PROFILE
MODEL PAPER - I

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P-partial

HIGHER SECONDARY SECOND YEAR

MATHEMATICS

MODEL QUESTION PAPER – 1

Time Allowed: 15 Min + 3.00 Hours] **[Maximum Marks:9**

Instructions: (a) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.

(b) Use **Blue** or **Black** ink to write and underline and pencil to draw diagrams.

PART – I

Note: (i) **All** questions are **compulsory.** $20\times1=2$

(ii) Choose the correct or most suitable answer from the given **four** alternatives. Write th option code and the corresponding answer.

1. If A is a 3×3 non-singular matrix such that $AA^T = A^T A$ and $B = A^{-1}A^T$, then $BB^T =$

   (1) A (2) B (3) I (4) B^T

2. The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \end{bmatrix}$ is

   (1) 1 (2) 2 (3) 4 (4) 3

3. Which of the following are correct statements?

   (i) $e^{-i\theta} = \cos\theta - i\sin\theta$ (ii) $e^{i\frac{\pi}{2}} = i$

   (iii) $e^{i(x+iy)} = e^{-y}(\cos x + i\sin x)$ (iv) $e^{-i(y-ix)} = e^{-x}(\cos y - i\sin y)$

   (1) (i) and (iv) only (2) (iii) only (3) (i), (ii) and (iii) (4) all

4. The value of $\sum_{i=1}^{13}\left(i^n + i^{n-1}\right)$ is

   (1) $1+i$ (2) i (3) 1 (4) 0

5. If α, β, and γ are the roots of $x^3 + px^2 + qx + r = 0$, then $\sum\frac{1}{\alpha}$ is

   (1) $-\frac{q}{r}$ (2) $-\frac{p}{r}$ (3) $\frac{q}{r}$ (4) $-\frac{q}{p}$

6. e^{ix} is a periodic function with period

   (1) 0 (2) π (3) 2π (4) 4π

7. The radius of the circle $3x^2+by^2+4bx-6by+b^2=0$ is

(1) 1 (2) 3 (3) $\sqrt{10}$ (4) $\sqrt{11}$

8. The length of latus rectum of the parabola $x^2=25y$ is

(1) 25 (2) $\frac{25}{4}$ (3) 100 (4) $\frac{25}{2}$

9. The volume of the parallelepiped with its edges represented by the vectors $\hat{i}+\hat{j},\hat{i}+2\hat{j},\hat{i}+\hat{j}+\pi\hat{k}$ is

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$ (3) π (4) $\frac{\pi}{4}$

10. The Cartesian form of the plane $\vec{r}.(2\hat{i}+2\hat{j}+5\hat{k})=1$ is

(1) $2x+2y+5z+1=0$ (2) $2x+2y+5z=1$ (3) $x+y+5z=1$ (4) $2x+2y+5z=-1$

11. The position of a particle moving along a horizontal line of any time t is given by $s(t)=3t^2-2t-8$. The time at which the particle is at rest is

(1) $t=0$ (2) $t=\frac{1}{3}$ (3) $t=1$ (4) $t=3$

12. The minimum value of the function $|3-x|+9$ is

(1) 0 (2) 3 (3) 6 (4) 9

13. The differential df of $f(x)=x+\tan x$ is

(1) $1+\sec^2 x$ (2) $(1+\sec^2 x)dx$ (3) $1+\tan x$ (4) $\sec^2 x\,dx$

14. If $f(2a-x)=f(x)$ then $\int_0^{2a} f(x)dx=$

(1) $2\int_0^a f(x)dx$ (2) $\int_{-a}^a f(x)dx$ (3) 0 (4) $\int_0^a f(x)dx$

15. $\int_0^{\frac{\pi}{2}}(\sec^2 x-\tan^2 x)dx=$

(1) $\frac{\pi}{4}$ (2) 0 (3) π (4) $\frac{\pi}{2}$

16. The order and degree of the differential equation $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^{1/3}+x^{1/4}=0$ are respectively

(1) 2, 3 (2) 3, 3 (3) 2, 6 (4) 2, 4

17. The differential equation of the family of curves $y=Ae^x+Be^{-x}$, where A and B are arbitrary constants, is

(1) $\frac{d^2y}{dx^2}+y=0$ (2) $\frac{d^2y}{dx^2}-y=0$ (3) $\frac{dy}{dx}+y=0$ (4) $\frac{dy}{dx}-y=0$

18. Which of the following are true in the case of $c.d.f\ F(x)$? (X is a discrete random variable)

(i) $0 \le F(x) \le 1$ (ii) $\lim_{x \to -\infty} F(x) = 0$ and $\lim_{x \to \infty} F(x) = 1$

(iii) $P[x_1 < X \le x_2] = F(x_2) - F(x_1)$ (iv) $P[X > x] = 1 - P[X \le x] = 1 - F(x)$

(1) (i) and (iv) only (2) (ii),(iii),(iv) only

(3) (i), (ii), (iii) only (4) all

19. If X is the random variable with distribution function $F(x)$ given by,

$$F(x) = \begin{cases} 0, & x < 0 \\ x, & 0 \le x < 1 \\ 1, & 1 \le x \end{cases}$$ then the p.d.f. $f(x)$ is

(1) $\begin{cases} 1 & 0 \le x < 1 \\ 0 & \text{otherwise} \end{cases}$ (2) $\begin{cases} 1 & 0 \le x < 1 \\ x & \text{otherwise} \end{cases}$ (3) $\begin{cases} x & 0 \le x < 1 \\ 1 & \text{otherwise} \end{cases}$ (4) 0 for all x

20. In the set $\mathbb{Q}$ define $a \odot b = a + b + ab$. Then the value of y in $3 \odot (y \odot 5) = 7$ is

(1) $\frac{2}{3}$ (2) $\frac{-2}{3}$ (3) $\frac{-3}{2}$ (4) 4

PART – II

Note: (i) Answer any **SEVEN** questions. $7 \times 2 = 1$

(ii) Question number **30 is compulsory.**

21. If $z_1 = 3 - 2i$ and $z_2 = 6 + 4i$, show that $\frac{z_1}{z_2} = \frac{5}{26} - \frac{6}{13}i$.

22. If α and β are the roots of the quadratic equation $2x^2 - 7x + 13 = 0$, find $\alpha^2 + \beta^2 + 3\alpha\beta$.

23. Find the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$.

24. Show that the vectors $\hat{i} + 2\hat{j} - 3\hat{k}, 2\hat{i} - \hat{j} + 2\hat{k}$ and $3\hat{i} + \hat{j} - \hat{k}$ are coplanar.

25. Find the tangent to the curve $y = x^4 + 2e^x$ at $(0,2)$.

26. Find df if $f(x) = x^2 + 3x$, $x = 3$ and $dx = 0.02$.

27. If $\sin x$ is the integrating factor of the linear differential equation $\frac{dy}{dx} + Py = Q$, then show th P is $\cot x$.

28. Suppose the amount of milk sold daily at a milk booth is distributed with a minimum of 20 litres and a maximum of 600 litres with probability density function $f(x) = \begin{cases} k & 200 \le x \le 600 \\ 0 & \text{otherwise} \end{cases}$

Find the value of k.

29. The p.m.f of a random variable X is

x	2	4	6	8	10
$f(x)$	$\frac{1}{36}$	$\frac{4}{36}$	$\frac{10}{36}$	$\frac{12}{36}$	$\frac{9}{36}$

Write the c.d.f in horizontal form.

30. Show that the solution of the differential equation $\frac{dy}{dx} = 2xy$ is $\log y = x^2 + c$.

PART – III

Note: (i) Answer any **SEVEN** questions. $7 \times 3 = 21$

(ii) Question number **40 is compulsory.**

31. If A is nonsingular, then prove that A^{-1} is also nonsingular and $(A^{-1})^{-1} = A$.

32. Find the matrix A for which $A\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix}$.

33. Show that the equation $z^3 + 2\bar{z} = 0$ has five solutions.

34. Find the exact number of real zeros and imaginary zeros of the polynomial $x^9 + 9x^7 + 7x^5 + 5x^3 + 3x$.

35. A circle of area 9π square units has two of its diameters along the lines $x + y = 5$ and $x - y = 1$. Find the equation of the circle.

36. Evaluate : $\lim_{x \to 0} \frac{1 - \cos x}{x^2}$.

37. A circular plate expands uniformly under the influence of heat. If it's radius increases from 10.5cm to 10.75cm, then find an approximate change in the area.

38. Show that $\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{1}{1 + \sqrt{\tan x}} dx = \frac{\pi}{8}$.

39. Show that the cube roots of unity under usual multiplication satisfies the closure axiom.

40. A force given by $3\hat{i} + 2\hat{j} - 4\hat{k}$ is applied at the point $(1, -1, 2)$. Find the moment of the force about the point $(2, -1, 3)$.

PART – IV

Note: Answer **all** the questions. $7 \times 5 = 35$

41. (a) Solve the system of linear equations $2x + 3y - z = 9,\ x + y + z = 9,\ 3x - y - z = -1$ by matrix inversion method.

(OR)

(b) Find the centre, foci, vertices of the ellipse $\frac{(x-1)^2}{25} + \frac{(y+2)^2}{16} = 1$.

42. (a) Solve : $\sin^{-1}\frac{5}{x}+\sin^{-1}\frac{12}{x}=\frac{\pi}{2}$.

(OR)

(b) Find the fourth roots of $\sqrt{3}+i$.

43. (a) Prove that $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$ by vector method.

(OR)

(b) Verify whether $((p\vee q)\wedge\neg p)\to q$ is tautology or contradiction or contingency.

44. (a) Find the area of the region bounded between the parabola $y^2=4ax$ and its latus rectum.

(OR)

(b) Salt is poured from a conveyer belt at a rate of 30 cubic metre per minute forming a conic pile with a circular base whose height and diameter of base are always equal. How fast the height of the pile increasing when the pile is 10 metre high?

45. (a) Write any five points to trace the curve $y=x\sqrt{4-x}$.

(OR)

(b) Find the volume of the solid generated by revolving the region enclosed by $x^2=1+y$ and $y=3$ about y-axis, using integration.

46. (a) An engineer designs a satellite dish with a parabolic cross section. The dish is 5m wide at the opening, and the focus is placed 1.2m from the vertex. Find the depth of the satellite dish at the vertex.

(OR)

(b) Solve the differential equation $xdy-ydx=\sqrt{x^2+y^2}dx$.

47. (a) The cumulative distribution function of a discrete random variable is given by

$$F(x)=\begin{cases}0 & -\infty<x<-1\\ 0.15 & -1\le x<0\\ 0.35 & 0\le x<1\\ 0.60 & 1\le x<2\\ 0.85 & 2\le x<3\\ 1 & 3\le x<\infty\end{cases}$$

Find (i) the probability mass function (ii) $P(X<1)$ and (iii) $P(X\ge 2)$.

(OR)

47. (b) Find the vector (parametric, non parametric) and Cartesian equations of the plane passing through the points $(-1,2,0)$, $(2,2-1)$ and parallel to the straight line $\frac{x-1}{1}=\frac{2y+1}{2}=\frac{z+1}{-1}$.

MODEL QUESTION PAPER – 1

ANSWERS

PART – I

Qn. No.	Option	Answer	Qn. No.	Option	Answer
1.	(3)	I	11.	(2)	$t=\frac{1}{3}$
2.	(1)	1	12.	(4)	9
3.	(4)	all	13.	(2)	$(1+\sec^2 x)dx$
4.	(1)	$1+i$	14.	(1)	$2\int_0^a f(x)dx$
5.	(1)	$-\frac{q}{r}$	15.	(4)	$\frac{\pi}{2}$
6.	(3)	2π	16.	(1)	2, 3
7.	(3)	$\sqrt{10}$	17.	(2)	$\frac{d^2y}{dx^2}-y=0$
8.	(1)	25	18.	(4)	all
9.	(3)	π	19.	(1)	$\begin{cases}1 & 0\le x<1\\ 0 & \text{otherwise}\end{cases}$
10.	(2)	$2x+2y+5z=1$	20.	(2)	$-\frac{2}{3}$

For writing the correct option and the answer – one mark.

PART – II

21. If $z_1 = 3-2i$ and $z_2 = 6+4i$, show that $\frac{z_1}{z_2} = \frac{5}{26} - \frac{6}{13}i$.

Solution

$$\frac{z_1}{z_2} = \frac{3-2i}{6+4i} = \frac{3-2i}{6+4i} \times \frac{6-4i}{6-4i}$$
$$= \frac{(18-8)+i(-12-12)}{6^2+4^2} = \frac{10-24i}{52} = \frac{10}{52} - \frac{24i}{52}$$
$$= \frac{5}{26} - \frac{6}{13}i.$$

22. If α and β are the roots of the quadratic equation $2x^2 - 7x + 13 = 0$, find $\alpha^2 + \beta^2 + 3\alpha\beta$.

Solution

$$\alpha + \beta = \frac{7}{2}\ ;\ \alpha\beta = \frac{13}{2}.$$
$$\alpha^2 + \beta^2 + 3\alpha\beta = (\alpha+\beta)^2 + \alpha\beta$$
$$= \left(\frac{7}{2}\right)^2 + \frac{13}{2}$$
$$= \frac{49}{4} + \frac{13}{2} = \frac{75}{4}$$

23. Find the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$.

Solution

$$\sin^{-1}\left(-\frac{1}{2}\right) = -\sin^{-1}\frac{1}{2} \text{ since } \frac{1}{2} \in [-1,1]$$
$$= -\frac{\pi}{6} \text{ since } \frac{-\pi}{6} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

24. Show that the vectors $\hat{i}+2\hat{j}-3\hat{k}, 2\hat{i}-\hat{j}+2\hat{k}$ and $3\hat{i}+\hat{j}-\hat{k}$ are coplanar.

Solution

$$\vec{a} = \hat{i}+2\hat{j}-3\hat{k},\ \vec{b} = 2\hat{i}-\hat{j}+2\hat{k},\ \vec{c} = 3\hat{i}+\hat{j}-\hat{k}$$
$$\left[\vec{a},\vec{b},\vec{c}\right] = \begin{vmatrix} 1 & 2 & -3 \\ 2 & -1 & 2 \\ 3 & 1 & -1 \end{vmatrix} = 0$$

Therefore, the three vectors are coplanar.

25. Find the tangent to the curve $y=x^4+2e^x$ at $(0,2)$.

Solution

$$y = x^4+2e^x$$
$$y' = 4x^3+2e^x$$
$$\text{At } (0,2), y' = 0+2=2$$
$$m = 2$$

Equation of tangent at $(0,2)$ is

$$y-2 = 2x$$
$$2x-y+2 = 0$$

26. Find df if $f(x)=x^2+3x$, $x=3$ and $dx=0.02$.

Solution

$$df = (2x+3)dx$$
$$df = [2(3)+3][0.02] = 9(0.02) = 0.18$$

27. If $\sin x$ is the integrating factor of the linear differential equation $\frac{dy}{dx}+Py=Q$, then show that P is $\cot x$.

