Example 8.10

Consider $g(x,y)=\frac{2x^2y}{x^2+y^2}$ if $(x,y)\neq(0,0)$ and $g(0,0)=0$. Show that g is continuous on $\mathbb{R}^2$.

Example 8.16

If $w(x,y,z)=x^2y+y^2z+z^2x, x,y,z\in\mathbb{R}$, find the differential dw.

Example 8.17

Let $U(x,y,z)=x^2-xy+3\sin z, x,y,z\in\mathbb{R}$. Find the linear approximation for U at $(2,-1,0)$.

EXERCISE 9.3

1. Evaluate the following definite integrals :

(ii) $\int_{-1}^{1}\frac{dx}{x^2+2x+5}$ (iii) $\int_{0}^{1}\sqrt{\frac{1-x}{1+x}}\,dx$

(iv) $\int_{0}^{\frac{\pi}{2}}e^x\left(\frac{1+\sin x}{1+\cos x}\right)dx$ (v) $\int_{0}^{\frac{\pi}{2}}\sqrt{\cos\theta}\sin^3\theta\,d\theta$ (vi) $\int_{0}^{1}\frac{1-x^2}{(1+x^2)^2}dx$

2. Evaluate the following integrals using properties of integration:

(i) $\int_{-5}^{5} x\cos\left(\frac{e^x-1}{e^x+1}\right)dx$ (iii) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin^2 x\, dx$

(iv) $\int_{0}^{2\pi} x\log\left(\frac{3+\cos x}{3-\cos x}\right)dx$ (x) $\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{1}{1+\sqrt{\tan x}}dx$

EXERCISE 9.4

Evaluate the following:

1. $\int_0^1 x^3 e^{-2x}\, dx$

2. $\int_0^1 \frac{\sin(3\tan^{-1}x)\tan^{-1}x}{1+x^2}dx$

3. $\int_0^{\frac{1}{\sqrt{2}}} \frac{e^{\sin^{-1}x}\sin^{-1}x}{\sqrt{1-x^2}}dx$

4. $\int_0^{\frac{\pi}{2}} x^2\cos 2x\, dx$

EXERCISE 9.5

Evaluate the following:

(1) $\int_0^{\frac{\pi}{2}} \frac{dx}{1+5\cos^2 x}$ (2) $\int_0^{\frac{\pi}{2}} \frac{dx}{5+4\sin^2 x}$

EXERCISE 9.6

1. Evaluate the following:

(iii) $\int_0^{\frac{\pi}{4}} \sin^6 2x\, dx$ (iv) $\int_0^{\frac{\pi}{6}} \sin^5 3x\, dx$

(v) $\int_0^{\frac{\pi}{2}} \sin^2 x\cos^4 x\, dx$ (vi) $\int_0^{2\pi} \sin^7\frac{x}{4}\, dx$ (vii) $\int_0^{\frac{\pi}{2}} \sin^3\theta\cos^5\theta\, d\theta$ (viii) $\int_0^1 x^2(1-x)^3\, dx$

EXERCISE 9.8

2. Find the area of the region bounded by $2x-y+1=0$, $y=-1$, $y=3$ and y-axis.

EXERCISE 9.9

1. Find the volume of the solid generated by revolving the region enclosed by $y=2x^2, y=0$ and $x=1$ about the x-axis, using integration.

Example 9.5

Evaluate : $\int_0^3 (3x^2-4x+5)dx$.

Example 9.6

Evaluate : $\int_0^1 \frac{2x+7}{5x^2+9}dx$.

Example 9.8

Evaluate : $\int_0^{\frac{\pi}{3}} \frac{\sec x \tan x}{1+\sec^2 x}dx$.

Example 9.9

Evaluate : $\int_0^9 \frac{1}{x+\sqrt{x}}\, dx$.

Example 9.23

If $f(x)=f(a+x)$, then $\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx$

Example 9.26

Evaluate : $\int_0^a \frac{f(x)}{f(x)+f(a-x)}dx.$

Example 9.29

Evaluate : $\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}dx$.

Example 9.31

Evaluate $\int_0^{\pi} x^2 \cos nx dx$, where n is a positive integer.

Example 9.32

Evaluate : $\int_0^1 e^{-2x}(2x^3-x-1)dx$.

Example 9.33

Evaluate : $\int_0^{2\pi} x^2 \sin nx dx$, where n is a positive integer.

Example 9.34

Evaluate : $\int_{-1}^1 e^{-2x}(1-x^2)dx$.

Example 9.35

Evaluate $\int_b^{\infty} \frac{1}{a^2+x^2}dx, a>0, b\in\mathbb{R}$.

Example 9.38

Evaluate $\int_0^{\frac{\pi}{2}} \begin{vmatrix} \cos^4 x & 7 \\ \sin^5 x & 3 \end{vmatrix} dx$.

Example 9.39

Find the values of the following:

(i) $\int_0^{\frac{\pi}{2}} \sin^5 x \cos^4 x\, dx$ (ii) $\int_0^{\frac{\pi}{2}} \sin^4 x \cos^6 x\, dx$.

Example 9.41

Evaluate $\int_0^1 x^5 \left(1-x^2\right)^5 dx$.

Example 9.42

Evaluate $\int_0^1 x^3 (1-x)^4 dx$.

Example 9.44

Evaluate $\int_0^\infty e^{-ax} x^n dx$, where $a > 0$.

Example 9.48

Find the area of the region bounded by the line $7x-5y=35$, $x-$axis and the lines $x=-2$ and $x=3$.

Example 9.62

Find the volume of a sphere of radius *a*.

Example 9.67

Find, by integration, the volume of the solid generated by revolving about *y*-axis the region bounded between the parabola $x = y^2 + 1$, the *y*-axis, and the lines $y=1$ and $y=-1$.

EXERCISE 10.3

1. Find the differential equation of the family of (i) all non-vertical lines in a plane (ii) all non-horizontal lines in a plane.
2. Form the differential equation of all straight lines touching the circle $x^2+y^2=r^2$.
3. Find the differential equation of the family of circles passing through the origin and having their centres on the x-axis.
4. Find the differential equation of the family of all the parabolas with latus rectum $4a$ and whose axes are parallel to the x-axis.
5. Find the differential equation of the family of parabolas with vertex at $(0,-1)$ and having axis along the y-axis.
6. Find the differential equations of the family of all the ellipses having foci on the y-axis and centre at the origin.
7. Find the differential equation corresponding to the family of curves represented by the equation $y = Ae^{8x} + Be^{-8x}$, where A and B are arbitrary constants.

EXERCISE 10.4

3. The slope of the tangent to the curve at any point is the reciprocal of four times the ordinate at that point. The curve passes through (2,5). Find the equation of the curve.
5. Show that $y = ax + \frac{b}{x}, x \neq 0$ is a solution of the differential equation $x^2y'' + xy' - y = 0$.

6. Show that $y = ae^{-3x} + b$, where a and b are arbitrary constants, is a solution of the differential equation $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} = 0$.

7. Show that the differential equation representing the family of curves $y^2 = 2a\left(x + a^{\frac{2}{3}}\right)$, where a is a positive parameter, is $\left(y^2 - 2xy\frac{dy}{dx}\right)^3 = 8\left(y\frac{dy}{dx}\right)^5$.

EXERCISE 10.5

4. Solve the following differential equations:

(i) $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$

(iii) $\sin\frac{dy}{dx} = a,\ y(0) = 1$ (iv) $\frac{dy}{dx} = e^{x+y} + x^3e^y$

EXERCISE 10.7

Solve the following Linear differential equations:

1. $\cos x\frac{dy}{dx} + y\sin x = 1$

Example 10.4

Find the differential equation of the family of circles passing through the points $(a,0)$ and $(-a,0)$.

Example 10.6

Find the differential equation of the family of all ellipses having foci on the x-axis and centre at the origin.

Example 10.9

Show that $y = 2(x^2-1) + ce^{-x^2}$ is a solution of the differential equation $\frac{dy}{dx} + 2xy - 4x^3 = 0$.

Example 10.10

Show that $y = a\cos(\log x) + b\sin(\log x), x > 0$ is a solution of the differential equation $x^2y'' + xy' + y = 0$.

Example 10.11

Solve $(1+x^2)\frac{dy}{dx} = 1 + y^2$.

Example 10.16

Solve : $\frac{dy}{dx} = (3x + y + 4)^2$.

Example 10.20

Solve $y^2 + x^2\frac{dy}{dx} = xy\frac{dy}{dx}$.

EXERCISE 11.1

1. Suppose X is the number of tails occurred when three fair coins are tossed once simultaneously. Find the values of the random variable X and number of points in its inverse images.
2. An urn contains 5 mangoes and 4 apples. Three fruits are taken at random. If the number of taken fruits are apple, then find the values of the random variable and number of points in its inverse images.
3. Two balls are chosen randomly from an urn containing 6 red and 8 black balls. Suppose that we win ₹15for each red ball selected and we lose ₹10 for each black ball selected. X denotes of winning amount, then find the values of X and number of points in its inverse images.
4. A six sided die is marked '2' on one face, '3' on two of its faces, and '4' on remaining three faces. The die is thrown twice. If X denotes the total score in two throws, find the values of the random variable and number of points in its inverse images.

EXERCISE 11.2

1. Three fair coins are tossed simultaneously. Find the probability mass function for number of heads occurred.

EXERCISE 11.3

1. The probability density function of X is given by $f(x)=\begin{cases} kxe^{-2x} & \text{for } x>0 \\ 0 & \text{for } x\le 0 \end{cases}$

   Find the value of k.

6. If X is the random variable with distribution function $F(x)$ given by,

$$F(x)=\begin{cases} 0, & x<0 \\ \frac{1}{2}\left(x^2+x\right) & 0\le x<1 \\ 1, & x\ge 1 \end{cases}$$

then find (ii) $P(0.3\le X\le 0.6)$.

EXERCISE 11.4

3. If μ and σ^2 are the mean and variance of the discrete random variable X, and $E(X+3)=10$ and $E(X+3)^2=116$, find μ and σ^2.
5. A commuter train arrives punctually at a station every half an hour. Everyday in the morning, a student leaves his home to the railway station. Let X denote the amount of time, in minutes, that the student waits for the train from the time he reaches the railway station. The pdf of X is

$$f(x)=\begin{cases} \frac{1}{30} & 0<x<30 \\ 0 & \text{elsewhere} \end{cases}$$

   Obtain and interpret the expected value of the random variable X.

8. A lottery with 600 tickets gives one prize of ₹200, four prices of ₹100, and six prizes of ₹50 If the ticket costs is ₹2, find the expected winning amount of a ticket.

EXERCISE 11.5

2. The probability that Q hits a target at any trial is $\frac{1}{4}$. He tries at the target 10 times. Find the probability that he hits the target (i) exactly 4 times (ii) at least one time.

5. A retailer purchases a certain kind of electronic device from a manufacturer. The manufacturer indicates that the defective rate of the device is 5%. The inspector of the retailer randomly picks 10 items from a shipment. What is the probability that there will be (i) at least one defective item (ii) exactly two defective items.

8. If $X \sim B(n, p)$ such that $4P(X = 4) = P(x = 2)$ and $n = 6$. Find the distribution, mean and standard deviation.

9. In a binomial distribution consisting of 5 independent trials, the probability of 1 and 2 successes are 0.4096 and 0.2048 respectively. Find the mean and variance of the distribution.

Example 11.1:

Suppose two coins are tossed once. If X denotes the number of tails, (i) write down the sample space (ii) find the inverse image of 1 (iii) the values of the random variable, and number of elements in its inverse images.

Example 11.2:

Suppose a pair of unbiased dice is rolled once. If X denotes the total score of two dice, write down (i) the sample space (ii) the values taken by the random variable X, (iii) the inverse image of 10, and (iv) the number of elements in inverse image of X.

Example 11.3

An urn contains 2 white balls and 3 red balls. A sample of 3 balls are chosen at random from the urn. If X denotes the number of red balls chosen, find the values taken by the random variable X and its number of if inverse images.

Example 11. 4:

Two balls are chosen randomly from an urn containing 6 white and 4 black balls. Suppose that we win ₹ 30 for each black ball selected and the loss ₹ 20 for each white ball selected. If X denotes the winning amount, then find the values of X and number of points in its inverse images.

Example 11.5

Two fair coins are tossed simultaneously (equivalent to a fair coin is tossed twice). Find the probability mass function for number of heads occurred.

Example 11.11

Find the constant C such that the function

$$f(x) = \begin{cases} Cx^2 & 1 < x < 4 \\ 0 & \text{Otherwise} \end{cases}$$

is a density function, and compute (i) $P(1.5 < X < 3.5)$ (ii) $P(X \le 2)$ (iii) $P(3 < X)$.

Example 11.13

If X is the random variable with distribution function $F(x)$ given by,

$$F(x)=\begin{cases}0, & x<0\\ x, & 0\le x<1\\ 1, & 1\le x\end{cases}$$

then find (i) the probability density function $f(x)$ (ii) $P(0.2\le X\le 0.7)$.

Example 11.15

Let X be random variable denoting the life time of an electrical equipment having probability density function

$$f(x)=\begin{cases}k\,e^{-2x} & \text{for } x>0\\ 0 & \text{for } x\le 0.\end{cases}$$

Find (i) the value of k (ii) Distribution function (iii) $P(X<2)$

(iv) calculate the probability that X is at least for four units of life time (v) $P(X=3)$.

Example 11.16

Suppose that $f(x)$ given below represents a probability mass function,

x	1	2	3	4	5	6
$f(x)$	c^2	$2c^2$	$3c^2$	$4c^2$	c	$2c$

Find (i) the value of c (ii) Mean and variance.

Example 11.19 (each 3 marks)

Find the binomial distribution function for each of the following.

(i) Five fair coins are tossed once and X denotes the number of heads.

(ii) A fair die is rolled 10 times and X denotes the number of times 4 appeared.

EXERCISE 12.2

6. Construct the truth table for the following statements. (each 3 marks)

   (i) $\neg p\wedge\neg q$ (ii) $\neg(p\wedge\neg q)$ (iii) $(p\vee q)\vee\neg q$ (iv) $(\neg p\rightarrow r)\wedge(p\leftrightarrow q)$

9. Prove that $q\rightarrow p\equiv\neg p\rightarrow\neg q$.

OTHER QUESTIONS TAKEN FROM THE TEXT BOOK

Theorem 1.1

For every square matrix A of order n, $A(\text{adj } A) = (\text{adj } A)A = |A| I_n$.

Theorem 1.2

If a square matrix has an inverse, then it is unique.

Theorem 1.3

Let A be square matrix of order n. Then, A^{-1} exists if and only if A is non-singular.

Chapter 2

If z_1 and z_2 are two non-zero complex numbers

(1) $\arg(z_1 z_2) = \arg z_1 + \arg z_2$

(2) $\arg\left(\dfrac{z_1}{z_2}\right) = \arg z_1 - \arg z_2$

Chapter 9

Property 6

$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

Property 7

$\int_0^{2a} f(x)dx = \int_0^a [f(x) + f(2a-x)]dx$.

Property 8

If $f(x)$ is an even function, then $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$.

Property 9

If $f(x)$ is an odd function, then $\int_{-a}^{a} f(x)dx = 0$.

Property 10

If $f(2a-x) = f(x)$, then $\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx$.

Property 11

If $f(2a-x) = -f(x)$, then $\int_0^{2a} f(x)dx = 0$.

Chapter 11

$Var(X) = E(X^2) - [E(X)]^2$

Part – IV
(5 Mark Questions)

EXERCISE 1.1

3. If $F(\alpha)=\begin{bmatrix}\cos\alpha & 0 & \sin\alpha\\ 0 & 1 & 0\\ -\sin\alpha & 0 & \cos\alpha\end{bmatrix}$, show that $[F(\alpha)]^{-1}=F(-\alpha)$.

14. If $A=\begin{bmatrix}0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 1 & 0\end{bmatrix}$, show that $A^{-1}=\frac{1}{2}(A^2-3I)$.

15. Decrypt the received encoded message $[2\ \ -3][20\ \ 4]$ with the encryption matrix $\begin{bmatrix}-1 & -1\\ 2 & 1\end{bmatrix}$ and the decryption matrix as its inverse, where the system of codes are described by the numbers 1-26 to the letters $A-Z$ respectively, and the number 0 to a blank space.

EXERCISE 1.2

3. Find the inverse of each of the following by Gauss-Jordan method: (each 5 marks)

(i) $\begin{bmatrix}2 & -1\\ 5 & -2\end{bmatrix}$ (ii) $\begin{bmatrix}1 & -1 & 0\\ 1 & 0 & -1\\ 6 & -2 & -3\end{bmatrix}$ (iii) $\begin{bmatrix}1 & 2 & 3\\ 2 & 5 & 3\\ 1 & 0 & 8\end{bmatrix}$

EXERCISE 1.3

1. Solve the following system of linear equations by matrix inversion method:
   (iii) $2x+3y-z=9,\ x+y+z=9,\ 3x-y-z=-1$
   (iv) $x+y+z-2=0,\ 6x-4y+5z-31=0,\ 5x+2y+2z=13$

2. If $A=\begin{bmatrix}-5 & 1 & 3\\ 7 & 1 & -5\\ 1 & -1 & 1\end{bmatrix}$ and $B=\begin{bmatrix}1 & 1 & 2\\ 3 & 2 & 1\\ 2 & 1 & 3\end{bmatrix}$, find the products AB and BA and hence solve the system of equations $x+y+2z=1, 3x+2y+z=7, 2x+y+3z=2$.

3. A man is appointed in a job with a monthly salary of certain amount and a fixed amount of annual increment. If his salary was ₹.19,800 per month at the end of the first month after 3 years of service and ₹.23,400 per month at the end of the first month after 9 years of service, find his starting salary and his annual increment. (Use matrix inversion method to solve the problem.)

4. 4 men and 4 women can finish a piece of work jointly in 3 days while 2 men and 5 women can finish the same work jointly in 4 days. Find the time taken by one man alone and that of one woman alone to finish the same work by using matrix inversion method.

5. The price of three commodities A, B and C are ₹.x, y and z per units respectively. A person P purchases 4 units of B and sells two units of A and 5 units of C. Person Q purchases 2 units of C and sells 3 units of A and one unit of B. Person R purchases one unit of A and sells 3 unit of B and one unit of C. In the process, P, Q and R earn ₹.15,000, ₹.1,000 and ₹.4,000 respectively. Find the prices per unit of A, B and C. (Use matrix inversion method to solve the problem.)

EXERCISE 1.4

1. Solve the following systems of linear equations by Cramer's rule:
   (iii) $3x+3y-z=11,\ 2x-y+2z=9,\ 4x+3y+2z=25$
   (iv) $\frac{3}{x}-\frac{4}{y}-\frac{2}{z}-1=0, \frac{1}{x}+\frac{2}{y}+\frac{1}{z}-2=0, \frac{2}{x}-\frac{5}{y}-\frac{4}{z}+1=0$

3. A chemist has one solution which is 50% acid and another solution which is 25% acid. How much each should be mixed to make 10 litres of a 40% acid solution ? (Use Cramer's rule to solve the problem).

4. A fish tank can be filled in 10 minutes using both pumps A and B simultaneously. However, pump B can pump water in or out at the same rate. If pump B is inadvertently run in reverse, then the tank will be filled in 30 minutes. How long would it take each pump to fill the tank by itself ?

5. A family of 3 people went out for dinner in a restaurant. The cost of two dosai, three idlies and two vadais is ₹.150. The cost of the two dosai, two idlies and four vadais is ₹.200. The cost of five dosai, four idlies and two vadais is ₹.250. The family has ₹. 350 in hand and they ate 3 dosai and six idlies and six vadais. Will they be able to manage to pay the bill within the amount they had ?

EXERCISE 1.5

1. Solve the following systems of linear equations by Gaussian elimination method:
   (i) $2x-2y+3z=2,\ x+2y-z=3,\ 3x-y+2z=1$
   (ii) $2x+4y+6z=22,\ 3x+8y+5z=27,\ -x+y+2z=2$

2. If ax^2+bx+c is divided by $x+3, x-5,$ and $x-1,$ the remainders are $21, 61$ and 9 respectively. Find a, b and c. (Use Gaussian elimination method.)

