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CLASS : X MARKS : 75

SUBJECT : MATHS TIME : 150 Min

I. ANSWER ALL THE QUESTIONS **1x10 = 10**

1. If $\vec{a}$ and $\vec{b}$ are parallel vectors, then $[\vec{a},\vec{c},\vec{b}]$ is equal to
   (1) 2 (2) -1 (3) 1 (4) 0

2. If $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}=0$, then the value of $[\vec{a},\vec{b},\vec{c}]$ is
   (1) $|\vec{a}|\,|\vec{b}|\,|\vec{c}|$ (2) $\frac{1}{3}|\vec{a}|\,|\vec{b}|\,|\vec{c}|$ (3) 1 (4) -1

3. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+\hat{j}$, $\vec{c}=\hat{i}$ and $(\vec{a}\times\vec{b})\times\vec{c}=\lambda\vec{a}+\mu\vec{b}$, then the value of $\lambda+\mu$ is
   (1) 0 (2) 1 (3) 6 (4) 3

4. The angle between the lines $\frac{x-2}{3}=\frac{y+1}{-2}, z=2$ and $\frac{x-1}{1}=\frac{2y+3}{3}=\frac{z+5}{2}$ is
   (1) $\frac{\pi}{6}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{2}$

5. The angle between the line $\vec{r}=(\hat{i}+2\hat{j}-3\hat{k})+t(2\hat{i}+\hat{j}-2\hat{k})$ and the plane $\vec{r}\cdot(\hat{i}+\hat{j})+4=0$ is
   (1) $0°$ (2) $30°$ (3) $45°$ (4) $90°$

6. Distance from the origin to the plane $3x-6y+2z+7=0$ is
   (1) 0 (2) 1 (3) 2 (4) 3

7. If the direction cosines of a line are $\frac{1}{c},\frac{1}{c},\frac{1}{c}$, then
   (1) $c=\pm 3$ (2) $c=\pm\sqrt{3}$ (3) $c>0$ (4) $0<c<1$

8. If the length of the perpendicular from the origin to the plane $2x+3y+\lambda z=1$, $\lambda>0$ is $\frac{1}{5}$, then the value of λ is
   (1) $2\sqrt{3}$ (2) $3\sqrt{2}$ (3) 0 (4) 1

9. If $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$, $\vec{b}=\hat{i}+2\hat{j}-5\hat{k}$, $\vec{c}=3\hat{i}+5\hat{j}-\hat{k}$, then a vector perpendicular to $\vec{a}$ and lies in the plane containing $\vec{b}$ and $\vec{c}$ is
   (1) $-17\hat{i}+21\hat{j}-97\hat{k}$ (2) $17\hat{i}+21\hat{j}-123\hat{k}$
   (3) $-17\hat{i}-21\hat{j}+97\hat{k}$ (4) $-17\hat{i}-21\hat{j}-97\hat{k}$

10. If $\vec{a},\vec{b},\vec{c}$ are non-coplanar, non-zero vectors such that $[\vec{a}, \vec{b}, \vec{c}]=3$, then $\{[\vec{a}\times\vec{b}, \vec{b}\times\vec{c}, \vec{c}\times\vec{a}]\}^2$ is equal to

(1) 81 (2) 9 (3) 27 (4)18

II. ANSWER ANY 10 QUESTIONS 10x2=20

11. A particle is acted upon by the forces $3\hat{i}-2\hat{j}+2\hat{k}$ and $2\hat{i}+\hat{j}-\hat{k}$ is displaced from the point $(1,3,-1)$ to the point $(4,-1,\lambda)$. If the work done by the forces is 16 units, find the value of λ.

12. Show that the vectors $\hat{i}+2\hat{j}-3\hat{k}$, $2\hat{i}-\hat{j}+2\hat{k}$ and $3\hat{i}+\hat{j}-\hat{k}$ are coplanar.

13. If $\vec{a}=\hat{i}-2\hat{j}+3\hat{k}$, $\vec{b}=2\hat{i}+\hat{j}-2\hat{k}$, $\vec{c}=3\hat{i}+2\hat{j}+\hat{k}$, find $\vec{a}\cdot(\vec{b}\times\vec{c})$.