Solution

$$e^{\int Pdx} = \sin x$$
$$\int Pdx = \log\sin x$$
$$P = \frac{1}{\sin x}\cos x = \cot x.$$

28. Suppose the amount of milk sold daily at a milk booth is distributed with a minimum of 200 litres and a maximum of 600 litres with probability density function $f(x)=\begin{cases} k & 200\le x\le 600 \\ 0 & \text{otherwise}\end{cases}$.

Find the value of k.

Solution

Since $f(x)$ is a probability density function, $\int_{-\infty}^{\infty} f(x)dx=1$, that is,

$$\int_{200}^{600} k\,dx = 1 \Rightarrow k[x]_{200}^{600}=1$$
$$k[600-200] = 1$$
$$k = \frac{1}{400}.$$

29. The p.m.f of a random variable X is

x	2	4	6	8	10
$f(x)$	$\frac{1}{36}$	$\frac{4}{36}$	$\frac{10}{36}$	$\frac{12}{36}$	$\frac{9}{36}$

Write the c.d.f in horizontal form.

Solution

x	2	4	6	8	10
$f(x)$	$\frac{1}{36}$	$\frac{4}{36}$	$\frac{10}{36}$	$\frac{12}{36}$	$\frac{9}{36}$
$F(x)$	$\frac{1}{36}$	$\frac{5}{36}$	$\frac{15}{36}$	$\frac{27}{36}$	$\frac{36}{36}=1$

30. Show that the solution of the differential equation $\frac{dy}{dx}=2xy$ is $\log y=x^2+c$.

Solution

$$\frac{dy}{y} = 2x\,dx$$

$$\log y = x^2+c$$

PART– III

31. If A is nonsingular, then prove that A^{-1} is also nonsingular and $(A^{-1})^{-1}=A$.

Solution

$$\left|A^{-1}\right| = \frac{1}{|A|}\neq 0 \text{ since } |A|\neq 0$$

A^{-1} is non singular

We know that $A^{-1}A=I$ and $\left(A^{-1}\right)\left(A^{-1}\right)^{-1}=I$

By Left Cancellation Law

$$\left(A^{-1}\right)^{-1} = A.$$

32. Find the matrix A for which $A\begin{bmatrix}5 & 3\\-1 & -2\end{bmatrix}=\begin{bmatrix}14 & 7\\7 & 7\end{bmatrix}$.

Solution

$$A = \begin{bmatrix}14 & 7\\7 & 7\end{bmatrix}\begin{bmatrix}5 & 3\\-1 & -2\end{bmatrix}^{-1} = 7\begin{bmatrix}2 & 1\\1 & 1\end{bmatrix}\frac{1}{(-10+3)}\begin{bmatrix}-2 & -3\\1 & 5\end{bmatrix}$$

$$= \begin{bmatrix}2 & 1\\1 & 1\end{bmatrix}\begin{bmatrix}2 & 3\\-1 & -5\end{bmatrix}$$

$$A = \begin{bmatrix}3 & 1\\1 & -2\end{bmatrix}$$

33. Show that the equation $z^3+2\bar{z}=0$ has five solutions.

Solution

$$z^3+2\bar{z} = 0$$

$$|z|^3 = |-2\bar{z}|$$

i.e., $$|z|(|z|^2-2) = 0$$

$$|z| = 0,\ z\bar{z}-2=0$$

$$|z| = 0,\ \bar{z}=\frac{2}{z}$$

$$z^3+2\bar{z} = 0 \Rightarrow z^3+\frac{4}{z}=0$$

$$z^4+4 = 0$$

Thus $|z| = 0$, and $z^4+4=0$.

Thus the equation has 5 solutions.

34. Find the exact number of real and imaginary zeros of the polynomial $x^9+9x^7+7x^5+5x^3+3x$.

Solution

The number of changes in sign is 0 in $f(x)$.

The number of changes in sign is 0 in $f(-x)$.

Therefore no positive zero and negative zeros.

But $x=0$ is a zero, which is real.

It has 8 imaginary zeros. Thus it has only one real zero and 8 imaginary zeros.

35. A circle of area 9π square units has two of its diameters along the lines $x+y=5$ and $x-y=1$. Find the equation of the circle.

Solution

Area $\pi r^2 = 9\pi \Rightarrow r=3$

Solving the equations of diameters $x+y=5$ and $x-y=1$, we get the centre $(3,2)$.

$\therefore$ The equation of the circle is $(x-3)^2+(y-2)^2 = 3^2$

$$(x-3)^2+(y-2)^2 = 9.$$

36. Evaluate : $\lim\limits_{x\to 0}\dfrac{1-\cos x}{x^2}$.

Solution

$$\lim_{x\to 0}\frac{1-\cos x}{x^2}\left(\frac{0}{0}\text{ form}\right) = \lim_{x\to 0}\frac{\sin x}{2x}\left(\frac{0}{0}\text{form}\right) = \lim_{x\to 0}\frac{\cos x}{2} = \frac{1}{2}.$$

37. A circular plate expands uniformly under the influence of heat. If it's radius increases from 10.5cm to 10.75cm, then find an approximate change in the area.

Solution

$$\text{Area of the circular plate } A = \pi r^2$$
$$\text{Approximate change in area } dA \simeq 2\pi r\, dr$$
$$dA = 2\pi(10.5)(0.25)$$
$$= 5.25\pi$$

$$r = 10.5$$
$$r+\Delta r = 10.75$$
$$\Delta r = 0.25$$

38. Show that $\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{1}{1+\sqrt{\tan x}}dx = \frac{\pi}{8}$.

Solution

$$\text{Let } I = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{1}{1+\sqrt{\tan x}}dx \quad \ldots (1)$$

$$I = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{1}{1+\sqrt{\tan\left(\frac{\pi}{8}+\frac{3\pi}{8}-x\right)}}dx$$

$$= \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{1}{1+\sqrt{\cot x}}dx \quad \ldots (2)$$

Adding (1) and (2) we get

$$2I = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{1+\sqrt{\tan x}}{1+\sqrt{\tan x}}dx = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} dx = [x]_{\frac{\pi}{8}}^{\frac{3\pi}{8}}$$

$$2I = \frac{3\pi}{8}-\frac{\pi}{8} = \frac{\pi}{4}$$

$$I = \frac{\pi}{8}.$$

39. Show that the cube roots of unity under usual multiplication satisfies the closure axion.

Solution

$$S = \{1, \omega, \omega^2\} \text{ where } \omega^3 = 1$$

$\cdot$	1	ω	ω^2
1	1	ω	ω^2
ω	ω	ω^2	1
ω^2	ω^2	1	ω

From the table, the closure axiom is satisfied.

40. A force given by $3\hat{i}+2\hat{j}-4\hat{k}$ is applied at the point $(1,-1,2)$. Find the moment of the force about the point $(2,-1,3)$.

Solution

$$\vec{r} = \left(\hat{i}-\hat{j}+2\hat{k}\right)-\left(2\hat{i}-\hat{j}+3\hat{k}\right)=-\hat{i}-\hat{k}$$

$$\vec{F} = 3\hat{i}+2\hat{j}-4\hat{k}$$

$$\vec{M} = \vec{r}\times\vec{F}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -1 & 0 & -1\\ 3 & 2 & -4\end{vmatrix}=2\hat{i}-7\hat{j}-2\hat{k}$$

PART – IV

41. (a) Solve the following system of linear equations by matrix inversion method:
$2x+3y-z=9,\ x+y+z=9,\ 3x-y-z=-1$.

Solution

$$2x+3y-z = 9$$
$$x+y+z = 9$$
$$3x-y-z = -1$$

$$\begin{bmatrix}2 & 3 & -1\\ 1 & 1 & 1\\ 3 & -1 & -1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}9\\ 9\\ -1\end{bmatrix}$$

$$AX = B$$
$$X = A^{-1}B$$

$$|A| = \begin{vmatrix}2 & 3 & -1\\ 1 & 1 & 1\\ 3 & -1 & -1\end{vmatrix}=2(-1+1)-3(-1-3)-1(-1-3)=16$$

$$\text{Cofactor of } A = \begin{bmatrix}0 & 4 & -4\\ -(-3-1) & (-2+3) & -(-2-9)\\ (3+1) & -(2+1) & (2-3\end{bmatrix}$$

$$adjA = \begin{bmatrix}0 & 4 & -4\\ 4 & 1 & 11\\ 4 & -3 & -1\end{bmatrix}^T=\begin{bmatrix}0 & 4 & 4\\ 4 & 1 & -3\\ -4 & 11 & -1\end{bmatrix}$$

$$X = A^{-1}B$$

$$= \frac{1}{16}\begin{bmatrix} 0 & 4 & 4 \\ 4 & 1 & -3 \\ -4 & 11 & -1 \end{bmatrix}\begin{bmatrix} 9 \\ 9 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{16}\begin{bmatrix} 32 \\ 48 \\ 64 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

$$\therefore x = 2, \; y = 3, \; z = 4.$$

41. (b) Find the centre, foci, vertices of the ellipse $\frac{(x-1)^2}{25} + \frac{(y+2)^2}{16} = 1$.

Solution :

$$\frac{X^2}{25} + \frac{Y^2}{16} = 1 \; ; \; X = x-1, \; Y = y+2$$

$$e = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$$

$$ae = 5 \times \frac{3}{5} = 3$$

X, Y		x, y
Centre $(0,0)$		$C(1,-2)$
Focus	$(ae,0)$ $(-ae,0)$ $(3,0)$ $(-3,0)$	$X = 3 \Rightarrow x = 4$ $Y = 0 \Rightarrow y = -2$ $S(4,-2)$ $X = -3 \Rightarrow x = -2$ $Y = 0 \Rightarrow y = -2$ $S'(-2,-2)$
Vertices	$(a,0)$ $(-a,0)$ $(5,0)$ $(-5,0)$	$X = 5 \Rightarrow x = 6$ $Y = 0 \Rightarrow y = -2$ $A(6,-2)$ $X = -5 \Rightarrow x = -4$ $A'(-4,-2)$

42. (a) Solve : $\sin^{-1}\frac{5}{x} + \sin^{-1}\frac{12}{x} = \frac{\pi}{2}$.

Solution

$$\sin^{-1}\left(\frac{5}{x}\right) + \sin^{-1}\left(\frac{12}{x}\right) = \frac{\pi}{2}$$

$$\sin^{-1}\left(\frac{5}{x}\right)+\sin^{-1}\left(\frac{12}{x}\right) = \sin^{-1}(1)$$

$$\sin^{-1}1-\sin^{-1}\frac{5}{x} = \sin^{-1}\frac{12}{x}$$

$$\sin^{-1}\left[1\sqrt{1-\frac{25}{x^2}}-\frac{5}{x}\sqrt{1-1}\right] = \sin^{-1}\frac{12}{x}$$

$$\frac{\sqrt{x^2-25}}{x} = \frac{12}{x}$$

$$\sqrt{x^2-25} = 12$$

$$x^2 = 169$$

$$x = \pm 13$$

$x=-13$ does not satisfy the equation and therefore $x=13$.

42. (b) Find the fourth roots of $\sqrt{3}+i$.

Solution

$$\sqrt{3}+i = r(\cos\theta+i\sin\theta)$$

$$r = 2,\ \theta=\frac{\pi}{6}$$

$$\left(\sqrt{3}+i\right) = 2\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$$

$$\left(\sqrt{3}+i\right)^{\frac{1}{4}} = 2^{\frac{1}{4}}\left[\cos\left(2k\pi+\frac{\pi}{6}\right)+i\sin\left(2k\pi+\frac{\pi}{6}\right)\right]^{\frac{1}{4}},\ k\in\mathbb{Z}$$

$$= 2^{\frac{1}{4}}\left[\cos\left(\frac{12k+1}{24}\right)\pi+i\sin\left(\frac{12k+1}{24}\right)\pi\right],\ k=0,1,2,3.$$

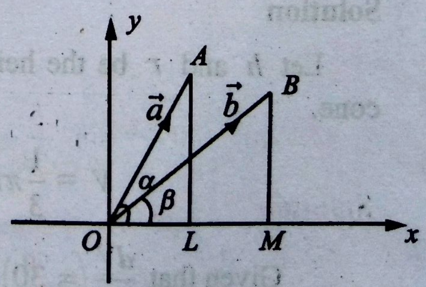
43. (a) Prove that $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$ by vector method.

Solution :

Let $\hat{a}=\overrightarrow{OA}$ and $\hat{b}=\overrightarrow{OB}$ be the unit vectors and which make angles α and β, respectively with positive x-axis. Draw AL and BM perpendicular to the x-axis.

A is $(\cos\alpha,\sin\alpha)$

B is $(\cos\beta,\sin\beta)$



Therefore, $\hat{a} = \overrightarrow{OA}=\overrightarrow{OL}+\overrightarrow{LA}$

$= \cos\alpha\hat{i}+\sin\alpha\hat{j}$

Similarly, $\hat{b} = \cos\beta\hat{i}+\sin\beta\hat{j}$

The angle between $\hat{a}$ and $\hat{b}$ is $\alpha-\beta$

By definition, $\hat{a}\cdot\hat{b} = |\hat{a}|\,|\hat{b}|\cos(\alpha-\beta) = \cos(\alpha-\beta)$ ... (1)

By their values, $\hat{a}\cdot\hat{b} = (\cos\alpha\hat{i}+\sin\alpha\hat{j})\cdot(\cos\beta\hat{i}+\sin\beta\hat{j})$

$= \cos\alpha\cos\beta+\sin\alpha\sin\beta$ ... (2)

From (1) and (2), we get $\cos(\alpha-\beta) = \cos\alpha\cos\beta+\sin\alpha\sin\beta$.

Note : The result can be proved by drawing a unit circle also.

43. (b) Verify whether compound proposition $((p\vee q)\wedge\neg p)\rightarrow q$ is tautology or contradiction or contingency.

Solution

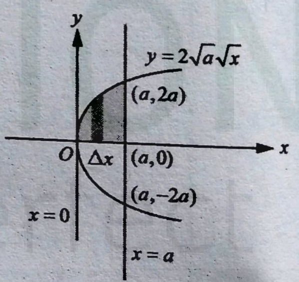
p	q	$r:(p\vee q)$	$s:\neg p$	$t:r\wedge s$	$t\rightarrow q$
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	T	T	T
F	F	F	T	F	T

It is a tautology.

44. (a) Find the area of the region bounded between the parabola $y^2=4ax$ and its latus rectum.

Solution.

The equation of the latus-rectum is $x=a$. The required area is bounded by $y^2=4ax$ and $x=a$. By symmetry, the required area is twice the area bounded in the first quadrant.



$$\therefore\ A = 2\int_0^a y\,dx = 2\int_0^a 2\sqrt{a}\sqrt{x}\,dx = 4\sqrt{a}\left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^a$$

$$= 4\sqrt{a}\times\frac{2}{3}a^{\frac{3}{2}} = \frac{8a^2}{3}.$$

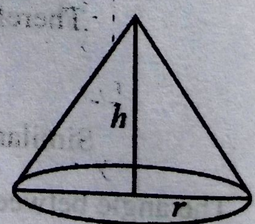
44. (b) Salt is poured from a conveyer belt at a rate of 30 cubic metre per minute forming a conical pile with a circular base whose height and diameter of base are always equal. How fast is the height of the pile increasing when the pile is 10 metre high?