3. An amount of ₹. 65,000 is invested in three bonds at the rates of $6\%, 8\%$ and 9% per annum respectively. The total annual income is ₹.4,800. The income from the third bond is ₹.600 more than that from the second bond. Determine the price of each bond. (Use Gaussian elimination method.)

4. A boy is walking along the path $y=ax^2+bx+c$ through the points $(-6,8), (-2,-12)$, and $(3,8)$. He wants to meet his friend at $P(7,60)$. Will he meet his friend? (Use Gaussian elimination method.)

EXERCISE 1.6

1. Test for consistency and if possible, solve the following systems of equations by rank method.

   (i) $x - y + 2z = 2,\quad 2x + y + 4z = 7,\quad 4x - y + z = 4$

   (ii) $3x + y + z = 2,\quad x - 3y + 2z = 1,\quad 7x - y + 4z = 5$

   (iv) $2x - y + z = 2,\quad 6x - 3y + 3z = 6,\quad 4x - 2y + 2z = 4$

2. Find the value of k for which the equations $kx - 2y + z = 1,\quad x - 2ky + z = -2,\quad x - 2y + kz = 1$ have

   (i) no solution (ii) unique solution (iii) infinitely many solution

3. Investigate the values of λ and μ the system of linear equations $2x + 3y + 5z = 9$, $7x + 3y - 5z = 8$, $2x + 3y + \lambda z = \mu$, have

   (i) no solution (ii) a unique solution (iii) an infinite number of solutions.

EXERCISE 1.7

1. Solve the following system of homogenous equations.

   (i) $3x + 2y + 7z = 0,\quad 4x - 3y - 2z = 0,\quad 5x + 9y + 23z = 0$

   (ii) $2x + 3y - z = 0,\quad x - y - 2z = 0,\quad 3x + y + 3z = 0$

2. Determine the values of λ for which the following system of equations $x + y + 3z = 0$, $4x + 3y + \lambda z = 0$, $2x + y + 2z = 0$ has

   (i) a unique solution (ii) a non-trivial solution.

Example 1.1

If $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$, verify that $A(\text{adj } A) = (\text{adj } A)A = |A| I_3$.

Example 1.12

If $A = \frac{1}{7}\begin{bmatrix} 6 & -3 & a \\ b & -2 & 6 \\ 2 & c & 3 \end{bmatrix}$ is orthogonal, find a, b and c, and hence A^{-1}.

Example 1.20

Find the inverse the non-singular matrix $A = \begin{bmatrix} 0 & 5 \\ -1 & 6 \end{bmatrix}$, by Gauss-Jordan method.

Example 1.21

Find the inverse of $A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ by Gauss-Jordan method.

Example 1.23

Solve the following system of equations, using matrix inversion method:

$2x_1 + 3x_2 + 3x_3 = 5,\quad x_1 - 2x_2 + x_3 = -4,\quad 3x_1 - x_2 - 2x_3 = 3$.

Example 1.24

If $A=\begin{bmatrix}-4 & 4 & 4\\ -7 & 1 & 3\\ 5 & -3 & -1\end{bmatrix}$ and $B=\begin{bmatrix}1 & -1 & 1\\ 1 & -2 & -2\\ 2 & 1 & 3\end{bmatrix}$, find the products AB and BA and hence solve the system of equations $x-y+z=4, x-2y-2z=9, 2x+y+3z=1$.

Example 1.25

Solve, by Cramer's rule, the system of equations

$x_1-x_2=3, 2x_1+3x_2+4x_3=17, x_2+2x_3=7$.

Example 1.26

In a T20 match, a team needed just 6 runs to win with 1 ball left to go in the last over. The last ball was bowled and the batsman at the crease hit it high up. The ball traversed along a path in a vertical plane and the equation of the path is $y=ax^2+bx+c$ with respect to a $xy-$ coordinate system in the vertical plane and the ball traversed through the points $(10,8),(20,16),(40,22)$, can you conclude that the team won the match? Justify your answer. (All distances are measured in metres and the meeting point of the plane of the path with the farthest boundary line is $(70,0)$.)

Example 1.27

Solve the following system of linear equations, by Gaussian elimination method :

$4x+3y+6z=25,\ x+5y+7z=13,\ 2x+9y+z=1$.

Example 1.28

The upward speed $v(t)$ of a rocket is approximated by $v(t)=at^2+bt+c$, where a,b, and c are constants. It has been found that the speed at times $t=3, t=6$, and $t=9$ seconds are respectively, 64, 133, and 208 miles per second respectively. Find the speed at time $t=15$ seconds.

Example 1.29

Test for consistency of the following system of linear equations and if possible solve: $x+2y-z=3,\ 3x-y+2z=1,\ x-2y+3z=3,\ x-y+z+1=0$.

Example 1.30

Test for consistency of the following system of linear equations and if possible solve:

$4x-2y+6z=8,\ x+y-3z=-1,\ 15x-3y+9z=21$.

Example 1.31

Test for consistency of the following system of linear equations and if possible solve: $x-y+z=-9,\ 2x-2y+2z=-18,\ 3x-3y+3z+27=0$.

Example 1.32

Test the consistency of the following system of linear equations

$x-y+z=-9,\ 2x-y+z=4,\ 3x-y+z=6, 4x-y+2z=7$.

Example 1.33

Find the condition on a,b and c so that the following system of linear equations has one parameter family of solutions: $x+y+z=a,\ x+2y+3z=b,\ 3x+5y+7z=c$.

Example 1.34

Investigate for what values of λ and μ the system of linear equations

$x+2y+z=7,\ x+y+\lambda z=\mu, x+3y-5z=5$

has (i) no solution (ii) a unique solution (iii) an infinite number of solutions.

Example 1.35

Solve the following system:

$x+2y+3z=0,\ 3x+4y+4z=0,\ 7x+10y+12z=0.$

Example 1.36

Solve the system: $x+3y-2z=0,\ 2x-y+4z=0,\ x-11y+14z=0.$

Example 1.37

Solve the system:

$x+y-2z=0, 2x-3y+z=0, 3x-7y+10z=0, 6x-9y+10z=0.$

Example 1.38

Determine the values of λ for which the following system of equations

$(3\lambda-8)x+3y+3z=0,\ 3x+(3\lambda-8)y+3z=0,\ 3x+3y+(3\lambda-8)z=0$

has a non-trivial solution.

Example 1.40

If the system of equations $px+by+cz=0, ax+qy+cz=0, ax+by+rz=0$ has a non-trivial solution and $p\neq a, q\neq b, r\neq c$, prove that $\frac{p}{p-a}+\frac{q}{q-b}+\frac{r}{r-c}=2.$

EXERCISE 2.5

7. If z_1, z_2, and z_3 are three complex numbers such that $|z_1|=1,|z_2|=2,|z_3|=3$ and $|z_1+z_2+z_3|=1$, show that $|9z_1z_2+4z_1z_3+z_2z_3|=6$.

EXERCISE 2.6

2. If $z=x+iy$ is a complex number such that $Im\left(\frac{2z+1}{iz+1}\right)=0$, show that the locus of z is $2x^2+2y^2+x-2y=0.$

EXERCISE 2.7

3. If $(x_1+iy_1)(x_2+iy_2)(x_3+iy_3)\cdots(x_n+y_n)=a+ib$, show that
   (i) $(x_1^2+y_1^2)(x_2^2+y_2^2)(x_3^2+y_3^2)\cdots(x_n^2+y_n^2)=a^2+b^2$
   (ii) $\sum_{r=1}^{n}\tan^{-1}\left(\frac{y_r}{x_r}\right)=\tan^{-1}\left(\frac{b}{a}\right)+k\pi, k\in\mathbb{Z}.$
5. If $\cos\alpha+\cos\beta+\cos\gamma=\sin\alpha+\sin\beta+\sin\gamma=0$, show that
   (i) $\cos3\alpha+\cos3\beta+\cos3\gamma=3\cos(\alpha+\beta+\gamma)$ and
   (ii) $\sin3\alpha+\sin3\beta+\sin3\gamma=3\sin(\alpha+\beta+\gamma)$.
6. If $z=x+iy$ and $\arg\left(\frac{z-i}{z+2}\right)=\frac{\pi}{4}$, show that $x^2+y^2+3x-3y+2=0$.

EXERCISE 2.8

3. Find the value of $\left(\frac{1+\sin\frac{\pi}{10}+i\cos\frac{\pi}{10}}{1+\sin\frac{\pi}{10}-i\cos\frac{\pi}{10}}\right)^{10}$.

4. If $2\cos\alpha = x+\frac{1}{x}$ and $2\cos\beta = y+\frac{1}{y}$, show that

(i) $\frac{x}{y}+\frac{y}{x}=2\cos(\alpha-\beta)$ (ii) $xy-\frac{1}{xy}=2i\sin(\alpha+\beta)$ (any 2 carry 5 marks)

(iii) $\frac{x^m}{y^n}-\frac{y^n}{x^m}=2i\sin(m\alpha-n\beta)$ (iv) $x^m y^n+\frac{1}{x^m y^n}=2\cos(m\alpha+n\beta)$

5. Solve the equation $z^3+27=0$.

6. If $\omega\neq 1$ is a cube root of unity, show that the roots of the equation $(z-1)^3+8=0$ are $-1, 1-2\omega, 1-2\omega^2$.

Example 2.12

If z_1, z_2 and z_3 are complex numbers such that $|z_1|=|z_2|=|z_3|=|z_1+z_2+z_3|=1$,

find the value of $\left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right|$.

Example 2.15

Let z_1, z_2 and z_3 be complex numbers such that $|z_1|=|z_2|=|z_3|=r>0$ and $z_1+z_2+z_3\neq 0$.

Prove that $\left|\frac{z_1z_2+z_2z_3+z_3z_1}{z_1+z_2+z_3}\right|=r$.

Example 2.18

Given the complex number $z=3+2i$, represent the complex numbers z, iz, and $z+iz$ on one Argand diagram. Show that these complex numbers form the vertices of an isosceles right triangle.

Example 2.27

If $z=x+iy$ and $\arg\left(\frac{z-1}{z+1}\right)=\frac{\pi}{2}$, show that $x^2+y^2=1$.

Example 2.30

Simplify $\left(\frac{1+\cos 2\theta+i\sin 2\theta}{1+\cos 2\theta-i\sin 2\theta}\right)^{30}$.

Example 2.31

Simplify (ii) $(-\sqrt{3}+3i)^{31}$.

Example 2.32

Find the cube roots of unity.

Example 2.33

Find the fourth roots of unity.

Example 2.34

Solve the equation $z^3 + 8i = 0$, where $z \in \mathbb{C}$.

Example 2.35

Find all cube roots of $\sqrt{3} + i$.

Example 2.36

Suppose z_1, z_2, and z_3 are the vertices of an equilateral triangle inscribed in the circle $|z| = 2$. If $z_1 = 1 + i\sqrt{3}$, then find z_2 and z_3.

Chapter 3
Theory of Equations

EXERCISE 3.1

4. Solve the equation $3x^3 - 16x^2 + 23x - 6 = 0$ if the product of two roots is 1.
5. Find the sum of squares of roots of the equation $2x^4 - 8x^3 + 6x^2 - 3 = 0$.
6. Solve the equation $x^3 - 9x^2 + 14x + 24 = 0$ if it is given that two of its roots are in the ratio 3:2.
10. If the equations $x^2 + px + q = 0$ and $x^2 + p'x + q' = 0$ have a common root, then show that $\frac{pq' - p'q}{q - q'} = \frac{q - q'}{p' - p}$.

EXERCISE 3.2

1. If k is real, discuss the nature of the roots of the polynomial equation $2x^2 + kx + k = 0$, in terms of k.
5. Prove that a straight line and parabola cannot intersect at more than two points.

EXERCISE 3.3

1. Solve the cubic equation : $2x^3 - x^2 - 18x + 9 = 0$ if sum of two of its roots vanishes.
2. Solve the equation $9x^3 - 36x^2 + 44x - 16 = 0$ if the roots form an arithmetic progression.
3. Solve the equation $3x^3 - 26x^2 + 52x - 24 = 0$ if its roots form a geometric progression.
4. Determine k and solve the equation $2x^3 - 6x^2 + 3x + k = 0$ if one of its roots is twice the sum of the other two roots.
5. Find all zeros of the polynomial $x^6 - 3x^5 - 5x^4 + 22x^3 - 39x^2 - 39x + 135$, if it is known that $1 + 2i$ and $\sqrt{3}$ are two of its zeros.
6. Solve the cubic equations : (i) $2x^3 - 9x^2 + 10x = 3$, (ii) $8x^3 - 2x^2 - 7x + 3 = 0$. (each 5 marks)
7. Solve the equation : $x^4 - 14x^2 + 45 = 0$.

EXERCISE 3.4

1. Solve : (i) $(x-5)(x-7)(x+6)(x+4) = 504$ (ii) $(x-4)(x-7)(x-2)(x+1) = 16$. (each 5)
2. Solve : $(2x-1)(x+3)(x-2)(2x+3) + 20 = 0$.

EXERCISE 3.5

1. Solve the following equations:
   (ii) $12x^3 + 8x = 29x^2 - 4$

2. Examine for the rational roots of (each 5 marks)
(i) $2x^3 - x^2 - 1 = 0$ (ii) $x^8 - 3x + 1 = 0$
3. Solve : $8x^{\frac{3}{2n}} - 8x^{\frac{-3}{2n}} = 63$.
4. Solve : $2\sqrt{\frac{x}{a}} + 3\sqrt{\frac{a}{x}} = \frac{b}{a} + \frac{6a}{b}$.
5. Solve the equations (each 5 marks)
(i) $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$ (ii) $x^4 + 3x^3 - 3x - 1 = 0$
6. Find all real numbers satisfying $4^x - 3(2^{x+2}) + 2^5 = 0$.
7. Solve the equation $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$ if it is known that $\frac{1}{3}$ is a solution.

EXERCISE 3.6

1. Discuss the maximum possible number of positive and negative roots of the polynomial equation $9x^9 - 4x^8 + 4x^7 - 3x^6 + 2x^5 + x^3 + 7x^2 + 7x + 2 = 0$.
2. Discuss the maximum possible number of positive and negative zeros of the polynomials $x^2 - 5x + 6$ and $x^2 - 5x + 16$. Also draw rough sketch of the graphs.
4. Determine the number of positive and negative roots of the equation $x^9 - 5x^8 - 14x^7 = 0$.

Example 3.5

Find the condition that the roots of cubic equation $x^3 + ax^2 + bx + c = 0$ are in the ratio $p:q:r$.

Example 3.6

From the equation whose roots are the squares of the roots of the cubic equation $x^3 + ax^2 + bx + c = 0$.

Example 3.7

If p is real, discuss the nature of the roots of the equation $4x^2 + 4px + p + 2 = 0$, in terms of p.

Example 3.14

Prove that a line cannot intersect a circle at more than two points.

Example 3.15

If $2+i$ and $3-\sqrt{2}$ are roots of the equation $x^6 - 13x^5 + 62x^4 - 126x^3 + 65x^2 + 127x - 140 = 0$ find all roots.

Example 3.16

Solve the equation $x^4 - 9x^2 + 20 = 0$.

Example 3.17

Solve the equation $x^3 - 3x^2 - 33x + 35 = 0$.

Example 3.18

Solve the equation $2x^3 + 11x^2 - 9x - 18 = 0$.

Example 3.19

Obtain the condition that the roots of $x^3 + px^2 + qx + r = 0$ are in A.P.

Example 3.20

Find the condition that the roots of $ax^3 + bx^2 + cx + d = 0$ are in geometric progression. Assume $a, b, c, d \neq 0$.

Example 3.21

If the roots of $x^3+px^2+qx+r=0$ are in H.P., prove that $9pqr=27r^2+2q^3$. Assume $p,q,r\neq 0$ (Remark : HP is not defined).

Example 3.22

It is known that the roots of the equation $x^3-6x^2-4x+24=0$ are in arithmetic progression. Find its roots.

Example 3.23

Solve the equation $(x-2)(x-7)(x-3)(x+2)+19=0$.

Example 3.24

Solve the equation $(2x-3)(6x-1)(3x-2)(x-2)-5=0$.

Example 3.26

Find the roots of $2x^3+3x^2+2x+3=0$.

Example 3.27

Solve the equation $7x^3-43x^2=43x-7$.

Example 3.28

Solve the following equation : $x^4-10x^3+26x^2-10x+1=0$.

Example 3.31 (each 5 marks)

Discuss the nature of the roots of the following polynomials:

(i) $x^{2018}+1947x^{1950}+15x^8+26x^6+2019$

(ii) $x^5-19x^4+2x^3+5x^2+11$

EXERCISE 4.2

6. Find the domain of

(i) $f(x)=\sin^{-1}\left(\frac{|x|-2}{3}\right)+\cos^{-1}\left(\frac{1-|x|}{4}\right)$.

EXERCISE 4.5

5. Prove that $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$.

6. If $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\pi$, show that $x+y+z=xyz$.

7. Prove that $\tan^{-1}x+\tan^{-1}\frac{2x}{1-x^2}=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right),|x|<\frac{1}{\sqrt{3}}$.

9. (i) $\sin^{-1}\frac{5}{x}+\sin^{-1}\frac{12}{x}=\frac{\pi}{2}$ (ii) $2\tan^{-1}x=\cos^{-1}\frac{1-a^2}{1+a^2}-\cos^{-1}\frac{1-b^2}{1-b^2}$, $a>0$, $b>0$

(iii) $2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec} x)$ (iv) $\cot^{-1}x-\cot^{-1}(x+2)=\frac{\pi}{12}$, $x>0$.

10. Find the number of solutions of the equation

$\tan^{-1}(x-1)+\tan^{-1}x+\tan^{-1}(x+1)=\tan^{-1}3x$.

Example 4.11

Prove that $\tan\left(\sin^{-1} x\right) = \frac{x}{\sqrt{1-x^2}}$, $-1 < x < 1$.

Example 4.18

Find the value of (iii) $\tan\left[\frac{1}{2}\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \frac{1}{2}\cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)\right]$.

Example 4.20

Evaluate $\sin\left[\sin^{-1}\left(\frac{3}{5}\right) + \sec^{-1}\left(\frac{5}{4}\right)\right]$.

Example 4.22

If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$ and $0 < x, y, z < 1$, then show that $x^2 + y^2 + z^2 + 2xyz = 1$.

Example 4.23

If $a_1, a_2, a_3, \ldots a_n$ is an arithmetic progression with common difference d, then prove that $\tan\left[\tan^{-1}\left(\frac{d}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{d}{1+a_2a_3}\right) + \ldots + \tan^{-1}\left(\frac{d}{1+a_{n-1}a_n}\right)\right] = \frac{a_n - a_1}{1+a_1a_n}$.

Example 4.24

Solve $\tan^{-1}\frac{1-x}{1+x} = \frac{1}{2}\tan^{-1} x$ for $x > 0$.

Example 4.28

Solve : $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$.

Example 4.29

Solve : $\cos\left(\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\right) = \sin\left\{\cot^{-1}\left(\frac{3}{4}\right)\right\}$.

Chapter 4

(Created from the Text Book)

(1) Draw $y = \sin x$ in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $y = \sin^{-1} x$ in $[-1, 1]$

(2) Draw $y = \cos x$ in $[0, \pi]$ and $y = \cos^{-1} x$ in $[-1, 1]$

(3) Draw $y = \tan x$ in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $y = \tan^{-1} x$ in $(-\infty, \infty)$

(4) Draw $y = \operatorname{cosec} x$ in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \setminus \{0\}$ and $y = \operatorname{cosec}^{-1} x$ in $\mathbb{R} \setminus (-1, 1)$

(5) Draw $y = \sec x$ in $[0, \pi] \setminus \left\{\frac{\pi}{2}\right\}$ and $y = \sec^{-1}$ in $\mathbb{R} \setminus (-1, 1)$

(6) Draw $y = \cot x$ in $(0, \pi)$ and $y = \cot^{-1}$ in $(-\infty, \infty)$

EXERCISE 5.1

6. Find the equation of the circle passing through the points $(1,0), (-1,0)$, and $(0,1)$.
9. Find the equation of the tangent and normal to the circle $x^2 + y^2 - 6x + 6y - 8 = 0$ at $(2, 2)$.

