42. a) Solve the equation $x^3 - 9x^2 + 14x + 24 = 0$ if it is given that two of its roots are in the ratio 3 : 2.

(OR)

b) Find the value of $\tan^{-1}(-1) + \cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$.

43. a) Cross - section of a nuclear cooling tower is in the shape of a hyperbola with equation $\frac{x^2}{30^2} - \frac{y^2}{44^2} = 1$. The tower is 150m tall and the distance from the top of the tower to the centre of the hyperbola is half the distance from the base of the tower to the centre of the hyperbola. Find the diameter of the top and base of the tower. (OR)

b) Find the shortest distance between the two given straight lines $\vec{r} = (2\hat{i} + 3\hat{j} + 4\hat{k}) + t(-2\hat{i} + \hat{j} - 2\hat{k})$ and $\frac{x-3}{2} = \frac{y}{-1} = \frac{z+2}{2}$.

44. a) A hollow cone with base radius a cm and height b cm is placed on a table. Show that the volume of the largest cylinder that can be hidden underneath is $\frac{4}{9}$ times volume of the cone. (OR)

b) If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$.

45. a) A watermelon has an ellipsoid shape which can be obtained by revolving an ellipse with major - axis 20cm and minor - axis 10cm about its major axis. Find its volume using integration. (OR)

b) Solve : $\frac{dy}{dx} + \frac{y}{x\log x} = \frac{\sin 2x}{\log x}$

46. a) Suppose that $f(x)$ given below represents a probability mass function.

x	1	2	3	4	5	6
f(x)	c^2	$2c^2$	$3c^2$	$4c^2$	c	2c

Find (i) the value of c (ii) mean and variance. (OR)

b) Using the truth table prove that $p \vee (q \vee r) \equiv (p \vee q) \vee r$.

47. a) Show that $\int_0^1 (\tan^{-1} x + \tan^{-1}(1-x))\,dx = \frac{\pi}{2} - \log 2$. (OR)

b) Find the local maximum and minimum of the function x^2y^2 on the lime $x + y = 10$.

★★★★★

Full Portion Test - 1

Standard XII

MATHEMATICS

Time : 3.00 hr Marks : 90

Instructions: 1) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.
2) Use Blue or Black ink to write and underline and pencil to draw diagrams.

PART - A 20x1=20

Note: i) Answer all the questions.
ii) Choose most suitable answer from the given four alternatives and write the option code with the corresponding answer.

1. If A is a non-singular matrix such that $A^{-1} = \begin{pmatrix} 1 & 2 \\ -3 & -4 \end{pmatrix}$, then $(A^T)^{-1} =$

a) $\begin{pmatrix} -1 & 2 \\ 3 & 4 \end{pmatrix}$ b) $\begin{pmatrix} 1 & 2 \\ -3 & -4 \end{pmatrix}$ c) $\begin{pmatrix} -4 & -2 \\ 3 & 1 \end{pmatrix}$ d) $\begin{pmatrix} 1 & -3 \\ 2 & -4 \end{pmatrix}$

2. If $A = \begin{pmatrix} 2 & 0 \\ 1 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 4 \\ 2 & 0 \end{pmatrix}$ then $|\text{adj}(AB)| =$

a) −40 b) −80 c) −60 d) −20

3. The principal argument of $(\sin 40° + i\cos 40°)^5$ is

a) −110° b) −70° c) 70° d) 110°

4. If $\frac{Z-1}{Z+1}$ is purely imaginary, then $|Z|$ is

a) $\frac{1}{2}$ b) 1 c) 2 d) 3

5. The solution of the equation $|Z| - Z = 1 + 2i$ is

a) $\frac{3}{2} - 2i$ b) $\frac{-3}{2} + 2i$ c) $2 - \frac{3}{2}i$ d) $2 + \frac{3}{2}i$

6. According to the rational root theorem, which number is not possible rational root of $4x^5 + 2x^4 - 10x^3 - 5$?

a) −1 b) $\frac{5}{4}$ c) $\frac{4}{5}$ d) 5

7. For $x \in R$, $\cot^{-1}(x) =$

a) $\frac{\pi}{2} - \cot^{-1} x$ b) $\pi - \cot^{-1} x$ c) $\pi + \cot^{-1} x$ d) $\frac{\pi}{2} + \cot^{-1} x$

8. If $x = \frac{1}{5}$, the value of $\cos(\cos^{-1} x + 2\sin^{-1} x)$ is

a) $-\sqrt{\frac{24}{25}}$ b) $\sqrt{\frac{24}{25}}$ c) $\frac{1}{5}$ d) $-\frac{1}{5}$

9. Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ is

a) 12 b) 6 c) $\sqrt{6}$ d) $\frac{3}{2}$

10. If $x+y=k$ is a normal to the parabola $y^2=12x$, then the value of k is

a) 3 b) -1 c) 1 d) 9

11. Distance from the origin to the plane $3x-6y+2z+14=0$ is

a) 0 b) 1 c) 2 d) 3

12. The point of inflection of the curve $y=(x-1)^3$ is

a) (0, 0) b) (0, 1) c) (1, 0) d) (1, 1)