Solution

Let h and r be the height and the base radius. Therefore $h=2r$. Let V be the volume of the cone.

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{12}\pi h^3$$

Given that $\frac{dV}{dt} = 30$



But $\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$

$$\frac{dh}{dt} = 4\frac{dV}{dt}\cdot\frac{1}{\pi h^2}$$

$$\frac{dh}{dt} = 4\times 30\times\frac{1}{100\pi}$$

$$= \frac{6}{5\pi} \text{ meter / minute.}$$

45. (a) Write any five points to trace the curve $y = x\sqrt{4-x}$.

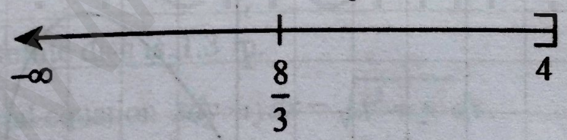
Solution

1. Domain $x \le 4$ i.e, $(-\infty, 4]$.
2. Range $\left(-\infty, \frac{16}{3\sqrt{3}}\right]$.
3. x-intercepts : Put $y = 0 \Rightarrow x = 0, 4$ are x-intercepts.
4. y-intercepts : Put $x = 0 \Rightarrow y = 0$ is the y-intercept.
5. It has no symmetrical property.
6. It has no asymptote
7. First derivative tests :

$$y' = x\cdot\frac{1}{2}(4-x)^{-\frac{1}{2}}(-1) + \sqrt{4-x}$$

$$= \frac{8-3x}{2\sqrt{4-x}}$$

$$y' = 0 \Rightarrow x = \frac{8}{3}$$



$\left(-\infty, \frac{8}{3}\right)$

Let $x = 0$

$y' > 0 \Rightarrow$ function is strictly increasing in $\left(-\infty, \frac{8}{3}\right)$.

$\left(\frac{8}{3}, 4\right)$

Let $x = 3$

$y' < 0 \Rightarrow$ function is strictly decreasing in $\left[\frac{8}{3}, 4\right]$ or $\left(\frac{8}{3}, 4\right)$.

8. Second derivative tests

$$f'' = \frac{2\sqrt{4-x}(-3)-(8-3x)\cdot 2\cdot\frac{1}{2}(4-x)^{-\frac{1}{2}}(-1)}{4(4-x)}$$

$$= \frac{-6\sqrt{4-x}+\frac{8-3x}{\sqrt{4-x}}}{4(4-x)}$$

$$= \frac{3x-16}{4(4-x)^{\frac{3}{2}}}$$

$$f''=0 \Rightarrow x=\frac{16}{3}$$

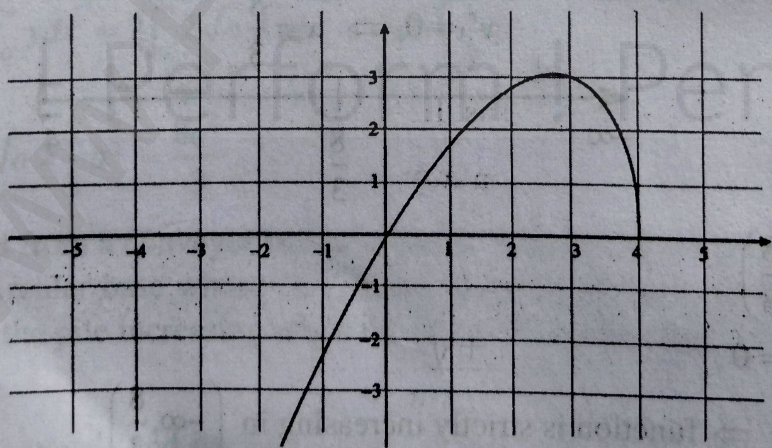
But $\frac{16}{3} \notin (-\infty,4]$

But it is concave downward in $(-\infty,4)$. No point of inflection.

When $x=\frac{8}{2}, f''<0$

$x=\frac{8}{3}$ gives a local maximum and the local maximum is $\frac{16}{3\sqrt{3}}$.

The curve is



Note : The curve is the positive branch of the curve $y^2=x^2(4-x)$

It gives a loop between 0 and 4 and open left wards.

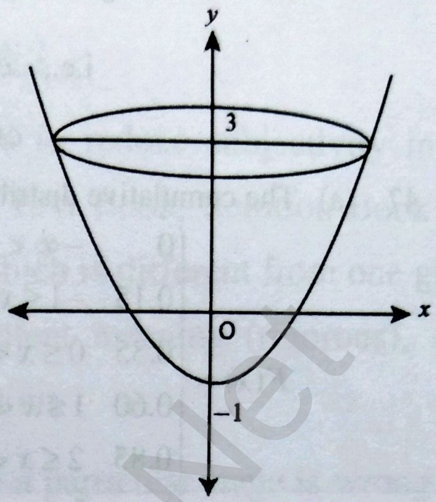
45. (b) Find the volume of the solid generated by revolving the region enclosed by $x^2=1+y$ and $y=3$ about y-axis, using integration.

Solution :

Find the limits in terms of y.

Put $x=0 \Rightarrow y=-1$

$\therefore$ The limits are $y=-1$ and $y=3$.



$$V=\pi\int_a^b x^2\,dy = \pi\int_{-1}^{3}(1+y)\,dy$$

$$= \pi\left[y+\frac{y^2}{2}\right]_{-1}^{3}$$

$$= \pi\left[\left(3+\frac{9}{2}\right)-\left(-1+\frac{1}{2}\right)\right]$$

$$= \pi[4+4]$$

$$= 8\pi .$$

46. (a) An engineer designs a satellite dish with a parabolic cross section. The dish is 5m wide at the opening, and the focus is placed 1.2m from the vertex. Find the depth of the satellite dish at the vertex.

Solution

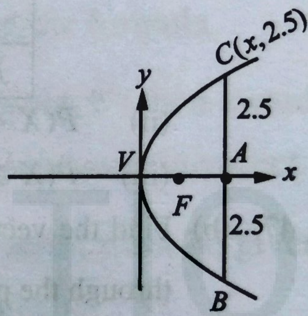
Given the focus is at a distance $1.2\,m$

$VF=1.2=a$

Equation of the parabola is $y^2=4ax$

i.e., $y^2=4.8x$

Let $VA=x$.



Therefore $C\ (x,2.5)$ which lies on the parabola $y^2=4.8x$.

$$\left(\frac{5}{2}\right)^2 = 4.8x \Rightarrow x=1.3$$

$\therefore$ The depth of the satellite dish is 1.3 m.

46. (b) Solve the differential equation $xdy-ydx=\sqrt{x^2+y^2}\,dx$.

Solution :

From the given equation, we have

$$\frac{dy}{dx} = \frac{y+\sqrt{x^2+y^2}}{x}$$

It is a homogeneous equation

Put $y = vx$

$$LHS = v+x\frac{dv}{dx}\ ;\ RHS = \frac{v+\sqrt{1+v^2}}{1}$$

$$\therefore\ v+x\frac{dv}{dx} = v+\sqrt{1+v^2} \quad \text{or} \quad \frac{dx}{x}=\frac{dv}{\sqrt{1+v^2}}$$

$$\log x + \log c = \log\left[v + \sqrt{1+v^2}\right]$$

$$\text{i.e., } cx = v + \sqrt{1+v^2}$$

$$cx^2 = y + \sqrt{x^2+y^2}.$$

47. (a) The cumulative distribution function of a discrete random variable is given by

$$F(x) = \begin{cases} 0 & -\infty < x < -1 \\ 0.15 & -1 \le x < 0 \\ 0.35 & 0 \le x < 1 \\ 0.60 & 1 \le x < 2 \\ 0.85 & 2 \le x < 3 \\ 1 & 3 \le x < \infty \end{cases}$$

Find (i) the probability mass function (ii) $P(X<1)$ and (iii) $P(X \ge 2)$.

Solution : (i) The equivalent table is

X	-1	0	1	2	3
$F(x)$	0.15	0.35	0.60	0.85	1
$f(x)=P[X=x]$	0.15	0.20	0.25	0.25	0.15

(ii) $P(X<1) = P(X=-1)+P(X=0)=0.35$

(iii) $P(X\ge 2) = P(X=2)+P(X=3)=0.40$

47. (b) Find the vector (parametric, non parametric) and Cartesian equations of the plane passing through the points $(-1,2,0)$, $(2,2-1)$ and parallel to the straight line $\frac{x-1}{1}=\frac{2y+1}{2}=\frac{z+1}{-1}$.

Solution

The straight line is $\frac{x-1}{1}=\frac{y+\frac{1}{2}}{1}=\frac{z+1}{-1}$.

The plane passes through the points $\vec{a}=-\hat{i}+2\hat{j}$, $\vec{b}=2\hat{i}+2\hat{j}-\hat{k}$ and parallel to $\vec{c}=\hat{i}+\hat{j}-\hat{k}$.

The vector equation of the plane in parametric form is $\vec{r}=\vec{a}+s\left(\vec{b}-\vec{a}\right)+t\vec{c}$, where $s,t\in\mathbb{R}$

$\vec{r}=\left(-\hat{i}+2\hat{j}\right)+s\left(3\hat{i}-\hat{k}\right)+t\left(\hat{i}+\hat{j}-\hat{k}\right)$, where $s,t\in\mathbb{R}$.

The vector equation of this plane in non-parametric form is $\left[\vec{r}-\vec{a}\ \vec{b}-\vec{a}\ \vec{c}\right]=0$,

where $\vec{a},\vec{b},\vec{c}$ are as mentioned earlier.

Cartesian form is $\begin{vmatrix} x+1 & y-2 & z \\ 3 & 0 & -1 \\ 1 & 1 & -1 \end{vmatrix} = 0$

$$x+2y+3z-3 = 0.$$

MARKING SCHEME - MATHEMATICS

GENERAL INSTRUCTIONS

The marking scheme provides general guidelines to reduce subjectivity in the marking. The answer given in the marking scheme are Text Book, Solution Book and COME book bound. If a student has given any answer which is different from one given in the marking scheme, but carries the prescribed content meaning (rigorous), such answers should be given full credit with suitable distribution.

There is no separate mark allotted for formulae. If a particular stage is wrong and if the candidate writes the appropriate formula then award 1 mark for the formula irrespective of stage marks. This mark is attached with that stage. This is done with the aim that a student who did the problem correctly without writing the formula should not be penalised. That is, mark should not be deducted for not writing the formula.

In the case of Part II, Part III and Part IV, if the solution is correct then award full mark directly. The stage mark is essential only if the part of the solution is incorrect.

Only a Rough sketch of the diagram is expected and full credit must be given for such diagrams that form a part and parcel of the solution of a problem.

In questions on Variable Separable, Homogeneous and Linear differential equations, full marks should be given for an equivalent answer and the students should not be penalised for not getting the final answer as mentioned in the marking scheme.

If the method is not mentioned in the question, then one may adopt any method.

Complicated radicals and combinations need not be simplified (for example $\sqrt{7}$, $\sqrt{82}$, $\frac{3}{\sqrt{2}}$, $10c_6$ etc.)

Important Note : In an answer to a question between any two particular stages of marks (greater than one) if a student starts a stage with correct step but reaches the next stage with a wrong result then suitable credits should be given to the related steps instead of denying the entire marks meant for that stage.

PART – II

21. $$\frac{z_1}{z_2} = \frac{3-2i}{6+4i} = \frac{3-2i}{6+4i} \times \frac{6-4i}{6-4i}$$
$$= \frac{5}{26} - \frac{6}{13}i.$$

22. $$\alpha+\beta = \frac{7}{2} \; ; \; \alpha\beta = \frac{13}{2}.$$
$$\alpha^2+\beta^2+3\alpha\beta = \frac{75}{4}$$

23. $$\sin^{-1}\left(-\frac{1}{2}\right) = -\sin^{-1}\frac{1}{2}$$
$$= -\frac{\pi}{6}$$

24. $$\left[\vec{a},\vec{b},\vec{c}\right] = \begin{vmatrix} 1 & 2 & -3 \\ 2 & -1 & 2 \\ 3 & 1 & -1 \end{vmatrix}$$
$$= 0$$
Therefore, the three vectors are coplanar.

25. At $(0,2), y' = 0+2=2$
$$m = 2$$
Equation of tangent at $(0,2)$ is
$$y-2 = 2x$$
(or)
$$2x-y+2 = 0$$

26. $$df = (2x+3)dx$$
$$df = 0.18$$

27. $$e^{\int Pdx} = \sin x$$
$$P = \frac{1}{\sin x}\cos x = \cot x.$$

28. $$\int_{200}^{600} k\, dx = 1$$ ... (1)

$$k = \frac{1}{400}.$$ ... (1)

29.

x	2	4	6	8	10
$f(x)$	$\frac{1}{36}$	$\frac{4}{36}$	$\frac{10}{36}$	$\frac{12}{36}$	$\frac{9}{36}$
$F(x)$	$\frac{1}{36}$	$\frac{5}{36}$	$\frac{15}{36}$	$\frac{27}{36}$	$\frac{36}{36}=1$

... (2)

30. $$\frac{dy}{y} = 2x\,dx$$ ... (1)

$$\log y = x^2 + c$$ ... (1)

PART– III

31. $$\left|A^{-1}\right| = \frac{1}{|A|} \neq 0$$ ... (1)

A^{-1} is non singular ... (1)

We know that $A^{-1}A = I$ and $\left(A^{-1}\right)\left(A^{-1}\right)^{-1} = I$

By Left Cancellation Law ... (1)

$$\left(A^{-1}\right)^{-1} = A.$$

32. $$A = \begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix}\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}^{-1}$$ ... (1)

$$= \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix}$$ ... (1)

$$A = \begin{bmatrix} 3 & 1 \\ 1 & -2 \end{bmatrix}.$$ ... (1)

33. $$z^3 + 2\bar{z} = 0$$ ... (1)

$$|z|^3 = |-2\bar{z}|$$

$$|z| = 0,\ z\bar{z} - 2 = 0$$ ... (1)

$$|z| = 0,\ \bar{z} = \frac{2}{z}$$ ... (1)

$$z^3 + 2\bar{z} = 0 \Rightarrow z^3 + \frac{4}{z} = 0$$

$$z^4 + 4 = 0$$

Thus $|z| = 0$ and $z^4 + 4 = 0$.