EXERCISE 5.2

4. Find the vertex, focus, equation of directrix and length of the latus rectum of the following:

(iv) $x^2-2x+8y+17=0$ (v) $y^2-4y-8x+12=0$

5. Identify the type of conic and find centre, foci, vertices and directrices of each of the following:

(i) $\frac{x^2}{25}+\frac{y^2}{9}=1$ (ii) $\frac{x^2}{3}+\frac{y^2}{10}=1$ (iii) $\frac{x^2}{25}-\frac{y^2}{144}=1$ (iv) $\frac{y^2}{16}-\frac{x^2}{9}=1$

8. Identify the type of conic and find centre, foci, vertices, and directrices of each of the following : (any three informations carry 5 marks)

(i) $\frac{(x-3)^2}{225}+\frac{(y-4)^2}{289}=1$ (ii) $\frac{(x+1)^2}{100}+\frac{(y-2)^2}{64}=1$

(iii) $\frac{(x+3)^2}{225}-\frac{(y-4)^2}{64}=1$ (iv) $\frac{(y-2)^2}{25}-\frac{(x+1)^2}{16}=1$

(v) $18x^2+12y^2-144x+48y+120=0$ (vi) $9x^2-y^2-36x-6y+18=0$

EXERCISE 5.4

1. Find the equations of the two tangents that can be drawn from $(5,2)$ to the ellipse $2x^2+7y^2=14$.
2. Find the equation of tangents to the hyperbola $\frac{x^2}{16}-\frac{y^2}{64}=1$ which are parallel to $10x-3y+9=0$.
3. Show that the line $x-y+4=0$ is a tangent to the ellipse $x^2+3y^2=12$. Also find the coordinates of the point of contact.
4. Find the equation of the tangent to the parabola $y^2=16x$ perpendicular to $2x+2y+3=0$.
5. Find the equation of tangent at $t=2$ to the parabola $y^2=8x$.
6. Find the equation of the tangent and normal to hyperbola $12x^2-9y^2=108$ at $\theta=\frac{\pi}{3}$. (Hint : Use parametric form)
7. Prove that the point of intersection of the tangents at 't_1' and 't_2' on the parabola $y^2=4ax$ is $\left[at_1t_2, a(t_1+t_2)\right]$.
8. If the normal at the point 't_1' on the parabola $y^2=4ax$ meets the parabola again at the point 't_2' then prove that $t_2=-\left(t_1+\frac{2}{t_1}\right)$.

EXERCISE 5.5

1. A bridge has a parabolic arch that is 10m height in the centre and 30m wide at the bottom. Find the height of the arch 6m from the centre, on either sides.
2. A tunnel through a mountain for a four lane highway is to have a elliptical opening. The total width of the highway (not the opening) is to be 16m, and the height at the edge of the road must be sufficient for a truck 4m high to clear if the highest point of the opening is to be 5m approximately. How wide must the opening be?

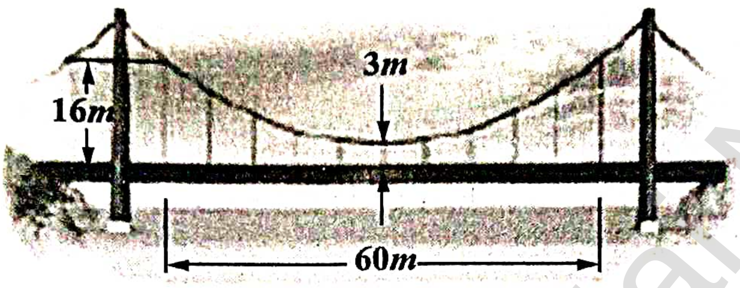
3. At a water fountain, water attains a maximum height of 4m at horizontal distance of 0.5m from its origin. If the path of water is a parabola, find the height of water at a horizontal distance of 0.75m from the point of origin.

4. An engineer designs a satellite dish with a parabolic cross section. The dish is 5m wide at the opening, and the focus is placed 1.2m from the vertex.

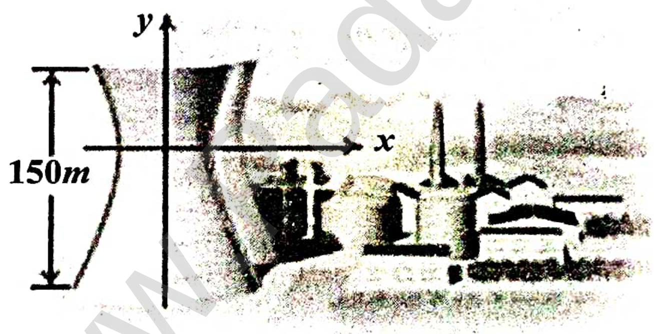
   (a) Position a coordinate system with the origin at the vertex and the x-axis on the parabola's axis of symmetry and find an equation of the parabola.

   (b) Find the depth of the satellite dish at the vertex.

5. Parabolic cable of 60m portion of the roadbed of a suspension bridge are positioned as shown below. Vertical cables are to be spaced every 6m along this portion of the roadbed. Calculate the lengths of first two of these vertical cables from the vertex.



6. Cross section of a Nuclear cooling tower is in the shape of a hyperbola with equation $\frac{x^2}{30^2}-\frac{y^2}{44^2}=1$. The tower is 150m tall and the distance from the top of the tower to the centre of the hyperbola is half the distance from the base of the tower to the centre of the hyperbola. Find the diameter of the top and base of the tower.



7. A rod of length 1.2m moves with its ends always touching the coordinate axes. The locus of a point P on the rod, which is 0.3m from the end in contact with x-axis is an ellipse. Find the eccentricity.

8. Assume that water issuing from the end of a horizontal pipe, 7.5m above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position 2.5m below the line of the pipe, the flow of water has curved outward 3m beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground?

9. On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4m when it is 6m away from the point of projection. Finally it reaches the ground 12m away from the starting point. Find the angle of projection.

10. Points A and B are 10km apart and it is determined from the sound of an explosion heard at these points at different times that the location of the explosion is 6km closer to A than B. Show that the location of the explosion is restricted to a particular curve and find an equation of it.

Example 5.10

Find the equation of the circle passing through the points $(1,1)$, $(2,-1)$, and $(3,2)$.

Example 5.19

Find the vertex, focus, directrix, and length of the latus rectum of the parabola $x^2-4x-5y-1=0$.

Example 5.21

Find the equation of the ellipse whose eccentricity is $\frac{1}{2}$, one of the foci is $(2,3)$ and a directrix is $x=7$. Find the length of the major and minor axes of the ellipse.

Example 5.22

Find the foci, vertices and length of major and minor axis of the conic $4x^2+36y^2+40x-288y+532=0$.

Example 5.23

For the ellipse $4x^2+y^2+24x-2y+21=0$, find the centre, vertices, and the foci. Also prove that the length of latus rectum is 2.

Example 5.26

Find the centre, foci and eccentricity of the hyperbola $11x^2-25y^2-44x+50y-256=0$.

Example 5.29

Find the equations of tangent and normal to the parabola $x^2+6x+4y+5=0$ at $(1,-3)$.

Example 5.30

Find the equations of tangent and normal to the ellipse $x^2+4y^2=32$ in cartesian form and parametric form when $\theta=\frac{\pi}{4}$.

Example 5.31

A semielliptical archway over a one-way road has a height of $3m$ and a width of $12m$. The truck has a width of $3m$ and a height of $2.7m$. Will the truck clear the opening of the archway?

Example 5.40

Two coast guard stations are located 600 km apart at points $A(0,0)$ and $B(0,600)$. A distress signal from a ship at P is received at slightly different times by two stations. It is determined that the ship is 200 km farther from station A than it is from station B. Determine the equation of hyperbola that passes through the location of the ship.

Example 5.41

Certain telescopes contain both parabolic mirror and a hyperbolic mirror. In the telescope shown in figure the parabola and hyperbola share focus F_1 which is $14m$ above the vertex of the parabola. The hyperbola's second focus F_2 is $2m$ above the parabola's vertex. The vertex of the hyperbolic mirror is $1m$ below F_1. Position a coordinate system with the origin at the centre of the hyperbola and with the foci on the y-axis. Then find the equation of the hyperbola.

EXERCISE 6.1

8. If G is the centroid of a ΔABC, prove that

(area of ΔGAB) =(area of ΔGBC) = (area of ΔGCA) = $\frac{1}{3}$ (area of ΔABC).

9. Using vector method, prove that $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$.

10. Prove by vector method that $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$.

EXERCISE 6.3

4. If $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$, $\vec{b}=3\hat{i}+5\hat{j}+2\hat{k}$, $\vec{c}=-\hat{i}-2\hat{j}+3\hat{k}$, verify that

(i) $(\vec{a}\times\vec{b})\times\vec{c}=(\vec{a}\cdot\vec{c})\vec{b}-(\vec{b}\cdot\vec{c})\vec{a}$ (ii) $\vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c}$

EXERCISE 6.5

2. Show that the lines $\vec{r}=(6\hat{i}+\hat{j}+2\hat{k})+s(\hat{i}+2\hat{j}-3\hat{k})$ and $\vec{r}=(3\hat{i}+2\hat{j}-2\hat{k})+t(2\hat{i}+4\hat{j}-5\hat{k})$ are skew lines and hence find the shortest distance between them.

4. Show that the lines $\frac{x-3}{3}=\frac{y-3}{-1}, z-1=0$ and $\frac{x-6}{2}=\frac{z-1}{3}, y-2=0$ intersect. Also find the point of intersection.

5. Show that the straight lines $x+1=2y=-12z$ and $x=y+2=6z-6$ are skew and hence find the shortest distance between them.

6. Find the parametric form of vector equation of the straight line passing through $(-1,2,1)$ and parallel to the straight line $\vec{r}=(2\hat{i}+3\hat{j}-\hat{k})+t(\hat{i}-2\hat{j}+\hat{k})$ and hence find the shortest distance between the lines.

7. Find the foot of the perpendicular drawn from the point $(5,4,2)$ to the line $\frac{x+1}{2}=\frac{y-3}{3}=\frac{z-1}{-1}$. Also, find the equation of the perpendicular.

EXERCISE 6.7

1. Find the non-parametric form of vector equation, and Cartesian equation of the plane passing through the point $(2,3,6)$ and parallel to the straight lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-3}{1}$ and $\frac{x+3}{2}=\frac{y-3}{-5}=\frac{z+1}{-3}$.

2. Find the parametric form of vector equation and Cartesian equation of the plane passing through the points $(2,2,1),(9,3,6)$ and perpendicular to the plane $2x+6y+6z=9$.

3. Find parametric form of vector equation and Cartesian equation of the plane passing through the points $(2,2,1),(1,-2,3)$ and parallel to the straight line passing through the points $(2,1,-3)$ and $(-1,5,-8)$.

4. Find the vector (parametric and non-parametric) and Cartesian equations of the plane passing through the point $(1,-2,4)$ and perpendicular to the plane $x+2y-3z=11$ and parallel to the line $\frac{x+7}{3}=\frac{y+3}{-1}=\frac{z}{1}$.

5. Find the vector (parametric and non-parametric) and Cartesian equations of the plane containing the line $\vec{r}=(\hat{i}-\hat{j}+3\hat{k})+t(2\hat{i}-\hat{j}+4\hat{k})$ and perpendicular to plane $\vec{r}\cdot(\hat{i}+2\hat{j}+\hat{k})=8$.

6. Find the vector (parametric and non-parametric)and Cartesian form of the equations of the plane passing through the three non-collinear points $(3,6,-2),(-1,-2,6)$, and $(6,4,-2)$.
7. Find the non-parametric form of vector equation, and Cartesian equations of the plane $\vec{r}=\left(6\hat{i}-\hat{j}+\hat{k}\right)+s\left(-\hat{i}+2\hat{j}+\hat{k}\right)+t\left(-5\hat{i}-4\hat{j}-5\hat{k}\right)$.

EXERCISE 6.8

1. Show that the straight lines $\vec{r}=\left(5\hat{i}+7\hat{j}-3\hat{k}\right)+s\left(4\hat{i}+4\hat{j}-5\hat{k}\right)$ and $\vec{r}=\left(8\hat{i}+4\hat{j}+5\hat{k}\right)+t\left(7\hat{i}+\hat{j}+3\hat{k}\right)$ are coplanar. Find the vector equation of the plane in which they lie.
2. Show that the lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{3}$ and $\frac{x-1}{-3}=\frac{y-4}{2}=\frac{z-5}{1}$ are coplanar. Also, find the cartesian equation of the plane containing these lines.
3. If the straight lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{m^2}$ and $\frac{x-3}{1}=\frac{y-2}{m^2}=\frac{z-1}{2}$ are coplanar, find the distinct real values of m .
4. If the straight lines $\frac{x-1}{2}=\frac{y+1}{\lambda}=\frac{z}{2}$ and $\frac{x+1}{5}=\frac{y+1}{2}=\frac{z}{\lambda}$ are coplanar, find λ and Cartesian equation of the plane containing these two lines.

EXERCISE 6.9

2. Find the equation of the plane passing through the line of intersection of the planes $x+2y+3z=2$ and $x-y+z=3$, and at a distance $\frac{2}{\sqrt{3}}$ from the point $(3,1-1)$.
7. Find the point of intersection of the line $x-1=\frac{y}{2}=z+1$ with the plane $2x-y+2z=2$. Also, find the angle between the line and the plane.
8. Find the coordinates of the foot of the perpendicular and length of the perpendicular from the point $(4,3,2)$ to the plane $x+2y+3z=2$.

Example 6.1(Cosine formulae)

With usual notations, in any triangle *ABC*, prove the following by vector method.

(i) $a^2=b^2+c^2-2bc\cos A$ (ii) $b^2=c^2+a^2-2ca\cos B$

(iii) $c^2=a^2+b^2-2ab\cos C$

Example 6.2

With usual notations, in any triangle *ABC*, prove the following by vector method.

(i) $a=b\cos C+c\cos B$ (ii) $b=c\cos A+a\cos C$

(iii) $c=a\cos B+b\cos A$

Example 6.3

By vector method, prove that $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$.

Example 6.5

Prove by vector method that $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$.

Example 6.6

If D is the midpoint of the side BC of a triangle ABC, then show by vector method that $|\overrightarrow{AB}|^2+|\overrightarrow{AC}|^2=2(|\overrightarrow{AD}|^2+|\overrightarrow{BD}|^2)$.

Example 6.7

Show that the altitudes of a triangle are concurrent by using vectors.

Example 6.8

In triangle ABC, the points D, E, F are the midpoints of the sides BC, CA and AB, respectively. Using vector method, show that the area of ΔDEF is equal to $\frac{1}{4}$ (area of ΔABC).

Example 6.16

Show that the four points $(6,-7,0)$, $(16,-19,-4)$, $(0,3,-6)$, $(2,-5,10)$ lie on a same plane.

Example 6.21 (each)

For any four vectors $\vec{a},\vec{b},\vec{c},\vec{d}$, verify

(i) $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=[\vec{a},\vec{b},\vec{d}]\vec{c}-[\vec{a},\vec{b},\vec{c}]\vec{d}$

(ii) $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=[\vec{a},\vec{c},\vec{d}]\vec{b}-[\vec{b},\vec{c},\vec{d}]\vec{a}$.

Example 6.22

If $\vec{a}=-2\hat{i}+3\hat{j}-2\hat{k}$, $\vec{b}=3\hat{i}-\hat{j}+3\hat{k}$, $\vec{c}=2\hat{i}-5\hat{j}+\hat{k}$, find $(\vec{a}\times\vec{b})\times\vec{c}$ and $\vec{a}\times(\vec{b}\times\vec{c})$. State whether they are equal.

Example 6.23

If $\vec{a}=\hat{i}-\hat{j}$, $\vec{b}=\hat{i}-\hat{j}-4\hat{k}$, $\vec{c}=3\hat{j}-\hat{k}$ and $\vec{d}=2\hat{i}+5\hat{j}+\hat{k}$, verify that

(i) $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=[\vec{a},\vec{b},\vec{d}]\vec{c}-[\vec{a},\vec{b},\vec{c}]\vec{d}$

(ii) $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=[\vec{a},\vec{c},\vec{d}]\vec{b}-[\vec{b},\vec{c},\vec{d}]\vec{a}$

Example 6.24

A straight line passes through the point $(1,2,-3)$ and parallel to $4\hat{i}+5\hat{j}-7\hat{k}$. Find (i) vector equation in parametric form (ii) vector equation in non-parametric form (iii) Cartesian equations of the straight line.

Example 6.25

The vector equation in parametric form of a line is $\vec{r}=(3\hat{i}-2\hat{j}+6\hat{k})+t(2\hat{i}-\hat{j}+3\hat{k})$. Find (i) the direction cosines of the straight line (ii) vector equation in non-parametric form of the line (iii) Cartesian equations of the line

Example 6.27

Find the vector parametric and Cartesian equations of a straight passing through the points $(-5,7,-4)$ and $(13,-5,2)$. Find the point where the straight line crosses the xy ,plane.

Example 6.37

Find the coordinates of the foot of the perpendicular drawn from the point $(-1,2,3)$ to the straight line $\vec{r}=(\hat{i}-4\hat{j}+3\hat{k})+t(2\hat{i}+3\hat{j}+\hat{k})$. Also, find the shortest distance from the point to the straight line.

Example 6.43

Find the non-parametric form of vector equation and Cartesian equation of the plane passing through the point $(0,1,-5)$ and parallel to the straight lines $\vec{r}=(\hat{i}+2\hat{j}-4\hat{k})+s(2\hat{i}+3\hat{j}+6\hat{k})$ and $\hat{r}=(\hat{i}-3\hat{j}+5\hat{k})+t(\hat{i}+\hat{j}-\hat{k})$.

Example 6.44

Find the vector parametric, vector non-parametric and Cartesian form of the equation of the plane passing through the points $(-1,2,0)$, $(2,2-1)$ and parallel to the straight line $\frac{x-1}{1}=\frac{2y+1}{2}=\frac{z+1}{-1}$.

Example 6.46

Show that the lines $\vec{r}=\left(-\hat{i}-3\hat{j}-5\hat{k}\right)+s\left(3\hat{i}+5\hat{j}+7\hat{k}\right)$ and $\vec{r}=\left(2\hat{i}+4\hat{j}+6\hat{k}\right)+t\left(\hat{i}+4\hat{j}+7\hat{k}\right)$ are coplanar. Also,find the vector equation in non-parametric form of the plane containing these lines.

Example 6.50

Find the distance of the point $(5,-5,-10)$ from the point of intersection of a straight line passing through the points $A(4,1,2)$ and $B(7,5,4)$ with the plane $x-y+z=5$.

Example 6.53

Find the equation of the plane passing through the intersection of the planes $\vec{r}\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)+1=0$ and $\vec{r}\cdot\left(2\hat{i}-3\hat{j}+5\hat{k}\right)=2$ and the point $(-1,2,1)$.

Example 6.54

Find the equation of the plane passing through the intersection of the planes $2x+3y-z+7=0$ and $x+y-2z+5=0$ and perpendicular to the plane $x+y-3z-5=0$.

Example 6.55

Find the image of the point whose position vector is $\hat{i}+2\hat{j}+3\hat{k}$ in the plane $\vec{r}\cdot\left(\hat{i}+2\hat{j}+4\hat{k}\right)=38$.

