PROFILE
MODEL PAPER - V

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HIGHER SECONDARY SECOND YEAR

MATHEMATICS

MODEL QUESTION PAPER – 5

Time Allowed: 15 Min + 3.00 Hours] **[Maximum Marks:90**

Instructions:
(a) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.
(b) Use **Blue** or **Black** ink to write and underline and pencil to draw diagrams.

PART – I

Note: (i) **All** questions are **compulsory.** $20 \times 1 = 20$

(ii) Choose the correct or most suitable answer from the given **four** alternatives. Write the option code and the corresponding answer.

1. If $P = \begin{bmatrix} 1 & x & 0 \\ 1 & 3 & 0 \\ 2 & 4 & -2 \end{bmatrix}$ is the adjoint of 3×3 matrix A and $|A| = 4$, then x is

(1) 15 (2) 12 (3) 14 (4) 11

2. Consider the statements :

A : A is symmetric $\Rightarrow$ $adjA$ is symmetric

B : $adj(AB) = adj(A)\cdot adj(B)$

Choose the correct option

(1) Both statements are correct (2) Neither statements are correct
(3) A is correct, B is incorrect (4) A is incorrect, B is correct

3. The principal argument of $\frac{3}{-1+i}$ is

(1) $-\frac{5\pi}{6}$ (2) $-\frac{2\pi}{3}$ (3) $-\frac{3\pi}{4}$ (4) $-\frac{\pi}{2}$

4. Identify the incorrect statement.

(1) $|z|^2 = 1 \Rightarrow \frac{1}{z} = \bar{z}$ (2) $\text{Re}(z) \le |z|$
(3) $||z_1| - |z_2|| \ge |z_1 + z_2|$ (4) $|z^n| = |z|^n$

5. The statement "A polynomial equation of degree n has exactly n roots which are either real or complex" is

(1) Fundamental theorem of Algebra (2) Rational root theorem
(3) Descartes rule (4) Complex conjugate root theorem

6. The length of the diameter of the circle which touches the x-axis at the point $(1,0)$ and passes through the point $(2,3)$ is

(1) $\frac{6}{5}$ (2) $\frac{5}{3}$ (3) $\frac{10}{3}$ (4) $\frac{3}{5}$

7. If $x+y=k$ is a normal to the parabola $y^2=12x$, then the value of k is

(1) 3 (2) -1 (3) 1 (4) 9

8. The distance from the origin to the plane $3x-6y+2z+7=0$ is

(1) 0 (2) 1 (3) 2 (4) 3

9. The angle between the lines $\frac{x-2}{3}=\frac{y+1}{-2}$, $z=2$ and $\frac{x-1}{1}=\frac{2y+3}{3}=\frac{z+5}{2}$ is

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{2}$

10. The tangent to the curve $y^2-xy+9=0$ is vertical when

(1) $y=0$ (2) $y=\pm\sqrt{3}$ (3) $y=\frac{1}{2}$ (4) $y=\pm 3$

11. $\lim_{x\to 0}\left(\cot x-\frac{1}{x}\right)$ is

(1) 0 (2) 1 (3) 2 (4) ∞

12. The approximate change in the volume V of a cube of side 10 metres caused by increasing the side by 1% is

(1) $0.3m^3$ (2) $300m^3$ (3) $3m^3$ (4) $30m^3$

13. The value of $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\left(\frac{2x^7-3x^5+7x^3-x}{\cos^2 x}\right)dx$ is

(1) 4 (2) 3 (3) 2 (4) 0

14. The value of $\int_0^{\frac{\pi}{6}}\cos^3 3x\,dx$ is

(1) $\frac{2}{3}$ (2) $\frac{2}{9}$ (3) $\frac{1}{9}$ (4) $\frac{1}{3}$

15. The order and the degree of the differential equation $x^2\frac{d^2y}{dx^2}+\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{2}}=0$ are

(1) $2,2$ (2) $2,\frac{1}{2}$ (3) $\frac{1}{2},2$ (4) $2,1$

6. The length of the diameter of the circle which touches the x-axis at the point $(1,0)$ and passes through the point $(2,3)$ is

(1) $\frac{6}{5}$ (2) $\frac{5}{3}$ (3) $\frac{10}{3}$ (4) $\frac{3}{5}$

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(1) 4 (2) 3 (3) 2 (4) 0

14. The value of $\int_0^{\frac{\pi}{6}}\cos^3 3x\,dx$ is

(1) $\frac{2}{3}$ (2) $\frac{2}{9}$ (3) $\frac{1}{9}$ (4) $\frac{1}{3}$

15. The order and the degree of the differential equation $x^2\frac{d^2y}{dx^2}+\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{2}}=0$ are

(1) 2,2 (2) $2,\frac{1}{2}$ (3) $\frac{1}{2},2$ (4) 2,1

16. Assume that a spherical rain drop evaporates at a rate proportional to its surface area. The of change of the radius (r) of the rain drop, then $\frac{dr}{dt}$ (k is the constant of proportionality $k<0$) is

(1) kr (2) k (3) $-k$ (4) $-kr$

17. For a continuous random variable X, which of the following is/are incorrect?

(i) $P[X=x]=0$ and $P[a<X<b]=F(b)-F(a)$

(ii) $P[X=x]=1$ and $P[a<X<b]=F(b)-F(a)$

(iii) $P[X=x]=0$ and $P[a\le X\le b]=P[a<X<b]$

(iv) $P[a<X<b]=P[a\le X<b]=P[a<X\le b]$ and $P[X=x]=0$

(1) (ii) and (iii) only (2) (ii) only (3) (i) and (ii) only (4) (iv) only

18. A random variable X has binomial distribution with $n=25$ and $p=0.8$ then standard deviati of X is

(1) 6 (2) 4 (3) 3 (4) 2

19. The additive inverses do not exist for some elements in the set

(1) $\mathbb{R}$ (2) $-1\le x\le 2$ (3) $\mathbb{Z}$ (4) Q

20. If a compound statement involves 3 simple statements, then the number of rows in the truth tab is

(1) 9 (2) 8 (3) 6 (4) 3

PART – II

Note: (i) Answer any **SEVEN** questions.

(ii) Question number **30 is compulsory.** $7\times2=14$

21. Find the rank of the matrix $\begin{bmatrix}-2 & 2 & -1\\ 0 & 5 & 1\\ 2 & 0 & 0\end{bmatrix}$

22. If $z=x+iy$, find $\text{Im}(3z+4\bar{z}-4i)$.

23. Find the period and amplitude of $y=\sin 7x$

24. If α, β are the roots of $17x^2+43x-73=0$, find the equation whose roots are $-\alpha$ and $-\beta$.

25. For any vector $\vec{a}$, prove that $\hat{i}\times(\hat{a}\times\hat{i})+\hat{j}\times(\vec{a}\times\hat{j})+\hat{k}\times(\vec{a}\times\hat{k})=2\vec{a}$.

26. State Rolle's theorem.

27. Evaluate : $\int_{-\log 2}^{\log 2} e^{-|x|}\, dx$.

28. If $E(X+5)=6$ then show that $E(X)=1$.

29. Examine the closure property of the operation $a*b=a+3ab-5b^2; \forall a,b\in\mathbb{Z}$

30. Show that the differential equation for the function $y^2=4ax$, where a is arbitrary, is $y=2y'x$.

PART– III

Note: (i) Answer any **SEVEN** questions. $7\times3=21$

(ii) Question number **40 is compulsory.**

31. If $\text{adj}(A)=\begin{bmatrix} 2 & -4 & 2 \\ -3 & 12 & -7 \\ -2 & 0 & 2 \end{bmatrix}$, find A.

32. If $\omega\neq1$ is a cube root of unity, show that $\dfrac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2}+\dfrac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}=-1$.

33. Find a polynomial equation of minimum degree with rational coefficients, having $2-\sqrt{3}$ as a root.

34. Find the domain of $\sin^{-1}\left(\dfrac{2+\sin x}{3}\right)$.

35. Find the local extremum of the function $f(x)=x^4+32x$.

36. Find the volume of a sphere of radius *a* through integration.

37. Solve the differential equation $\cos x\dfrac{dy}{dx}+y\sin x=1$.

38. Find the constant C such that the function $f(x)=\begin{cases} Cx^2 & 1<x<4 \\ 0 & \text{Otherwise} \end{cases}$

is a density function, and compute $P(X\leq2)$.

39. Construct the truth table for $(p\vee q)\vee\neg q$

40. If $f(x,y)=\dfrac{x^2+y^2+xy}{x^2-y^2}$ then show that $x\dfrac{\partial f}{\partial x}+y\dfrac{\partial f}{\partial y}=0$.

PART – IV

Note: Answer **all** the questions. $7\times5=35$

41. (a) Test for consistency of the following system of linear equations and if possible solve: $x+2y-z=3,\ 3x-y+2z=1,\ x-2y+3z=3,\ x-y+z+1=0$.

(OR)

(b) The velocity v, of a parachute falling vertically satisfies the equation $v\frac{dv}{dx} = g\left(1-\frac{v^2}{k^2}\right)$, where g and k are constants. If v and x are both initially zero, find v in terms of x.

42\. (a) A rod of length 1.2m moves with its ends always touching the coordinate axes. The locus of a point P on the rod, which is 0.3m from the end in contact with x-axis. Prove that the locus is an ellipse and hence find the eccentricity.

(OR)

(b) Prove that $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$.

43\. (a) Identify the type of conic $\frac{(y-2)^2}{25} - \frac{(x+1)^2}{16} = 1$. Find the centre, foci and vertices.

(OR)

(b) Show that the two curves $x^2 - y^2 = r^2$ and $xy = c^2$, where c, r are constants, cut orthogonally.

44\. (a) Let $g(x, y) = 2y + x^2, x = 2r - s, y = r^2 + 2s, r, s \in \mathbb{R}$. Find $\frac{\partial g}{\partial r}, \frac{\partial g}{\partial s}$.

(OR)

(b) Find the area of the region bounded by the parabola $y^2 = x$ and the line $y = x - 2$.

45\. (a) For the complex number $z = 3 + 2i$, plot z, iz, and $z + iz$ in the Argand plane. Show that these complex numbers form the vertices of an isosceles right angled triangle.

(OR)

(b) The rate of increase in the number of bacteria in a certain bacteria culture is proportional to the number present. Given that the number triples in 5 hours, find how many bacteria will be present after 10 hours?

46\. (a) Two balls are drawn in succession without replacement from an urn containing four red balls and three black balls. Let X be the possible outcomes of drawing red balls. Find the probability mass function and mean for X.

(OR)

(b) Show that $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\cot x}} = \frac{\pi}{12}$.