Thus the equation has 5 solutions. ... (1)

34. No positive zeros and negative zeros. ... (1)

$x = 0$ is a zero, which is real. ... (1)

It has 8 imaginary zeros. ... (1)

35. Area $\pi r^2 = 9\pi \Rightarrow r = 3$ ... (1)

Centre $(3, 2)$. ... (1)

$\therefore$ The equation of the circle is $(x-3)^2 + (y-2)^2 = 3^2$

(or)

$$(x-3)^2 + (y-2)^2 = 9.$$ ... (1)

36. $\lim_{x \to 0} \frac{1 - \cos x}{x^2} \left(\frac{0}{0} \text{ form}\right)$ ... (1)

$$= \lim_{x \to 0} \frac{\sin x}{2x} \left(\frac{0}{0} \text{form}\right)$$ ... (1)

$$= \lim_{x \to 0} \frac{\cos x}{2} = \frac{1}{2}.$$ ... (1)

37. $A = \pi r^2$

$$dA \simeq 2\pi r\, dr$$ ... (1)

$$dA = 2\pi(10.5)(0.25)$$

$$= 5.25\pi$$ ... (2)

38. Let $I = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{1}{1+\sqrt{\tan x}}\,dx$

$$I = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{1}{1+\sqrt{\tan\left(\frac{\pi}{8}+\frac{3\pi}{8}-x\right)}}\,dx = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{1}{1+\sqrt{\cot x}}\,dx \quad \text{... (1)}$$

$$2I = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{1+\sqrt{\tan x}}{1+\sqrt{\tan x}}\,dx = \int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} dx = [x]_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \quad \text{... (1)}$$

$$2I = \frac{3\pi}{8} - \frac{\pi}{8}$$

$$I = \frac{\pi}{8}. \quad \text{... (1)}$$

39. $S = \{1, \omega, \omega^2\}$ where $\omega^3 = 1$ ... (1)

.	1	ω	ω^2
1	1	ω	ω^2
ω	ω	ω^2	1
ω^2	ω^2	1	ω

From the table, the closure property is satisfied. ... (2)

40. $\vec{r} = (\hat{i}-\hat{j}+2\hat{k})-(2\hat{i}-\hat{j}+3\hat{k}) = -\hat{i}-\hat{k}$ ... (1)

$$\vec{M} = \vec{r}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 0 & -1 \\ 3 & 2 & -4 \end{vmatrix} \quad \text{... (1)}$$

$$= 2\hat{i}-7\hat{j}-2\hat{k} \quad \text{... (1)}$$

PART – IV

41. (a)

$$2x+3y-z = 9$$
$$x+y+z = 9$$
$$3x-y-z = -1$$

$$\begin{bmatrix} 2 & 3 & -1 \\ 1 & 1 & 1 \\ 3 & -1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 9 \\ -1 \end{bmatrix} \quad \text{... (1)}$$

$$AX = B$$

$$x^2 = 169 \quad \text{... (2)}$$
$$x = \pm 13$$

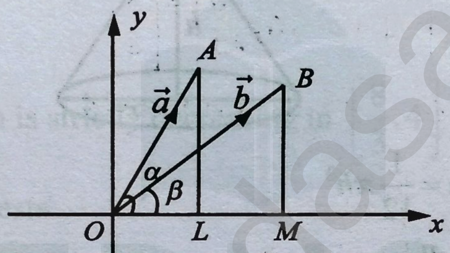
$x = -13$ does not satisfy the equation and therefore $x = 13$. ... (1)

42. (b) $$\left(\sqrt{3}+i\right) = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) \quad \text{... (2)}$$

$$\left(\sqrt{3}+i\right)^{\frac{1}{4}} = 2^{\frac{1}{4}}\left[\cos\left(2k\pi+\frac{\pi}{6}\right)+i\sin\left(2k\pi+\frac{\pi}{6}\right)\right]^{\frac{1}{4}}, k\in\mathbb{Z} \quad \text{... (1)}$$

$$= 2^{\frac{1}{4}}\left[\cos\left(\frac{12k+1}{24}\right)\pi + i\sin\left(\frac{12k+1}{24}\right)\pi\right], k = 0,1,2,3. \quad \text{... (2)}$$

43. (a) Rough diagram ... (1)



$$\hat{a} = \cos\alpha\hat{i} + \sin\alpha\hat{j}$$
$$\hat{b} = \cos\beta\hat{i} + \sin\beta\hat{j} \quad \text{... (1)}$$
$$\hat{a}\cdot\hat{b} = |\hat{a}|\,|\hat{b}|\cos(\alpha-\beta) = \cos(\alpha-\beta) \quad \text{... (1)}$$
$$\hat{a}\cdot\hat{b} = (\cos\alpha\hat{i}+\sin\alpha\hat{j})\cdot(\cos\beta\hat{i}+\sin\beta\hat{j})$$
$$= \cos\alpha\cos\beta + \sin\alpha\sin\beta \quad \text{... (1)}$$
$$\cos(\alpha-\beta) = \cos\alpha\cos\beta+\sin\alpha\sin\beta. \quad \text{... (1)}$$

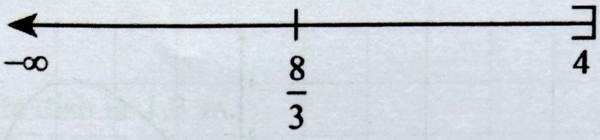
Note : The result can be proved by drawing a unit circle also.

43. (b) Second, third, fourth, fifth column - each one mark ... (4)

p	q	$r:(p\vee q)$	$s:\neg p$	$t:r\wedge s$	$t\to q$
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	T	T	T
F	F	F	T	F	T

It is a tautology. ... (1)

$$y' = 0 \Rightarrow x = \frac{8}{3}$$



$\left(-\infty, \frac{8}{3}\right)$

Let $x = 0$

$y' > 0 \Rightarrow$ function is strictly increasing in $\left(-\infty, \frac{8}{3}\right)$.

$\left(\frac{8}{3}, 4\right)$

Let $x = 3$

$y' < 0 \Rightarrow$ function is strictly decreasing in $\left[\frac{8}{3}, 4\right]$ or $\left(\frac{8}{3}, 4\right)$.

8. Second derivative tests ... (1)

$$f'' = \frac{2\sqrt{4-x}(-3) - (8-3x)\cdot 2 \cdot \frac{1}{2}(4-x)^{-\frac{1}{2}}(-1)}{4(4-x)}$$

$$= \frac{-6\sqrt{4-x)} + \frac{8-3x}{\sqrt{4-x}}}{4(4-x)}$$

$$= \frac{3x-16}{4(4-x)^{\frac{3}{2}}}$$

$$f'' = 0 \Rightarrow x = \frac{16}{3}$$

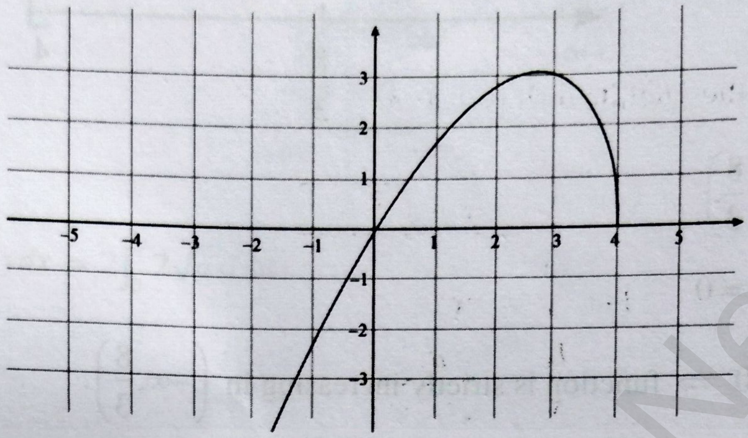
which is $\notin (-\infty, 4]$

But it is concave downward in $(-\infty, 4)$. No point of inflection.

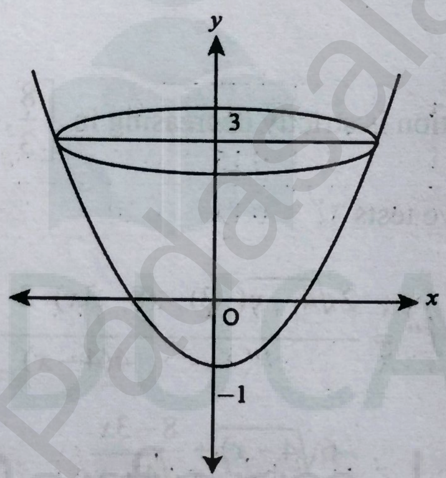
When $x = \frac{8}{2}, f'' < 0$

$x = \frac{8}{3}$ gives a local maximum and the local maximum is $\frac{16}{3\sqrt{3}}$.

The curve is



45. (b) Rough diagram ... (1)

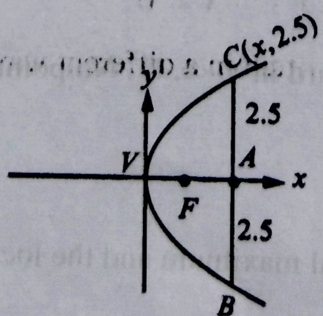


The limits are $y = -1$ and $y = 3$. ... (1)

$$V = \pi\int_a^b x^2\, dy = \pi\int_{-1}^{3}(1+y)\,dy$$ ... (1)

$$= 8\pi\,.$$ ... (2)

46. (a) Rough diagram ... (1)



$VF = 1.2 = a$ ... (1)

Equation of the parabola is $y^2 = 4ax$

i.e., $y^2 = 4.8x$ ... (1)

The depth of the satellite dish is 1.3 m. ... (2)

46. (b) $\frac{dy}{dx} = \frac{y+\sqrt{x^2+y^2}}{x}$ ... (1)

Put $y = vx$ ... (1)

$\frac{dx}{x} = \frac{dv}{\sqrt{1+v^2}}$ ... (1)

$cx^2 = y+\sqrt{x^2+y^2}$ (or) any equivalent form ... (2)

47. (a)

(i)

X	-1	0	1	2	3
$F(x)$	0.15	0.35	0.60	0.85	1
$f(x)=P[X=x]$	0.15	0.20	0.25	0.25	0.15

... (3)

(ii) $P(X<1) = P(X=-1)+P(X=0)=0.35$ ... (1)

(iii) $P(X\geq 2) = P(X=2)+P(X=3)=0.40$ ... (1)

47. (b) The vector equation is $\vec{r}=(-\hat{i}+2\hat{j})+s(3\hat{i}-\hat{k})+t(\hat{i}+\hat{j}-\hat{k})$, where $s,t\in\mathbb{R}$. ... (2)

Non-parametric form is $[\vec{r}-\vec{a}\ \vec{b}-\vec{a}\ \vec{c}]=0$,

where $\vec{a}=-\hat{i}+2\hat{j},\ \vec{b}=2\hat{i}+2\hat{j}-\hat{k}\ \ \vec{c}=\hat{i}+\hat{j}-\hat{k}$. ... (1)

Cartesian form is $\begin{vmatrix} x+1 & y-2 & z \\ 3 & 0 & -1 \\ 1 & 1 & -1 \end{vmatrix} = 0$

$x+2y+3z-3 = 0$. ... (2)

Note : The vector equation can be written in a different way.

PROFILE
MODEL PAPER - II

Part I		Part II		Part III		Part IV	
1	Exercise 1.8 (16)	21	Chapter – 2 Created	31	Example 1.18	41 a	Chapter – 1 Created
2	Chapter – 1 Created (3)	22	Example 3.3	32	Example 1.12 (modified)	41 b	Example 5.40
3	Chapter – 2 Created	23	Example 4.9 (i)	33	Example 2.4	42 a	Exercise 4.2 (6 – i)
4	Exercise 2.9 (12)	24	Exercise 6.2 (8)	34	Exercise 3.2 (4)	42 b	Exercise 2.6 (2)
5	Exercise 3.7 (1)	25	Example 7.47	35	Exercise 5.2 (1 – iv)	43 a	Example 11.22
6	Chapter – 4 Created	26	Exercise 7.3 (3 – ii)	36	Exercise 7.2 (4)	43 b	Exercise 6.7 (4)
7	Exercise 5.6 (9)	27	Example 10.3	37	Exercise 7.4 (1 – iv)	44 a	Exercise 5.2 (4 – v)
8	Chapter – 5 Created	28	Example 11.6 (modified)	38	Exercise 9.3 (2 – iii)	44 b	Exercise 7.1 (9 - i)
9	Exercise 6.10 (4)	29	Exercise 11.3 (4 – i)	39	Exercise 12.1 (10)	45 a	Exercise 7.8 (6)
10	Chapter – 6 Created (1)	30	Chapter – 10 Created	40	Chapter – 6 Created	45 b	Exercise 9.9 (4)
11	Chapter – 7 Created (1)					46 a	Exercise 9.8 (10)
12	Chapter – 7 Created					46 b	Exercise 10.5 (3)
13	Exercise 8.8 (11)					47 a	Exercise 6.1 (10)
14	Exercise 9.10 (4)					47 b	Exercise 12.2 (15)
15	Chapter – 9 Created						
16	Exercise 10.9 (3)						
17	Exercise 10.9 (2)						
18	Chapter – 11 Created						
19	Chapter – 11 Created						
20	Exercise 12.3 (7)						

HIGHER SECONDARY SECOND YEAR
MATHEMATICS
MODEL QUESTION PAPER – 2

Time Allowed: 15 Min + 3.00 Hours] **[Maximum Marks:90**

Instructions:
(a) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.
(b) Use **Blue** or **Black** ink to write and underline and pencil to draw diagrams.

PART – I

Note: (i) **All** questions are **compulsory.** $20 \times 1 = 20$

(ii) Choose the correct or most suitable answer from the given **four** alternatives. Write the option code and the corresponding answer.

1. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $\lambda A^{-1} = A$, then λ is

(1) 17 (2) 14 (3) 19 (4) 21

2. Which of the following is incorrect?

(1) $adj(adj\,A) = |A|^{n-2} A$ (2) $|adjA| = A^{n-1}$

(3) $|adj(adjA)| = |A|^{(n-1)^2}$ (4) $(adjA)^T = adj(A^T)$

3. The conjugate of $i^{16} + i^{17} + i^{18} + i^{19}$ is

(1) 0 (2) $-i$ (3) i (4) 1

4. If z is a complex number such that $z \in \mathbb{C} \setminus \mathbb{R}$ and $z + \frac{1}{z} \in \mathbb{R}$, then $|z|$ is

(1) 0 (2) 1 (3) 2 (4) 3

5. A zero of $x^3 + 64$ is

(1) 0 (2) 4 (3) $4i$ (4) -4

6. The principal value of $\sin^{-1}\left(\frac{1}{2}\right)$ is

(1) $\frac{\pi}{4}$ (2) $-\frac{\pi}{6}$ (3) $\frac{\pi}{6}$ (4) $\frac{\pi}{3}$

7. The radius of the circle passing through the point $(6,2)$ two of whose diameters are $x + y = 6$ and $x + 2y = 4$ is

(1) 10 (2) $2\sqrt{5}$ (3) 6 (4) 4

8. The length of the conjugate axis of the hyperbola $\frac{x^2}{9}-\frac{y^2}{25}=1$ is

(1) 10 (2) 6 (3) 5 (4) 3

9. If $\vec{a},\vec{b},\vec{c}$ are three unit vectors such that $\vec{a}$ is perpendicular to $\vec{b}$, and is parallel to $\vec{c}$ th $\vec{a}\times(\vec{b}\times\vec{c})$ is equal to

(1) $\vec{a}$ (2) $\vec{b}$ (3) $\vec{c}$ (4) $\vec{0}$

10. Which one is meaningful?

(1) $(\vec{a}\times\vec{b})\times(\vec{b}\cdot\vec{c})$ (2) $\vec{a}\times(5+\vec{b})$ (3) $(\vec{a}\cdot\vec{b})\times(\vec{c}\cdot\vec{d})$ (4) $(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d})$

11. If $ax^2+by^2=1$ and $a_1x^2+b_1y^2=1$ intersect each other orthogonally then which one incorrect?

(1) $\frac{1}{a}-\frac{1}{b}=\frac{1}{a_1}-\frac{1}{b_1}$ (2) $\frac{1}{a}-\frac{1}{a_1}=\frac{1}{b}-\frac{1}{b_1}$ (3) $\frac{1}{a}+\frac{1}{b_1}=\frac{1}{b}+\frac{1}{a_1}$ (4) $\frac{1}{a}-\frac{1}{b_1}=\frac{1}{b}-\frac{1}{a_1}$

12. The minimum value of $f(x)=x^2+x$ in N is

(1) ∞ (2) 3 (3) 2 (4) 1

13. If $f(x)=\frac{x}{x+1}$, then its differential is

(1) $\frac{-1}{(x+1)^2}dx$ (2) $\frac{1}{(x+1)^2}dx$ (3) $\frac{1}{x+1}dx$ (4) $\frac{-1}{x+1}dx$

14. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sin^2 x\cos x\,dx$ is

(1) $\frac{3}{2}$ (2) $\frac{1}{2}$ (3) 0 (4) $\frac{2}{3}$

15. $\int_0^{\pi} x\cos x\,dx=$

(1) $x\sin x$ (2) 2π (3) 0 (4) −2

16. The order and degree of the differential equation $\sqrt{\sin x}(dx+dy)=\sqrt{\cos x}(dx-dy)$ is

(1) 1, 2 (2) 2, 2 (3) 1, 1 (4) 2, 1

17. The differential equation representing the family of curves $y=A\cos(x+B)$, where A and B are parameters, is

(1) $\frac{d^2y}{dx^2}-y=0$ (2) $\frac{d^2y}{dx^2}+y=0$ (3) $\frac{d^2y}{dx^2}=0$ (4) $\frac{d^2x}{dy^2}=0$

18. If the probability mass function $f(x)$ of a random variable X is

x	1	2	3	4
$f(x)$	$\frac{1}{12}$	$\frac{5}{12}$	$\frac{5}{12}$	$\frac{1}{12}$

Then $F(3)$ is

(1) $\frac{11}{12}$ (2) $\frac{5}{12}$ (3) 1 (4) 0

19. The probability density function of X is given by $f(x)=\begin{cases} k & 1\le x\le 5 \\ 0 & \text{otherwise} \end{cases}$

Then the value of k is

(1) 4 (2) $\frac{1}{4}$ (3) $\frac{3}{4}$ (4) 5

20. If $a*b=\sqrt{a^2+b^2}$ on the real numbers then $*$ is

(1) commutative but not associative (2) associative but not commutative

(3) both commutative and associative (4)neither commutative nor associative

PART – II

Note: (i) Answer any **SEVEN** questions. $7\times2=14$

(ii) Question number **30 is compulsory.**

21. Show that the points representing complex numbers $7+9i, -3+7i, 3+3i$ form a right angled triangle in the Argand plane.

22. If α, β, γ are the roots of the equation $x^3+px^2+qx+r=0$, show that $\sum\frac{1}{\beta\gamma}=\frac{p}{r}$.

23. Find the principal value of $\tan^{-1}(-\sqrt{3})$.

24. $\vec{a}=\hat{i}-\hat{k}, \vec{b}=x\hat{i}+\hat{j}+(1-x)\hat{k}, \hat{c}=y\hat{i}+x\hat{j}+(1+x-y)\hat{k}$, show that $[\vec{a},\vec{b},\vec{c}]$ is independent of x and y.

25. Prove that the function $f(x)=x^2-2x-3$ is strictly increasing in $(2,\infty)$.

26. Explain why Lagrange's mean value theorem is not applicable to $f(x)=|3x+1|, x\in[-1,3]$

27. Form the differential equation by eliminating the arbitrary constants A and B from $y=A\cos x+B\sin x$.

28. A pair of fair dice is rolled once. Find the probability to get two 4's.

29. The probability density function of X is given by $f(x)=\begin{cases} ke^{-\frac{x}{3}} & \text{for } x>0 \\ 0 & \text{for } x\leq 0 \end{cases}$

show that the value of k is $\frac{1}{3}$.

30. Show that the integrating factor of the differential equation $\frac{dy}{dx}=\frac{x+y+1}{x+1}$ is $\frac{1}{x+1}$.

PART– III

Note: (i) Answer any **SEVEN** questions. 7×3=2

(ii) Question number **40 is compulsory.**

31. Find the rank of the matrix $\begin{bmatrix} 2 & -2 & 4 & 3 \\ -3 & 4 & -2 & -1 \\ 6 & 2 & -1 & 7 \end{bmatrix}$.

32. Show that $A=\frac{1}{7}\begin{bmatrix} 6 & -3 & 2 \\ -3 & -2 & 6 \\ 2 & 6 & 3 \end{bmatrix}$ is orthogonal and hence find A^{-1}.

33. Show that $\left(\frac{1+i}{1-i}\right)^3-\left(\frac{1-i}{1+i}\right)^3=-2i$.

34. Find a polynomial equation of minimum degree with rational coefficients, having $\sqrt{5}-\sqrt{3}$ as root.

35. Show that the equation of the parabola whose end points of latus rectum are (4,–8) and (4,8 open rightward and the vertex is (0, 0), is $y^2=16x$.

36. Find the points on the curve $y^2-4xy=x^2+5$ for which the tangent is horizontal.

37. Write the Maclaurin's series expansion of the function $\log(1-x); -1\leq x<1$

38. Evaluate : $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin^2 x\,dx$

39. Let A be $Q-\{1\}$. Define $*$ on A by $x*y=x+y-xy$. Is $*$ a binary on A?

40. Find the direction cosines and torque of the force $2\hat{i}+\hat{j}-\hat{k}$ if it acts about the point (2,0,- and through the origin.

PART – IV

Note: Answer all the questions. 7×5=3

41. (a) Can you solve the following system by using Cramers method. If yes, solve the system.
$x+y+z-2=0,\ 6x-4y+5z-31=0,\ 5x+2y+2z=13$

(OR)

MODEL QUESTION PAPER – II

ANSWERS

PART – I

Qn. No.	Option	Answer	Qn. No.	Option	Answer
1.	(3)	19	11.	(4)	$\frac{1}{a}-\frac{1}{b_1}=\frac{1}{b}-\frac{1}{a_1}$
2.	(2)	$\lvert adjA\rvert = A^{n-1}$	12.	(3)	2
3.	(1)	0	13.	(2)	$\frac{1}{(x+1)^2}dx$
4.	(2)	1	14.	(4)	$\frac{2}{3}$
5.	(4)	–4	15.	(4)	–2
6.	(3)	$\frac{\pi}{6}$	16.	(3)	1, 1
7.	(2)	$2\sqrt{5}$	17.	(2)	$\frac{d^2y}{dx^2}+y=0$
8.	(1)	10	18.	(1)	$\frac{11}{12}$
9.	(2)	$\vec{b}$	19.	(2)	$\frac{1}{4}$
10.	(4)	$(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d})$	20.	(3)	both commutative and associative

For writing the correct option and the answer – one mark.

PART – II

21. Show that the points representing complex numbers $7+9i, -3+7i, 3+3i$ form a right angled triangle.

Solution

Let $A(7+9i), B(-3+7i), C(3+3i)$ be the vertices

$$AB = |(7+9i)-(-3+7i)| = |10+2i| = \sqrt{104}$$

$$BC = |(-3+7i)-(3+3i)| = |-6+4i| = \sqrt{52}$$

$$CA = |(3+3i)-(7+9i)| = |-4-6i| = \sqrt{52}$$

$$AB^2 = BC^2 + CA^2$$

$\therefore$ ABC is a right angled triangle.

22. If α, β, γ are the roots of the equation $x^3+px^2+qx+r=0$, show that $\sum \frac{1}{\beta\gamma} = \frac{p}{r}$.

Solution

Since α, β, and γ are the roots of the equation $x^3+px^2+qx+r=0$, we have

$$\alpha+\beta+\gamma = -p \text{ and } \alpha\beta\gamma = -r.$$

Now

$$\sum \frac{1}{\beta\gamma} = \frac{1}{\beta\gamma}+\frac{1}{\gamma\alpha}+\frac{1}{\alpha\beta} = \frac{\alpha+\beta+\gamma}{\alpha\beta\gamma} = \frac{-p}{-r} = \frac{p}{r}$$

23. Find the principal value of $\tan^{-1}(-\sqrt{3})$.

Solution

$\tan^{-1}(-\sqrt{3}) = \tan^{-1}\left(\tan\left(-\frac{\pi}{3}\right)\right) = -\frac{\pi}{3}$, since $-\frac{\pi}{3} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ or $\tan^{-1}(-\sqrt{3}) = -\tan^{-1}(\sqrt{3}) = -\frac{\pi}{3}$.

24. $\vec{a} = \hat{i} - \hat{k}, \vec{b} = x\hat{i} + \hat{j} + (1-x)\hat{k}, \vec{c} = y\hat{i} + x\hat{j} + (1+x-y)\hat{k}$, show that $[\vec{a}, \vec{b}, \vec{c}]$ is independent of x and y.

Solution

$$[\vec{a}, \vec{b}, \vec{c}] = \begin{vmatrix} 1 & 0 & -1 \\ x & 1 & 1-x \\ y & x & 1+x-y \end{vmatrix} = 1[(1+x-y)-x(1-x)] - 1[x^2-y]$$

$$= 1+x-y-x+x^2-x^2+y=1.$$

Therefore, $\left[\vec{a}, \hat{b}, \vec{c}\right]$ is independent of x and y.

25. Prove that the function $f(x)=x^2-2x-3$ is strictly increasing in $(2,\infty)$.

Solution

Since $f(x)=x^2-2x-3$, $f'(x)=2x-2>0\,\forall x\in(2,\infty)$. Hence $f(x)$ is strictly increasing $(2,\infty)$.

26. Explain why Lagrange's mean value theorem is not applicable to $f(x)=|3x+1|, x\in[-1,3]$.

Solution

$f(x)$ is not differentiable at $x=-\frac{1}{3}\in(-1,3)$.

27. Form the differential equation by eliminating the arbitrary constants A and B fr $y=A\cos x+B\sin x$.

Solution

$$y' = -A\sin x+B\cos x$$
$$y'' = -A\cos x-B\sin x$$
$$y'' = -[A\cos x+B\sin x]$$
$$y'' = -y \quad \text{i.e.,} \quad y''+y=0$$

28. A pair of fair dice is rolled once. Find the probability to get two fours.

Solution

Let X be the random variable taking the number of 4^s. X can take the values $0,1,2$.

The sample space S contains 6×6 elements.

To get two 4^s, means $X=2$.

$X=2$ means getting $(4,4)$ only.

$\therefore\ P[X=2]=\frac{1}{36}$.

29. The probability density function of X is given by $f(x)=\begin{cases} ke^{-\frac{x}{3}} & \text{for } x>0 \\ 0 & \text{for } x\le 0 \end{cases}$

show that the value of k is $\frac{1}{3}$.

Solution

Since $f(x)$ is a probability density function, $\int_{-\infty}^{\infty} f(x)\,dx=1$.

30. Show th

Solution

31. Find the

Solution

The las

$$\int_0^\infty ke^{-\frac{x}{3}}dx = 1$$

$$k\left[-3e^{-\frac{x}{3}}\right]_0^\infty = 1 \Rightarrow -3k[0-1]=1 \Rightarrow k=\frac{1}{3}.$$

30. Show that the integrating factor of the differential equation $\frac{dy}{dx}=\frac{x+y+1}{x+1}$ is $\frac{1}{x+1}$.

Solution

$$\frac{dy}{dx}=1+\frac{y}{1+x}$$

$$\frac{dy}{dx}-\frac{1}{1+x}y=1$$

$$P=-\frac{1}{1+x}$$

$$\int Pdx=-\log(1+x)$$

$$e^{\int Pdx}=e^{-\log(1+x)}=e^{\log(1+x)^{-1}}$$

$$I.F.=\frac{1}{1+x}.$$

PART– III

31. Find the rank of the matrix $\begin{bmatrix}2 & -2 & 4 & 3\\ -3 & 4 & -2 & -1\\ 6 & 2 & -1 & 7\end{bmatrix}$.

Solution

$$\begin{bmatrix}2 & -2 & 4 & 3\\ -3 & 4 & -2 & -1\\ 6 & 2 & -1 & 7\end{bmatrix} \sim \begin{bmatrix}2 & -2 & 4 & 3\\ 0 & 2 & 8 & 7\\ 0 & 8 & -13 & -2\end{bmatrix}\begin{matrix}R_1\\ R_2\to 2R_2+3R_1\\ R_3\to R_3-3R_1\end{matrix}$$

$$\sim\begin{bmatrix}2 & -2 & 4 & 3\\ 0 & 2 & 8 & 7\\ 0 & 0 & -45 & -30\end{bmatrix}\begin{matrix}R_1\\ R_2\\ R_3\to R_3-4R_2\end{matrix}$$

The last equivalent matrix is in row-echelon form. It has three non-zero rows. So, $\rho(A)=3$.

32. Show that $A=\frac{1}{7}\begin{bmatrix}6 & -3 & 2\\ -3 & -2 & 6\\ 2 & 6 & 3\end{bmatrix}$ is orthogonal and hence find A^{-1}.

Solution

Here $A=A^T$ and hence it is orthogonal.

Since it is orthogonal, $A^{-1}=A^T$.

$$\therefore\ A^{-1}=\frac{1}{7}\begin{bmatrix}6 & -3 & 2\\ -3 & -2 & 6\\ 2 & 6 & 3\end{bmatrix}.$$

33. Show that $\left(\frac{1+i}{1-i}\right)^3-\left(\frac{1-i}{1+i}\right)^3=-2i$.

Solution

$$\frac{1+i}{1-i}=\frac{(1+i)(1+i)}{(1-i)(1+i)}=\frac{1+2i-1}{1+1}=i$$

$$\frac{1-i}{1+i}=\left(\frac{1+i}{1-i}\right)^{-1}=\frac{1}{i}=-i$$

Therefore, $\left(\frac{1+i}{1-i}\right)^3-\left(\frac{1-i}{1-i}\right)^3=i^3-(-i)^3=-i-i=-2i$

34. Find a polynomial equation of minimum degree with rational coefficients, having $\sqrt{5}-\sqrt{3}$ as root.

Solution

Since $\sqrt{5}-\sqrt{3}$ is a root, $\sqrt{5}+\sqrt{3}$ is also a root. Further $-\sqrt{5}+\sqrt{3}, -\sqrt{5}-\sqrt{3}$ are also the roots

The polynomial connected to the zeros $\sqrt{5}-\sqrt{3}, \sqrt{5}+\sqrt{3}$ is $x^2-2\sqrt{5}x+2$.