14. Prove that $[\vec{a}\times\vec{b}, \vec{b}\times\vec{c}, \vec{c}\times\vec{a}]=[\vec{a},\vec{b},\vec{c}]^2$.

15. Find the acute angle between the lines $\vec{r}=(\hat{i}+2\hat{j}+4\hat{k})+t(2\hat{i}+2\hat{j}+\hat{k})$ and the straight line passing through the points $(5,1,4)$ and $(9,2,12)$.

16. Show that the points $(2,3,4), (-1,4,5)$ and $(8,1,2)$ are collinear.

17. Determine whether the pair of straight lines $\vec{r}=(2\hat{i}+6\hat{j}+3\hat{k})+t(2\hat{i}+3\hat{j}+4\hat{k})$, $\vec{r}=(2\hat{j}-3\hat{k})+s(\hat{i}+2\hat{j}+3\hat{k})$ are parallel. Find the shortest distance between them.

18. Find the intercepts cut off by the plane $\vec{r}\cdot(6\hat{i}+4\hat{j}-3\hat{k})=12$ on the coordinate axes.

19. Show that the lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{3}$ and $\frac{x-1}{-3}=\frac{y-4}{2}=\frac{z-5}{1}$ are coplanar.Also, find the plane containing these lines.

20. Find the acute angle between the planes $\vec{r}\cdot\left(2\hat{i}+2\hat{j}+2\hat{k}\right)=11$ and $4x-2y+2z=15$.

21. Find the angle between the straight line $\vec{r}=\left(2\hat{i}+3\hat{j}+\hat{k}\right)+t\left(\hat{i}-\hat{j}+\hat{k}\right)$ and the plane $2x-y+z=5$.

22. Find the distance of a point $(2,5,-3)$ from the plane $\vec{r}\cdot\left(6\hat{i}-3\hat{j}+2\hat{k}\right)=5$.

III. ANSWER ANY 9 QUESTIONS 9x5=45

23. A) By vector method, prove that $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$.

B) Prove by vector method that $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$,

24. With usual notations, in any triangle *ABC*, prove by vector method that $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$.

25. Show that the lines $\vec{r}=\left(-\hat{i}-3\hat{j}-5\hat{k}\right)+s\left(3\hat{i}+5\hat{j}+7\hat{k}\right)$ and $\vec{r}=\left(2\hat{i}+4\hat{j}+6\hat{k}\right)+t\left(\hat{i}+4\hat{j}+7\hat{k}\right)$ are coplanar. Also,find the non-parametric form of vector equation of the plane containing these lines.

26. Find the non-parametric form of vector equation, and Cartesian equation of the plane passing through the point $(0,1,-5)$ and parallel to the straight lines $\vec{r}=(\hat{i}+2\hat{j}-4\hat{k})+s(2\hat{i}+3\hat{j}+6\hat{k})$ and $\vec{r}=(\hat{i}-3\hat{j}+5\hat{k})+t(\hat{i}+\hat{j}-\hat{k})$.

27. For any four vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$, we have

$$(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}) = [\vec{a},\vec{b},\vec{d}]\vec{c}-[\vec{a},\vec{b},\vec{c}]\vec{d}=[\vec{a},\vec{c},\vec{d}]\vec{b}-[\vec{b},\vec{c},\vec{d}]\vec{a}.$$

28. Find the parametric form of vector equation and Cartesian equations of the straight line passing through the point $(-2,3,4)$ and parallel to the straight line $\frac{x-1}{-4}=\frac{y+3}{5}=\frac{8-z}{6}$.

29. If the two lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-m}{2}=z$ intersect at a point, find the value of m.

30. Find the direction cosines of the normal to the plane and length of the perpendicular from the origin to the plane $\vec{r}\cdot(3\hat{i}-4\hat{j}+12\hat{k})=5$.

31. Find the vector parametric, vector non-parametric and Cartesian form of the equation of the plane passing through the points $(-1,2,0)$, $(2,2-1)$ and parallel to the straight line $\frac{x-1}{1}=\frac{2y+1}{2}=\frac{z+1}{-1}$.

32. If the straight lines $\frac{x-1}{2}=\frac{y+1}{\lambda}=\frac{z}{2}$ and $\frac{x+1}{5}=\frac{y+1}{2}=\frac{z}{\lambda}$ are coplanar, find λ and equations of the planes containing these two lines.

33. Find the coordinates of the point where the straight line $\vec{r}=\left(2\hat{i}-\hat{j}+2\hat{k}\right)+t\left(3\hat{i}+4\hat{j}+2\hat{k}\right)$ intersects the plane $x-y+z-5=0$.