47\. (a) Find the vector and Cartesian equations of the plane passing through the points $(3,6,-2), (-1,-2,6)$, and $(6,4,-2)$.

(OR)

(b) Find the dimensions of the rectangle with maximum area that can be inscribed in a circle of radius 10 cm.

MODEL QUESTION PAPER – V

ANSWERS

PART – I

Qn. No.	Option	Answer	Qn. No.	Option	Answer
1.	(4)	11	11.	(1)	0
2.	(3)	A is correct, B is incorrect	12.	(4)	$30m^3$
3.	(3)	$\frac{-3\pi}{4}$	13.	(4)	0
4.	(3)	$\lvert \lvert z_1 \rvert - \lvert z_2 \rvert \rvert \geq \lvert z_1 + z_2 \rvert$	14.	(2)	$\frac{2}{9}$
5.	(1)	Fundamental theorem of Algebra	15.	(1)	2, 2
6.	(3)	$\frac{10}{3}$	16.	(2)	k
7.	(4)	9	17.	(2)	(ii) only
8.	(2)	1	18.	(4)	2
9.	(4)	$\frac{\pi}{2}$	19.	(2)	$-1 \leq x \leq 2$
10.	(4)	$y = \pm 3$	20.	(2)	8

For writing the correct option and the answer – one mark..

PART – II

21. Find the rank of the matrix $\begin{bmatrix} -2 & 2 & -1 \\ 0 & 5 & 1 \\ 2 & 0 & 0 \end{bmatrix}$.

Solution

$$\begin{bmatrix} -2 & 2 & -1 \\ 0 & 5 & 1 \\ 2 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} -2 & 2 & -1 \\ 0 & 5 & 1 \\ 0 & 2 & -1 \end{bmatrix} R_3 \to R_3 + R_1$$

$$\sim \begin{bmatrix} -2 & 2 & -1 \\ 0 & 5 & 1 \\ 0 & 0 & -7 \end{bmatrix} R_3 \to 5R_3 - 2R_2$$

$$\text{rank} = 3$$

$$\text{(or)} \quad 2\begin{vmatrix} 2 & -1 \\ 5 & 1 \end{vmatrix} \neq 0$$

$$\text{rank} = 3.$$

22. If $z = x + iy$, find $\text{Im}(3z + 4\bar{z} - 4i)$.

Solution

$$\text{Im}(3z + 4\bar{z} - 4i) = \text{Im}[3(x+iy) + 4(x-iy) - 4i]$$
$$= \text{Im}(3x + 3iy + 4x - 4iy - 4i)$$
$$= \text{Im}[7x + i(-y-4)] = -y - 4$$

23. Find the period and amplitude of $y = \sin 7x$

Solution

If $y = a\sin bx$ then the amplitude and period are $|a|$ and $\dfrac{2\pi}{|b|}$ respectively.

$\text{Amp} = 1, \ \text{period} = \dfrac{2\pi}{7}$.

24. If α, β are the roots of $17x^2 + 43x - 73 = 0$, find the equation whose roots are $-\alpha$ and $-\beta$.

Solution

$$\alpha + \beta = \frac{-43}{17}$$

$$\alpha\beta = \frac{-73}{17}$$

$$-\alpha - \beta = \frac{43}{17}$$

$$(-\alpha)(-\beta) = \frac{-73}{17}$$

The required equation is $x^2 - \frac{43}{17}x - \frac{73}{17} = 0$ (or) $17x^2 - 43x - 73 = 0$.

25. For any vector $\vec{a}$, prove that $\hat{i} \times (\hat{a} \times \hat{i}) + \hat{j} \times (\vec{a} \times \hat{j}) + \hat{k} \times (\vec{a} \times \hat{k}) = 2\vec{a}$.

Solution

Now $\hat{i} \times (\vec{a} \times \hat{i}) = (\hat{i} \cdot \hat{i})\vec{a} - (\hat{i} \cdot \vec{a})\hat{i} = \vec{a} - (\vec{a} \cdot \hat{i})\hat{i}$

similarly $\hat{j} \times (\vec{a} \times \hat{j}) = \vec{a} - (\vec{a}.\hat{j})\hat{j}$

$\hat{k} \times (\vec{a} \times \hat{k}) = \vec{a} - (\vec{a} \cdot \hat{k})\hat{k}$

L.H.S $= 3\vec{a} - \left[(\vec{a} \cdot \vec{i})\vec{i} + (\vec{a} \cdot \vec{j})\vec{j} + (\vec{a} \cdot \vec{k})\vec{k}\right]$

But we know that $\vec{a} = (\vec{a} \cdot \hat{i})\hat{i} + (\vec{a} \cdot \hat{j})\hat{j} + (\vec{a} \cdot \hat{k})\hat{k}$

$\therefore \hat{i} \times (\vec{a} \times \hat{i}) + \hat{j} \times (\vec{a} \times \hat{j}) + \hat{k} \times (\vec{a} \times \hat{k}) = 3\vec{a} - \vec{a} = 2\vec{a}$.

26. State Rolles theorem.

Solution

Let $f(x)$ be continuous on a closed interval $[a,b]$, differentiable on the open interval (a,b) and $f(a) = f(b)$. Then there is at least one point $c \in (a,b)$ where $f'(c) = 0$.

27. Evaluate : $\int_{-\log 2}^{\log 2} e^{-|x|}\,dx$.

Solution

Let $f(x) = e^{-|x|}dx$ then $f(-x) = e^{-|-x|} = e^{-|x|} = f(x)$

So $f(x)$ is an even function.

Hence $\int_{-\log 2}^{\log 2} e^{-|x|}\,dx = 2\int_0^{\log 2} e^{-|x|}\,dx = 2\int_0^{\log 2} e^{-x}\,dx = 2(-e^{-x})_0^{\log 2} = 2(-e^{-\log 2} + e^0)$

$= 2\left(-\frac{1}{2} + 1\right) = 1$.

28. If $E(X+5) = 6$ then show that $E(X) = 1$.

Solution

$E(X+5) = E(X) + 5 = 6$

$E(X) = 1$

29. Examine the closure property of the operation $a * b = a + 3ab - 5b^2; \forall a, b \in \mathbb{Z}$

Solution

Since $\times$ is binary on $\mathbb{Z}$, $a, b \in \mathbb{Z} \Rightarrow a \times b = ab \in \mathbb{Z}$ and $b \times b = b^2 \in \mathbb{Z}$ ... (1)

Also $3ab, 5b^2 \in \mathbb{Z}$.

$a * b = (a + 3ab - 5b^2) \in \mathbb{Z}$.

Since $a * b$ belongs to $\mathbb{Z}$, $*$ is binary on $\mathbb{Z}$.

(or)

Since a and b are integers $a + 3ab - 5b^2$ is also an integer.

$\therefore$ $*$ satisfies closure axiom.

30. Show that the differential equation for the function $y^2 = 4ax$, where a is arbitrary, is $y = 2y'x$

Solution

$$y^2 = 4ax \quad \text{... (1)}$$

$$2yy' = 4a$$

$$(1) \Rightarrow y^2 = 2yy'x \Rightarrow y = 2y'x.$$

PART– III

31. If $\text{adj}(A) = \begin{bmatrix} 2 & -4 & 2 \\ -3 & 12 & -7 \\ -2 & 0 & 2 \end{bmatrix}$, find A.

Solution

$$|adjA| = 2[24-0] - (-4)[-6-14] + 2[0+24] = 16$$

$$adj(adjA) = \begin{bmatrix} 24 & 20 & 24 \\ -(-8-0) & (4+4) & -(0-8) \\ (28-24) & -(-14+6) & (24-12) \end{bmatrix}^T$$

$$= \begin{bmatrix} 24 & 20 & 24 \\ 8 & 8 & 8 \\ 4 & 8 & 12 \end{bmatrix}^T = \begin{bmatrix} 24 & 8 & 4 \\ 20 & 8 & 8 \\ 24 & 8 & 12 \end{bmatrix} = 4\begin{bmatrix} 6 & 2 & 1 \\ 5 & 2 & 2 \\ 6 & 2 & 3 \end{bmatrix}$$

$$A = \pm \frac{1}{\sqrt{|adjA|}}(adjA)$$

$$= \pm \frac{1}{\sqrt{16}}(4)\begin{bmatrix} 6 & 2 & 1 \\ 5 & 2 & 2 \\ 6 & 2 & 3 \end{bmatrix} = \pm \begin{bmatrix} 6 & 2 & 1 \\ 5 & 2 & 2 \\ 6 & 2 & 3 \end{bmatrix}.$$

32. If $\omega \neq 1$ is a cube root of unity, show that $\dfrac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2}+\dfrac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2}=-1$.

Solution

$$LHS = \frac{\omega\left[a\omega^2+b+c\omega\right]}{\left(b+c\omega+a\omega^2\right)}+\frac{\omega^2\left[a\omega+b\omega^2+c\right]}{\left[a\omega+b\omega^2+c\right]}=\omega+\omega^2=-1$$

33. Find a polynomial equation of minimum degree with rational coefficients, having $2-\sqrt{3}$ as a root.

Solution

Since $2-\sqrt{3}$ is a root and the coefficients are rational numbers, $2+\sqrt{3}$ is also a root.

Sum $=4$, Product $=1$.

A required polynomial equation is

x^2-(Sum of the roots) $x+$ Product of the roots $=0$

$x^2-4x+1=0$ is a required equation.

34. Find the domain of $\sin^{-1}\left(\dfrac{2+\sin x}{3}\right)$.

Solution

By definition, the domain of $y=\sin^{-1}x$ is $-1\leq x\leq 1$.

$\therefore\ -1\leq\dfrac{2+\sin x}{3}\leq 1$ which is same as $-3\leq 2+\sin x\leq 3$.

So, $-5\leq\sin x\leq 1$ reduces to $-1\leq\sin x\leq 1$, which gives $-\dfrac{\pi}{2}\leq x\leq\dfrac{\pi}{2}$

Thus, the domain of $\sin^{-1}\left(\dfrac{2+\sin x}{3}\right)$ is $\left[-\dfrac{\pi}{2},\dfrac{\pi}{2}\right]$.

35. Find the local extremum of the function $f(x)=x^4+32x$.

Solution

We have,

$$f'(x) = 4x^3+32=0$$
$$\Rightarrow x = -2$$

and $f''(x) = 12x^2$.

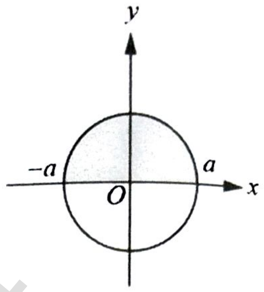
As $f''(-2)>0$, the function has local minimum at $x=-2$.

The local minimum value is $f(-2)=-48$.