EXERCISE 7.1

2. A camera is accidentally knocked off an edge of a cliff 400 ft. high. The camera falls a distance of $s=-16t^2$ in t seconds.
   (i) How long does the camera fall before it hits the ground?
   (ii) What is the average velocity with which the camera falls during the last 2 seconds?
   (iii) What is the instantaneous velocity of the camera when it hits the ground?
3. A particle moves along a line according to the law $s(t)=2t^3-9t^2+12t-4$, where $t\geq 0$.
   (i) At what times the particle changes direction?
   (ii) Find the total distance travelled by the particle in the first 4 seconds.
   (iii) Find the particle's acceleration each time the velocity is zero.
7. A beacon makes one revolution every 10 seconds. It is located on a ship which is anchored 5 km from a straight shore line. How fast is the beam moving along the shore line when it makes an angle of 45° with the shore?
8. A conical water tank with vertex down of 12 metres height has a radius of 5 metres at the top. If water flows into the tank at a rate 10 cubic m/min, how fast is the depth of the water increases when the water is 8 metres deep?

9. A ladder 17 metre long is leaning against the wall. The base of the ladder is pulled away from the wall at a rate of 5 m/s. When the base of the ladder is 8 metres from the wall.
   (i) How fast is the top of the ladder moving down the wall?
   (ii) At what rate, the area of the triangle formed by the ladder, wall, and the floor, is changing?

10. A police jeep, approaching an orthogonal intersection from the northern direction, is chasing a speeding car that has turned and moving straight east. When the jeep is 0.6 km north of the intersection and the car is 0.8 km to the east. The police determine with a radar that the distance between the jeep and the car is increasing at 20 km/hr. If the jeep is moving at 60 km/hr at the instant of measurement, what is the speed of the car?

EXERCISE 7.2

9. Find the angle between the curves $xy = 2$ and $x^2 + 4y = 0$.

10. Show that the two curves $x^2 - y^2 = r^2$ and $xy = c^2$ where c, r are constants, cut orthogonally.

EXERCISE 7.4

1. Write the Maclaurin series expansion of the following functions:
   (v) $\tan^{-1}(x)$; $-1 \le x \le 1$ (vi) $\cos^2 x$

EXERCISE 7.5

Evaluate the following limits, if necessary use I'Hôpital Rule :

8. $\lim\limits_{x \to 0^+} x^x$

9. $\lim\limits_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$

10. $\lim\limits_{x \to \frac{\pi}{2}} (\sin x)^{\tan x}$

11. $\lim\limits_{x \to 0^+} (\cos x)^{\frac{1}{x^2}}$

12. If an initial amount A_0 of money is invested at an interest rate r compounded n times a year, the value of the investment after t years is $A = A_0\left(1 + \frac{r}{n}\right)^{nt}$. If the interest is compounded continuously, (that is as $n \to \infty$), show that the amount after t years is $A = A_0 e^{rt}$.

EXERCISE 7.6

1. Find the absolute extrema of the following functions on the given closed interval.
   (ii) $f(x) = 3x^4 - 4x^3$; $[-1, 2]$

2. Find the intervals of monotonicities and hence find the local extremum for the following functions:
   (i) $f(x) = 2x^3 + 3x^2 - 12x$
   (ii) $f(x) = \frac{x}{x-5}$
   (iii) $f(x) = \frac{e^x}{1 - e^x}$
   (iv) $f(x) = \frac{x^3}{3} - \log x$
   (v) $f(x) = \sin x \cos x + 5,\ x \in \left[0, \frac{\pi}{2}\right]$

EXERCISE 7.7

1. Find intervals of concavity and points of inflexion for the following functions:
   (i) $f(x)=x(x-4)^3$ (ii) $f(x)=\sin x+\cos x,\ 0<x<2\pi$
   (iii) $f(x)=\frac{1}{2}\left(e^x-e^{-x}\right)$
2. Find the local extrema for the following functions using second derivative test:
   (i) $f(x)=-3x^5+5x^3$ (ii) $f(x)=x\log x$
3. For the function $f(x)=4x^3+3x^2-6x+1$ find the intervals of monotonicity, local extrema, intervals of concavity and points of inflection.

EXERCISE 7.8

1. Find two positive numbers whose sum is 12 and their product is maximum.
3. Find the smallest possible value of x^2+y^2 given that $x+y=10$.
4. A garden is to be laid out in a rectangular area and protected by wire fence. What is the largest possible area of the fenced garden with 40 metres of wire.
5. A rectangular page is to contain 24cm^2 of print. The margins at the top and bottom of the page are 1.5 cm and the margins at other sides of the page is 1 cm. What should be the dimensions of the page so that the area of the paper used is minimum?
6. A farmer plans to fence a rectangular pasture adjacent to a river. The pasture must contain 1,80,000 sq.mtrs in order to provide enough grass for herds. No fencing is needed along the river. What is the length of the minimum needed fencing material?
7. Find the dimensions of the rectangle with maximum area that can be inscribed in a circle of radius 10 cm.
8. Prove that among all the rectangles of the given perimeter, the square has the maximum area.
9. Find the dimensions of the largest rectangle that can be inscribed in a semi circle of radius r cm.
10. A manufacturer wants to design an open box having a square base and a surface area of 108 sq.cm. Determine the dimensions of the box for the maximum volume.
11. The volume of a cylinder is given by the formula $V=\pi r^2 h$. Find the greatest value of V if $r+h=6$.
12. A hollow cone with base radius a cm and height b cm is placed on a table. Show that the volume of the largest cylinder that can be hidden underneath is $\frac{4}{9}$ times volume of the cone.

EXERCISE 7.9

2. Sketch the graphs of the following functions :
   (I) $y=-\frac{1}{3}(x^3-3x+2)$.
   (II) $y=x\sqrt{4-x}$.
   (III) $y=\frac{x^2+1}{x^2-4}$.
   (IV) $y=\frac{1}{1+e^{-x}}$

Note : The above 4 sub-divisions have more than 10 stages apart from the diagram. Hence any 5 points is enough for 5 marks.

Classification of Questions – 5 marks

Example 7.1

For the function $f(x)=x^2, x\in[0,2]$, compute the average rate of changes in the subintervals $[0,0.5],[0.5,1],[1,1.5],[1.5,2]$ and the instantaneous rate of changes at the points $x=0.5,1,1.5,2$.

Example 7.6

A particle moves along a horizontal line such that its position at any time $t\geq 0$ is given by $s(t)=t^3-6t^2+9t+1$, where s is measured in metres and t in seconds.

(1) At what time the particle is at rest.

(2) At what time the particle changes direction.

(3) Find the total distance travelled by the particle in the first 2 seconds.

Example 7.7

If we blow air into a balloon of spherical shape at a rate of $1000cm^3$ per second. At what rate the radius of the baloon changes when the radius is 7cm? Also compute the rate at which the surface area changes.

Example 7.8

The price of a product is related to the number of units' available (supply) by the equation $Px+3P-16x=234$, where P is the price of the product per unit in Rupees and x is the number of units. Find the rate at which the price is changing with respect to time when 90 units are available and the supply is increasing at a rate of 15 units/week.

Example 7.9

Salt is poured from a conveyer belt at a rate of 30 cubic metre per minute forming a conical pile with a circular base whose height and diameter of base are always equal. How fast is the height of the pile increasing when the pile is 10 metre high?

Example 7.10 (Two variable related rate problem)

A road running north to south crosses a road going east to west at the point P. Car A is driving north along the first road, and car B is driving east along the second road. At a particular time car A is 10 kilometres to the north of P and traveling at 80 km/hr, while car B is 15 kilometres to the east of P and traveling at 100 km/hr. How fast is the distance between the two cars changing?

Example 7.14

Find the acute angle between $y=x^2$ and $y=(x-3)^2$.

Example 7.15

Find the acute angle between the curves $y=x^2$ and $x=y^2$ at their point of intersections $(0,0)$ and $(1,1)$.

Example 7.17

If the curves $ax^2+by^2=1$ and $cx^2+dy^2=1$ intersect each other orthogonally if,

$$\frac{1}{a}-\frac{1}{b}=\frac{1}{c}-\frac{1}{d}.$$

Example 7.18

Prove that the ellipse $x^2+4y^2=8$ and the hyperbola $x^2-2y^2=4$ intersect orthogonally.

Example 7.30

Expand $\log(1+x)$ as a Maclaurin's series upto 4 non-zero terms for $-1<x\leq 1$.

Example 7.31

Expand $\tan x$ in ascending powers of x upto 5^{th} power for $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

Example 7.32

Write the Taylor series expansion of $\frac{1}{x}$ about $x = 2$ by finding the first three non-zero terms.

Example 7.43

Using the l'Hôpital rule prove that, $\lim_{x \to 0^+} (1+x)^{\frac{1}{x}} = e$.

Example 7.44

Evaluate : $\lim_{x \to \infty} (1+2x)^{\frac{1}{2\log x}}$.

Example 7.45

Evaluate : $\lim_{x \to 1^+} x^{\frac{1}{1-x}}$.

Example 7.51

Find the intervals of monotonicity and hence find the local extrema for the function $f(x) = x^{\frac{2}{3}}$.

Example 7.53

Discuss the monotonicity and local extrema of the function

$f(x) = \log(1+x) - \frac{x}{1+x}, x > -1$ and hence find the domain where, $\log(1+x) > \frac{x}{1+x}$.

Example 7.54

Find the intervals of monotonicity and local extrema of the function $f(x) = x\log x + 3x$.

Example 7.55

Find the intervals of monotonicity and local extrema of the function $f(x) = \frac{1}{1+x^2}$.

Example 7.56

Find the intervals of monotonicity and local extrema of the function $f(x) = \frac{x}{1+x^2}$.

Example 7.57

Determine the intervals of concavity of the curve $f(x) = (x-1)^3 \cdot (x-5), x \in \mathbb{R}$ and, points of inflection if any.

Example 7.58

Determine the intervals of concavity of the curve $y = 3 + \sin x$.

Example 7.59

Find the local extremum of the function $f(x) = x^4 + 32x$.

Example 7.60

Find the local extrema of the function $f(x) = 4x^6 - 6x^4$.

Example 7.61

Find the local maximum and minimum of the function x^2y^2 on the line $x + y = 10$.

Example 7.62

A 12 square unit piece of thin material is to be made an open box by cutting small squares from the four corners and folding the sides up. Find the length of the side of the square to be removed when the volume is maximum?

Example 7.63

Find the points on the unit circle $x^2 + y^2 = 1$ nearest and farthest from $(1,1)$.

Example 7.64

A steel plant is capable of producing x tonnes per day of low-grade steel and y tonnes per day of a high-grade steel, where $y = \frac{40-5x}{10-x}$. If the fixed market price of low-grade steel is half that of high-grade steel, then what should be optimal productions in low-grade steel and high-grade steel in order to have maximum receipts (gains).

Example 7.65

Prove that among all the rectangles of the given area square has the least perimeter.

Note :

The following 4 examples have more than 10 stages apart from diagram. Hence any 5 points is enough for 5 marks.

Example 7.69

Sketch the curve $y = f(x) = x^2 - x - 6$.

Example 7.70

Sketch the curve $y = f(x) = x^3 - 6x - 9$.

Example 7.71

Sketch the curve $y = \frac{x^2 - 3x}{(x-1)}$.

Example 7.72

Sketch the graph of the function $y = \frac{3x}{x^2 - 1}$.

EXERCISE 8.1

1. Let $f(x) = \sqrt[3]{x}$. Find the linear approximation at $x = 27$. Use the linear approximation to approximate $\sqrt[3]{27.2}$.

EXERCISE 8.2

5. The trunk of a tree has diameter 30cm. During the following year, the circumference grew 6cm.

   (i) Approximately, how much did the tree's diameter grow?

   (ii) What is the percentage increase in area of the tree's cross-section?

EXERCISE 8.4

2. For each of the following functions find the f_x, f_y, and show that $f_{xy} = f_{yx}$. (each 5 marks)

   (i) $f(x, y) = \frac{3x}{y + \sin x}$ (ii) $f(x, y) = \tan^{-1}\left(\frac{x}{y}\right)$ (iii) $f(x, y) = \cos(x^2 - 3xy)$

3. If $U(x,y,z)=\frac{x^2+y^2}{xy}+3z^2y$, find $\frac{\partial U}{\partial x},\frac{\partial U}{\partial y}$, and $\frac{\partial U}{\partial z}$.

5. For each of the following functions find the g_{xy}, g_{xx}, g_{yy} and g_{yx}.

   (i) $g(x,y)=xe^y+3x^2y$ (ii) $g(x,y)=\log(5x+3y)$

   (iii) $g(x,y)=x^2+3xy-7y+\cos(5x)$ (each 5 marks)

6. Let $w(x,y,z)=\frac{1}{\sqrt{x^2+y^2+z^2}},(x,y,z)\neq(0,0,0)$. Show that $\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}=0$.

7. If $V(x,y)=e^x(x\cos y-y\sin y)$, then prove that $\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}=0$.

9. If $v(x,y,z)=x^3+y^3+z^3+3xyz$, show that $\frac{\partial^2 v}{\partial y\partial z}=\frac{\partial^2 v}{\partial z\partial y}$.

EXERCISE 8.6

1. If $u(x,y)=x^2y+3xy^4, x=e^t$ and $y=\sin t$, find $\frac{du}{dt}$ and evaluate it at $t=0$.

2. If $v(x,y)=x\sin(xy^2), x=\log t$ and $y=e^t$, find $\frac{dv}{dt}$.

3. If $w(x,y,z)=x^2+y^2+z^2, x=e^t, y=e^t\sin t$ and $z=e^t\cos t$, find $\frac{dw}{dt}$.

4. Let $U(x,y,z)=xyz, x=e^{-t}, y=e^{-t}\cos t, z=\sin t, t\in\mathbb{R}$. Find $\frac{dU}{dt}$.

5. If $w(x,y)=6x^3-3xy+2y^2, x=e^s, y=\cos s, s\in\mathbb{R}$, find $\frac{dw}{ds}$, and evaluate at $s=0$,

6. If $z(x,y)=x\tan^{-1}(xy), x=t^2, y=se^t, s,t\in\mathbb{R}$. Find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ at $s=t=1$.

7. Let $u(x,y)=e^x\sin y$, where $x=st^2, y=s^2t, s,t\in\mathbb{R}$. Find $\frac{\partial u}{\partial s},\frac{\partial u}{\partial t}$ and evaluate them at $(1,1)$.

8. Let $z(x,y)=x^3-3x^2y^3$, where $x=se^t, y=se^{-t}, s,t\in\mathbb{R}$. Find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$.

9. $W(x,y,z)=xy+yz+zx, x=u-v, y=uv, z=u+v, u,v\in\mathbb{R}$. Find $\frac{\partial W}{\partial u},\frac{\partial W}{\partial v}$, and evaluate them at $\left(\frac{1}{2},1\right)$.

EXERCISE 8.7

2. Prove that $f(x,y)=x^3-2x^2y+3xy^2+y^3$ is homogeneous; what is the degree? Verify Euler's Theorem for f.

3. Prove that $g(x,y)=x\log\left(\frac{y}{x}\right)$ is homogeneous; what is the degree? Verify Euler's Theorem for g.

5. If $v(x,y)=\log\left(\frac{x^2+y^2}{x+y}\right)$, prove that $x\frac{\partial v}{\partial x}+y\frac{\partial v}{\partial y}=1$.

6. If $w(x,y,z)=\log\left(\frac{5x^3y^4+7y^2xz^4-75y^3z^4}{x^2+y^2}\right)$, find $x\frac{\partial w}{\partial x}+y\frac{\partial w}{\partial y}+z\frac{\partial w}{\partial z}$.

Example 8.4

A right circular cylinder has radius $r=10$ cm. and height $h=20$ cm. Suppose that the radius of the cylinder is increased from 10 cm to 10.1 cm and the height does not change. Estimate the change in the volume of the cylinder. Also, calculate the relative error and percentage error.

Example 8.11

Let $f(x,y)=0$ if $xy\neq 0$ and $f(x,y)=1$ if $xy=0$.

(i) Calculate : $\frac{\partial f}{\partial x}(0,0), \frac{\partial f}{\partial y}(0,0)$.

(ii) Show that f is not continuous at $(0,0)$.

Example 8.12

Let $F(x,y)=x^3y+y^2x+7$ for all $(x,y)\in\mathbb{R}^2$. Calculate $\frac{\partial F}{\partial x}(-1,3)$ and $\frac{\partial F}{\partial y}(-2,1)$.

Example 8.13

Let $f(x,y)=\sin(xy^2)+e^{x^3+5y}$ for all $(x,y)\in\mathbb{R}^2$. Calculate $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ and $\frac{\partial^2 f}{\partial y\partial x}$, and $\frac{\partial^2 f}{\partial x\partial y}$,

Example 8.14

Let $w(x,y)=xy+\frac{e^y}{y^2+1}$ for all $(x,y)\in\mathbb{R}^2$. Calculate $\frac{\partial^2 w}{\partial y\partial x}$ and $\frac{\partial^2 w}{\partial x\partial y}$.

Example 8.15

Let $u(x,y)=e^{-2y}\cos(2x)$ for all $(x,y)\in\mathbb{R}^2$. Prove that u is a harmonic function in $\mathbb{R}^2$.

Example 8.18

Verify $\frac{dF}{dt}=\frac{\partial F}{\partial x}\frac{dx}{dt}+\frac{\partial F}{\partial y}\frac{dy}{dt}$ for the function $F(x,y)=x^2-2y^2+2xy$ where $x(t)=\cos t, y(t)=\sin t, t\in[0,2\pi]$.

Example 8.19

Let $g(x,y)=x^2-yx+\sin(x+y), x(t)=e^{3t}, y(t)=t^2, t\in\mathbb{R}$. Find $\frac{dg}{dt}$.

Example 8.20

Let $g(x,y)=2y+x^2, x=2r-s, y=r^2+2s, r,s\in\mathbb{R}$. Find $\frac{\partial g}{\partial r}, \frac{\partial g}{\partial r}$.

Example 8.22

If $u=\sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, Show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{2}\tan u$.

EXERCISE 9.1

1. Find an approximate value of $\int_{1}^{1.5} x\,dx$ by applying the left-end rule with the partition $\{1.1, 1.2, 1.3, 1.4, 1.5\}$.
2. Find an approximate value of $\int_{1}^{1.5} x^2\,dx$ by applying the right-end rule with the partition $\{1.1, 1.2, 1.3, 1.4, 1.5\}$.
3. Find an approximate value of $\int_{1}^{1.5} (2-x)\,dx$ by applying the mid-point rule with the partition $\{1.1, 1.2, 1.3, 1.4, 1.5\}$.

EXERCISE 9.2

1. Evaluate the following integrals as the limit of sums:

(i) $\int_{0}^{1} (5x+4)\,dx$ (ii) $\int_{1}^{2} (4x^2-1)\,dx$

EXERCISE 9.3

2. Evaluate the following integrals using properties of integration:

(ii) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (x^5 + x\cos x + \tan^3 x + 1)\,dx$

(v) $\int_{0}^{2\pi} \sin^4 x \cos^3 x\,dx$ (vi) $\int_{0}^{1} |5x-3|\,dx$

(vii) $\int_{0}^{\sin^2 x} \sin^{-1}\sqrt{t}\,dt + \int_{0}^{\cos^2 x} \cos^{-1}\sqrt{t}\,dt$ (viii) $\int_{0}^{1} \frac{\log(1+x)}{1+x^2}\,dx$

(ix) $\int_{0}^{\pi} \frac{x\sin x}{1+\sin x}\,dx$

(xi) $\int_{0}^{\pi} x\left[\sin^2(\sin x) + \cos^2(\cos x)\right]dx$

EXERCISE 9.7

1. Evaluate the following

(ii) $\int_{0}^{\frac{\pi}{2}} \frac{e^{-\tan x}}{\cos^6 x}\,dx$

2. If $\int_{0}^{\infty} e^{-\alpha x^2} x^3\,dx = 32, \alpha > 0$, find α.

EXERCISE 9.8

3. Find the area of the region bounded by the curve $2 + x - x^2 + y = 0$, x-axis, $x = -3$ and $x = 3$.
4. Find the area of the region bounded by the line $y = 2x + 5$ and the parabola $y = x^2 - 2x$.