13. The value of $\sin^{-1}(\cos x)$; $0\le x\le\pi$ is

a) $\pi-x$ b) $x-\frac{\pi}{2}$ c) $\frac{\pi}{2}-x$ d) $x-\pi$

14. If $u(x, y)=e^{x^3+y^3}$, then $\frac{\partial u}{\partial x}$ is equal to

a) $e^{x^3+y^3}$ b) $3xu$ c) x^3u d) y^3u

15. The value of $\int_{-1}^{2}|x|\,dx$ is

a) $\frac{1}{2}$ b) $\frac{3}{2}$ c) $\frac{5}{2}$ d) $\frac{7}{2}$

16. The general solution of the differential equation $\frac{dy}{dx}=\frac{y}{x}$ is

a) $xy=k$ b) $y=k\log x$ c) $y=kx$ d) $\log y=kx$

17. The degree of the differential equation $y(x)=1+\frac{dy}{dx}+\frac{1}{1.2}\left(\frac{dy}{dx}\right)^2+\frac{1}{1.2.3}\left(\frac{dy}{dx}\right)^3+\ldots$ is

a) 2 b) 3 c) 1 d) 4

18. Let X have a Bernoulli distribution with mean 0.4, then the variance of $(2x-3)$ is

a) 0.24 b) 0.48 c) 0.6 d) 0.96

19. If $f(x)=\begin{cases}2x & 0\le x<a\\ 0 & \text{otherwise}\end{cases}$ is a probability density function of a random variable, then the value of a is

a) 1 b) 2 c) 3 d) 4

20. The proposition $p\wedge(\neg p\vee q)$ is

a) a tautology b) a contradiction
c) logically equivalent to $p\wedge q$ d) logically equivalent to $p\vee q$

PART - B 7x2=14

Note: i) Answer any seven questions.
ii) Question No. 30 is compulsory.

21. Using cramer's rule solve the equations $5x-2y+16=0$, $x+3y-7=0$.

22. Find the modulus of the complex number $(1-i)^{10}$.

23. Find the domain of $\cos^{-1}\left(\frac{2+\sin x}{3}\right)$.

24. Find the vertices, foci for the hyperbola $9x^2-16y^2=144$.

25. Show that the lines $\frac{x-1}{4}=\frac{2-y}{6}=\frac{z-4}{12}$ and $\frac{x-3}{-2}=\frac{y-3}{3}=\frac{5-z}{6}$ are parallel.

26. Find the local extremum of the function $f(x)=x^4+32x$

27. Let $V(x, y, z)=xy+yz+zx$, $x, y, z\in R$. Find the differential dV

28. Evaluate: $\int_0^{\pi/2}\sin^{10}x\,dx$

29. Find the order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3=\sqrt{1+\left(\frac{dy}{dx}\right)}$.

30. Write the statement in words corresponding to $\sim p$, $p\wedge q$, $p\vee q$ and $q\vee\sim p$, where P is 'it is cold' and q is 'It is raining'

PART - C 7x3=21

Note: i) Answer any seven questions.
ii) Question No. 40 is compulsory.

31. Using matrix inversion method solve the equations $5x+2y=3$, $3x+2y=5$.

32. If $|Z|=1$, show that $2\le|z^2-3|\le4$.

33. If p and q are the roots of the equation $\ell x^2+nx+n=0$, show that $\sqrt{\frac{p}{q}}+\sqrt{\frac{q}{p}}+\sqrt{\frac{n}{\ell}}=0$

34. Simplify: $\tan^{-1}\frac{x}{y}-\tan^{-1}\frac{x-y}{x+y}$

35. Find the equations of tangents to the hyperbola $\frac{x^2}{16}-\frac{y^2}{64}=1$. Which are parallel to $10x-3y+9=0$

36. Find the length of the perpendicular from the point $(1, -2, 3)$ to the plane $x-y+z=5$.

37. Find the asymptotes of the curve $f(x)=\frac{x^2-6x-1}{x+3}$.

38. If $w(x, y)=x^3-3xy+2y^2$, $x, y\in R$, find the linear approximation for w at $(1, -1)$.

39. Evaluate: $\int_{\frac{1}{2}}^{1}\frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}dx$

40. The probability density function of X is give by $f(x)=\begin{cases}kxe^{-2x} & \text{for } x>0\\ 0 & \text{for } x\le0\end{cases}$. Then find the value of k.

PART - D 7x5=35

Note: Answer all the questions.

41. a) A chemist has one solution which is 50% acid and another solution which is 25% acid. How much each should be mixed to make 10 litres of a 40% acid solution? (Use cramer's rule to solve the problem)

(OR)

b) Suppose z_1, z_2 and z_3 are the vertices of an equilateral triangle inscribed in the circle $|z|=2$. If $z_1=1+i\sqrt{3}$, then find z_2 and z_3.