36. Find the volume of a sphere of radius a through integration.

Solution

By revolving the upper semicircular region enclosed between the circle $x^2 + y^2 = a^2$ and the x-axis, we get a sphere of radius a.



The boundaries of the region are $y = \sqrt{a^2 - x^2}$, x-axis, the lines $x = -a$ and $x = a$.

$$\text{Hence, } V = \pi\int_{-a}^{a} y^2 dx = \pi\int_{-a}^{a}\left(a^2 - x^2\right)dx$$

$$= 2\pi\int_{0}^{a}\left(a^2 - x^2\right)dx.$$

$$= 2\pi\left(a^2 x - \frac{x^3}{3}\right)_0^a = 2\pi\left(a^3 - \frac{a^3}{3}\right) = \frac{4}{3}\pi a^3.$$

37. Solve the differential equation $\cos x\frac{dy}{dx} + y\sin x = 1$.

Solution

The given differential equation is $\cos x\frac{dy}{dx} + y\sin x = 1$

$$\frac{dy}{dx} + \frac{\sin x}{\cos x}y = \frac{1}{\cos x} \Rightarrow \frac{dy}{dx} + y\tan x = \sec x$$

This is a linear differential equation in y, where $P = \tan x$, $Q = \sec x$

$$\text{Integrating Factor I.F} = e^{\int Pdx} = e^{\int \tan x\, dx} = e^{\log|\sec x|} = \sec x$$

The general solution of the given differential equation is

$$ye^{\int Pdx} = \int Qe^{\int Pdx}dx + c$$

$$y\sec x = \int(\sec x)(\sec x)dx + c$$

$$y\sec x = \tan x + c$$

38. Find the constant C such that the function $f(x) = \begin{cases} Cx^2 & 1 < x < 4 \\ 0 & \text{Otherwise} \end{cases}$ is a density function, and compute $P(X \le 2)$.

Solution

Since the given function is a probability density function,

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$$\text{That is } \int_{1}^{4} f(x)dx = 1$$

From the given information

$$\int_{1}^{4} Cx^2\, dx = 1$$

$$C\left[\frac{x^3}{3}\right]_1^4 = 1, \quad \Rightarrow C\left[\frac{64-1}{3}\right]=1 \quad \Rightarrow 21C=1 \quad \Rightarrow C=\frac{1}{21}$$

$$P(X \le 2) = \int_{-\infty}^{1} f(x)\,dx + \int_{1}^{2} f(x)\,dx = \int_{1}^{2}\frac{1}{21}x^2\,dx$$

$$= \frac{1}{21}\left[\frac{x^3}{3}\right]_1^2$$

$$= \frac{1}{21}\left(\frac{2^3-1^3}{3}\right)=\frac{7}{63}.$$

39. Construct the truth table for $(p\vee q)\vee\neg q$

Solution :

$(p\vee q)\vee\neg q$

p	q	$r:(p\vee q)$	$s:\neg q$	$r\vee s$
T	T	T	F	T
T	F	T	T	T
F	T	T	F	T
F	F	F	T	T

40. If $f(x,y)=\dfrac{x^2+y^2+xy}{x^2-y^2}$ then show that $x\dfrac{\partial f}{\partial x}+y\dfrac{\partial f}{\partial y}=0$.

Solution

$$f(tx,ty) = \frac{t^2x^2+t^2y^2+t^2xy}{t^2x^2-t^2y^2}=\frac{x^2+y^2+xy}{x^2-y^2}=t^{\circ}f(x,y)$$

f is homogenous and the degree is 0.

By Eulers theorem, $x\dfrac{\partial f}{\partial x}+y\dfrac{\partial f}{\partial y}=n\cdot f=0\cdot f=0$

PART – IV

41. (a) Test for consistency of the following system of linear equations and if possible solve: $x+2y-z=3,\ 3x-y+2z=1,\ x-2y+3z=3,\ x-y+z+1=0$.

Solution

Here the number of unknowns is 3.

The matrix form of the system is $AX=B$, where

$$A = \begin{bmatrix}1&2&-1\\3&-1&2\\1&-2&3\\1&-1&1\end{bmatrix}, X=\begin{bmatrix}x\\y\\z\end{bmatrix}, B=\begin{bmatrix}3\\1\\3\\-1\end{bmatrix}.$$

$$[A\ B] = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 3 & -1 & 2 & 1 \\ 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -7 & 5 & -8 \\ 0 & -4 & 4 & 0 \\ 0 & -3 & 2 & -4 \end{bmatrix} \begin{matrix} R_1 \\ R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - R_1 \\ R_4 \to R_4 - R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -7 & 5 & -8 \\ 0 & 0 & 8 & 32 \\ 0 & 0 & -4 & -16 \end{bmatrix} \begin{matrix} R_1 \\ R_2 \\ R_3 \to 7R_3 - 4R_2 \\ R_4 \to 4R_4 - 3R_3 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -7 & 5 & -8 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 1 & 4 \end{bmatrix} \begin{matrix} R_1 \\ R_2 \\ R_3 \to R_3/8 \\ R_4 \to R_4/-4 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -7 & 5 & -8 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} R_1 \\ R_2 \\ R_3 \\ R_4 \to R_4 - R_3 \end{matrix}$$

It is in echelon form and

$$\rho(A) = \rho([A\ B]) = 3 = \text{number of unknowns}$$

The system reduces to

$$x + 2y = 3 \ ;$$
$$-7y + 5z = -8$$
$$z = 4 \quad \Rightarrow \quad y = 4,\ x = -1$$

i.e., $x = -1,\ y = 4,\ z = 4$.

41. (b) The velocity v, of a parachute falling vertically satisfies the equation $v\dfrac{dv}{dx} = g\left(1 - \dfrac{v^2}{k^2}\right)$, where g and k are constants. If v and x are both initially zero, find v in terms of x.

Solution

$$\text{Given that, } v\frac{dv}{dx} = g\left(1 - \frac{v^2}{k^2}\right) \quad \text{or} \quad v\frac{dv}{dx} = \frac{g}{k^2}\left(k^2 - v^2\right)$$

In this equation the variables are separable. On separating the variables, we have

$$\frac{vdv}{k^2 - v^2} = \frac{g}{k^2}dx$$

On integrating both sides, we get

$$-\frac{1}{2}\int \frac{-2vdv}{k^2 - v^2} dv = \frac{g}{k^2}\int dx$$

$$-\frac{1}{2}\log\left(k^2-v^2\right) = \frac{g}{k^2}x+c \qquad \ldots(1)$$

It is given that, initially both v and x are zero. i.e., when $x=0$, $v=0$

Therefore, from (1) we have $-\frac{1}{2}\log k^2 = c \qquad \ldots(2)$

Using (1) and (2), we get

$$-\frac{1}{2}\log\left(k^2-v^2\right) = \frac{g}{k^2}x-\frac{1}{2}\log k^2$$

$$\frac{1}{2}\log k^2-\frac{1}{2}\log\left(k^2-v^2\right) = \frac{g}{k^2}x$$

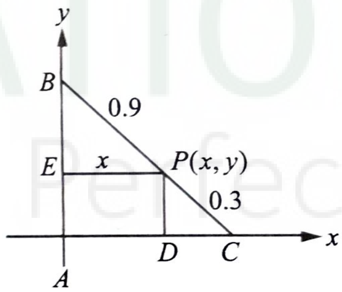
$$\frac{1}{2}\log\left(\frac{k^2}{k^2-v^2}\right) = \frac{gx}{k^2}$$

$$\frac{k^2}{k^2-v^2} = e^{\frac{2gx}{k^2}}$$

$$k^2-v^2 = k^2e^{-\frac{2gx}{k^2}} \text{ or } v^2=k^2\left(1-e^{-\frac{2gx}{k^2}}\right)$$

42. (a) A rod of length 1.2m moves with its ends always touching the coordinate axes. The locus of a point P on the rod, which is 0.3m from the end in contact with x-axis. Prove that the locus is an ellipse and hence find the eccentricity.

Solution



Let BC be the rod and $P(x, y)$ be a point on the rod which is 0.3m from the end in contact with x-axis.

$CP=0.3\text{ m}$, $PB=0.9\text{ m}$

Triangles BEP and PDC are similar.

$$\therefore \frac{BE}{PD} = \frac{BP}{PC}=\frac{EP}{DC}$$

$$\frac{BE}{y} = \frac{0.9}{0.3}=\frac{x}{DC}$$

$$DC = \frac{1}{3}x$$

$$BE = 3y$$

$$AC = x+\frac{1}{3}x=\frac{4}{3}x$$

$$AB = y+3y=4y$$

$$AB^2+AC^2 = BC^2 \Rightarrow 16y^2+\frac{16}{9}x^2=1.44$$

$$\Rightarrow \frac{x^2}{\left(\frac{9}{16}\right)} + \frac{y^2}{\left(\frac{1}{16}\right)} = 1.44 \text{ which is an ellipse.}$$

$$e = \sqrt{1-\frac{b^2}{a^2}} = \sqrt{1-\frac{(1.44)\frac{1}{16}}{(1.44)\frac{9}{16}}} = \sqrt{1-\frac{1}{9}} = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}.$$

Note : This problem can be solved by using trigonometry also.

42. (b) Prove that $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$.

Solution

$$\text{L.H.S} = \tan^{-1}x+\tan^{-1}y+\tan^{-1}z$$

$$= \tan^{-1}\left[\frac{x+y}{1-xy}\right]+\tan^{-1}z = \tan^{-1}\left[\frac{\frac{x+y}{1-xy}+z}{1-\frac{x+y}{1-xy}\cdot z}\right] = \tan^{-1}\left[\frac{\frac{x+y+z-xyz}{1-xy}}{\frac{1-xy-(xz+yz)}{1-xy}}\right]$$

$$= \tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right] = \text{R.H.S}$$

43. (a) Identify the type of conic $\frac{(y-2)^2}{25}-\frac{(x+1)^2}{16}=1$. Find the centre, foci and vertices.