5. Find the area of the region bounded between the curves $y=\sin x$ and $y=\cos x$ and the lines $x=0$ and $x=\pi$.
6. Find the area of the region bounded by $y=\tan x$, $y=\cot x$ and the lines $x=0$, $x=\frac{\pi}{2}$, $y=0$.
7. Find the area of the region bounded by the parabola $y^2=x$ and the line $y=x-2$.
8. Father of a family wishes to divide his square field bounded by $x=0$, $x=4$, $y=4$ and $y=0$ along the curve $y^2=4x$ and $x^2=4y$ into three equal parts for his wife, daughter and son. Is it possible to divide? If so, find the area to be divided among them.
9. The curve $y=(x-2)^2+1$ has a minimum point at P. A point Q on the curve is such that the slope of PQ is 2. Find the area bounded by the curve and the chord PQ.
10. Find the area of the region common to the circle $x^2+y^2=16$ and the parabola $y^2=6x$.

EXERCISE 9.9

2. Find the volume of the solid generated by revolving the region enclosed by $y=e^{-2x}, y=0, x=0$ and $x=1$ about x-axis, using integration.
3. Find the volume of the solid generated by revolving the region enclosed by $x^2=1+y$ and $y=3$ about y-axis, using integration.
4. The region enclosed between the graphs of $y=x$ and $y=x^2$ is denoted by R, find the volume generated by R when R is rotated through 360° about x-axis, using integration.
5. Find the volume of the container which is in the shape of a right circular concial frustum as shown in the given figure by using integration.
6. A watermelon is ellipsoid in model. Its major and minor axes are 20 cm and 10 cm respectively. Find its volume by revolving the area about its major axis.

Example 9.1

Estimate the value of $\int_0^{0.5} x^2dx$ using the Riemann sum corresponding to 5 subintervals of equal width and applying (i) left-end rule (ii) right-end rule (iii) the mid-point rule.

Example 9.2

Evaluate $\int_0^1 x\,dx$, as a limit of a sum.

Example 9.3

Evaluate $\int_0^1 x^3dx$, as a limit of a sum.

Example 9.4

Evaluate $\int_1^4 (2x^2+3)\,dx$, as a limit of a sum.

Example 9.10

Evaluate: $\int_1^2 \frac{x}{(x+1)(x+2)}dx$.

Example 9.11

Evaluate : $\int_0^{\frac{\pi}{2}} \frac{\cos\theta}{(1+\sin\theta)(2+\sin\theta)}d\theta$.

Example 9.12

Evaluate : $\int_{0}^{\frac{1}{\sqrt{2}}} \frac{\sin^{-1} x}{(1-x^2)^{\frac{3}{2}}} dx$.

Example 9.13

Evaluate : $\int_{0}^{\frac{\pi}{2}} \left(\sqrt{\tan x} + \sqrt{\cot x}\right) dx$.

Example 9.14

Evaluate : $\int_{0}^{1.5} \left[x^2\right] dx$, where $[x]$ is the greatest integer function.

Example 9.15

Evaluate : $\int_{-4}^{4} |x+3|\ dx$.

Example 9.16

Show that $\int_{0}^{\frac{\pi}{2}} \frac{dx}{4+5\sin x} = \frac{1}{3}\log_e 2$.

Example 9.17

Prove that $\int_{0}^{\frac{\pi}{4}} \frac{\sin 2x\, dx}{\sin^4 x + \cos^4 x} = \frac{\pi}{4}$.

Example 9.18

Prove that $\int_0^{\frac{\pi}{4}} \frac{dx}{a^2 \sin^2 x + b^2 \cos^2 x} = \frac{1}{ab} \tan^{-1}\left(\frac{a}{b}\right)$, where $a,\ b > 0$.

Example 9.19

Evaluate $\int_0^{\frac{\pi}{4}} \frac{1}{\sin x + \cos x} dx$.

Example 9.21

Evaluate $\int_0^{\pi} \frac{x}{1+\sin x} dx$.

Example 9.27

Prove that $\int_0^{\frac{\pi}{4}} \log(1+\tan x) dx = \frac{\pi}{8} \log 2$.

Example 9.28

Show that $\int_0^1 \left(\tan^{-1} x + \tan^{-1}(1-x)\right) dx = \frac{\pi}{2} - \log_e 2$.

Example 9.30

Evaluate $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx$.

Classification of Questions – 5 marks

Example 9.36

Evaluate $\int_0^{\frac{\pi}{2}} \frac{dx}{4\sin^2 x + 5\cos^2 x}$.

Example 9.40

Evaluate $\int_0^{2a} x^2\sqrt{2ax - x^2}\,dx$.

Example 9.43

Prove that $\int_0^{\infty} e^{-x}x^n dx = n!$, where n is a positive integer.

Example 9.45

Show that $\Gamma(n) = 2\int_0^{\infty} e^{-x^2}x^{2n-1}dx$.

Example 9.46

Evaluate $\int_0^{\infty} \frac{x^n}{n^x}dx$, where n is a positive integer ≥ 2.

Example 9.49

Find the area of the region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Example 9.50

Find the area of the region bounded between the parabola $y^2 = 4ax$ and its latus rectum.

Example 9.51

Find the area of the region bounded by the y-axis and the parabola $x = 5 - 4y - y^2$.

Example 9.52

Find the area of the region bounded by $x-$axis, the sine curve $y = \sin x$, the lines $x = 0$ and $x = 2\pi$.

Example 9.53

Find the area of the region bounded by $x-$axis, the curve $y = |\cos x|$, the lines $x = 0$ and $x = \pi$.

Example 9.54

Find the area of the regioun bounded between the parabolas $y^2 = 4x$ and $x^2 = 4y$.

Example 9.55

Find the area of the region bounded between the parabola $x^2 = y$ and the curve $y = |x|$.

Example 9.56

Find the area of the region bounded by $y = \cos x, y = \sin x$, the lines $x = \frac{\pi}{4}$ and $x = \frac{5\pi}{4}$.

Example 9.57

The region enclosed by the circle $x^2 + y^2 = a^2$ is divided into two segments by the line $x = h$. Find the area of the smaller segment.

Example 9.58

Find the area of the region in the first quadrant bounded by the parabola $y^2 = 4x$, the line $x + y = 3$ and y-axis.

Example 9.59

Find, by integration, the area of the region bounded by the lines $5x - 2y = 15$, $x + y + 4 = 0$ and the x-axis.

Example 9.60

Using integration find the area of the region bounded by triangle ABC, whose vertices A, B and C are $(-1,1)$, $(3,2)$ and $(0,5)$ respectively.

Example 9.61

Using integration, find the area of the region which is bounded by x-axis, the tangent and normal to the circle $x^2 + y^2 = 4$ drawn at $(1, \sqrt{3})$.

Example 9.63

Find the volume of a right-circular cone of base radius r and height h.

Example 9.64

Find the volume of the spherical cap of height h cut of from a sphere of radius r.

Example 9.65

Find the volume of the solid formed by revolving the region bounded by the parabola $y = x^2$, x-axis, ordinates $x = 0$ and $x = 1$ about the x-axis.

Example 9.66

Find the volume of the solid formed by revolving the region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$ about the major axis.

Example 9.68

Find, by integration, the volume of the solid generated by revolving about y-axis the region bounded between the curve $y = \frac{3}{4}\sqrt{x^2 - 16}$, $x \geq 4$, the y-axis, and the lines $y = 1$ and $y = 6$.

Example 9.69

Find, by integration, the volume of the solid generated by revolving about y-axis the region bounded by the curves $y = \log x$, $y = 0$, $x = 0$ and $y = 2$.

EXERCISE 10.5

1. If F is the constant force generated by the motor of an automobile of mass M, its velocity v is given by $M\frac{dv}{dt} = F - kv$, where k is a constant. Express v in terms of t given that $v = 0$ when $t = 0$.

2. The velocity v, of a parachute falling vertically satisfies the equation $v\frac{dv}{dx} = g\left(1 - \frac{v^2}{k^2}\right)$, where g and k are constants. If v and x are both initially zero, find v in terms of x.

3. Find the equation of the curve whose slope is $\frac{y-1}{x^2+x}$ and which passes through the point $(1,0)$.

4. Solve the following differential equations:

(ii) $ydx+(1+x^2)\tan^{-1}x\,dy=0$

(v) $(e^y+1)\cos x\,dx+e^y\sin x\,dy=0$

(vi) $(ydx-xdy)\cot\left(\frac{x}{y}\right)=ny^2dx$

(vii) $\frac{dy}{dx}-x\sqrt{25-x^2}=0$

(viii) $x\cos y\,dy=e^x(x\log x+1)dx$

(ix) $\tan y\frac{dy}{dx}=\cos(x+y)+\cos(x-y)$

(x) $\frac{dy}{dx}=\tan^2(x+y)$

EXERCISE 10.6

Solve the following differential equations:

1. $\left[x+y\cos\left(\frac{y}{x}\right)\right]dx=x\cos\left(\frac{y}{x}\right)dy$

2. $(x^3+y^3)dy-x^2ydx=0$

3. $ye^{\frac{x}{y}}dx=\left(xe^{\frac{x}{y}}+y\right)dy$

4. $2xydx+(x^2+2y^2)dy=0$

5. $(y^2-2xy)dx=(x^2-2xy)dy$

6. $x\frac{dy}{dx}=y-x\cos^2\left(\frac{y}{x}\right)$

7. $\left(1+3e^{\frac{y}{x}}\right)dy+3e^{\frac{y}{x}}\left(1-\frac{y}{x}\right)dx=0$, given that y = 0 when $x=1$.

8. $(x^2+y^2)dy=xy\,dx$. It is given that $y(1)=1$ and $y(x_0)=e$. Find the value of x_0.

EXERCISE 10.7

Solve the following Linear differential equations:

2. $(1-x^2)\frac{dy}{dx}-xy=1$

3. $\frac{dy}{dx}+\frac{y}{x}=\sin x$

4. $(x^2+1)\frac{dy}{dx}+2xy=\sqrt{x^2+4}$

5. $(2x-10y^3)dy+ydx=0$

6. $x\sin x\frac{dy}{dx}+(x\cos x+\sin x)y=\sin x$

7. $\left(y-e^{\sin^{-1}x}\right)\frac{dx}{dy}+\sqrt{1-x^2}=0$

8. $\frac{dy}{dx}+\frac{y}{(1-x)\sqrt{x}}=1-\sqrt{x}$

9. $(1+x+xy^2)\frac{dy}{dx}+(y+y^3)=0$

10. $\frac{dy}{dx}+\frac{y}{x\log x}=\frac{\sin 2x}{\log x}$

11. $(x+a)\frac{dy}{dx}-2y=(x+a)^4$

12. $\frac{dy}{dx}=\frac{\sin^2 x}{1+x^3}-\frac{3x^2}{1+x^3}y$

13. $x\frac{dy}{dx}+y=x\log x$

14. $x\frac{dy}{dx}+2y-x^2\log x=0$

15. $\frac{dy}{dx}+\frac{3y}{x}=\frac{1}{x^2}$, given that $y=2$ when $x=1$

EXERCISE 10.8

1. The rate of increase in the number of bacteria in a certain bacteria culture is proportional to the number present. Given that the number triples in 5 hours, find how many bacteria will be present after 10 hours?
2. Find the population of a city at any time t, given that the rate of increase of population is proportional to the population at that instant and that in a period of 40 years the population increased from 3,00,000 to 4,00,000.
3. The equation of electromotive force for an electric circuit containing resistance and self-inductance is $E = Ri + L\frac{di}{dt}$, where E is the electromotive force given to the circuit, R the resistance and L, the coefficient of induction. Find the current i at time t when $E = 0$.
4. The engine of a motor boat moving at 10 m/s is shut off. Given that the retardation at any subsequent time (after shutting off the engine) equal to the velocity at that time. Find the velocity after 2 seconds of switching off the engine.
5. Suppose a person deposits 10,000 Indian rupees in a bank account at the rate of 5% per annum compounded continuously. How much money will be in his bank account 18 months later?
6. Assume that the rate at which radioactive nuclei decay is proportional to the number of such nuclei that are present in a given sample. In a certain sample 10% of the original number of radioactive nuclei have undergone disintegration in a period of 100 years.What percentage radioactive nuclei will remain after 1000 years? (Take the initial amount as A_0)
7. Water at temperature $100^\circ C$ cools in 10 minutes to $80^\circ C$ in a room temperature of $25^\circ C$. Find
   (i) The temperature of water after 20 minutes
   (ii) The time when the temperature is $40^\circ C$ $\left[\log_e \frac{11}{15} = -0.3101;\ \log_e 5 = 1.6094\right]$.
8. At 10.00 A.M. a woman took a cup of hot instant coffee from her microwave oven and placed it on a nearby Kitchen counter to cool. At this instant the temperature of the coffee was $180^\circ F$, and 10 minutes later it was $160^\circ F$. Assume the constant temperature of the kitchen was $70^\circ F$. What was the temperature of the coffee at 10.15A.M.?
9. A pot of boiling water at $100^\circ C$ is removed from a stove at time $t = 0$ and left to cool in the kitchen. After 5minutes, the water temperature has decreased to $80^\circ C$, and another 5 minutes later it has dropped to $65^\circ C$. Determine the temperature of the kitchen.
10. A tank initially contains 50 litres of pure water. Starting at time $t = 0$ a brine containing with 2 grams of dissolved salt per litre flows into the tank at the rate of 3 litres per minute. The mixture is kept uniform by stirring and the well-stirred mixture simultaneously flows out of the tank at the same rate. Find the amount of salt present in the tank at any time $t > 0$.

Example 10.12

Find the particular solution of $(1+x^3)dy - x^2 y dx = 0$ satisfying the condition $y(1) = 2$.

Example 10.13

Solve $y' = \sin^2(x - y + 1)$.

Classification of Questions – 5 marks

Example 10.14

Solve : $\frac{dy}{dx}=\sqrt{4x+2y-1}$.

Example 10.15:

Solve $\frac{dy}{dx}=\frac{x-y+5}{2(x-y)+7}$.

Example 10.17

Solve $\left(x^2-3y^2\right)dx+2xydy=0$.

Example 10.18

Solve $\left(y+\sqrt{x^2+y^2}\right)dx-xdy=0, \quad y(1)=0$.

Example 10.19

Solve $(2x+3y)dx+(y-x)dy=0$.

Example 10.21

Solve $\left(1+2e^{x/y}\right)dx+2e^{x/y}\left(1-\frac{x}{y}\right)dy=0$.

Example 10.23

Solve $\left[y(1-x\tan x)+x^2\cos x\right]dx-xdy=0$.

Example 10.24

Solve : $\frac{dy}{dx}+2y\cot x=3x^2\operatorname{cosec}^2 x$.

Example 10.25

Solve $\left(1+x^3\right)\frac{dy}{dx}+6x^2y=1+x^2$.

Example 10.26

Solve $ye^y dx=\left(y^3+2xe^y\right)dy$.

Example 10.27

The growth of a population is proportional to the number present. If the population of a colony doubles in 50 years, in how many years will the population become triple?

Example 10.28

A radioactive isotope has an initial mass $200mg$, which two years later is $50mg$. Find the expression for the amount of the isotope remaining at any time. What is its half-life? (half-life means the time taken for the radioactivity of a specified isotope to fall to half its original value).

Example 10.29

In a murder investigation, a corpse was found by a detective at exactly 8 P.M. Being alert, the detective also measured the body temperature and found it to be 70°F. Two hours later, the detective measured the body temperature again and found it to be 60°F. If the room temperature is 50°F, and assuming that the body temperature of the person before death was 98.6°F, at what time did the murder occur?

$\left[\log(2.43)=0.88789; \quad \log(0.5)=-0.69315\right]$

Example 10.30

A tank contains 1000 litres of water in which 100 grams of salt is dissolved. Brine (*Brine is a high-concentration solution of salt (usually sodium chloride) in water*) runs in a rate of 10 litres per minute, and each litre contains 5grams of dissolved salt. The mixture of the tank is kept uniform by stirring. Brine runs out at 10 litres per minute. Find the amount of salt at any time t.

EXERCISE 11.2

2. A six sided dic is marked '1' on one face, '3' on two of its faces, and '5' on remaining three faces. The dic is thrown twice. If X denotes the total score in two throws, find
   (i) the probability mass function (ii) the cumulative distribution function
   (iii) $P(4 \leq X \leq 10)$ (ii) $P(X \geq 6)$
3. Find the probability mass function and cumulative distribution function of number of girl child in families with 4 children, assuming equal probabilities for boys and girls.
4. Suppose a discrete random variable can only take the values $0, 1,$ and 2.
   The probability mass function is defined by
   $$f(x) = \begin{cases} \frac{x^2+1}{k}, & \text{for } x = 0,1,2 \\ 0 & \text{otherwise} \end{cases}$$
   Find (i) the value of k (ii) cumulative distribution function (iii) $P(X \geq 1)$.
5. The cumulative distribution function of a discrete random variable is given by
   $$F(x) = \begin{cases} 0 & -\infty < x < -1 \\ 0.15 & -1 \leq x < 0 \\ 0.35 & 0 \leq x < 1 \\ 0.60 & 1 \leq x < 2 \\ 0.85 & 2 \leq x < 3 \\ 1 & 3 \leq x < \infty \end{cases}$$
   Find (i) the probability mass function (ii) $P(X < 1)$ and (iii) $P(X \geq 2)$.
6. A random variable X has the following probability mass function:

X	1	2	3	4	5
$f(x)$	k^2	$2k^2$	$3k^2$	$2k$	$3k$

   Find (i) the value of k (ii) $P(2 \leq X < 5)$ (iii) $P(3 < X)$
7. The cumulative distribution function of a discrete random variable is given by
   $$F(x) = \begin{cases} 0 & \text{for } -\infty < x < 0 \\ \frac{1}{2} & \text{for } 0 \leq x < 1 \\ \frac{3}{5} & \text{for } 1 \leq x < 2 \\ \frac{4}{5} & \text{for } 2 \leq x < 3 \\ \frac{9}{10} & \text{for } 3 \leq x < 4 \\ 1 & \text{for } 4 \leq x < \infty \end{cases}$$
   Find (i) the probability mass function (ii) $P(X < 3)$ and (iii) $PX \geq 2)$.

EXERCISE 11.3

3. Suppose the amount of milk sold daily at a milk booth is distributed with a minimum of 200 litres and a maximum of 600 litres with probability density function

$$f(x)=\begin{cases} k & 200\le x\le 600 \\ 0 & \text{otherwise} \end{cases}$$

Find (i) the value of k (ii) the distribution function
(iii) the probability that daily sales will fall between 300 litres and 500 litres?

4. The probability density function of X is given by $f(x)=\begin{cases} ke^{-\frac{x}{3}} & \text{for } x>0 \\ 0 & \text{for } x\le 0 \end{cases}$

Find (i) the value of k (ii) the distribution function (iii) $P(X<3)$
(iv) $P(5\le X)$ (v) $P(X\le 4)$

5. If X is the random variable with probability density function,

$$f(x)=\begin{cases} x+1, & -1\le x<0 \\ -x+1, & 0\le x<1 \\ 0 & \text{otherwise} \end{cases}$$

then find (i) the distribution function $F(x)$ (ii) $P(-0.5\le X\le 0.5)$

6. If X is the random variable with distribution function,

$$F(x)=\begin{cases} 0, & x<0 \\ \frac{1}{2}(x^2+x) & 0\le x<1 \\ 1, & x\ge 1 \end{cases}$$

then find (i) the probability density function $f(x)$ (ii) $P(0.3\le X\le 0.6)$.

EXERCISE 11.4

2. Two balls are drawn in succession without replacement from an urn containing four red balls and three black balls. Let X be the possible outcomes of drawing red balls. Find the probability mass function and mean for X.

4. Four fair coins are tossed once. Find the probability mass function, mean and variance for number of heads occurred.

7. The probability density function of the random variable X is given by

$$f(x)=\begin{cases} 16xe^{-4x} & \text{for } x>0 \\ 0 & \text{for } x\le 0 \end{cases}$$

find the mean and variance of X.

EXERCISE 11.5

6. If the probability that a fluorescent light has a useful life of at least 600 hours is 0.9, find the probabilities that among 12 such lights
   (i) exactly 10 will have a useful life of at least 600 hours;
   (ii) at least 11 will have a useful life of at least 600 hours.
   (iii) atleast 2 will not have a useful life of at least 600 hours.