The polynomial connected to the zeros $-\sqrt{5}+\sqrt{3}$ and $-\sqrt{5}-\sqrt{3}$ is $x^2+2\sqrt{5}x+2$.

The required equation is $\left(x^2+2\sqrt{5}x+2\right)\left(x^2-2\sqrt{5}x+2\right)=0$.

$x^4-16x^2+4=0$.

35. Show that the equation of the parabola whose end points of latus rectum are (4,–8) and (4,8) open rightward and the vertex is (0, 0), is $y^2=16x$.

Solution

Vertex at $(0,0)$.

The distance between the end points is the length of latus rectum

$$4a = 16$$

Since the vertex is $(0,0)$ and the type is open rightward,

the equation is $y^2 = 4ax$

$$y^2 = 16x.$$

36. Find the points on the curve $y^2 - 4xy = x^2 + 5$ for which the tangent is horizontal.

Solution

Since the tangent is horizontal, $m = 0$

$$y^2 - 4xy = x^2 + 5$$

$$2yy' - (4xy' + 4y) = 2x$$

$$y' = \frac{x+2y}{y-2x}$$

$$m = 0 \Rightarrow x = -2y$$

$$y^2 - 4(-2y)y = 4y^2 + 5$$

$$5y^2 = 5$$

$$y = \pm 1$$

$$y = 1 \Rightarrow x = -2(1) = -2$$

$$y = -1 \Rightarrow x = -2(-1) = 2.$$

$\therefore$ The points are $(2,-1)$ and $(-2,1)$.

37. Write the Maclaurin series expansion of the function $\log(1-x); -1 \le x < 1$.

Solution:

$$f(x) = \log(1-x) \Rightarrow f(0) = 0$$

$$f'(x) = \frac{1}{1-x}(-1) = (-1)(1-x)^{-1} \Rightarrow f'(0) = -1$$

$$f''(x) = (-1)(-1)(1-x)^{-2}(-1) \Rightarrow f''(0) = -1$$

$$f'''(x) = (-1)^3(-2)(1-x)^{-3}(-1) \Rightarrow f'''(0) = -2$$

$$\log(1-x) = 0 + \frac{x}{\lfloor 1}(-1) + \frac{x^2}{\lfloor 2}(-1) + \frac{x^3}{\lfloor 3}(-2) + \cdots$$

$$\log(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \cdots$$

38. Evaluate : $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin^2 x\,dx$

Solution

Let $f(x) = \sin^2 x$

Since $\sin x$ is odd function, $\sin^2 x$ is even function.

$$\therefore \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin^2 x\,dx = 2\int_0^{\frac{\pi}{4}} \sin^2 x\,dx = 2\int_0^{\frac{\pi}{4}}\left(\frac{1-\cos 2x}{2}\right)dx$$

$$= \int_0^{\frac{\pi}{4}}(1-\cos 2x)\,dx = \left[x-\frac{\sin 2x}{2}\right]_0^{\frac{\pi}{4}}$$

$$= \left[\frac{\pi}{4}-\left(\frac{\sin\frac{\pi}{2}}{2}\right)-[0-0]\right] = \frac{\pi}{4}-\frac{1}{2}=\frac{\pi-2}{4}.$$

39. Let A be $Q-\{1\}$. Define $*$ on A by $x*y=x+y-xy$. Is $*$ a binary on A?

Solution

Let $x,y\in Q-\{1\}$.

Since x and y are rational, $x+y-xy$ is also a rational. To prove binary, it is to pro $x+y-xy\neq 1$,

If not, assume $x+y-xy = 1$

$\Rightarrow x-1+y-xy = 0$

$\Rightarrow (x-1)(1-y) = 0$

$\Rightarrow$ either $x = 1$ or $y=1$ which is a contradiction.

$\therefore x+y-xy\in Q-\{1\}$

Therefore $*$ is binary on A.

40. Find the direction cosines and torque of the force $2\hat{i}+\hat{j}-\hat{k}$ if it acts about the point $(2,0,-$ and through the origin.

Solution

$$\vec{r} = \vec{0}-(2\hat{i}-\hat{k}) = -2\hat{i}+\hat{k}$$

$$\vec{F} = 2\hat{i}+\hat{j}-\hat{k}$$

$$\vec{r}\times\vec{F} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -2 & 0 & 1\\ 2 & 1 & -1\end{vmatrix} = -\hat{i}-2\hat{k}$$

Torque is $-\hat{i}-2\hat{k}$

direction cosines are $\frac{-1}{\sqrt{5}},0,\frac{-2}{\sqrt{5}}$.

PART – IV

41. (a) Can you solve the following system by using Cramers method. If yes, solve the system.
$x+y+z-2=0,\ 6x-4y+5z-31=0,\ 5x+2y+2z=13.$

Solution

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 6 & -4 & 5 \\ 5 & 2 & 2 \end{vmatrix} = 27$$

Since $\Delta \neq 0$, we can apply Cramer's rule.

$$\Delta x = \begin{vmatrix} 2 & 1 & 1 \\ 31 & -4 & 5 \\ 13 & 2 & 2 \end{vmatrix} = 81$$

$$\Delta y = \begin{vmatrix} 1 & 2 & 1 \\ 6 & 31 & 5 \\ 5 & 13 & 2 \end{vmatrix} = -54$$

$$\Delta z = \begin{vmatrix} 1 & 1 & 2 \\ 6 & -4 & 31 \\ 5 & 2 & 13 \end{vmatrix} = 27$$

$$x = \frac{\Delta x}{\Delta} = \frac{81}{27} = 3$$

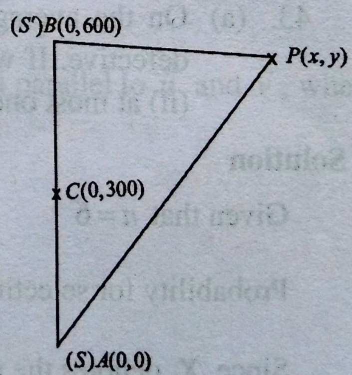
$$y = \frac{\Delta y}{\Delta} = \frac{-54}{27} = -2$$

$$z = \frac{\Delta z}{\Delta} = \frac{27}{27} = 1$$

$$x = 3,\ y = -2,\ z = 1.$$

41. (b) Two coast guard stations are located 600 km apart at points $A(0,0)$ and $B(0,600)$. A distress signal from a ship at P is received at slightly different times by two stations. It is determined that the ship is 200 km farther from station A than it is from station B. Determine the equation of hyperbola that passes through the location of the ship.

Solution



Since centre is the midpoint of S and S', C is $(0,300)$ and the type is II.

The equation is $\frac{(y-300)^2}{a^2} - \frac{(x-0)^2}{b^2} = 1$

It is given that $SP \sim S'P = 2a = 200$

$$a = 100,\ a^2 = 10000$$

$$SS' = 2ae = 600$$

$$ae = 300$$

Therefore X follows a binomial distribution denoted by $X \sim B\left(6, \frac{1}{5}\right)$.

This gives
$$f(x) = \binom{6}{x}\left(\frac{1}{5}\right)^{x}\left(\frac{4}{5}\right)^{6-x}, \quad x = 0,1,2,\ldots,6.$$

(i) Probability for two defective products is

$$P(X=2) = f(2) = \binom{6}{2}\left(\frac{1}{5}\right)^{2}\left(\frac{4}{5}\right)^{6-2} = 15\left(\frac{4^4}{5^6}\right)$$

(ii) Probability for at most one defective products is

$$P(X \le 1) = P(X=0) + P(X=1)$$
$$= \binom{6}{0}\left(\frac{1}{5}\right)^{0}\left(\frac{4}{5}\right)^{6-0} + \binom{6}{1}\left(\frac{1}{5}\right)^{1}\left(\frac{4}{5}\right)^{6-1}$$
$$= \left(\frac{4}{5}\right)^{6} + (6)\left(\frac{4^5}{5^6}\right) = 2\left(\frac{4}{5}\right)^{5}$$

Probability for at most one defective products is $2\left(\frac{4}{5}\right)^{5}$.

(iii) Probability for at least two defective products is

$$P(X \ge 2) = 1 - P(X<2) = 1 - P(X \le 1) = 1 - 2\left(\frac{4}{5}\right)^{5}$$

Probability for at least two defective products is $1 - 2\left(\frac{4}{5}\right)^{5}$.

43. (b) Find the vector and cartesian equations of the plane passing through the point $(1,-2,4)$ and perpendicular to the plane $x+2y-3z=11$ and parallel to the line $\frac{x+7}{3} = \frac{y+3}{-1} = \frac{z}{1}$.

Solution

The plane is passing through a point whose position vector is $\vec{a}$ and parallel to $\vec{u}$ and $\vec{v}$, where

$$\vec{a} = \hat{i} - 2\hat{j} + 4\hat{k}$$
$$\vec{u} = \hat{i} + 2\hat{j} - 3\hat{k}$$
$$\vec{v} = 3\hat{i} - \hat{j} + \hat{k}$$

Vector equation of the plane is

$$\vec{r} = \vec{a} + s\vec{u} + t\vec{v}$$
$$\vec{r} = \left(\hat{i} - 2\hat{j} + 4\hat{k}\right) + s\left(\hat{i} + 2\hat{j} - 3\hat{k}\right) + t\left(3\hat{i} - \hat{j} + \hat{k}\right)$$

Cartesian equation

$$(x_1, y_1, z_1) = (1,-2,4)$$
$$(l_1, m_1, n_1) = (1,2,-3)$$
$$(l_2, m_2, n_2) = (3,-1,1)$$

Cartesian equation of required plane is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

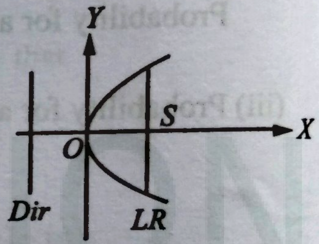
$$\begin{vmatrix} x-1 & y+2 & z-4 \\ 1 & 2 & -3 \\ 3 & -1 & 1 \end{vmatrix} = 0$$

$$\Rightarrow -(x-1)-10(y+2)-7(z-4) = 0$$
$$x+10y+7z-9 = 0$$

44. (a) Find the vertex, focus, equation of directrix and length of the latus rectum of the parabo $y^2-4y-8x+12=0$.

Solution

$$y^2-4y-8x+12 = 0$$
$$y^2-4y = 8x-12$$
$$(y-2)^2-4 = 8x-12$$
$$(y-2)^2 = 8x-8$$
$$(y-2)^2 = 8(x-1)$$
$$(y-2)^2 = 4\times 2(x-1)$$



$$Y^2 = 4aX \text{ where } a=2,\ X=x-1,\ Y=y-2$$

X,Y	x,y
Vertex : $(0,0)$	$C\ (1,2)$
Focus : $(a,0)$ i.e., $(2,0)$	$X=2 \Rightarrow x=3$ $Y=0 \Rightarrow y=2$ Focus is $S(3,2)$
Directrix : $X=-a$ i.e., $X=-2$	$X=-2 \Rightarrow x=-1$
Length of L.R :	$4a=8$

$$b^2 = a^2e^2 - a^2 = 90000 - 10000 = 80000$$

The required equation is

$$\frac{(y-300)^2}{10000} - \frac{x^2}{80000} = 1.$$

42. (a) Find the domain of $f(x) = \sin^{-1}\left(\frac{|x|-2}{3}\right) + \cos^{-1}\left(\frac{1-|x|}{4}\right)$

Solution

The domain of $\sin^{-1}x$ and $\cos^{-1}x$ is $[-1,1]$.

$$-1 \le \frac{|x|-2}{3} \le 1 \text{ and } -1 \le \frac{1-|x|}{4} \le 1$$

$-3 \le |x|-2 \le 3$ $\qquad$ $-4 \le 1-|x| \le 4$

$-1 \le |x| \le 5$ and $\qquad$ $-5 \le -|x| \le 3$

$|x| \le 5$ ... (1) $\qquad$ $-3 \le |x| \le 5$ ... (2)

From (1) and (2)

$|x| \le 5$

The domain is $-5 \le x \le 5$

42. (b) If $z = x+iy$ is a complex number such that $Im\left(\frac{2z+1}{iz+1}\right) = 0$, show that $2x^2+2y^2+x-2y=0$.

Solution

$$\frac{2z+1}{iz+1} = \frac{2(x+iy)+1}{i(x+iy)+1} = \frac{(2x+1)+i2y}{(1-y)+ix} \times \frac{(1-y)-ix}{(1-y)-ix}$$

$$\text{Im}\frac{(2z+1)}{iz+1} = 0 \Rightarrow \frac{-x(2x+1)+2y(1-y)}{(1-y)^2+x^2} = 0$$

$$-2x^2-x+2y-2y^2 = 0$$

$$2x^2+2y^2+x-2y = 0.$$

43. (a) On the average, 20% of the products manufactured by ABC Company are found to be defective. If we select 6 of these products at random, find (i) two products are defective (ii) at most one product is defective (iii) at least two products are defective.

Solution

Given that $n=6$

Probability for selecting a defective product is $\frac{20}{100}$, that is $p = \frac{1}{5}$.

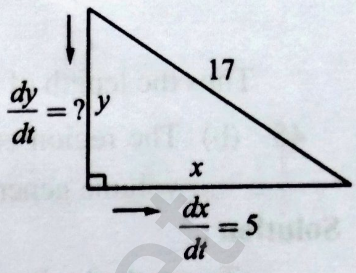
Since X denotes the number defective products, X can take on the values $0,1,2,...,6$

The probability for defective (success) is $p = \frac{1}{5}$ and for failure $q = 1-p = \frac{4}{5}$, and $n=6$

44. (b) A ladder 17 metre long is leaning against the wall. The base of the ladder is pulled away from the wall at a rate of 5 m/s. When the base of the ladder is 8 metres from the wall, how fast is the top of the ladder moving down the wall?

Solution

Let x be the distance between the wall and the base, y be the distance between top of the ladder and bottom of the wall.



$$x^2 + y^2 = 17^2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$$

$$= -\frac{5x}{y}$$

When $x = 8$

$$x^2 + y^2 = 17^2$$

$$8^2 + y^2 = 17^2$$

$$y^2 = 17^2 - 8^2 = 225$$

$$y = 15$$

When $x = 8$

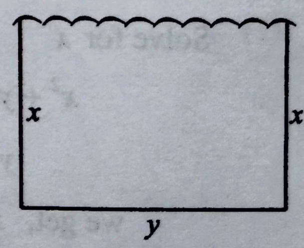
$$\frac{dy}{dt} = \frac{-5(8)}{15} = \frac{-8}{3} \text{ m/s.}$$

i.e., The ladder is sliding down at the rate of $\frac{8}{3}$ *m/s.*

45. (a) A farmer plans to fence a rectangular pasture adjacent to a river. The pasture must contain 1,80,000 sq.mtrs in order to provide enough grass for herds. No fencing is needed along the river. What is the length of the minimum needed fencing material?