Solution

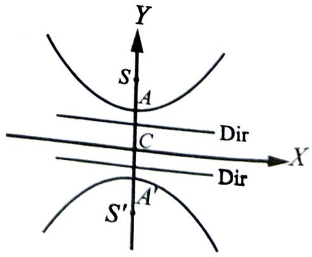
$\frac{(y-2)^2}{25}-\frac{(x+1)^2}{16}=1$ represents a hyperbola with transverse axis parallel to y-axis and the type is II.

$$\frac{Y^2}{25}-\frac{X^2}{16}=1, \text{ where } X=x+1,\ Y=y-2$$

$$a^2=25,\ b^2=16$$

$$e = \sqrt{1+\frac{b^2}{a^2}} = \sqrt{1+\frac{16}{25}} = \frac{\sqrt{41}}{5}$$

$$ae = 5\times\frac{\sqrt{41}}{5} = \sqrt{41}$$



$$= \tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right] = \text{R.H.S}$$

43. (a) $\frac{(y-2)^2}{25} - \frac{(x+1)^2}{16} = 1$ represents a hyperbola

$$a^2 = 25,\ b^2 = 16$$

$$e = \sqrt{1+\frac{b^2}{a^2}} = \sqrt{1+\frac{16}{25}} = \frac{\sqrt{41}}{5}$$

$$ae = 5\times\frac{\sqrt{41}}{5} = \sqrt{41}$$

Centre $C(-1,2)$

Vertices $A(-1,7)$ and $A'(-1,-3)$

Foci $S\left(-1,\sqrt{41}+2\right)$ and $S'\left(-1,-\sqrt{41}+2\right)$

43. (b) Let (x_1, y_1) be the point of intersection.

$$\text{At}\,(x_1,y_1), y' = \frac{x_1}{y_1} = m_1$$

$$xy = c^2$$

$$\text{At}\ (x_1,y_1), y' = \frac{-y_1}{x_1} = m_2$$

$$m_1m_2 = -1$$

The two curves cut orthogonally.

44. (a) $\frac{\partial g}{\partial r} = \frac{\partial g}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial g}{\partial y}\frac{\partial y}{\partial r}$

$$= 4(2r-s)+4r = 12r-4s$$

$$\frac{\partial g}{\partial s} = \frac{\partial g}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial g}{\partial y}\frac{\partial y}{\partial s}$$

$$= -2(2r-s)+4 = 2s-4r+4$$

X,Y	x,y
Centre : $(0,0)$	$C(-1,2)$
Vertices : $(0,\pm a)$ i.e., $(0,5)$ $(0,-5)$	$X=0 \Rightarrow x=-1$ $Y=5 \Rightarrow y=7$ $A(-1,7)$ $Y=-5 \Rightarrow y-2=-5$ $\Rightarrow y=-3$ $A'(-1,-3)$
Foci : $(0,\pm ae)$ $\left(0,\pm\sqrt{41}\right)$	$X=0 \Rightarrow x=-1$ $Y=\sqrt{41} \Rightarrow y=\sqrt{41}+2$ $S\left(-1,\sqrt{41}+2\right)$ $Y=-\sqrt{41} \Rightarrow y=-\sqrt{41}+2$ $S'\left(-1,-\sqrt{41}+2\right)$

43. (b) Show that the two curves $x^2-y^2=r^2$ and $xy=c^2$ where c,r are constants, cut orthogonally.

Solution :

Let (x_1, y_1) be the point of intersection.

$$x^2-y^2 = r^2$$
$$2x-2yy' = 0$$
$$y' = \frac{-2xy}{-2y} = \frac{x}{y}$$
$$\text{At}\,(x_1,y_1), y' = \frac{x_1}{y_1} = m_1$$
$$xy = c^2$$
$$xy'+y = 0$$
$$y' = \frac{-y}{x}$$
$$\text{At}\,(x_1,y_1), y' = \frac{-y_1}{x_1} = m_2$$

Now, $m_1 m_2 = \left(\frac{x_1}{y_1}\right)\left(\frac{-y_1}{x_1}\right)$

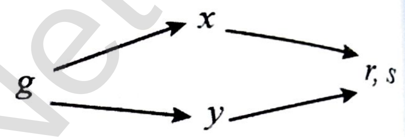
$m_1 m_2 = -1$

$\Rightarrow$ The two curves cut orthogonally.

44. (a) Let $g(x,y) = 2y + x^2, x = 2r - s, y = r^2 + 2s, r, s \in \mathbb{R}$. Find $\frac{\partial g}{\partial r}, \frac{\partial g}{\partial s}$.

Solution

Hence we find



$$\frac{\partial g}{\partial x} = 2x, \frac{\partial g}{\partial y} = 2, \frac{\partial x}{\partial r} = 2, \frac{\partial x}{\partial s} = -1, \frac{\partial y}{\partial r} = 2r \text{ and } \frac{\partial y}{\partial s} = 2.$$

$$\frac{\partial g}{\partial r} = \frac{\partial g}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial g}{\partial y}\frac{\partial y}{\partial r} = (2x)(2) + 2(2r) = 4x + 4r$$

$$= 4(2r - s) + 4r = 12r - 4s$$

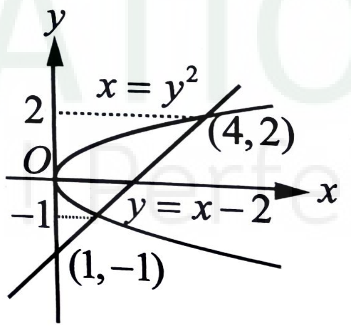
$$\frac{\partial g}{\partial s} = \frac{\partial g}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial g}{\partial y}\frac{\partial y}{\partial s} = 2x(-1) + 2(2) = -2x + 4$$

$$= -2(2r - s) + 4 = 2s - 4r + 4$$

44. (b) Find the area of the region bounded by the parabola $y^2 = x$ and the line $y = x - 2$.

Solution :

To get point of intersection we should solve the equations.



$$y^2 = x$$
$$y = x - 2$$

Solve for y

$$y = y^2 - 2$$

or $y^2 - y - 2 = 0$

$(y-2)(y+1) = 0$

$y = 2$ and $y = -1$.

From the diagram, the required area $A = \int_{-1}^{2} (x_1 - x_2)\, dy$

where x_1 is x from the line ; x_2 is x from the parabola

$$A = \int_{-1}^{2} \left[(y+2) - y^2\right] dy = \left[\frac{y^2}{2} + 2y - \frac{y^3}{3}\right]_{-1}^{2} = \left(\frac{4}{2} + 4 - \frac{8}{3}\right) - \left[\frac{1}{2} - 2 + \frac{1}{3}\right] = \frac{9}{2}$$

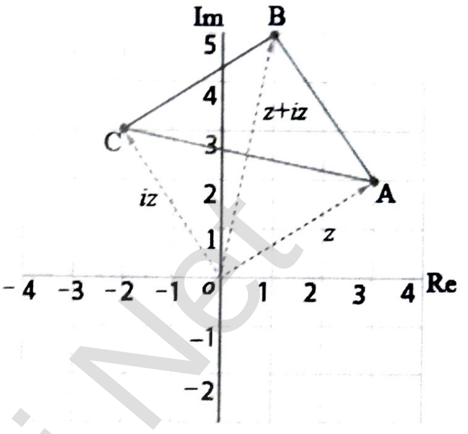
45. (a) For the complex number $z = 3+2i$, plot z, iz, and $z+iz$ in the Argand plane. Show that these complex numbers form the vertices of an isosceles right angled triangle.

Solution

Given that $z = 3+2i$.

Therefore, $iz = i(3+2i) = -2+3i$

$z + iz = (3+2i) + i(3+2i) = 1+5i$



Let A, B, and C be z, $z+iz$, and iz respectively.

$AB^2 = |(z+iz)-z|^2 = |-2+3i|^2 = 13$

$BC^2 = |iz-(z+iz)|^2 = |-3-2i|^2 = 13$

$CA^2 = |z-iz|^2 = |5-i|^2 = 26$

Since $AB^2 + BC^2 = CA^2$ and $AB = BC$, ΔABC is an isosceles right triangle

45. (b) The rate of increase in the number of bacteria in a certain bacteria culture is proportional to the number present. Given that the number triples in 5 hours, find how many bacteria will be present after 10 hours?

Solution

Let x be the number of bacteria at any time t.

Therefore we have $\frac{dx}{dt} \alpha\, x$

$$\Rightarrow \frac{dx}{dt} = kx \text{, where } k \text{ is the constant of proportionality.}$$

$$\Rightarrow \frac{dx}{x} = kdt$$

On integrating both sides, we get $\log|x| = kt + c \Rightarrow x = ce^{kt}$ ... (1)

When $t = 0$, let $x = x_0$.

Therefore, from (1), we have $c = x_0$.

$$x = x_0 e^{kt} \quad \text{... (2)}$$

When $t = 5$, $x = 3x_0$ from (2), we have

$$3x_0 = x_0 e^{5k}$$

$$e^{5k} = 3$$

When $t = 10$, we have $x = x_0 e^{10k} = x_0\left(e^{5k}\right)^2 = 9x_0$

Hence after 10 hours the number of bacteria is 9 times the original number of bacteria.

46. (a) Two balls are drawn in succession without replacement from an urn containing four red balls and three black balls. Let X be the possible outcomes of drawing red balls. Find the probability mass function and mean for X.

Solution

Let X be the possible outcomes drawing red balls. Then X takes values $0,1,2$. The probabilities are given by

$$P(X=0) = \frac{3}{7}\times\frac{2}{6}=\frac{1}{7}$$

$$P(X=1) = \frac{4}{7}\times\frac{3}{6}+\frac{3}{7}\times\frac{4}{6}=\frac{4}{7}$$

$$P(X=2) = \frac{4}{7}\times\frac{3}{6}=\frac{2}{7}.$$

Therefore the probability mass function is

x	0	1	2
$f(x)$ $P[X=x]$	$\frac{1}{7}$	$\frac{4}{7}$	$\frac{2}{7}$

$$\text{Mean} = \sum xf(x) = \left(0\times\frac{1}{7}\right)+\left(1\times\frac{4}{7}\right)+\left(2\times\frac{2}{7}\right)=\frac{8}{7}.$$

46. (b) Show that $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{dx}{1+\sqrt{\cot x}}=\frac{\pi}{12}$.

Solution Let $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{dx}{1+\sqrt{\cot x}}=\frac{\pi}{12}$ ... (1)

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{1}{\sqrt{1+\cot\left(\frac{\pi}{6}+\frac{\pi}{3}-x\right)}}dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{1}{\sqrt{1+\tan x}}dx \quad \text{... (2)}$$

Adding (1) and (2) we get

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{1+\sqrt{\cot x}}{1+\sqrt{\cot x}}dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}}dx$$

$$=[x]_{\frac{\pi}{6}}^{\frac{\pi}{3}} = \frac{\pi}{3}-\frac{\pi}{6} = \frac{\pi}{6}$$

$$I = \frac{\pi}{12}.$$

47. (a) Find the vector and Cartesian equations of the plane passing through the points $(3,6,-2),(-1,-2,6)$, and $(6,4,-2)$.