7. The mean and standard deviation of a binomial variate X are respectively 6 and 2. Find (i) the probability mass function (ii) $P(X=3)$ (iii) $P(X \geq 2)$.

Example 11.7

If the probability mass function $f(x)$ of a random variable X is

x	1	2	3	4
$f(x)$	$\frac{1}{12}$	$\frac{5}{12}$	$\frac{5}{12}$	$\frac{1}{12}$

find (i) its cumulative distribution function, hence find (ii) $P(X \leq 3)$ and, (iii) $P(X \geq 2)$

Example 11.8

A six sided die is marked '1' on one face, '2' on two of its faces, and '3' on remaining three faces. The die is rolled twice. If X denotes the total score in two throws.

(i) Find the probability mass function

(ii) Find the cumulative distribution function

(iii) Find $P(3 \leq X < 6)$ (iv) Find $P(X \geq 4)$

Example 11.9

Find the probability mass function $f(x)$ of the discrete random variable X whose cumulative distribution function

$$F(x) = \begin{cases} 0 & -\infty < x < -2 \\ 0.25 & -2 \leq x < -1 \\ 0.60 & -1 \leq x < 0 \\ 0.90 & 0 \leq x < 1 \\ 1 & 1 \leq x < \infty \end{cases}$$

Also find (i) $P(X < 0)$ and (ii) $P(X \geq -1)$.

Example 11.10

A random variable X has the following probability mass function.

x	1	2	3	4	5	6
$f(x)$	k	$2k$	$6k$	$5k$	$6k$	$10k$

Find (i) $P(2 < X < 6)$ (ii) $P(2 \leq X < 5)$ (iii) $P(X \leq 4)$ (iv) $P(3 < X)$.

Example 11.12

If X is the random variable with probability density function,

$$f(x) = \begin{cases} x-1, & 1 \leq x < 2 \\ -x+3, & 2 \leq x < 3 \\ 0 & \text{otherwise} \end{cases}$$

find (i) the distribution function $F(x)$

(ii) $P(1.5 \leq X \leq 2.5)$

Example 11.14

The probability density function of X is given by $f(x) = \begin{cases} k & 1 \leq x \leq 5 \\ 0 & \text{otherwise} \end{cases}$

find (i) distribution function (ii) $P(X < 3)$ (iii) $P(2 < X < 4)$ (iv) $P(3 \leq X)$

Example 11.17

Two balls are chosen randomly from an urn containing 8 white and 4 black balls. Suppose that we win Rs 20 for each black ball selected and we lose Rs10 for each white ball selected. Find the expected winning amount and variance.

Example 11.18

Find the mean and variance of a random variable X, whose probability density function is

$$f(x)=\begin{cases}\lambda e^{-\lambda x} & \text{for } x\geq 0\\ 0 & \text{otherwise}\end{cases}$$

Example 11.20

A multiple choice examination has ten questions, each question has four distractors with exactly one correct answer. Suppose a student answers by guessing and if X denotes the number of correct answers, find (i) binomial distribution (ii) probability that the student will get seven correct answers (iii) at least one correct answer.

Example 11.22

On the average, 20% of the products manufactured by ABC Company are found to be defective. If we select 6 of these products at random and X denote the number of defective products find (i) two products are defective (ii) at most one product is defective (iii) at least two products are defective.

EXERCISE 12.1

5. (i) Define an operation $*$ on $\mathbb{Q}$ as follows: $a*b=\left(\dfrac{a+b}{2}\right);a,b\in\mathbb{Q}$. Examine the closure, commutative, associative properties satisfied by $*$ on $\mathbb{Q}$.

   (ii) Define an operation $*$ on $\mathbb{Q}$ as follows: $a*b=\left(\dfrac{a+b}{2}\right);a,b\in\mathbb{Q}$. Examine the existence of identity and the existence of inverse for the operation $*$ on $\mathbb{Q}$.

9. Let $M=\left\{\begin{pmatrix} x & x\\ x & x\end{pmatrix}:x\in R-\{0\}\right\}$ and let $*$ be the matrix multiplication. Determine whether M is closed under $*$. If so, examine the closure, commutative, associative, existence of identity and inverse properties.

10. Let A be $Q-\{1\}$. Define $*$ on A by $x*y=x+y-xy$. Is $*$ a binary on A. If so, examine the closure, commutative, associative, the existence of identity and existence of inverse properties.

EXERCISE 12.2

7. Verify whether the following compound propositions are tautologies or contradictions or contingency

   (i) $(p\wedge q)\wedge\neg(p\vee q)$ (ii) $((p\vee q)\wedge\neg p)\rightarrow q$

   (iii) $(p\rightarrow q)\leftrightarrow(\neg p\rightarrow q)$ (iv) $((p\rightarrow q)\wedge(q\rightarrow r))\rightarrow(p\rightarrow r)$

8. Show that (i) $\neg(p\wedge q)\equiv\neg p\vee\neg q$ (ii) $\neg(p\rightarrow q)\equiv p\wedge\neg q$.

10. Show that $p\rightarrow q$ and $q\rightarrow p$ are not equivalent.

11. Show that $\neg(p\leftrightarrow q)\equiv p\leftrightarrow\neg q$.

12. Check whether the statement $p \to (q \to p)$ is a tautology or a contradiction without using the truth table.
13. Using truth table check whether the statements $\neg(p \vee q) \vee (\neg p \wedge q)$ and $\neg p$ are logically equivalent.
14. Prove $p \to (q \to r) \equiv (p \wedge q) \to r$ without using truth table.
15. Prove that $p \to (\neg q \vee r) \equiv \neg p \vee (\neg q \vee r)$ using truth table.

Example 12.2

Verify the (i) closure property, (ii) commutative property, (iii) associative property (iv) existence of identity and (v) existence of inverse for the arithmetic operation + on $\mathbb{Z}$.

Example 12.3

Verify the (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity and (v) existence of inverse for the arithmetic operation − on $\mathbb{Z}$.

Example 12.4

Verify the (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity and (v) existence of inverse for the arithmetic operation + on $\mathbb{Z}_e$ = the set of all even integers.

Example 12.5

Verify the (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity and (v) existence of inverse for the arithmetic operation + on $\mathbb{Z}_o$ = the set of all odd integers.

Example 12.6

Verify (i) closure property, (ii) commutative property, and (iii) associative property of the following operation on the given set.

$(a * b) = a^b; \forall a, b \in \mathbb{N}$ (exponentiation property).

Example 12.7

Verify (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity, and (v) existence of inverse for following operation on the given set.

$m * n = m + n - mn; m, n \in \mathbb{Z}$

Example 12.9

Verify (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity, and (v) existence of inverse for the operation $+_5$ on $\mathbb{Z}_5$ using multiplication table corresponding to addition modulo 5.

Example 12.10

Verify (i) closure property, (ii) commutative property, (iii) associative property, (iv) existence of identity, and (v) existence of inverse for the operation $\times_{11}$ on a subset $A = \{1, 3, 4, 5, 9\}$ of the set of remainders $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

Example 12.16

Construct the truth table for $(p \bar{\vee} q) \wedge (p \bar{\vee} \neg q)$.

Example 12.18

Establish the equivalence property connecting the bi-conditional with conditional:
$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

Example 12.19

Using the equivalence property, show that $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$.

OBJECTIVE TYPE QUESTIONS
(Created from the Text Book)

CHAPTER 1

1. Which of the following are corret?

(i) $|A^{-1}| = \frac{1}{|A|}$ (ii) $(A^T)^{-1} = (A^{-1})^T$ (iii) $(\lambda A^{-1}) = \frac{1}{\lambda} A^{-1}, \lambda \neq 0$

(1) (i) only (2) (i) and (ii) only (3) (i) and (iii) only (4) all

2. Which of the following are incorrect?

(i) A is non singular and $AB = AC \Rightarrow B = C$

(ii) A is non singular and $BA = CA \Rightarrow B = C$

(iii) A and B are non singular of same order then $(AB)^{-1} = B^{-1}A^{-1}$

(iv) A is non singular then $A = (A^{-1})^{-1}$

(1) none (2) (i) and (ii) (3) (ii) and (iii) (4) (iii) and (iv)

3. Which of the following is incorrect?

(1) $adj(adj\,A) = |A|^{n-2} A$ (2) $|adjA| = A^{n-1}$

(3) $|adj(adjA)| = |A|^{(n-1)^2}$ (4) $(adjA)^T = adj(A^T)$

4. A is of order n, $\lambda \neq 0$ then $adj(\lambda A) =$

(1) $\lambda^{n-1} adj(A)$ (2) $\lambda^{n-2} adj(A)$ (3) $\frac{1}{\lambda} adj(A)$ (4) $\lambda^n adj(A)$

5. If A is a n, non singular matrix then $[adj(A)]^{-1}$ is

(1) $\neq adj(A^{-1})$ and $= \frac{1}{|A|} A$ (2) $= adj(A^{-1})$ and $\neq \frac{1}{|A|} A$

(3) $\neq adj(A^{-1})$ and $\neq \frac{1}{|A|} A$ (4) $= adj(A^{-1})$ and $= \frac{1}{|A|} A$

6. Consider the statements :

A : A is symmetric $\Rightarrow$ $adjA$ is symmetric

B : $adj(AB) = adj(A) \cdot adj(B)$

Choose the correct option

(1) Both statements are correct (2) Neither statements are correct

(3) A is correct, B is incorrect (4) A is incorrect, B is correct

7. A is orthogonal and consider the statements and select the suitable option :

A : $A^{-1} = A^T$

B : $AA^T = A^T A = I$

(1) A and B are true (2) A only true

(3) B only true (4) both are false

8. Which of the following are correct in the case of a rank of a matrix A of order $m\times n$?
   (i) rank of I_n is n
   (ii) A is of order $m\times n$ then $\rho(A)\leq \min(m,n)$
   (iii) The necessary and sufficient condition to find inverse of an $n\times n$ matrix is $\rho(A)=n$
   (1) all (2) (i) and (iii) (3) (ii) and (iii) only (4) (iii) and (iv)
9. In the case of Cramer's rule which of the following are correct?
   (i) $\Delta=0$ (ii) $\Delta\neq 0$
   (iii) the system has unique solution (iv) the system has infinitely many solutions
   (1) (i) and (iv) (2) (ii) and (iii) (3) all (4) none
10. If ρ represents the rank and, A and B are $n\times n$ matrices, then
   (1) $\rho(A+B)=\rho(A)+\rho(B)$ (2) $\rho(AB)=\rho(A)\rho(B)$
   (3) $\rho(A-B)=\rho(A)-\rho(B)$ (4) $\rho(A+B)\leq n$

CHAPTER 2

1. If $\sqrt{-1}=i$ and $n\in\mathbb{N}$ then
   (1) $i^{4n+3}=-i$ (2) $i^{8n+2}=1$ (3) $i^{100n+4}=-1$ (4) $i^{4n+5}=1$
2. Which of the statement is incorrect if $i=\sqrt{-1}$ and z is any complex number?
   (1) iz is obtained by rotating z in the anti clockwise direction through an angle $\frac{\pi}{2}$
   (2) iz is obtained by rotating z in the clockwise direction through an angle $\frac{\pi}{2}$
   (3) $-z$ is obtained by rotating z in the anti clockwise direction through an angle π.
   (4) $-iz$ is obtained by rotating z in the clockwise direction through an angle $\frac{\pi}{2}$.
3. Find the correct statements.
   (i) Conjugate of the sum of two complex numbers is equal to the sum of their conjugates.
   (ii) Conjugate of the difference of two complex numbers is equal to the difference of their conjugates.
   (iii) Conjugate of the product of two complex numbers is equal to the product of their conjugates.
   (iv) Conjugate of the quotient of two complex numbers is equal to the quotient of their conjugates.
   (1) all (2) (i) and (iii) only (3) (i) and (iv) only (4) (ii), (iii), (iv) only
4. Identify the incorrect statement.
   (1) $|z|^2=1 \Rightarrow \frac{1}{z}=\bar{z}$ (2) $\text{Re}(z)\leq |z|$
   (3) $||z_1|-|z_2||\geq |z_1+z_2|$ (4) $|z^n|=|z|^n$

5. If $|z-z_1|=|z-z_2|$, the locus of z is

(1) the perpendicular bisector of line joining z_1 and z_2

(2) a line parallel to the line joining the points z_1 and z_2

(3) a circle, where z_1 and z_2 are the end points of a diameter

(4) a line joining z_1 and z_2.

6. Which of the following are correct statements?

(i) $e^{-i\theta}=\cos\theta-i\sin\theta$ (ii) $e^{i\frac{\pi}{2}}=i$

(iii) $e^{i(x+iy)}=e^{-y}(\cos x+i\sin x)$ (iv) $e^{-i(y-ix)}=e^{-x}(\cos y-i\sin y)$

(1) (i) and (iv) only (2) (iii) only (3) (i), (ii) and (iii) (4) all

7. Which of the following are correct?

(i) $\arg(z_1+z_2)=\arg(z_1)+\arg(z_2)$ (ii) $\arg(z_1-z_2)=\arg(z_1)-\arg(z_2)$

(iii) $\arg(z_1z_2)=\arg(z_1)+\arg(z_2)$ (iv) $\arg\left(\frac{z_1}{z_2}\right)=\arg(z_1)-\arg(z_2)$

(1) (i), (ii) and (iv) (2) all (3) (iii) and (iv) (4) (i) and (ii)

8. Which of the following are incorrect?

(i) $(\cos\theta+i\sin\theta)^m=\cos m\theta+i\sin m\theta$ if m is a negative integer

(ii) $(\sin\theta+i\cos\theta)^n=\cos n\left(\frac{\pi}{2}-\theta\right)+i\sin n\left(\frac{\pi}{2}-\theta\right)$

(iii) $(\cos\theta-i\sin\theta)^{-m}=\cos m\theta+i\sin m\theta$ if m is a negative integer

(iv) $(\cos\theta-i\sin\theta)^n=\cos n\theta-i\sin n\theta$

(1) none (2) (i) and (iv) (3) (i) and (ii) (4) (iii) and (iv)

9. In the case n^{th} roots of unity, identify the correct statements.

(i) the roots are in G.P

(ii) sum of the roots is zero

(iii) Product of the roots is $(-1)^{n+1}$

(iv) the roots are lying on a unit circle

(1) (i) and (ii) only (2) (ii) and (iii) only (3) all (4) (i), (ii) and (iii)

10. $cis\frac{28}{5}\pi$ is equal to

(1) $cis\left(-\frac{2\pi}{5}\right)$ (2) $cis\left(\frac{2\pi}{5}\right)$ (3) $cis\left(\frac{3\pi}{5}\right)$ (4) $cis\left(-\frac{3\pi}{5}\right)$

CHAPTER 3

1. The statement "A polynomial equation of degree n has exactly n roots which are either real or complex" is
   (1) Fundamental theorem of Algebra (2) Rational root theorem
   (3) Descartes rule (4) Complex conjugate root theorem

2. Identify the correct answer regarding the statements
   Statement A : If a complex number z_0 is a root of $p(x)=0$ then $\overline{z}_0$ is also a root.
   Statement B : For a polynomial equation with real coefficients, complex (imaginary) roots occur in conjugate pairs
   (1) Both are true (2) Both are false
   (3) A is false, B is true (4) A is false, B is true

3. If $p+\sqrt{q}$ and $-i\sqrt{q}$ are the roots of a polynomial equation with rational coefficients then the least possible degree of the equation is
   (1) 2 (2) 1 (3) 3 (4) 4

4. If $\frac{p}{q}$ (where p and q are co-primes), is a root of a polynomial equation $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0 = 0$, then identify the correct option.
   Statement A : p is a factor of a_0 and q is a factor of a_n.
   Statement B : q is a factor of a_0 and p is a factor of a_n.
   (1) both are not true (2) both are true
   (3) A is correct but B is false (4) A is incorrect but B is correct

5. A polynomial $p(x)$ of degree n is said to be a reciprocal polynomial if
   (1) either $p(x)=x^n p\left(\frac{1}{x}\right)$ or $p(x)=-x^n p\left(\frac{1}{x}\right)$
   (2) $p(x)=x^n p\left(\frac{1}{x}\right)$ and $p(x)=-x^n p\left(\frac{1}{x}\right)$
   (3) either $p(x)=p\left(\frac{1}{x}\right)$ or $p(x)=p\left(-\frac{1}{x}\right)$
   (4) $p(x)=p\left(\frac{1}{x}\right)$ and $p(x)=p\left(-\frac{1}{x}\right)$

6. Regarding Descarte's Rule, which of the following are true, where s_1, s_2 are the number of sign changes in $p(x)$ and $p(-x)$ respectively.
   (i) the number of positive zeros $> s_1$ (ii) the number of positive zeros $\leq s_1$
   (iii) the number of negative zeros $\leq s_2$ (iv) the total number of zeros $= s_1 + s_2$
   (1) (ii) and (iii) only (2) (i) and (iv) (3) all (4) none

CHAPTER 4

1. e^{x} is a periodic function with period
(1) 0 (2) π (3) 2π (4) 4π

2. $\sin^2 x + \cos x$ is
(1) an odd function (2) an even function
(3) neither odd nor even (4) either even or odd

3. If $y = a\sin bx$ then the amplitude and period are respectively
(1) $a, \frac{2\pi}{b}$ (2) $|b|, \frac{2\pi}{|a|}$ (3) $|a|, \frac{2\pi}{|b|}$ (4) $b, \frac{2\pi}{a}$

4. $\sin(\sin^{-1} x) = x$ if
(1) $|x| \le 1$ (2) $|x| \ge 1$ (3) $|x| < 1$ (4) $|x| \le \frac{\pi}{2}$

5. $\sin^{-1}(\sin x) = x$ if
(1) $|x| \le \frac{\pi}{2}$ (2) $|x| < \frac{\pi}{2}$ (3) $|x| \ge \frac{\pi}{2}$ (4) $|x| \le 1$

6. $\cos(\cos^{-1} x) = x$ if
(1) $|x| < 1$ (2) $|x| \le 1$ (3) $|x| \ge 1$ (4) $|x| = 0$

7. $\cos^{-1}(\cos x) = x$ if
(1) $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ (2) $0 < x \le \pi$ (3) $0 \le x \le \pi$ (4) $-1 \le x \le 1$

8. The amplitude and period of $y = a\tan bx$ are respectively
(1) $|a|, \frac{\pi}{|b|}$ (2) $a, \frac{\pi}{b}$ (3) not defined, $\frac{\pi}{|b|}$ (4) not defined, $\frac{\pi}{b}$

9. The domain of $\text{cosec}^{-1} x$ function is
(1) $\mathbb{R} \setminus (-1,1)$ (2) $\mathbb{R} \setminus \{-1,1\}$ (3) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (4) $\mathbb{R} - \{0\}$

10. The domain of secant function and $\sec^{-1} x$ function are respectively
(1) $[0\ \pi] \setminus \left\{\frac{\pi}{2}\right\}$ and $\mathbb{R} \setminus (-1,1)$ (2) $\mathbb{Z} \setminus (-1,1)$ and $(0\ \pi] \setminus \left\{\frac{\pi}{2}\right\}$
(3) $[0\ \pi] \setminus \left\{\frac{\pi}{2}\right\}$ and $\{-1,1\}$ (4) $\mathbb{Z} \setminus \{-1,1\}$ and $(0\ \pi] \setminus \left\{\frac{\pi}{2}\right\}$

CHAPTER 5

1. If the point (a,b) satisfies the inequality $x^2 + y^2 + 2gx + 2fy + c < 0$ then (a,b)
(1) lies within the circle (2) lie on the circle
(3) lie outside the circle (4) can't be determined

2. The number of tangents to the circle from inside the circle is

(1) 2 real (2) 0 (3) 2 imaginary (4) can't be determined

3. Which of the following are correct about parabola?

(i) axis of the parabola is axis of symmetry

(ii) vertex is the point of intersection of the axis and the parabola

(iii) latus rectum is a focal chord perpendicular to the axis

(iv) length of latus rectum is 4 times the distance between focus and vertex

(1) all (2) (i) and (ii) only (3) (iii) and (iv) only (4) (i), (ii) and (iii) only

4. For the equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ if $B^2 - 4AC = 0$,

(1) $e = 1$ and represents parabola (2) $e = 0$ and represents parabola

(3) $e = 1$ and represents a circle (4) $e = 0$ and represents a circle

5. For the parabola $(x-h)^2 = -4a(y-k)$, the equation of the directrix is

(1) $y = k$ (2) $y = a$ (3) $x = k + a$ (4) $y = k + a$

6. For the ellipse $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1,\ a < b$

(1) $e = \sqrt{1 - \frac{b^2}{a^2}} < 1$ (2) $e = \sqrt{1 - \frac{b^2}{a^2}} > 1$

(3) $e = \sqrt{1 - \frac{a^2}{b^2}} < 1$ (4) $e = \sqrt{1 + \frac{a^2}{b^2}} < 1$

7. Which of the statements are correct?

(i) The sum of the focal distances of any point on the ellipse is equal to length of major axis.