Solution

Let x and y be the dimensions.



$$xy = 180,000$$

Minimize $P = 2x + y$

$$P(x) = 2x + \frac{180000}{x}$$

$$P'(x) = 2 - \frac{180000}{x^2}$$

$$P''(x) = \frac{2 \times 180000}{x^3}$$

$$P'(x) = 0 \Rightarrow x^2 = 90000 \Rightarrow x = 300,$$

since x can't be negative.

When $x = 300$, $P''(x) > 0$

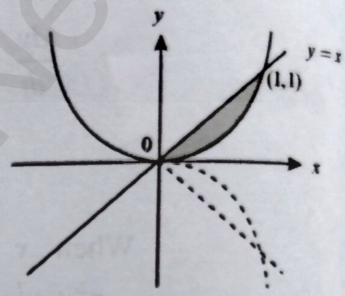
Hence when $x = 300$ the perimeter is minimum.

When $x = 300, y = 600$.

Thus the length of the fencing is $2x + y = 1200$ m.

45. (b) The region enclosed between the graphs of $y = x$ and $y = x^2$ is denoted by R, find volume generated by R when R is rotated through $360°$ about x-axis, using integratio

Solution :



To get the limits solve $y = x$ and $y = x^2$.

$$\text{Hence } x^2 = x$$
$$x^2 - x = 0$$
$$x(x-1) = 0$$
$$x = 0, x = 1$$

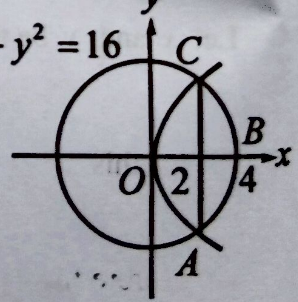
$\therefore$ The limits are $x = 0$, $x = 1$.

Here, the upper curve is $y = x(y_1)$ and the lower curve is $y = x^2(y_2)$

$$\text{Required volume} = \pi\int_a^b \left(y_1^2 - y_2^2\right)dx$$
$$= \pi\int_0^1 \left(x^2 - (x^2)^2\right)dx$$
$$= \pi\left(\frac{x^3}{3} - \frac{x^5}{5}\right)_0^1$$
$$= \pi\left[\frac{2}{15}\right]$$
$$= \frac{2\pi}{15}.$$

46. (a) Find the area of the region common to the circle $x^2 + y^2 = 16$ and the parabola $y^2 = 6x$.

Solution :



Solve for x

$$x^2 + y^2 = 16$$
$$y^2 = 6x$$

we get, $x = 2$.

From the diagram, the required area $OABC$ is twice the area OBC due to symmetrical properties of $x^2 + y^2 = 16$ and $y^2 = 6x$ about x-axis. Therefore the required area is sum of two segments i.e., the area bounded by the parabola, $x = 0, x = 2$ and the area bounded by the circle, $x = 2, x = 4$ and x-axis.

Thus the required area $A = 2\left[\int_0^2 y_1 dx + \int_2^4 y_2 dx\right]$

Where y_1 is y from parabola

y_2 is y from circle

$$\therefore A = 2\int_0^2 \sqrt{6x}\,dx + 2\int_2^4 \sqrt{16-x^2}\,dx = 2\sqrt{6}\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^2 + 2\left[\frac{x}{2}\sqrt{4^2-x^2} + \frac{4^2}{2}\sin^{-1}\frac{x}{4}\right]_2^4$$

$$= \frac{8\sqrt{12}}{3} - 2\sqrt{12} + 8\pi - \frac{8\pi}{3} = \frac{4}{3}\left(4\pi + \sqrt{3}\right)$$

46. (b) Find the equation of the curve whose slope is $\frac{y-1}{x^2+x}$ and which passes through the point $(1,0)$.

Solution

Given that slope of the curve is $\frac{y-1}{x^2+x}$

Therefore, $\frac{dy}{dx} = \frac{y-1}{x^2+x}$

In this equation the variables are separable. On separating the variables, we have

$$\frac{dy}{y-1} = \frac{dx}{x(x+1)} \Rightarrow \frac{dy}{y-1} = \left(\frac{1}{x} - \frac{1}{x+1}\right)dx$$

$$\log|y-1| = \log|x| - \log|x+1| + \log|c|$$

$$\log|y-1| = \log\frac{|cx|}{|x+1|} \Rightarrow y-1 = \frac{cx}{x+1} \quad \dots (1)$$

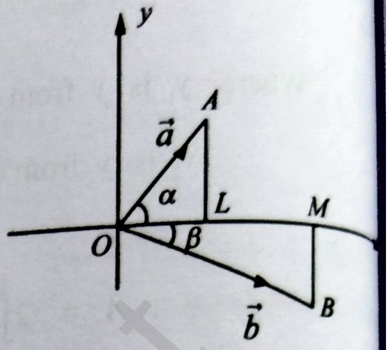
Since this curve passes through (1, 0), we have $-1 = \frac{c}{2}$ or $c = -2$.

Therefore from (1), we have $y-1 = \frac{-2x}{x+1} \Rightarrow y = \frac{-2x}{x+1} + 1$ or $y = \frac{1-x}{1+x}$, which is the required solution.

Note : Here $\log|x|$ means $\log x$ since $\log x$ is defined only for positive values and need not put absolute value sign every time.

47. (a) Prove that $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$.

Solution



Let $\hat{a}=\overrightarrow{OA}$ and $\hat{b}=\overrightarrow{OB}$ be the unit vectors and which make angles α and β, respectively with positive x-axis, where A and B are as shown in the diagram. Draw AL and BM perpendicular to the x-axis.

A is $(\cos\alpha,\sin\alpha)$ and B is $(\cos\beta,\sin\beta)$

Therefore, $\hat{a} = \overrightarrow{OA}=\overrightarrow{OL}+\overrightarrow{LA}=\cos\alpha\hat{i}+\sin\alpha\hat{j}$

Similarly, $\hat{b} = \overrightarrow{OB}=\cos\beta\hat{i}-\sin\beta\hat{j}$

The angle between $\hat{a}$ and $\hat{b}$ is $\alpha+\beta$

By definition, $\hat{b}\times\hat{a} = |\hat{b}|\,|\hat{a}|\sin(\alpha+\beta)\hat{k}=\sin(\alpha+\beta)\hat{k}$ ... (1)

By their values, $\hat{b}\times\hat{a} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \cos\beta & -\sin\beta & 0\\ \cos\alpha & \sin\alpha & 0\end{vmatrix}=(\sin\alpha\cos\beta+\cos\alpha\sin\beta)\hat{k}$ ... (2)

From (1) and (2), we get

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta+\cos\alpha\sin\beta.$$

Note : The result can be proved by drawing a unit circle also.

47. (b) Prove that $p\to(\neg q\vee r)\equiv\neg p\vee(\neg q\vee r)$.

Solution

p	q	r	$\neg p$	$\neg q$	$s:\neg q\vee r$	$t:p\to s$	$v:\neg p\vee s$
T	T	T	F	F	T	T	T
T	T	F	F	F	F	F	F
T	F	T	F	T	T	T	T
F	T	T	T	F	T	T	T
T	F	F	F	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

From the table they are equivalent.

MARKING SCHEME - MATHEMATICS

GENERAL INSTRUCTIONS

The marking scheme provides general guidelines to reduce subjectivity in the marking. The answer given in the marking scheme are Text Book, Solution Book and COME book bound. If a student has given any answer which is different from one given in the marking scheme, but carries the prescribed content meaning (rigorous), such answers should be given full credit with suitable distribution.

There is no separate mark allotted for formulae. If a particular stage is wrong and if the candidate writes the appropriate formula then award 1 mark for the formula irrespective of stage marks. This mark is attached with that stage. This is done with the aim that a student who did the problem correctly without writing the formula should not be penalised. That is, mark should not be deducted for not writing the formula.

In the case of Part II, Part III and Part IV, if the solution is correct then award full mark directly. The stage mark is essential only if the part of the solution is incorrect.

Only a Rough sketch of the diagram is expected and full credit must be given for such diagrams that form a part and parcel of the solution of a problem.

In questions on Variable Separable, Homogeneous and Linear differential equations, full marks should be given for an equivalent answer and the students should not be penalised for not getting the final answer as mentioned in the marking scheme.

If the method is not mentioned in the question, then one may adopt any method.

Complicated radicals and combinations need not be simplified (for example $\sqrt{7}$, $\sqrt{82}$, $\frac{3}{\sqrt{2}}$, $10c_6$ etc.)

Important Note : In an answer to a question between any two particular stages of marks (greater than one) if a student starts a stage with correct step but reaches the next stage with a wrong result then suitable credits should be given to the related steps instead of denying the entire marks meant for that stage.

PART – II

21. Let $A(7+9i), B(-3+7i), C(3+3i)$ be the vertices

$$AB = \sqrt{104}$$
$$BC = \sqrt{52}$$
$$CA = \sqrt{52}$$
$$AB^2 = BC^2 + CA^2$$

ABC is a right angled triangle.

22. $$\alpha+\beta+\gamma = -p \text{ and } \alpha\beta\gamma = -r.$$

$$\sum\frac{1}{\beta\gamma} = \frac{1}{\beta\gamma}+\frac{1}{\gamma\alpha}+\frac{1}{\alpha\beta} = \frac{\alpha+\beta+\gamma}{\alpha\beta\gamma} = \frac{-p}{-r} = \frac{p}{r}$$

23. $$\tan^{-1}\left(-\sqrt{3}\right) = \tan^{-1}\left(\tan\left(-\frac{\pi}{3}\right)\right)$$
$$= -\frac{\pi}{3}$$

(or)

$$\tan^{-1}\left(-\sqrt{3}\right) = -\tan^{-1}\left(\sqrt{3}\right)$$
$$= -\frac{\pi}{3}$$

24. $$\left[\vec{a},\vec{b},\vec{c}\right] = \begin{vmatrix} 1 & 0 & -1 \\ x & 1 & 1-x \\ y & x & 1+x-y \end{vmatrix}$$
$$= 1$$

Therefore, $\left[\vec{a}, \hat{b}, \vec{c}\right]$ is independent of x and y.

25. $f(x) = x^2 - 2x - 3$, $f'(x) = 2x - 2 > 0 \forall x \in (2,\infty)$. ... (1)

$f(x)$ is strictly increasing in $(2,\infty)$. ... (1)

26. $f(x)$ is not differentiable at $x = -\frac{1}{3} \in (-1,3)$. ... (2)

27. $$y' = -A\sin x + B\cos x$$ ... (1)

$$y'' = -A\cos x - B\sin x$$

$$y'' = -[A\cos x + B\sin x]$$ ... (1)

$$y'' = -y \quad \text{i.e.,} \quad y'' + y = 0$$

28. The sample space S contains 6×6 elements. ... (1)

$X = 2$ means getting $(4,4)$ only.

$\therefore \quad P[X=2] = \frac{1}{36}$ .... (1)

29. $$\int_0^\infty k e^{-\frac{x}{3}} dx = 1$$ ... (1)

$$k\left[-3e^{-\frac{x}{3}}\right]_0^\infty = 1 \Rightarrow k = \frac{1}{3}$$ ... (1)

30. $$P = -\frac{1}{1+x}$$ ... (1)

$$\int P dx = -\log(1+x)$$

$$e^{\int P dx} = e^{-\log(1+x)} = e^{\log(1+x)^{-1}}$$

$$= \frac{1}{1+x}.$$ ... (1)

PART– III

31. $$\begin{bmatrix} 2 & -2 & 4 & 3 \\ -3 & 4 & -2 & -1 \\ 6 & 2 & -1 & 7 \end{bmatrix} \sim \begin{bmatrix} 2 & -2 & 4 & 3 \\ 0 & 2 & 8 & 7 \\ 0 & 8 & -13 & -2 \end{bmatrix} \begin{matrix} R_1 \\ R_2 \to 2R_2 + 3R_1 \\ R_3 \to R_3 - 3R_1 \end{matrix}$$ ... (1)

$$\sim \begin{bmatrix} 2 & -2 & 4 & 3 \\ 0 & 2 & 8 & 7 \\ 0 & 0 & -45 & -30 \end{bmatrix} \begin{matrix} R_1 \\ R_2 \\ R_3 \to R_3 - 4R_2 \end{matrix}$$ ... (1)

$$\rho(A) = 3.$$ ... (1)

Note : One may get a different echelon form depends on the transformations.

32. Here $A = A^T$ and hence it is orthogonal. ... (1)

Since it is orthogonal, $A^{-1} = A^T$. ... (1)

$$\therefore\ A^{-1} = \frac{1}{7}\begin{bmatrix} 6 & -3 & 2 \\ -3 & -2 & 6 \\ 2 & 6 & 3 \end{bmatrix}.$$ ... (1)

33.
$$\frac{1+i}{1-i} = i$$ ... (1)
$$\frac{1-i}{1+i} = -i$$ ... (1)
$$\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1-i}\right)^3 = -2i$$ ... (1)

34. $\sqrt{5}-\sqrt{3}$ is a root, $\sqrt{5}+\sqrt{3}$ is also a root. Further $-\sqrt{5}+\sqrt{3}, -\sqrt{5}-\sqrt{3}$ are also the roots. ... (1)

The polynomial connected to the zeros $\sqrt{5}-\sqrt{3}, \sqrt{5}+\sqrt{3}$ is $x^2-2\sqrt{5}x+2$.

The polynomial connected to the zeros $-\sqrt{5}+\sqrt{3}$ and $-\sqrt{5}-\sqrt{3}$ is $x^2+2\sqrt{5}x+2$. ... (1)

The required equation is $\left(x^2+2\sqrt{5}x+2\right)\left(x^2-2\sqrt{5}x+2\right)=0$.