Solution

$$\vec{a} = 3\hat{i}+6\hat{j}-2\hat{k}$$
$$\vec{b} = -\hat{i}-2\hat{j}+6\hat{k}$$
$$\vec{c} = 6\hat{i}+4\hat{j}-2\hat{k}$$

Vector equation is

$$\vec{r} = (1-s-t)\vec{a}+s\vec{b}+t\vec{c}$$
$$\vec{r} = (1-s-t)\left(3\hat{i}+6\hat{j}-2\hat{k}\right)+s\left(-\hat{i}-2\hat{j}+6\hat{k}\right)+t\left(6\hat{i}+4\hat{j}-2\hat{k}\right)s,t\in\mathbb{R}.$$

Cartesian equation is $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$

$$\begin{vmatrix} x-3 & y-6 & z+2 \\ -4 & -8 & 8 \\ 3 & -2 & 0 \end{vmatrix} = 0$$

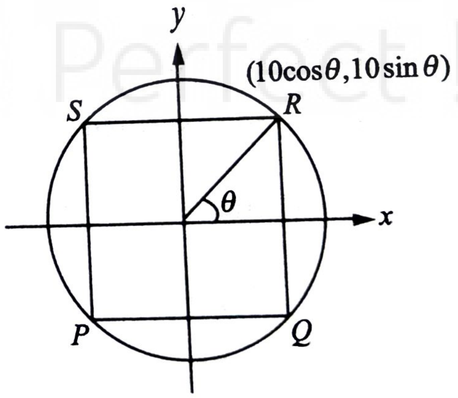
$$(x-3)(16)-(y-6)(-24)+(z+2)32 = 0$$
$$2x+3y+4z-16 = 0.$$

Note : Vector equation can be written in a different format also.

47. (b) Find the dimensions of the rectangle with maximum area that can be inscribed in a circle of radius 10 cm.

Solution

From the diagram the area of the rectangle $PQRS$ is $20\cos\theta\cdot 20\sin\theta$



$$A = 200\sin 2\theta$$
$$A' = 400\cos 2\theta$$
$$A'' = -800\sin 2\theta$$
$$A' = 0 \Rightarrow \cos 2\theta = 0 \Rightarrow 2\theta = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{4}$$

When $\theta = \frac{\pi}{4}$, $A''<0$

Hence when $\theta = \frac{\pi}{4}$, the area is maximum.

The dimensions are $20\cos\theta, 20\sin\theta$.

i.e., $20\cdot\frac{1}{\sqrt{2}}, 20\cdot\frac{1}{\sqrt{2}}$ i.e., $10\sqrt{2}, 10\sqrt{2}$ *cm.*

MARKING SCHEME - MATHEMATICS

GENERAL INSTRUCTIONS

The marking scheme provides general guidelines to reduce subjectivity in the marking. The answer given in the marking scheme are Text Book, Solution Book and COME book bound. If a student has given any answer which is different from one given in the marking scheme, but carries the prescribed content meaning (rigorous), such answers should be given full credit with suitable distribution.

There is no separate mark allotted for formulae. If a particular stage is wrong and if the candidate writes the appropriate formula then award 1 mark for the formula irrespective of stage marks. This mark is attached with that stage. This is done with the aim that a student who did the problem correctly without writing the formula should not be penalised. That is, mark should not be deducted for not writing the formula.

In the case of Part II, Part III and Part IV, if the solution is correct then award full mark directly. The stage mark is essential only if the part of the solution is incorrect.

Only a Rough sketch of the diagram is expected and full credit must be given for such diagrams that form a part and parcel of the solution of a problem.

In questions on Variable Separable, Homogeneous and Linear differential equations, full marks should be given for an equivalent answer and the students should not be penalised for not getting the final answer as mentioned in the marking scheme.

If the method is not mentioned in the question, then one may adopt any method.

Complicated radicals and combinations need not be simplified (for example $\sqrt{7}$, $\sqrt{82}$, $\frac{3}{\sqrt{2}}$, $10c_6$ etc.)

Important Note : In an answer to a question between any two particular stages of marks (greater than one) if a student starts a stage with correct step but reaches the next stage with a wrong result then suitable credits should be given to the related steps instead of denying the entire marks meant for that stage.

PART – II

21. $$\begin{bmatrix} -2 & 2 & -1 \\ 0 & 5 & 1 \\ 2 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} -2 & 2 & -1 \\ 0 & 5 & 1 \\ 0 & 2 & -1 \end{bmatrix} R_3 \to R_3 + R_1$$

$$\sim \begin{bmatrix} -2 & 2 & -1 \\ 0 & 5 & 1 \\ 0 & 0 & -7 \end{bmatrix} R_3 \to 5R_3 - 2R_2 \quad \text{... (1)}$$

rank $= 3$ ... (1)

(or)

$$2\begin{vmatrix} 2 & -1 \\ 5 & 1 \end{vmatrix} \neq 0 \quad \text{... (1)}$$

rank $= 3$. ... (1)

22. $$\text{Im}(3z + 4\bar{z} - 4i) = \text{Im}[3(x+iy) + 4(x-iy) - 4i] \quad \text{... (1)}$$

$$= \text{Im}[7x + i(-y-4)] = -y-4 \quad \text{... (1)}$$

23. Amp $= 1$. ... (1)

period $= \dfrac{2\pi}{7}$ ... (1)

24. $$\alpha + \beta = \frac{-43}{17}$$

$$\alpha\beta = \frac{-73}{17}$$

$$-\alpha - \beta = \frac{43}{17}$$

$$(-\alpha)(-\beta) = \frac{-73}{17} \quad \text{... (1)}$$

$$x^2 - \frac{43}{17}x - \frac{73}{17} = 0 \quad \text{(or)} \quad 17x^2 - 43x - 73 = 0. \quad \text{... (1)}$$

25. $$\hat{i} \times (\vec{a} \times \hat{i}) = (\hat{i} \cdot \hat{i})\vec{a} - (\hat{i} \cdot \vec{a})\hat{i} = \vec{a} - (\vec{a} \cdot \hat{i})\hat{i}$$

$$\hat{j} \times (\vec{a} \times \hat{j}) = \vec{a} - (\vec{a}.\hat{j})\hat{j}$$

$$\hat{k} \times (\vec{a} \times \hat{k}) = \vec{a} - (\vec{a} \cdot \hat{k})\hat{k}$$

$$\text{L.H.S} = 3\vec{a} - \left[\left(\vec{a}\cdot\hat{i}\right)\vec{i} + \left(\vec{a}\cdot\vec{j}\right)\vec{j} + \left(\vec{a}\cdot\vec{k}\right)\vec{k}\right]$$ ... (1)

But we know that $\vec{a} = \left(\vec{a}\cdot\hat{i}\right)\hat{i} + \left(\vec{a}\cdot\hat{j}\right)\hat{j} + \left(\vec{a}\cdot\hat{k}\right)\hat{k}$

$$\therefore \hat{i}\times\left(\vec{a}\times\hat{i}\right) + \hat{j}\times\left(\vec{a}\times\hat{j}\right) + \hat{k}\times\left(\vec{a}\times\hat{k}\right) = 3\vec{a} - \vec{a} = 2\vec{a}.$$ ... (1)

26. Statement : Rolles theorem.

Let $f(x)$ be continuous in the closed interval $[a,b]$, differentiable in the open interval (a,b) and $f(a) = f(b)$. Then there is at least one point $c \in (a,b)$ where $f'(c) = 0$.

To write the theorem without affecting inner meaning (need not follow word by word). ... (2)

27.
$$\int_{-\log 2}^{\log 2} e^{-|x|}\,dx = 2\int_0^{\log 2} e^{-|x|}\,dx = 2\int_0^{\log 2} e^{-x}\,dx$$ ... (1)

$$= 1.$$ ... (1)

28.
$$E(X+5) = E(X)+5 = 6$$ ... (1)

$$E(X) = 1$$ ... (1)

29. $a*b = (a+3ab-5b^2) \in \mathbb{Z}$.

in $\mathbb{Z}$, closure property is true. ... (1)

... (1)

(or)

Since a and b are integers $a+3ab-5b^2$ is also an integer.

$\therefore$ * satisfies closure property. ... (1)

... (1)

30.
$$y^2 = 4ax$$
$$2yy' = 4a$$
$$y^2 = 2yy'x \Rightarrow y = 2y'x.$$ ... (1)

... (1)

PART– III

31.
$$|adjA| = 16$$ ... (1)

$$adj(adjA) = 4\begin{bmatrix} 6 & 2 & 1 \\ 5 & 2 & 2 \\ 6 & 2 & 3 \end{bmatrix}$$ ... (1)

$$= \pm \begin{bmatrix} 6 & 2 & 1 \\ 5 & 2 & 2 \\ 6 & 2 & 3 \end{bmatrix}.$$ ... (1)

32. $$LHS = \frac{\omega\left[a\omega^2+b+c\omega\right]}{\left(b+c\omega+a\omega^2\right)}+\frac{\omega^2\left[a\omega+b\omega^2+c\right]}{\left[a\omega+b\omega^2+c\right]}$$ ... (1)

$$= \omega+\omega^2 = -1$$ ... (2)

33. $2+\sqrt{3}$ is also a root. ... (1)

Sum $=4$, Product $=1$. ... (1)

$x^2-4x+1=0$ ... (1)

34. The domain of $y=\sin^{-1}x$ is $-1\le x\le 1$.

$-1\le \frac{2+\sin x}{3}\le 1$ which is same as $-3\le 2+\sin x\le 3$. ... (2)

So, $-5\le \sin x\le 1$ reduces to $-1\le \sin x\le 1$, which gives $-\frac{\pi}{2}\le x\le \frac{\pi}{2}$ (OR) ... (1)

Thus, the domain of $\sin^{-1}\left(\frac{2+\sin x}{3}\right)$ is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$.

35. $f'(x) = 4x^3+32=0$ ... (1)

$\Rightarrow x = -2$

$f''(x) = 12x^2$.

As $f''(-2)>0$, the function has local minimum at $x=-2$. ... (1)

The local minimum value is $f(-2)=-48$. ... (1)

36. $$V = \pi\int_{-a}^{a} y^2dx = \pi\int_{-a}^{a}\left(a^2-x^2\right)dx$$ ... (1)

$$= \frac{4}{3}\pi a^3.$$ ... (2)

37. $$\frac{dy}{dx}+y\tan x=\sec x$$

I.F $=\sec x$ ... (1)

$y\sec x = \tan x+c$ ... (1)

38. $$C = \frac{1}{21}$$

$$P(X \le 2) = \int_{-\infty}^{1} f(x)\,dx + \int_{1}^{2} f(x)\,dx = \int_{1}^{2} \frac{1}{21} x^2\,dx$$

$$= \frac{7}{63}.$$

39. $(p \vee q) \vee \neg q$

p	q	$r:(p \vee q)$	$s:\neg q$	$r \vee s$
T	T	T	F	T
T	F	T	T	T
F	T	T	F	T
F	F	F	T	T

Third, fourth, fifth column – each one mark ...