(ii) The difference of the focal distances of any point on the hyperbola is equal to the length of its transverse axis

(iii) The values of a and b decide the type of ellipse.

(iv) The values of a and b do not decide the type of the hyperbola

(1) (i) and (ii) only (2) all (3) (i) and (iii) only (4) (i) and (iv) only

8. In the general equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, if $A = C = F$ and $B = D = E = 0$ then the curve represents

(1) parabola (2) hyperbola (3) circle or ellipse (4) none of the above

9. If $y = mx + c$ is a tangent to the parabola $y^2 = 4ax$ then the point of contact is

(1) $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$ (2) $\left(\frac{-a}{m^2}, \frac{2a}{m}\right)$ (3) $\left(\frac{a}{m^2}, \frac{-2a}{m}\right)$ (4) $\left(\frac{-a}{m^2}, \frac{-2a}{m}\right)$

10. Equation of any tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is of the form

(1) either $y=mx+\sqrt{a^2m^2-b^2}$ or $y=mx-\sqrt{a^2m^2-b^2}$

(2) either $y=mx+\sqrt{a^2m^2-b^2}$ and $y=mx-\sqrt{a^2m^2-b^2}$

(3) either $y=mx+\sqrt{a^2m^2+b^2}$ or $y=mx-\sqrt{a^2m^2+b^2}$

(4) either $y=mx+\sqrt{a^2m^2+b^2}$ and $y=mx-\sqrt{a^2m^2+b^2}$

11. $y=mx+c$ is a tangent to the parabola $y^2=4ax$ then

(1) $c=\frac{a}{m}$ (2) $c=\frac{m}{a}$ (3) $c^2=a^2m^2+m^2$ (4) $m=c$

12. If $y=mx+c$ is a tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ then

(1) $c^2=a^2m^2+b^2$ (2) $b^2=c^2+a^2m^2$ (3) $c^2=a^2m^2+m^2$ (4) $c^2=a^2m^2-b^2$

13. The point of contact of the tangent $y=mx+c$ and the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is

(1) $\left(\frac{a^2m}{c},\frac{b^2}{c}\right)$ (2) $\left(\frac{a^2m}{c},\frac{-b^2}{c}\right)$ (3) $\left(-\frac{a^2m}{c},\frac{b^2}{c}\right)$ (4) $\left(\frac{-a^2m}{c},\frac{-b^2}{c}\right)$

CHAPTER 6

1. Which one is meaningful?

(1) $(\vec{a}\times\vec{b})\times(\vec{b}\cdot\vec{c})$ (2) $\vec{a}\times(5+\vec{b})$ (3) $(\vec{a}\cdot\vec{b})\times(\vec{c}\cdot\vec{d})$ (4) $(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d})$

2. With usual notation which one is not equal to $\vec{a}\cdot(\vec{b}\times\vec{c})$?

(1) $-\vec{a}\cdot(\vec{c}\times\vec{b})$ (2) $\vec{c}\cdot(\vec{b}\times\vec{a})$ (3) $-\vec{b}\cdot(\vec{c}\times\vec{a})$ (4) $(\vec{c}\times\vec{a})\cdot\vec{b}$

3. Identify the correct statements.

(i) If three vectors are coplanar then their scalar triple product is 0.

(ii) If scalar triple product of three vectors is 0 then they are coplanar

(iii) If $\vec{p}=x_1\vec{a}+y_1\vec{b}+z_1\vec{c}$

$\vec{q}=x_2\vec{a}+y_2\vec{b}+z_2\vec{c}$

$\vec{r}=x_3\vec{a}+y_3\vec{b}+z_3\vec{c}$, and $\vec{a},\vec{b},\vec{c}$ are coplanar then $\vec{p},\vec{q},\vec{r}$ are coplanar

(iv) $\vec{a},\vec{b},\vec{c}$ and $\vec{d}$ are coplanar then $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=\vec{0}$

(1) (i) and (ii) only (2) all (3) (i) and (ii) only (4) (i), (ii) and (iii) only

4. The non-parametric form of vector equation of a straight line passing through a point whose position vector is $\vec{a}$ and parallel to $\vec{u}$ is

(1) $\vec{r}=\vec{a}+t\vec{u}$ (2) $\vec{r}=\vec{u}+t\vec{a}$ (3) $(\vec{r}-\vec{u})\times\vec{a}=\vec{0}$ (4) $(\vec{r}-\vec{a})\times\vec{u}=\vec{0}$

5. Which one of the following is insufficient to find the equation of a straight line?

(1) two points on the line

(2) one point on the line and direction ratios of one parallel line

(3) one point on the line and direction ratios of its perpendicular line

(4) a perpendicular line and a parallel line in Cartesian form.

6. Which of the following statement is incorrect?

(1) if two lines are coplanar then their direction ratios must be same

(2) two coplanar lines must lie in a plane

(3) skew lines are neither parallel nor intersecting

(4) if two lines are parallel or intersecting then they are coplanar

7. The shortest distance between the two skew lines $\vec{r}=\vec{a}+t\vec{u}$ and $\vec{r}=\vec{b}+t\vec{u}$ is

(1) $\dfrac{\left|(\vec{b}-\vec{a})\cdot(\vec{u}\times\vec{v})\right|}{|\vec{u}\times\vec{v}|}$ (2) $\dfrac{\left|(\vec{b}-\vec{a})\cdot(\vec{u}\times\vec{v})\right|}{\vec{u}\times\vec{v}}$

(3) $\dfrac{\left|(\vec{b}-\vec{a})\cdot(\vec{u}\times\vec{v})\right|}{|\vec{a}\times\vec{b}|}$ (4) $\dfrac{\left|(\vec{b}-\vec{a})\cdot(\vec{u}\times\vec{v})\right|}{|\vec{a}|}$

8. The non-parametric form of a vector equation passing through a point whose position vector is $\vec{a}$ and parallel to two vectors $\vec{u}$ and $\vec{v}$ is

(1) $[\vec{r}-\vec{u},\vec{u},\vec{v}]=0$ (2) $[\vec{r}-\vec{a},\vec{u},\vec{v}]=0$ (3) $[\vec{r}-\vec{v},\vec{u},\vec{v}]=0$ (4) $[\vec{r}-\vec{u},\vec{a},\vec{v}]=0$

9. The non-parametric form of a vector equation passing through two points whose position vectors are $\vec{a}$ and $\vec{b}$ and parallel to $\vec{u}$ is

(1) $[\vec{r}-\vec{u}\ \ \vec{b}-\vec{a}\ \ \vec{u}]=0$ (2) $[\vec{r}-\vec{a}\ \ \vec{u}-\vec{a}\ \ \vec{u}]=0$

(3) $[\vec{r}-\vec{u}\ \ \vec{b}-\vec{a}\ \ \vec{u}]=0$ (4) $[\vec{r}-\vec{a}\ \ \vec{b}-\vec{a}\ \ \vec{u}]=0$

10. Which of the following is/are false, in the case of a plane passing through three points whose position vectors are $\vec{a},\vec{b}$ and $\vec{c}$?

(i) $[\vec{r}-\vec{a}\ \ \vec{b}-\vec{a}\ \ \vec{c}-\vec{a}]=0$ (ii) $[\vec{r}-\vec{a}\ \ \vec{a}-\vec{b}\ \ \vec{c}-\vec{a}]=0$

(iii) $[\vec{r}-\vec{a}\ \ \vec{b}-\vec{a}\ \ \vec{a}-\vec{c}]=0$ (iv) $[\vec{r}-\vec{a}\ \ \vec{a}-\vec{b}\ \ \vec{a}-\vec{c}]=0$

(1) (ii) and (iii) (2) (iii) and (iv) (3) all (4) none

11. With usual notations which of the following are correct?

(i) angle between $\vec{r}\cdot\vec{n}_1 = p_1$ and $\vec{r}\cdot\vec{n}_2 = p_2$ is related by $\cos\theta = \left|\frac{\vec{n}_1\cdot\vec{n}_2}{|\vec{n}_1|\,|\vec{n}_2|}\right|$

(ii) angle between $\vec{r} = \vec{a} + t\vec{u}$ and the plane $\vec{r}\cdot\vec{n} = p$ is related by $\sin\theta = \frac{|\vec{u}\cdot\vec{n}|}{|\vec{u}|\,|\vec{n}|}$

(iii) the distance between a point with position vector $\vec{u}$ and the plane $\vec{r}\cdot\vec{n} = p$ is $\frac{|\vec{u}\cdot\vec{n} - p|}{|\vec{n}|}$

(iv) the angle between $\vec{r} = \vec{a} + s\vec{u}$ and $\vec{r} = \vec{b} + t\vec{v}$ is related by $\cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}|\,|\vec{v}|}$

(1) all (2) (ii) and (iii) only (3) (i) and (iv) only (4) (i), (ii) and (iv) only

12. Suppose you are given two lines which are lying in the required plane. In how many ways can find the equation of the plane?

(1) 1 (2) 2 (3) 3 (4) 4

13. What will be happened when finding the distance between two skew lines becomes zero?

(1) they are intersecting lines (2) they are perpendicular lines
(3) parallel lines (4) neither parallel nor intersecting

14. The shortest distance between $\vec{r} + \vec{a} + s\vec{u}$ and $\vec{r} = \vec{b} + t\vec{u}$ is

(1) $\frac{|(\vec{b}-\vec{a})\times\vec{u}|}{|\vec{u}|}$ (2) $\frac{(\vec{b}-\vec{a})\times\vec{u}}{|\vec{u}|}$ (3) $\frac{(\vec{b}-\vec{a})\times\vec{u}}{\vec{a}}$ (4) $\frac{(\vec{b}-\vec{a})\times\vec{u}}{\vec{b}}$

CHAPTER 7

1. If $ax^2 + by^2 = 1$ and $a_1x^2 + b_1y^2 = 1$ intersect each other orthogonally then which one is incorrect?

(1) $\frac{1}{a} - \frac{1}{b} = \frac{1}{a_1} - \frac{1}{b_1}$ (2) $\frac{1}{a} - \frac{1}{a_1} = \frac{1}{b} - \frac{1}{b_1}$ (3) $\frac{1}{a} + \frac{1}{b_1} = \frac{1}{b} + \frac{1}{a_1}$ (4) $\frac{1}{a} - \frac{1}{b_1} = \frac{1}{b} - \frac{1}{a_1}$

2. "Let $f(x)$ be continuous on $[a,b]$ and differentiable in (a,b). If $f(a) = f(b)$ then there exists atleast one point $c \in (a,b)$ such that $f'(c) = 0$". This statement is

(1) Intermediate value theorem (2) Rolles theorem
(3) Lagrange mean value theorem (4) Taylors theorem

3. Lagrange mean value theorem becomes Rolles theorem if

(1) $f(b) = f(a)$ (2) $f'(b) = f'(a)$ (3) $f(a) = 0$ (4) $f(b) = 0$

4. For the function $f(x) = \sin x,\ x \in \left[0, \frac{\pi}{2}\right]$, Rolles theorem is not applicable, since

(1) not continuous in $\left[0, \frac{\pi}{2}\right]$ (2) not differentiable in $\left(0, \frac{\pi}{2}\right)$
(3) $f(0) \neq f\left(\frac{\pi}{2}\right)$ (4) $f'(x)$ does not exist at $x = 0$

5. Lagrange mean value theorem, constant c for the function $y=\cos x$ in $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ is

(1) 1 (2) −1 (3) not exist (4) 0

6. Rolles constant c for the function $f(x)=|x|, x\in[-1\ 1]$ is

(1) 0 (2) 1 (3) −1 (4) not existing

7. The Maclaurin's series is obtained from the Taylors series by putting

(1) $x=a$ (2) $x=0$ (3) $a=0$ (4) $a=n$

8. l'H ôpital's Rule is not applicable for the limit tends to

(1) $\frac{0}{0}$ (2) $\infty-\infty$ (3) $\frac{\infty}{\infty}$ (4) 1°

9. "If $f(x)$ is continuous on $[a,b]$ then f has both absolute maximum and absolute minimum in $[a\ b]$". This statement is

(1) Extreme value theorem (2) Intermediate value theorem

(3) Lagrange mean value theorem (4) Taylors theorem

10. For the function $f(x)$, critical numbers are obtained by solving :

(1) $f'(x)=0$ if $f'(x)$ exists; and the values of x for which $f'(x)$ does not exist

(2) $f'(x)=0$ if $f'(x)$ does not exist; and the values for which $f'(x)$ exists

(3) $f'(x)=0$ if $f'(x)$ does not exist; and the values for which $f'(x)$ does not exist

(4) $f'(x)=0$ if $f'(x)$ exists; and the values for which $f'(x)$ exists

11. Let c be a critical number for $f(x)$ then which of the following is incorrect?

(i) $f'(x)$ changes from negative to positive through c then $f(x)$ has a local minimum

(ii) $f'(x)$ changes from positive to negative through c then $f(x)$ has a local maximum

(iii) $f''(c)$ exists and $f''(c)$ changes sign through c then $(c, f(c))$ is a point of inflection.

(iv) $f''(c)$ exists at the point of inflection then $f''(c)=0$

(1) all (2) (i) and (ii) only (3) (i) only (4) (i), (ii) and (iv) only

12. If c is a critical point and $f'(c)=0$, further $f''(c)$ exists then which is incorrect?

(1) f has a relative maximum at c if $f''(c)<0$

(2) f has a relative minimum at c if $f''(c)>0$

(3) $f''(c)=0$, there is no information regarding relative maxima

(4) f has a relative maximum at c if $f''(c)>0$

13. The vertical asymptote of $f(x)=\frac{1}{x}$ is

(1) $x=0$ (2) $y=0$ (3) $x=c$ (4) $y=c$

14. The horizontal asymptote of $f(x)=\frac{1}{x}$ is

(1) $y=0$ (2) $x=0$ (3) $x=c$ (4) $y=c$

15. The slant asymptote of $f(x)=\frac{x^2-6x+7}{x+5}$ is

(1) $x+y+11=0$ (2) $x+y-11=0$ (3) $x=-5$ (4) $y=x-11$

16. The vertical asymptotes of $f(x)=\frac{2x^2-8}{x^2-16}$ are

(1) $y=\pm 4$ (2) does not exist (3) $x=\pm 16$ (4) $x=\pm 4$

17. The horizontal asymptote of $f(x)=\frac{2x^2-8}{x^2-6}$ is

(1) $x=2$ (2) $y=2$ (3) $y=\pm 4$ (4) $y=4$

18. The vertical asymptotes of $f(x)=\frac{x^2}{x^2-1}$ are

(1) $x=\pm 1$ (2) $y=\pm 1$ (3) $x=0$ (4) $y=0$

19. The horizontal asymptote of $f(x)=\frac{x^2}{x^2-1}$ is

(1) $x=1$ (2) $x=\pm 1$ (3) $y=1$ (4) $y=\pm 1$

20. The vertical asymptote of $f(x)=\frac{x^2}{x+1}$ is

(1) $x=-1$ (2) $x=1$ (3) $y=1$ (4) $y=-1$

21. The slant asymptote of $f(x)=\frac{x^2}{x+1}$ is

(1) $y=x+1$ (2) $y=x-1$ (3) $x=y-1$ (4) $x=y$

22. The vertical asymptotes of $f(x)=\frac{3x}{\sqrt{x^2+2}}$

(1) x^2-2 (2) does not exist (3) $x=\sqrt{2}$ (4) $x=-\sqrt{2}$

23. The horizontal asymptotes of $f(x)=\frac{3x}{\sqrt{x^2+2}}$ are

(1) $y=\pm 3$ (2) $x=\pm 2$ (3) $y=\pm 2$ (4) $y=0$

24. The vertical asymptote of $f(x)=\frac{x^2-6x-1}{x+3}$ is

(1) $x=-3$ (2) $x=3$ (3) does not exist (4) $x=\pm 3$

25. The slant asymptote of $f(x)=\frac{x^2-6x-1}{x+3}$ is

(1) $y=x-9$ (2) $y=x+9$ (3) $x=y$ (4) $x+y=0$

26. The vertical asymptote of $f(x)=\frac{x^2+6x-4}{3x-6}$ is

(1) $x=2$ (2) $x=3$ (3) $y=2$ (4) $y=3$

27. The slant asymptote of $f(x)=\frac{x^2+6x-4}{3x-6}$ is

(1) $y=\frac{x}{3}-\frac{8}{3}$ (2) $y=\frac{x}{3}+\frac{8}{3}$ (3) $x=\frac{y}{3}+\frac{8}{3}$ (4) $y=\frac{x}{3}+8$

CHAPTER 8

1. Identify the incorrect statements

(i) absolute error = | Actual value − app. value |

(ii) relative error = $\frac{\text{absolute error}}{\text{actual value}}$

(iii) percentage error = relative error ×100

(iv) absolute error has unit of measurement but relative error and percentage errors are units free

(1) all (2) (i) and (ii) only (3) (i), (ii), (iii) only (4) none

2. If $f(x)>0$ for all x and $g(x)=\log(f(x))$ then dg is

(1) $\frac{1}{f(x)}f'(x)dx$ (2) $\frac{1}{x}$ (3) $\frac{1}{f(x)}dx$ (4) $\frac{1}{x}dx$

3. If f and g are differentiable functions, then $d(fg)$ is

(1) $fdg+gdf$ (2) $f\cdot df-g\cdot dg$ (3) $f\cdot df+gdg$ (4) $fdg-gdf$

4. Let $A=\{(x,y)/x,y\in\mathbb{R}\}$ and $f:A\to\mathbb{R}^2$, $f_{xy}=f_{yx}$ only if

(1) f_{xy},f_{yx} exist and continuous in A (2) f_x,f_y exist and continuous in A

(3) f_{xx},f_{yy} exist and continuous in A (4) f_{xy},f_{xx} exist and continuous in A

5. Let $A=\{(x,y)/x,y\in\mathbb{R}\}$ A function $f:A\to\mathbb{R}^2$ is said to be harmonic if

(1) $\frac{\partial^2u}{\partial x^2}+\frac{\partial^2u}{\partial y^2}=0\ \forall(x,y)\in A$ (2) $\frac{\partial^2u}{\partial x^2}-\frac{\partial^2u}{\partial y^2}=0\ \forall(x,y)\in A$

(3) $\frac{\partial^2u}{\partial x^2}\div\frac{\partial^2u}{\partial y^2}=0\ \forall(x,y)\in A$ (4) $\frac{\partial^2u}{\partial x^2}\times\frac{\partial^2u}{\partial y^2}=0\ \forall(x,y)\in A$

6. If w is a function of x and y; and x and y are functions of t, then which of the following is undefined?

(1) $\frac{\partial w}{\partial x}$ (2) $\frac{\partial w}{\partial y}$ (3) $\frac{\partial x}{\partial t}$ (4) $\frac{dy}{dt}$

7. If w is a function of x and y; and x and y are functions of s and t then which of following are correct?

(i) $\frac{dw}{dt}$ is not defined (ii) $\frac{dx}{ds}$ is not defined

(iii) $\frac{dy}{dt}$ is not defined (iv) $\frac{\partial w}{\partial t}=\frac{\partial w}{\partial x}\frac{\partial x}{\partial t}+\frac{\partial w}{\partial y}\frac{\partial y}{\partial t}$