$x^4-16x^2+4=0$. ... (1)

35. The distance between the end points is the length of latus rectum
$$4a = 16$$ ... (1)

Since the vertex is $(0,0)$ and the type is open rightward,
$$\text{the equation is } y^2 = 4ax$$
$$y^2 = 16x.$$ ... (2)

36.
$$m = 0$$
$$y^2 - 4xy = x^2 + 5$$
$$2yy' - (4xy' + 4y) = 2x$$
$$y' = \frac{x+2y}{y-2x}$$ ... (1)
$$m = 0 \Rightarrow x = -2y$$
$$y = \pm 1$$
$$y = 1 \Rightarrow x = -2(1) = -2$$

$$y = -1 \Rightarrow x = -2(-1) = 2.$$

The points are $(2,-1)$ and $(-2,1)$. ... (2)

37.
$$f(0) = 0$$
$$f'(0) = -1$$
$$f''(0) = -1$$
$$f'''(0) = -2 \quad \text{... (2)}$$
$$\log(1-x) = 0+\frac{x}{\lfloor 1}(-1)+\frac{x^2}{\lfloor 2}(-1)+\frac{x^3}{\lfloor 3}(-2)+\cdots$$
$$\log(1-x) = -x-\frac{x^2}{2}-\frac{x^3}{3}-\cdots \quad \text{... (1)}$$

38.
$$\therefore \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin^2 x\,dx = 2\int_0^{\frac{\pi}{4}} \sin^2 x\,dx \quad \text{... (1)}$$
$$= \frac{\pi}{4}-\frac{1}{2} \text{ (or) } \frac{\pi-2}{4} \quad \text{... (2)}$$

39. Let $x, y \in Q-\{1\}$.

Since x and y are rational, $x+y-xy$ is also a rational. ... (1)

Assume $x+y-xy = 1$

$$\Rightarrow x-1+y-xy = 0$$
$$\Rightarrow (x-1)(1-y) = 0$$
$$\Rightarrow \text{ either } x = 1 \text{ or } y=1 \text{ which is a contradiction.}$$
$$\therefore x+y-xy \in Q-\{1\} \quad \text{... (1)}$$

Closure property is satisfied. ... (1)

40.
$$\vec{r} = -2\hat{i}+\hat{k} \quad \text{... (1)}$$
$$\vec{F} = 2\hat{i}+\hat{j}-\hat{k}$$
$$\vec{r}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & 1 \\ 2 & 1 & -1 \end{vmatrix} \quad \text{... (1)}$$
$$= -\hat{i}-2\hat{k}$$

Torque is $-\hat{i}-2\hat{k}$

direction cosines are $\frac{-1}{\sqrt{5}}, 0, \frac{-2}{\sqrt{5}}$. ... (1)

PART – IV

41. (a) $\Delta = 27$

Since $\Delta \neq 0$, we can apply Cramer's rule. ... (1)

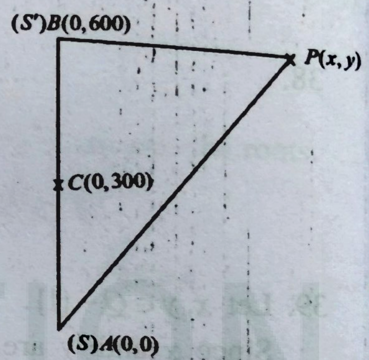
$$x = \frac{\Delta x}{\Delta} = \frac{81}{27} = 3$$ ... (1)

$$y = \frac{\Delta y}{\Delta} = \frac{-54}{27} = -2$$ ... (1)

$$z = \frac{\Delta z}{\Delta} = \frac{27}{27} = 1$$ ... (1)

$$x = 3,\ y = -2,\ z = 1.$$ ... (1)

41. (b)



C is $(0,300)$ ... (1)

The equation is $\frac{(y-300)^2}{a^2} - \frac{(x-0)^2}{b^2} = 1$ ... (1)

It is given that $SP \sim S'P = 2a = 200$

$a = 100,\ a^2 = 10000$ ... (1)

$SS' = 2ae = 600$

$ae = 300$

$b^2 = 80000$ ... (1)

The required equation is

$$\frac{(y-300)^2}{10000} - \frac{x^2}{80000} = 1.$$ ... (1)

42. (a) The domain of $\sin^{-1} x$ and $\cos^{-1} x$ is $[-1,1]$. ... (1)

$$-1 \leq \frac{|x|-2}{3} \leq 1 \text{ and } -1 \leq \frac{1-|x|}{4} \leq 1$$ ... (1)

$-3 \leq |x|-2 \leq 3$ $-4 \leq 1-|x| \leq 4$

$-1 \leq |x| \leq 5$ and $-5 \leq -|x| \leq 3$

$|x| \leq 5$ and $-3 \leq |x| \leq 5$ ... (2)

$|x| \leq 5$ (or) The domain is $-5 \leq x \leq 5$ ... (1)

42 (b) $$\frac{2z+1}{iz+1} = \frac{2(x+iy)+1}{i(x+iy)+1} = \frac{(2x+1)+i2y}{(1-y)+ix} \times \frac{(1-y)-ix}{(1-y)-ix}$$ ... (2)

$$\text{Im}\frac{(2z+1)}{iz+1} = 0 \Rightarrow \frac{-x(2x+1)+2y(1-y)}{(1-y)^2+x^2} = 0 \quad \text{... (2)}$$

$$-2x^2-x+2y-2y^2 = 0$$

$$2x^2+2y^2+x-2y = 0. \quad \text{... (1)}$$

43. (a) $p=\frac{1}{5}, q=\frac{4}{5}$, and $n=6$ ... (1)

$$f(x) = \binom{6}{x}\left(\frac{1}{5}\right)^x\left(\frac{4}{5}\right)^{6-x}, \quad x=0,1,2,...,6. \quad \text{... (1)}$$

(i) $P(X=2) = 15\left(\frac{4^4}{5^6}\right)$ (or) any other form ... (1)

(ii) $P(X\le 1) = 2\left(\frac{4}{5}\right)^5$ (or) any other form ... (1)

(iii) $P(X\ge 2) = 1-2\left(\frac{4}{5}\right)^5$ ... (1)

43. (b)

$$\vec{a} = \hat{i}-2\hat{j}+4\hat{k}$$

$$\vec{u} = \hat{i}+2\hat{j}-3\hat{k}$$

$$\vec{v} = 3\hat{i}-\hat{j}+\hat{k}$$

Vector equation of the plane is

$$\vec{r} = \left(\hat{i}-2\hat{j}+4\hat{k}\right)+s\left(\hat{i}+2\hat{j}-3\hat{k}\right)+t\left(3\hat{i}-\hat{j}+\hat{k}\right) \quad \text{... (2)}$$

Cartesian equation

$$\begin{vmatrix} x-1 & y+2 & z-4 \\ 1 & 2 & -3 \\ 3 & -1 & 1 \end{vmatrix} = 0 \quad \text{... (2)}$$

$$x+10y+7z-9 = 0 \quad \text{... (1)}$$

44. (a) $y^2-4y-8x+12 = 0$

$$(y-2)^2 = 8(x-1) \quad \text{... (1)}$$

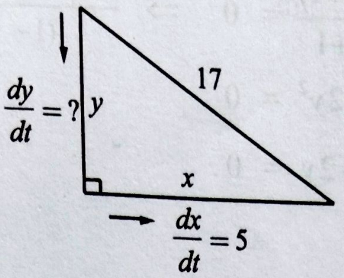
Centre is $C(1,2)$ ... (1)

Focus is $S(3,2)$ ... (1)

Directrix is $x=-1$ ... (1)

Length of LR is 8 ... (1)

44. (b) Rough diagram ... (1)



$$x^2 + y^2 = 17^2$$

$$\frac{dy}{dt} = -\frac{5x}{y}$$ ... (1)

... (1)

When $x = 8$

$y = 15$

When $x = 8$

$$\frac{dy}{dt} = \frac{-5(8)}{15} = \frac{-8}{3} \text{ m/s.}$$ ... (2)

(or) The ladder is sliding down at the rate of $\frac{8}{3}$ *m/s.*

45. (a) Let x and y be the dimensions.

$$xy = 180,000$$

Minimize $P = 2x + y$

$$P(x) = 2x + \frac{180000}{x}$$ ... (1)

$$P'(x) = 2 - \frac{180000}{x^2}$$

$$P''(x) = \frac{2 \times 180000}{x^3}$$

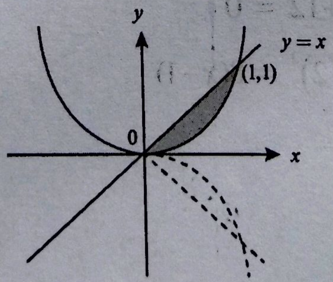
$$P'(x) = 0 \Rightarrow x^2 = 90000 \Rightarrow x = 300$$ ... (1)

When $x = 300,\ P''(x) > 0$ ... (1)

When $x = 300, y = 600.$ ... (1)

Thus the length of the fencing is $2x + y = 1200$ m. ... (1)

45. (b) Rough diagram ... (1)



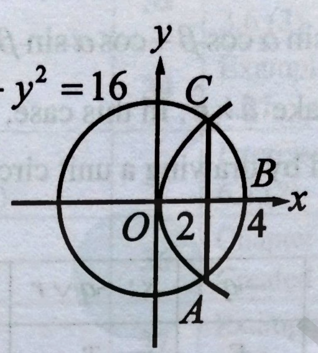
The limits are $x = 0,\ x = 1$. ... (1)

Required volume $= \pi\int_a^b \left(y_1^2 - y_2^2\right)dx$

$= \pi\int_0^1 \left(x^2 - \left(x^2\right)^2\right)dx$ ... (2)

$= \frac{2\pi}{15}$ . ... (1)

46. (a) Rough diagram ... (1)



$A = 2\int_0^2 \sqrt{6x}\,dx + 2\int_2^4 \sqrt{16 - x^2}\,dx$ ... (2)

$= \frac{4}{3}\left(4\pi + \sqrt{3}\right)$ (or) any other form. ... (2)

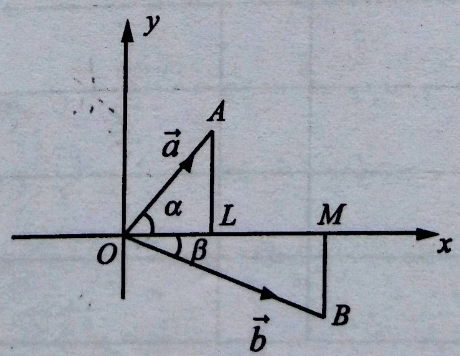
46. (b) $\frac{dy}{dx} = \frac{y-1}{x^2+x}$ ... (1)

$\frac{dy}{y-1} = \frac{dx}{x(x+1)}$ ... (1)

$y - 1 = \frac{cx}{x+1}$ ... (1)

$y - 1 = \frac{-2x}{x+1}$ (or) $y = \frac{1-x}{1+x}$ ... (2)

47. (a) Rough diagram ... (1)



$$\hat{a} = \cos\alpha\hat{i} + \sin\alpha\hat{j}$$

$$\hat{b} = \cos\beta\hat{i} - \sin\beta\hat{j}$$ ... (1)

$$\hat{b}\times\hat{a} = |\hat{b}|\,|\hat{a}|\sin(\alpha+\beta)\hat{k} = \sin(\alpha+\beta)\hat{k}$$ ... (1)

$$\hat{b}\times\hat{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \cos\beta & -\sin\beta & 0 \\ \cos\alpha & \sin\alpha & 0 \end{vmatrix} = (\sin\alpha\cos\beta + \cos\alpha\sin\beta)\hat{k}$$ ... (1)

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta .$$ ... (1)

Note : (1) Instead $\hat{b}\times\hat{a}$ one may take $\hat{a}\times\hat{b}$. In this case, instead of $\hat{k}$ we will get $-\hat{k}$.

(2) The result can be proved by drawing a unit circle also.

47. (b)

p	q	r	$\neg p$	$\neg q$	$s:\neg q \vee r$	$t: p \to s$	$v: \neg p \vee s$
T	T	T	F	F	T	T	T
T	T	F	F	F	F	F	F
T	F	T	F	T	T	T	T
F	T	T	T	F	T	T	T
T	F	F	F	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

4th and 5th column ... (1)

6th column ... (1)

7th column ... (1)

8th column ... (1)

From the table they are equivalent. ... (1)

$$X = A^{-1}B$$

$$|A| = \begin{vmatrix} 2 & 3 & -1 \\ 1 & 1 & 1 \\ 3 & -1 & -1 \end{vmatrix} = 16$$ ... (1)

$$adjA = \begin{bmatrix} 0 & 4 & -4 \\ 4 & 1 & 11 \\ 4 & -3 & -1 \end{bmatrix}^T = \begin{bmatrix} 0 & 4 & 4 \\ 4 & 1 & -3 \\ -4 & 11 & -1 \end{bmatrix}$$

$$X = A^{-1}B$$

$$= \frac{1}{16}\begin{bmatrix} 0 & 4 & 4 \\ 4 & 1 & -3 \\ -4 & 11 & -1 \end{bmatrix}\begin{bmatrix} 9 \\ 9 \\ -1 \end{bmatrix}$$ ... (2)

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{16}\begin{bmatrix} 32 \\ 48 \\ 64 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$$

$$\therefore x = 2, \ y = 3, \ z = 4.$$ ... (1)

41. (b) $$e = \sqrt{1-\frac{16}{25}} = \frac{3}{5}$$

$$ae = 5\times\frac{3}{5} = 3$$

Centre $C(1,2)$ ... (1)

Foci $S(4,-2)$ and $S'(-2,-2)$ ... (2)

Vertices $A(6,-2)$ and $A'(-4,-2)$ ... (2)

42. (a) $$\sin^{-1}\left(\frac{5}{x}\right)+\sin^{-1}\left(\frac{12}{x}\right) = \frac{\pi}{2}$$

$$\sin^{-1}\left(\frac{5}{x}\right)+\sin^{-1}\left(\frac{12}{x}\right) = \sin^{-1}(1)$$ ... (1)

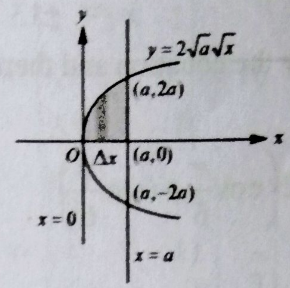
$$\sin^{-1}1 - \sin^{-1}\frac{5}{x} = \sin^{-1}\frac{12}{x}$$ ... (1)

$$\sin^{-1}\left[1\sqrt{1-\frac{25}{x^2}} - \frac{5}{x}\sqrt{1-1}\right] = \sin^{-1}\frac{12}{x}$$

$$\frac{\sqrt{x^2-25}}{x} = \frac{12}{x}$$

$$\sqrt{x^2-25} = 12$$

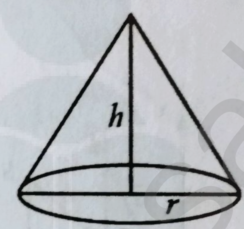
44. (a) Rough diagram



$$A = 2\int_0^a y\,dx = 2\int_0^a 2\sqrt{a}\sqrt{x}\,dx$$ ...(2)

$$= \frac{8a^2}{3}.$$ ...(2)

44. (b) Rough diagram ...(1)



$$V = \frac{1}{3}\pi r^2 h = \frac{1}{12}\pi h^3$$ ...(1)

$$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$$ ...(1)

$$\frac{dh}{dt} = \frac{6}{5\pi} \text{ meter / minute.}$$ ...(2)

45. (a) Each step (or) sub heading – one mark. Maximum five marks

1. Domain $x \le 4$ i.e, $(-\infty, 4]$ ...(1)
2. Range $\left(-\infty, \frac{16}{3\sqrt{3}}\right]$ . ...(1)
3. x-intercepts : Put $y=0 \Rightarrow x=0,4$ are x-intercepts. ...(1)
4. y-intercepts : Put $x=0 \Rightarrow y=0$ is the y-intercept. ...(1)
5. It has no symmetrical property. ...(1)
6. It has no asymptote ...(1)
7. First derivative tests : ...(1)

$$y' = x\cdot\frac{1}{2}(4-x)^{-\frac{1}{2}}(-1) + \sqrt{4-x}$$

$$= \frac{8-3x}{2\sqrt{4-x}}$$