40. f is homogenous and the degree is 0. ...

$$x \cdot \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = 0$$ ...

PART – IV

41. (a)

$$[A\ B] = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 3 & -1 & 2 & 1 \\ 1 & -2 & 3 & 3 \\ 1 & -1 & 1 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -7 & 5 & -8 \\ 0 & -4 & 4 & 0 \\ 0 & -3 & 2 & -4 \end{bmatrix} \begin{matrix} R_1 \\ R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - R_1 \\ R_4 \to R_4 - R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -7 & 5 & -8 \\ 0 & 0 & 8 & 32 \\ 0 & 0 & -4 & -16 \end{bmatrix} \begin{matrix} R_1 \\ R_2 \\ R_3 \to 7R_3 - 4R_2 \\ R_4 \to 4R_4 - 3R_3 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -7 & 5 & -8 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 1 & 4 \end{bmatrix} \begin{matrix} R_1 \\ R_2 \\ R_3 \to R_3 / 8 \\ R_4 \to R_4 / -4 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -7 & 5 & -8 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} R_1 \\ R_2 \\ R_3 \\ R_4 \to R_4 - R_3 \end{matrix}$$ ... (2)

$$\rho(A) = \rho([A\ B]) = 3 = \text{number of unknown}$$ ... (1)

$$x = -1, \quad y = 4, \quad z = 4.$$ ... (2)

41. (b)

$$\frac{v\,dv}{k^2 - v^2} = \frac{g}{k^2}dx$$ ... (1)

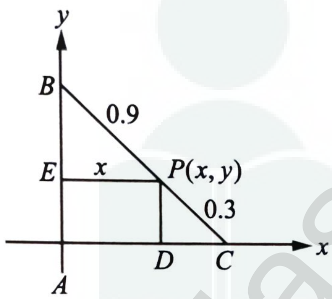
$$-\frac{1}{2}\log\left(k^2 - v^2\right) = \frac{g}{k^2}x + c$$ ... (1)

when $x = 0$, $v = 0$

$$-\frac{1}{2}\log k^2 = c$$ ... (1)

$$k^2 - v^2 = k^2 e^{-\frac{2gx}{k^2}} \text{ or } v^2 = k^2\left(1 - e^{-\frac{2gx}{k^2}}\right)$$ ... (2)

42. (a) Rough diagram ... (2)



$$\Rightarrow \frac{x^2}{\left(\frac{9}{16}\right)} + \frac{y^2}{\left(\frac{1}{16}\right)} = 1.44 \text{ which is an ellipse.}$$ ... (2)

$$e = \frac{2\sqrt{2}}{3}.$$ ... (1)

Note : This problem can be solved using trignometry also.

42. (b) $\text{L.H.S} = \tan^{-1}x + \tan^{-1}y + \tan^{-1}z$

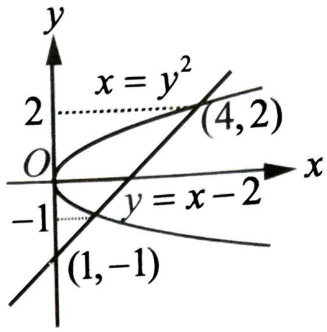
$$= \tan^{-1}\left[\frac{x+y}{1-xy}\right] + \tan^{-1}z$$ ... (1)

$$= \tan^{-1}\left[\frac{\frac{x+y}{1-xy} + z}{1 - \frac{x+y}{1-xy}\cdot z}\right]$$ ... (2)

$$= \tan^{-1}\left[\frac{\frac{x+y+z-xyz}{1-xy}}{\frac{1-xy-(xz+yz)}{1-xy}}\right]$$ ... (1)

... (1)

44. (b) Rough diagram



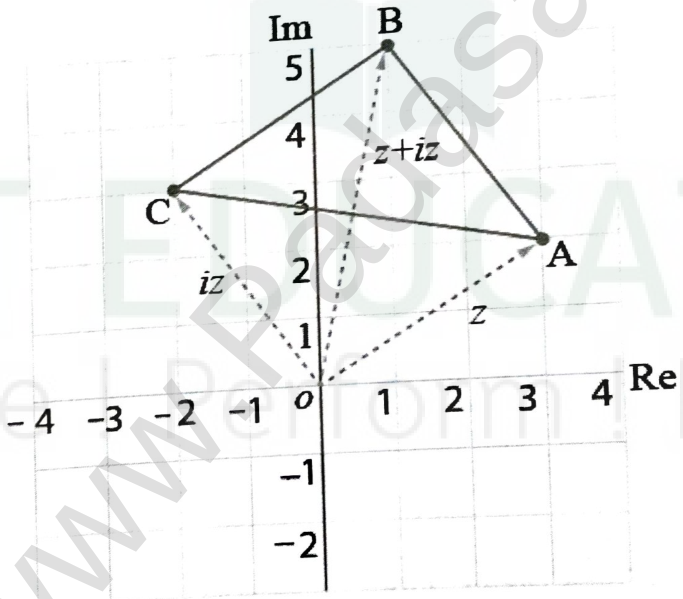
... (1)

The limits are $y = 2$ and $y = -1$. ... (1)

$$A = \int_{-1}^{2} \left[(y+2) - y^2\right] dy$$ ... (2)

$$= \frac{9}{2}$$ ... (2)

45. (a) Rough diagram



Given that $z = 3+2i$.

Therefore, $iz = i(3+2i) = -2+3i$

$z + iz = (3+2i) + i(3+2i) = 1+5i$

Let $A, B,$ and C be $z,\ z+iz,$ and iz respectively.

$$AB^2 = |(z+iz) - z|^2 = |-2+3i|^2 = 13$$

$$BC^2 = |iz - (z+iz)|^2 = |-3-2i|^2 = 13$$

$$CA^2 = |z - iz|^2 = |5-i|^2 = 26$$ ... (2)

$AB^2 + BC^2 = CA^2$ and $AB = BC$, ΔABC is an isosceles right triangle ... (1)

45. (b) $$\Rightarrow \frac{dx}{dt} = kx \quad \cdots (1)$$

$$x = ce^{kt} \quad \cdots (1)$$

When $t = 0$, let $x = x_0$.

$$c = x_0.$$

$$x = x_0 e^{kt} \quad \cdots (1)$$

When $t = 5$, $x = 3x_0$

$$3x_0 = x_0 e^{5k}$$

$$e^{5k} = 3 \quad \cdots (1)$$

When $t = 10$

$$x = x_0 e^{10k} = x_0\left(e^{5k}\right)^2 = 9x_0 \quad \cdots (1)$$

After 10 hours the number of bacteria is 9 times the original number of bacteria.

46. (a) The probability mass function is ... (3)

x	0	1	2
$f(x)$ $P[X = x]$	$\frac{1}{7}$	$\frac{4}{7}$	$\frac{2}{7}$

Mean $= \frac{8}{7}$. ... (2)

46. (b) $$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\cot x}} = \frac{\pi}{12}$$

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{\sqrt{1+\cot\left(\frac{\pi}{6}+\frac{\pi}{3}-x\right)}}dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{\sqrt{1+\tan x}}dx \quad \text{... (1)}$$

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1+\sqrt{\cot x}}{1+\sqrt{\cot x}}dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} dx \quad \text{... (2)}$$

$$= \frac{\pi}{6} \quad \text{... (1)}$$

$$I = \frac{\pi}{12}. \quad \text{... (1)}$$

47. (a)

$$\vec{a} = 3\hat{i} + 6\hat{j} - 2\hat{k}$$

$$\vec{b} = -\hat{i} - 2\hat{j} + 6\hat{k}$$

$$\vec{c} = 6\hat{i} + 4\hat{j} - 2\hat{k}$$

Vector equation is

$$\vec{r} = (1-s-t)\left(3\hat{i}+6\hat{j}-2\hat{k}\right)+s\left(-\hat{i}-2\hat{j}+6\hat{k}\right)+t\left(6\hat{i}+4\hat{j}-2\hat{k}\right) s,t \in \mathbb{R}.$$ ... (2)

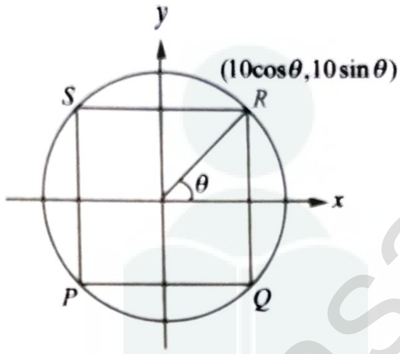
Cartesian equation is

$$\begin{vmatrix} x-3 & y-6 & z+2 \\ -4 & -8 & 8 \\ 3 & -2 & 0 \end{vmatrix} = 0$$ ... (2)

$$2x+3y+4z-16 = 0.$$ ... (1)

Note : The vector equation can be written in a different format also.

47. (b) Rough diagram ... (1)



The area of the rectangle is

$$A = 200\sin 2\theta$$ ... (1)

$$A' = 400\cos 2\theta$$

$$A'' = -800\sin 2\theta$$

$$A' = 0 \Rightarrow \cos 2\theta = 0 \Rightarrow 2\theta = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{4}$$

When $\theta = \frac{\pi}{4}$, $A'' < 0$

When $\theta = \frac{\pi}{4}$, the area is maximum. ... (2)

The dimensions are $10\sqrt{2}\ cm,\ 10\sqrt{2}\ cm$. ... (1)

PROFILE

MODEL PAPER - VI

Part I		Part II		Part III		Part IV	
1	Exercise 1.8 (19)	21	Exercise 1.1 (9)	31	Exercise 1.6 (1 – iii)	41 a	Example 1.24
2	Chapter – 1 Created (9)	22	Example 2.7	32	Example 2.16	41 b	Example 11.12
3	Exercise 2.9 (15)	23	Example 3.2	33	Exercise 3.5 (1 - i)	42 a	Exercise 5.5 (1)
4	Chapter – 2 Created	24	Exercise 5.3 (5)	34	Exercise 5.2 (2 – iii)	42 b	Example 4.20
5	Exercise 3.7 (5)	25	Chapter – 7 Created	35	Exercise 6.1 (12)	43 a	Example 5.10
6	Exercise 4.6 (15)	26	Exercise 8.2 (9)	36	Example 5.16	43 b	Example 6.3
7	Chapter – 5 Created	27	Example 10.2	37	Exercise 8.1 (6)	44 a	Exercise 7.1 (8)
8	Exercise 5.6 (14)	28	Exercise 11.5 (4)	38	Example 9.26	44 b	Example 12.18
9	Exercise 6.10 (17)	29	Exercise 12.1 (2)	39	Chapter – 10 Created	45 a	Example 9.68
10	Chapter – 6 Created (6)	30	Chapter – 9 Created	40	Chapter – 4 Created	45 b	Exercise 10.6 (8)
11	Exercise 7.10 (13)					46 a	Chapter – 2 Created
12	Chapter – 7 Created					46 b	Example 8.22
13	Chapter – 9 Created (3)					47 a	Chapter – 6 Created
14	Exercise 9.10 (9)					47 b	Chapter – 7 Created
15	Chapter – 10 Created (23)						
16	Chaper – 10 Created (28)						
17	Exercise 11.6 (7)						
18	Exercise 11.6 (16)						
19	Exercise 12.3 (1)						
20	Chapter – 12 Created (9)						

HIGHER SECONDARY SECOND YEAR

MATHEMATICS

MODEL QUESTION PAPER – 6

Time Allowed: 15 Min + 3.00 Hours] **[Maximum Marks:90**

Instructions: (a) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.