(1) all (2) (i) and (iii) only (3) (iii) & (iv) only (4) (i),(ii) and (iii) only

CHAPTER 9

1. If $f(x)$ is a continuous function on $[a,b]$ and $F(x)$ is an anti derivative of $f(x)$ then by second fundamental theorem of Integral Calculus $\int_a^b f(x)dx=$

(1) $F(b)-F(a)$ (2) $F'(b)-F'(a)$ (3) $F(a)-F(b)$ (4) 0

2. If $f(x)$ is a continuous function on $[a, b]$ and $F(x)=\int_a^x f(u)du,\ a<x<b$ then by fundamental theorem of integral calculus $\frac{d}{dx}F(x)=$

(1) $F'(x)$ (2) $f(x)$ (3) $f'(x)$ (4) $f(x)+c$

3. $\int_a^b f(a+b-x)dx=$

(1) $f(a)-f(b)$ (2) $\int_b^a f(x)dx$ (3) 0 (4) $\int_a^b f(x)dx$

4. $\int_0^{2a} f(x)dx=$

(1) 0 (2) $2\int_0^a f(x)dx$

(3) a (4) $\int_0^a f(x)dx+\int_0^a f(2a-x)dx$

5. If $f(2a-x)=f(x)$ then $\int_0^{2a} f(x)dx=$

(1) $2\int_0^a f(x)dx$ (2) $\int_{-a}^a f(x)dx$ (3) 0 (4) $\int_0^a f(x)dx$

6. If $f(2a-x)=-f(x)$ then $\int_0^{2a} f(x)dx=$

(1) $2\int_0^a f(x)dx$ (2) $\int_{-a}^a f(x)dx$ (3) 0 (4) $\int_0^a f(x)dx$

7. $\int_a^b [f(x)-f(a+b-x)]dx=$

(1) $f(b)(-f(a)$ (2) 0 (3) $f(b)-f(a)$ (4) 1

8. $\int_0^a \frac{f(x)}{f(x)+f(a-x)}dx=$

(1) 0 (2) a (3) $\frac{a}{2}$ (4) $2a$

9. If $I_{m,n}=\int_0^{\frac{\pi}{2}}\sin^m x\cos^n x\,dx$ then $I_{m,n}=$ (here $n\geq 2$)

(1) $\frac{n-1}{m+n}I_{m,n-2}$ (2) $\frac{n+1}{m+n}I_{m,n-2}$ (3) $\frac{n-1}{m+n}I_{m,n-1}$ (4) $\frac{n}{m+n}I_{m,n-2}$

10. If $I_{m,n}=\int_0^1 x^m(1-x)^n dx$ then $I_{m,n}=$ (here $n\geq 1$)

(1) $\frac{n}{m+n+1},I_{m,n-1}$ (2) $\frac{m}{m+n+1},I_{m,n-1}$ (3) $\frac{n}{m-n+1},I_{m,n-1}$ (4) $\frac{n}{m+n-1},I_{m,n-1}$

11. The values of $\int_0^\infty e^{-x}x^n dx$ and $\int_0^\infty e^{-x}x^{n-1}dx$ are respectively

(1) $m!$ and $(n-1)!$ (2) $(n+1)!$ and $(n-1)!$

(3) $n!$ and $(n-1)!$ (4) $n!$ and $(n+1)!$

CHAPTER – 10

1. Consider the statements

A : The order of a differential equation (D.E) is the highest order derivative present in the D.E

B : In the polynomial form of $D.E$., the degree of the D.E is the integral power of the highest order derivative.

Identify the correct option

(1) both are correct (2) both are false

(3) A is true, B is false (4) A is false, B is true

2. Formation of a differential equation is

(1) eliminating arbitrary constants from the given relationship by minimum number of differentiations

(2) eliminating constants from the given relationship by minimum number of differentiations

(3) eliminating arbitrary constants from the given relationship by maximum number of differentiations

(4) eliminating constants from the given relationship

3. Consider the statements :

A: The general solution of a differential equation is the solution which contains as many arbitrary constants as the order of the D.E

B: Giving particular values to the arbitrary constants in the general solution of the D.E is the particular solution.

(1) both are correct (2) both are incorrect

(3) A is correct, B is incorrect (4) A is incorrect, B is correct

4. An equation of the form $f_1(x)g_1(y)dx + f_2(x)g_2(y)dy = 0$ is called

(1) linear differential equation
(2) homogeneous
(3) linear differential equation of first order
(4) variable separable

5. A differential equation is said to be homogeneous if

(1) $\frac{dy}{dx} = g\left(\frac{y}{x}\right)$ (2) $\frac{dy}{dx} = g(x+y)$ (3) $\frac{dy}{dx} = g(xy)$ (4) $\frac{dy}{dx} = g(x-y)$

6. A first order linear differential equation is of the form

(1) $\frac{dy}{dx} + Py = Q$, where P and Q are functions of y

(2) $\frac{dx}{dy} + Py = Q$, where P and Q are functions of y

(3) $\frac{dy}{dx} + Px = Q$, where P and Q are functions of y

(4) $\frac{dy}{dx} + Py = Q$, where P and Q are functions of x

(or) $\frac{dx}{dy} + Px = Q$, where P and Q are functions of y

7. The integrating factor of $\frac{dy}{dx} + Py = Q$ is (P and Q are functions of x)

(1) $e^{\int Pdy}$ (2) $e^{\int Pdx}$ (3) $e^{\int Qdy}$ (4) $e^{\int Pdx}$

8. The integrating factor of $\frac{dx}{dy} + Px + Q$ is (P and Q are functions of y)

(1) $e^{\int Pdy}$ (2) $e^{\int Pdx}$ (3) $e^{\int Qdy}$ (4) $e^{\int Pdx}$

9. Assume that a population (x) grows or decays at a rate directly proportional to the amount population present at that time i.e. $\frac{dx}{dt} = kx$, then

(1) $k < 0$ if it is a growth problem
(2) $k > 0$ if it is a decay problem
(3) $k < 0$ if it is a decay problem and $k > 0$ if it is a growth problem
(4) $k = 0$

10. The Newtons law of cooling ($T -$ temperature of a body at any time t, T_m temperature of the surrounding medium) says

(1) $\frac{dT}{dt} \alpha (T - T_m)$ (2) $\frac{dT}{dt} = T - T_m$ always

(3) $\frac{dT}{dt} = k(T - T_m), k$ is constant of proportionality (4) $\frac{dt}{dT} = k(T - T_m)$

11. The order and degree of the differential equation $\frac{dy}{dx}=x+y+5$ are

(1) 0,0 (2) 0,1 (3) 1,0 (4) 1,1

12. The order and degree of the differential equation $\left(\frac{d^4y}{dx^4}\right)^3+4\left(\frac{dy}{dx}\right)^7+6y=5\cos 3x$ are

(1) 12,7 (2) 4,3 (3) 3,4 (4) 7,12

13. The order and degree of the differential equation $\frac{d^2y}{dx^2}+3\left(\frac{dy}{dx}\right)^2=x^2\log\left(\frac{d^2y}{dx^2}\right)$ are

(1) 2, not defined (2) 3,2 (3) 2,3 (4) 2,2

14. The order and degree of the differential equation $3\left(\frac{d^2y}{dx^2}\right)=\left[4+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}$ are

(1) $2,\frac{3}{2}$ (2) 2,2 (3) $\frac{3}{2},3$ (4) 3,2

15. The order and degree of the differential equation $dy+(xy-\cos x)dx=0$ are

(1) 1,1 (2) 1,0 (3) 0,0 (4) 0,1

16. The order and degree of the differential equation $\frac{dy}{dx}+xy=\cot x$ are

(1) 1,0 (2) 1,1 (3) 0,1 (4) 0,0

17. The order and degree of the differential equation $\left(\frac{d^3y}{dx^3}\right)^{\frac{2}{3}}-3\frac{d^2y}{dx^2}+5\frac{dy}{dx}+4=0$ are

(1) 2,2 (2) 3,3 (3) 2,3 (4) 3,2

18. The order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2+\left(\frac{dy}{dx}\right)^2=x\sin\left(\frac{d^2y}{dx^2}\right)$ are

(1) 2, not defined (2) 2,2 (3) 2,1 (4) 1,2

19. The order and degree of the differential equation $\sqrt{\frac{dy}{dx}}-4\frac{dy}{dx}-7x=0$ are

(1) 2,1 (2) 1,1 (3) 1,2 (4) 2,2

20. The order and degree of the differential equation $y\left(\frac{dy}{dx}\right)=\frac{x}{\left(\frac{dy}{dx}\right)+\left(\frac{dy}{dx}\right)^3}$ are

(1) 1,4 (2) 4,1 (3) 1,3 (4) 3,1

21. The order and degree of the differential equation $x^2\frac{d^2y}{dx^2}+\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{2}}=0$ are

(1) 2,1 (2) 2,2 (3) 1,2 (4) 1,1

22. The order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3=\sqrt{1+\left(\frac{dy}{dx}\right)}$ are

(1) 2,6 (2) 6,2 (3) 2,3 (4) 2,4

23. The order and degree of the differential equation $\frac{d^2y}{dx^2}=xy+\cos\left(\frac{dy}{dx}\right)$ are

(1) 2,1 (2) 1,2 (3) 2, not defined (4) 1,1

24. The order and degree of the differential equation $\frac{d^2y}{dx^2}+5\frac{dy}{dx}+\int ydx=x^3$ are

(1) 3,2 (2) 1,2 (3) 2,1 (4) 3,1

25. The order and degree of the differential equation $x=e^{xy\left(\frac{dy}{dx}\right)}$ are

(1) 1,1 (2) 0,1 (3) 1,0 (4) 2,1

26. Radium decays at a rate proportional to the amount Q present. The corresponding differential equation is (k is the constant of proportionality)

(1) $\frac{dQ}{dt}=k$ (2) $\frac{dQ}{dt}=Q$ (3) $\frac{dQ}{dt}=-k$ (4) $\frac{dQ}{dt}=kQ$

27. The population P of a city increases at a rate proportional to the product of population and the difference between 5,00,000 and the population. The corresponding differential equation (k is the constant of proportionality)

(1) $\frac{dP}{dt}=P(50000-P)$ (2) $\frac{dP}{dt}=k(50000-P)$ (3) $\frac{dP}{dt}=kP(500000-P)$ (4) $\frac{dP}{dt}=kP$

28. For a certain substance, the rate of change of vapor pressure P with respect to temperature T is proportional to the vapor pressure and inversely proportional to the square of the temperature. The corresponding differential equation is (k is the constant of proportionality)

(1) $\frac{dP}{dT}=\frac{P}{T^2}$ (2) $\frac{dP}{dT}=k\frac{P}{T}$ (3) $\frac{dP}{dT}=k\frac{P}{T^2}$ (4) $\frac{dP}{dT}=kP$

29. A saving amount (x) pays 8% interest per year, compounded continuously. In addition, the income from another investment is credited to the amount continuously at the rate of 400 per year. Then $\frac{dx}{dt}=$

(1) $\frac{8}{100}x+400$ (2) $\frac{8}{100}x$ (3) $8x+400$ (4) $\frac{1}{100}x+400$

30. Assume that a spherical rain drop evaporates at a rate proportional to its surface area. The rate of change of the radius (r) of the rain drop $\frac{dr}{dt}=$

(k is the constant of proportionality and $k>0$).

(1) kr (2) k (3) $-k$ (4) $-kr$

CHAPTER – 11

1. A random variable X is a function from

(1) $S \to \mathbb{R}$ (2) $\mathbb{R} \to S$ (3) $S \to \mathbb{N}$ (4) $\mathbb{N} \to S$

2. $X : S \to \mathbb{R}$ is said to be discrete random variable if

(1) range of X is countable (2) range of X is uncountable

(3) range of X is $\mathbb{N}$ (4) range of X is $\mathbb{R}$

3. $P[X=x_k], k=1,2,\cdots n$ is called a probability mass function if

(1) $P[X=x_k]\geq 0$ and $\sum_k P[X=x_k]=1$ (2) $P[X=x_k]>0$ and $\sum_k P[X=x_k]=1$

(3) $P[X=x_k]=0$ and $\sum_k P[X=x_k]=1$ (4) $P[X=x_k]\geq 0$ and $\sum_k P[X=x_k]=0$

4. Let X be a discrete random variable and taking the values $x_1, x_2 \cdots x_n$ with $p.m.f$ $P[X=x_k]$. The cumulative distribution function $F(x)$ is defined as

(1) $P[X \leq x]$ (2) $1-P[X\leq x]$ (3) $P[X<x]$ (4) $1-P[X<x]$

5. Which of the following are true in the case of $c.d.f$ $F(x)$? (X is a discrete random variable)

(i) $0 \leq F(x) \leq 1$ (ii) $\lim_{x\to-\infty} F(x)=0$ and $\lim_{x\to\infty} F(x)=1$

(iii) $P[x_1 < X \leq k_2] = F(x_2)-F(x_1)$ (iv) $P[X>x]=1-P[X\leq x]=1-F(x)$

(1) (i) and (iv) only (2) (ii),(iii),(iv) only

(3) (i), (ii), (iii) only (4) all

6. Let X be a continuous random variable. The function $f(x)$ is said to be a $p.d.f$ if

(1) $f(x)>0$ and $\int_a^b f(x)dx=0$ (2) $f(x)\geq 0$ and $\int_a^b f(x)dx=1$

(3) $f(x)>0$ and $\int_a^b f(x)dx=1$ (4) $f(x)\geq 0$ and $\int_a^b f(x)dx=0$

7. For a continuous random variable, which of the following is/are incorrect?

(i) $P[X=x]=0$ and $P[a<X<b]=F(b)-F(a)$

(ii) $P[X=x]=1$ and $P[a<X<b]=F(b)-F(a)$

(iii) $P[X=x]=0$ and $P[a\leq X\leq b]=P[a<X<b]$

(iv) $P[a<X<b]=P[a\leq X<b]=P[a<X\leq b]$ and $P[X=x]=0$

(1) (ii) and (iii) only (2) (ii) only (3) (i) and (ii) only (4) (iv) only

8. With usual notations, which of the following are correct?

(i) $Var(X) = E(X^2) - [E(X)]^2$

(ii) $Var(aX + b) = a^2 Var(X)$

(iii) $E(aX + b) = aE(X) + b$

(iv) $E(X) = \int_{-\infty}^{\infty} f(x)dx$ if X is continuous

(1) all (2) (i), (ii), (iii) only (3) (i), (ii), (iv) only (4) (ii), (iii), (iv) only

9. If X is a Bernoulli's random variable which follows Bernoulli's distribution with parameter then

(1) $\mu = p, \sigma = pq$ (2) $\mu = pq, \sigma = p$ (3) $\mu = pq, \sigma = q$ (4) $\mu = p, \sigma^2 = pq$

10. If $X \sim B(n, p)$ then

(1) $\mu = np, \sigma^2 = np(1-p)$ (2) $\mu = nq, \sigma = np(1-p)$

(3) $\mu = np, \sigma = np(1-p)$ (4) $\mu = npq \quad \sigma = npq$

CHAPTER 12

1. Which of the following is not a binary operation on $\mathbb{R}$?

(1) + (2) − (3) ÷ (4) ×

2. The operation '−' is binary on

(1) $\mathbb{N}$ (2) $Q \setminus \{0\}$ (3) $\mathbb{R} \setminus \{0\}$ (4) Q

3. The operation '÷' is binary on

(1) $\mathbb{R} \setminus \{0\}$ (2) $\mathbb{C}$ (3) $\mathbb{R}$ (4) $\mathbb{Z}$

4. The additive inverse do not exists for some elements in the set

(1) $\mathbb{R}$ (2) $-1 \le x \le 2$ (3) $\mathbb{Z}$ (4) Q

5. The multiplicative inverse exists for each element in the set

(1) $-2 \le x \le 2$ (2) $\mathbb{Z}$ (3) $\mathbb{R} \setminus \{0\}$ (4) $\mathbb{C}$

6. The identity element under addition exists in

(1) $\mathbb{N}$ (2) $\mathbb{C} \setminus \{0\}$ (3) $(0 \quad \infty)$ (4) $-3 \le x \le 3$

7. The properties closure, associative, identity, inverse and commutative under addition satisfy the set

(1) $\mathbb{R}$ (2) $\mathbb{N}$ (3) $\{1, -1, 0\}$ (4) $Q \setminus \{0\}$

8. The fourth roots of unity under multiplication satisfies the properties

(1) closure only (2) closure and associative only

(3) closure, associative and identity (4) closure, associative identity and inverse

9. Which one of the following is correct?

(1) $[3]+_4[2]=[5]$

(2) $[0]+_{10}[12]=[0]$

(3) $[4]\times_5[3]=[12]$

(4) $[5]\times_6[4]=[2]$

10. Which of the following is not true?

(1) A Boolean matrix is a real matrix whose entries are either 0 or 1

(2) The product $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$ is a Boolean matrix

(3) All identity matrices I_n are Boolean matrices

(4) $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\vee\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}=\begin{bmatrix}2 & 0\\0 & 2\end{bmatrix}$

Chapter – 1

Qn.No.	1	2	3	4	5	6	7	8	9	10
Key	(4)	(1)	(2)	(1)	(4)	(3)	(4)	(1)	(2)	(4)

Chapter – 2

Qn.No.	1	2	3	4	5	6	7	8	9	10
Key	(1)	(2)	(1)	(3)	(1)	(4)	(3)	(1)	(3)	(1)

Chapter – 3

Qn.No.	1	2	3	4	5	6				
Key	(1)	(3)	(4)	(3)	(1)	(1				

Chapter – 4

Qn.No.	1	2	3	4	5	6	7	8	9	10
Key	(3)	(2)	(3)	(1)	(1)	(2)	(1)	(3)	(1)	(1)

Chapter – 5

Qn.No.	1	2	3	4	5	6	7	8	9	10
Key	(1)	(3)	(1)	(1)	(4)	(3)	(2)	(4)	(1)	(1)
Qn.No.	11	12	13							
Key	(1)	(1)	(3)							

Chapter – 6

Qn.No.	1	2	3	4	5	6	7	8	9	10
Key	(4)	(3)	(2)	(4)	(4)	(1)	(1)	(2)	(4)	(3)
Qn.No.	11	12	13	14						
Key	(1)	(4)	(1)	(1)						

Chapter – 7

Qn.No.	1	2	3	4	5	6	7	8	9	10
Key	(4)	(2)	(1)	(3)	(4)	(4)	(3)	(4)	(1)	(1)
Qn.No.	11	12	13	14	15	16	17	18	19	20
Key	(1)	(4)	(1)	(1)	(4)	(4)	(2)	(1)	(3)	(1)
Qn.No.	21	22	23	24	25	26	27			
Key	(2)	(2)	(1)	(1)	(1)	(1)	(2)			

Chapter – 8

Qn.No.	1	2	3	4	5	6	7			
Key	(4)	(1)	(1)	(1)	(1)	(3)	(1)			

Chapter – 9

Qn.No.	1	2	3	4	5	6	7	8	9	10
Key	(1)	(2)	(2)	(4)	(1)	(3)	(2)	(3)	(1)	(1)
Qn.No.	11									
Key	(3)									

Chapter – 10

Qn.No.	1	2	3	4	5	6	7	8	9	10
Key	(1)	(1)	(1)	(4)	(1)	(4)	(4)	(1)	(3)	(3)
Qn.No.	11	12	13	14	15	16	17	18	19	20
Key	(4)	(2)	(1)	(2)	(1)	(1)	(4)	(1)	(3)	(1)
Qn.No.	21	22	23	24	25	26	27	28	29	30
Key	(2)	(1)	(3)	(4)	(1)	(4)	(3)	(3)	(1)	(3)

Chapter – 11

Qn.No.	1	2	3	4	5	6	7	8	9	10
Key	(1)	(1)	(1)	(1)	(4)	(2)	(2)	(2)	(4)	(1)

Chapter – 12

Qn.No.	1	2	3	4	5	6	7	8	9	10
Key	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(4)	(4)	(4)