(b) Use **Blue** or **Black** ink to write and underline and pencil to draw diagrams.

PART – I

Note: (i) **All** questions are **compulsory.** $20\times1=20$

(ii) Choose the correct or most suitable answer from the given **four** alternatives. Write the option code and the corresponding answer.

1. If $x^a y^b = e^m, x^c y^d = e^n, \Delta_1 = \begin{vmatrix} m & b \\ n & d \end{vmatrix}, \Delta_2 = \begin{vmatrix} a & m \\ c & n \end{vmatrix}, \Delta_3 = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$, then the values of x and y are respectively,

   (1) $e^{(\Delta_2/\Delta_1)}, e^{(\Delta_3/\Delta_1)}$ (2) $\log(\Delta_1/\Delta_3), \log(\Delta_2/\Delta_3)$

   (3) $\log(\Delta_2/\Delta_1), \log(\Delta_3/\Delta_1)$ (4) $e^{(\Delta_1/\Delta_3)}, e^{(\Delta_2/\Delta_3)}$

2. In the case of Cramer's rule which of the following are correct?

   (i) $\Delta = 0$ (ii) $\Delta \neq 0$

   (iii) the system has unique solution (iv) the system has infinitely many solutions

   (1) (i) and (iv) (2) (ii) and (iii) (3) all (4) none

3. If $z = x + iy$ is a complex number such that $|z+2| = |z-2|$, then the locus of z is

   (1) real axis (2) imaginary axis (3) ellipse (4) circle

4. $\arg\left(\frac{3}{-1-i}\right) =$

   (1) $-\frac{5\pi}{6}$ (2) $-\frac{2\pi}{3}$ (3) $\frac{3\pi}{4}$ (4) $-\frac{\pi}{2}$

5. Which number is not possible rational root of $4x^7 + 2x^4 - 10x^3 - 5 = 0$ (According to Rational Root Theorem) ?

   (1) -1 (2) $\frac{5}{4}$ (3) $\frac{4}{5}$ (4) 5

6. $\cot^{-1}\left(\sqrt{\sin\alpha}\right) + \tan^{-1}\left(\sqrt{\sin\alpha}\right) = u$, then $\cos 2u$ is equal to

   (1) $\tan^2\alpha$ (2) 0 (3) -1 (4) $\tan 2\alpha$

7. For the parabola $(x-h)^2 = -4a(y-k)$, the equation of the directrix is

   (1) $y = k$ (2) $y = a$ (3) $x = k + a$ (4) $y = k + a$

8. Two tangents are drawn to the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ parallel to the straight line $2x - y = 1$. One of the points of contact of tangents on the hyperbola is

(1) $\left(\frac{9}{2\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$ (2) $\left(\frac{-9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ (3) $\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ (4) $\left(3\sqrt{3} - 2\sqrt{2}\right)$

9. The angle between the line $\vec{r} = \left(\hat{i} + 2\hat{j} - 3\hat{k}\right) + t\left(2\hat{i} + \hat{j} - 2\hat{k}\right)$ and the plane $\vec{r}\cdot\left(\hat{i} + \hat{j}\right) + 4 = 0$ is

(1) $0°$ (2) $30°$ (3) $45°$ (4) $90°$

10. Which of the following statement is incorrect?

(1) if two lines are coplanar then their direction ratios must be same

(2) two coplanar lines must lie in a plane

(3) skew lines are neither parallel nor intersecting

(4) if two lines are parallel or intersecting then they are coplanar

11. The number given by the Mean value theorem for the function $\frac{1}{x}$, $x \in [1, 9]$ is

(1) 2 (2) 2.5 (3) 3 (4) 3.5

12. The slope of the tangent to the curve $f(x) = 2\cos 4x$ at $x = \frac{\pi}{12}$ is

(1) $-4\sqrt{3}$ (2) -4 (3) $\frac{\sqrt{3}}{12}$ (4) $4\sqrt{3}$

13. $\int_a^b f(a + b - x)dx =$

(1) $f(a) - f(b)$ (2) $\int_b^a f(x)dx$ (3) 0 (4) $\int_a^b f(x)dx$

14. The value of $\int_0^1 x(1-x)^{99} dx$ is

(1) $\frac{1}{11000}$ (2) $\frac{1}{10100}$ (3) $\frac{1}{10010}$ (4) $\frac{1}{10001}$

15. The order and degree of $\frac{d^2y}{dx^2} = xy + \cos\left(\frac{dy}{dx}\right)$ are

(1) 2, does not exist (2) 2,1 (3) 1,2 (4) 2,2

16. For a certain substance, the rate of change of vapor pressure P with respect to temperature T is proportional to the vapor pressure and inversely proportional to the square of the temperature. The corresponding differential equation is (k is constant of proportionality)

(1) $\frac{dP}{dT} = k\cdot\frac{T^2}{P}$ (2) $\frac{dT}{dP} = k\cdot T^2$ (3) $\frac{dP}{dT} = k\cdot\frac{P}{T^2}$ (4) $\frac{dP}{dT} = k\cdot P$

17. If the function $f(x)=\frac{1}{12}$ for $a<x<b$, represents a probability density function of a continuous random variable X, then which of the following cannot be the value of a and b?
(1) 0 and 12 (2) 5 and 17 (3) 7 and 19 (4) 16 and 24

18. If $f(x)=\begin{cases}2x & 0\le x\le a\\ 0 & \text{otherwise}\end{cases}$ is a probability density function of a random variable, then the value of a is
(1) 1 (2) 2 (3) 3 (4) 4

19. A binary operation on a set S is a function from
(1) $S\rightarrow S$ (2) $(S\times S)\rightarrow S$ (3) $S\rightarrow(S\times S)$ (4) $(S\times S)\rightarrow(S\times S)$

20. Which one of the following is correct?
(1) $[3]+_4[2]=[5]$ (2) $[0]+_{10}[12]=[0]$
(3) $[4]\times_5[3]=[12]$ (4) $[5]\times_6[4]=[2]$

PART – II

Note: (i) Answer any **SEVEN** questions. $7\times2=14$
(ii) Question number **30 is compulsory.**

21. If $\text{adj}(A)=\begin{bmatrix}0 & -2 & 0\\ 6 & 2 & -6\\ -3 & 0 & 6\end{bmatrix}$, find A^{-1}.

22. Find z^{-1}, if $z=(2+3i)(1-i)$.

23. If α and β are the roots of the quadratic equation $2x^2-7x+13=0$, construct a quadratic equation whose roots are α^2 and β^2.

24. Identify the type of conic $11x^2-25y^2-44x+50y-256=0$

25. Evaluate the limit $\lim\limits_{x\to0}\left(\frac{\sin mx}{\sin nx}\right)$.

26. The relation between number of words y a person learns in x hours is given by $y=52\sqrt{x}, 0\le x\le9$. What is the approximate number of words learned when x changes from 1 to 1.1 hour?

27. Find the differential equation for the family of all straight lines passing through the origin.

28. The probability that a certain kind of component will survive a electrical test is $\frac{3}{4}$. Find the probability that exactly 3 of the 5 components tested survive.

29. On $\mathbb{Z}$, define $\otimes$ by $(m \otimes n) = m^n + n^m : \forall m, n \in \mathbb{Z}$. Is $\otimes$ binary on $\mathbb{Z}$?

30. Show that the area of the region bounded by $y = \sin x, x = 0$ and $x = \pi$ is 2.

PART– III

$7 \times 3 = 21$

Note: (i) Answer any **SEVEN** questions.

(ii) Question number **40 is compulsory.**

31. Test for consistency and if possible, solve the system of equations $2x + 2y + z = 5, \quad x - y + z = 1, \quad 3x + y + 2z = 4$, by rank method.

32. Show that the equation $z^2 = \bar{z}$ has four solutions.

33. Solve the equation $\sin^2 x - 5\sin x + 4 = 0$

34. Find the equation of the ellipse whose length of latus rectum is 8, eccentricity is $\frac{3}{5}$, the centre is $(0,0)$ and the major axis is on x-axis.

35. Forces of magnitudes $5\sqrt{2}$ and $10\sqrt{2}$ units acting in the directions $3\hat{i} + 4\hat{j} + 5\hat{k}$ and $10\hat{i} + 6\hat{j} - 8\hat{k}$, respectively, act on a particle which is displaced from the point with position vector $4\hat{i} - 3\hat{j} - 2\hat{k}$ to the point with position vector $6\hat{i} + \hat{j} - 3\hat{k}$. Find the work done by the forces.

36. Find the equation of the parabola with focus $\left(-\sqrt{2}, 0\right)$ and directrix $x = \sqrt{2}$.

37. The time T, taken for a complete oscillation of a single pendulum with length l, is given by the equation $T = 2\pi\sqrt{\frac{l}{g}}$, where g is a constant. Find the approximate percentage error in the calculated value of T corresponding to an error of 2 percent in the value of t.

38. Evaluate : $\int_0^a \frac{f(x)}{f(x) + f(a-x)} dx.$

39. Solve the following differential equation : $\frac{dy}{dx} = xy\, e^x$

40. Find the value of $\tan^{-1}(-1) + \cos^{-1}\left(\frac{1}{2}\right) + \sec^{-1}(-2)$.

PART – IV

Note: Answer **all** the questions. $7 \times 5 = 35$

41. (a) If $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, find the products AB and BA and hence solve the system of equations $x - y + z = 4, x - 2y - 2z = 9, 2x + y + 3z = 1$.

(OR)

(b) If X is the random variable with probability density function

$$f(x)=\begin{cases} x-1, & 1\le x<2 \\ -x+3, & 2\le x<3 \\ 0 & \text{otherwise} \end{cases},$$

find the distribution function $F(x)$.

42. (a) A bridge has a parabolic arch that is 10m height in the centre and 30m wide at the bottom. Find the height of the arch 6m from the centre, on any one side.

(OR)

(b) Evaluate $\sin\left[\sin^{-1}\left(\frac{3}{5}\right)+\sec^{-1}\left(\frac{5}{4}\right)\right]$.

43. (a) Find the equation of the circle passing through the points $(1,1)$, $(2,-1)$, and $(3,2)$.

(OR)

(b) Prove that $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$, by using vectors.

44. (a) A conical water tank with vertex down of 12 metres height has a radius of 5 metres at the top. If water flows into the tank at a rate 10 cubic m/min, how fast is the depth of the water increases when the water is 8 metres deep?

(OR)

(b) Show that $p\leftrightarrow q\equiv(p\to q)\wedge(q\to p)$.

45. (a) Find the volume of the solid generated by revolving the region bounded by the curve $y=\frac{3}{4}\sqrt{x^2-16}$, $x\ge 4$, the y-axis, and the lines $y=1$ and $y=6$, about y-axis.

(OR)

(b) Solve $(x^2+y^2)dy=xy\,dx$. Find the value of x_0 when $y(1)=1$ and $y(x_0)=e$.

46 (a) Find all the values of $\left(-\sqrt{3}-i\right)^{\frac{1}{3}}$.

(OR)

(b) If $u=\sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{2}\tan u$.

47. (a) Find the vector and Cartesian equations of the plane passing through the point $(-2,-2,4)$ and perpendicular to the planes $6x+4y-6z=5$ and $10x-8y+2z=1$.

(OR)

(b) Examine the local extrema and points of inflexion for the curve $f(x)=\frac{4}{3}x^3-8x^2+16x+5$. If so, find them.

MODEL QUESTION PAPER – VI

ANSWERS

PART – II

Qn. No.	Option	Answer	Qn. No.	Option	Answer
1.	(4)	$e^{(\Delta_1/\Delta_3)}, e^{(\Delta_2/\Delta_3)}$	11.	(3)	3
2.	(2)	(ii) and (iii)	12.	(1)	$-4\sqrt{3}$
3.	(2)	Imaginary axis	13.	(4)	$\int_a^b f(x)dx$
4.	(3)	$\frac{3\pi}{4}$	14.	(2)	$\frac{1}{10100}$
5.	(3)	$\frac{4}{5}$	15.	(1)	2, does not exist
6.	(3)	-1	16.	(3)	$\frac{dP}{dT} = k\frac{P}{T^2}$
7.	(4)	$y = k + a$	17.	(4)	16 and 24
8.	(3)	$\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$	18.	(1)	1
9.	(3)	45°	19.	(2)	$(S \times S) \rightarrow S$
10.	(1)	if two lines are coplanar then their direction ratios must be same	20.	(4)	$[5] \times_6 [4] = [2]$

For writing the correct option and the answer – one mark.

PART – II

21. If $adj(A)=\begin{bmatrix}0&-2&0\\6&2&-6\\-3&0&6\end{bmatrix}$, find A^{-1}.

Solution

$$|adjA| = \begin{vmatrix}0&-2&0\\6&2&-6\\-3&0&6\end{vmatrix} = 2(36-18)=36$$

$$A^{-1} = \pm\frac{1}{\sqrt{(adjA)}}(adjA)$$

$$= \pm\frac{1}{\sqrt{36}}\begin{bmatrix}0&-2&0\\6&2&-6\\-3&0&6\end{bmatrix} = \pm\frac{1}{6}\begin{bmatrix}0&-2&0\\6&2&-6\\-3&0&6\end{bmatrix}.$$

22. Find z^{-1}, if $z=(2+3i)(1-i)$.

Solution

$$z = (2+3i)(1-i)=(2+3)+(3-2)i=5+i$$

$$z^{-1} = \frac{1}{z}=\frac{1}{5+i}.$$

$$= \frac{(5-i)}{(5+i)(5-i)}=\frac{5-i}{5^2+1^2}=\frac{5}{26}-i\frac{1}{26}$$

$$z^{-1} = \frac{5}{26}-i\frac{1}{26}.$$

23. If α and β are the roots of the quadratic equation $2x^2-7x+13=0$, construct a quadratic equation whose roots are α^2 and β^2.

Solution

Since α and β are the roots of the quadratic equation, we have

$$\alpha+\beta = \frac{7}{2} \text{ and } \alpha\beta=\frac{13}{2}.$$

Thus, to construct a new quadratic equation,

$$\text{Sum of the roots} = \alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=\frac{-3}{4}.$$

$$\text{Product of the roots} = \alpha^2\beta^2 = (\alpha\beta)^2 = \frac{169}{4}$$

Thus a required quadratic equation is $x^2 + \frac{3}{4}x + \frac{169}{4} = 0$.

$$\text{(or)} \quad 4x^2 + 3x + 169 = 0$$

24. Identify the type of conic $11x^2 - 25y^2 - 44x + 50y - 256 = 0$.

Solution

$$11x^2 - 25y^2 - 44x + 50y - 256 = 0$$

$$B^2 - 4AC = 0 - 4(11)(-25) > 0$$

It represents a hyperbola.

25. Evaluate the limit $\lim_{x\to 0}\left(\frac{\sin mx}{\sin nx}\right)$.

Solution

$$\lim_{x\to 0}\frac{\sin mx}{\sin nx} = \lim_{x\to 0}\frac{\cos mx \cdot m}{\cos nx \cdot n} = \frac{m}{n}$$

26. The relation between number of words y a person learns in x hours is given by $y = 52\sqrt{x}, 0 \le x \le 9$. What is the approximate number of words learned when x changes from 1 to 1.1 hour?

Solution

$$y = 52\sqrt{x}$$

$$dy = 52\cdot\frac{1}{2\sqrt{x}}dx \simeq \frac{26}{\sqrt{1}}(0.1) = 2.6 \simeq 3 \text{ words.}$$

27. Find the differential equation for the family of all straight lines passing through the origin.

Solution

The family of straight lines passing through the origin is $y = mx$, where m is an arbitrary constant.

$$\frac{dy}{dx} = m$$

Replace m with $\frac{dy}{dx}$, we get $y = x\frac{dy}{dx}$.

This is the required differential equation.

28. The probability that a certain kind of component will survive a electrical test is $\frac{3}{4}$. Find the probability that exactly 3 of the 5 components tested survive.

Solution

Here $n = 5, p = \frac{3}{4}$ and $q = \frac{1}{4}$

$$P(X=3) = \binom{5}{3}\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right)^2.$$

29. On $\mathbb{Z}$, define $\otimes$ by $(m \otimes n) = m^n + n^m : \forall m, n \in \mathbb{Z}$. Is $\otimes$ binary on $\mathbb{Z}$?

Solution

When n is a negative integer then m^n is not an integer and hence $m^n + n^m$ need not be in $\mathbb{Z}$.

Hence $\otimes$ is not binary on $\mathbb{Z}$.

30. Show that the area of the region bounded by $y = \sin x, x = 0$ and $x = \pi$ is 2.

Solution

The required area lies in the first quadrant and hence the area

$$= \int_0^\pi y\,dx = \int_0^\pi \sin x\,dx = [-\cos x]_0^\pi = 2$$

PART– III

31. Test for consistency and if possible, solve the system of equations $2x+2y+z=5, \quad x-y+z=1, \quad 3x+y+2z=4$, by rank method.

Solution

$2x+2y+z=5, x-y+z=1, \; 3x+y+2z=4$

The augmented matrix $[A \quad B] = \begin{bmatrix} 2 & 2 & 1 & 5 \\ 1 & -1 & 1 & 1 \\ 3 & 1 & 2 & 4 \end{bmatrix}$

$$\sim \begin{bmatrix} 1 & -1 & 1 & 1 \\ 2 & 2 & 1 & 5 \\ 3 & 1 & 2 & 4 \end{bmatrix} R_1 \leftrightarrow R_2$$

$$\sim \begin{bmatrix} 1 & -1 & 1 & 1 \\ 0 & 4 & -1 & 3 \\ 0 & 4 & -1 & 1 \end{bmatrix} \begin{matrix} \\ R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & -1 & 1 & 1 \\ 0 & 4 & -1 & 3 \\ 0 & 0 & 0 & -2 \end{bmatrix} R_3 \to R_3 - R_1$$

$\Rightarrow \rho(A) = 2, \; \rho([A \; B]) = 3$

$\therefore$ The system is inconsistent.

32. Show that the equation $z^2 = \bar{z}$ has four solutions.

Solution

We have

$$z^2 = \bar{z}$$

$$\Rightarrow |z|^2 = |z|$$

$$\Rightarrow |z|(|z|-1) = 0,$$

$$\Rightarrow |z| = 0, \text{ or } |z| = 1.$$

$|z| = 0 \Rightarrow z = 0$ is a solution, $|z| = 1 \Rightarrow z\bar{z} = 1 \Rightarrow \bar{z} = \frac{1}{z}$.

Given $z^2 = \bar{z} \Rightarrow z^2 = \frac{1}{z} \Rightarrow z^3 = 1$.

It has 3 solutions. Totally the equation has four solutions.

33. Solve the equation $\sin^2 x - 5\sin x + 4 = 0$

Solution

Let $\sin x = y$

The given equation becomes $y^2 - 5y + 4 = 0$

i.e., $(y-4)(y-1) = 0$

i.e., $y = 4, 1$

i.e., $\sin x = 4$ which is not possible and $\sin x = 1 \Rightarrow x = \frac{\pi}{2}$

The general solution is $x = (-1)^n \frac{\pi}{2} + n\pi, \ n \in Z$

34. Find the equation of the ellipse whose length of latus rectum is 8, eccentricity is $\frac{3}{5}$, the centre is $(0,0)$ and the major axis is on x-axis.

Solution

Since the major axis on x-axis and the centre is $(0,0)$, the equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Given that, $\frac{2b^2}{a} = 8$,

$$\frac{b^2}{a} = 4 \Rightarrow b^2 = 4a$$

But $b^2 = a^2(1-e^2)$

$$4a = a^2\left(1 - \frac{9}{25}\right)$$

$$\Rightarrow a = \frac{25}{4}$$

$$a^2 = \frac{625}{16}$$

$$b^2 = 4a = 25.$$

Thus the equation of the ellipse is $\dfrac{x^2}{\frac{625}{16}} + \dfrac{y^2}{25} = 1$.

35. Forces of magnitudes $5\sqrt{2}$ and $10\sqrt{2}$ units acting in the directions $3\hat{i}+4\hat{j}+5\hat{k}$ and $10\hat{i}+6\hat{j}-8\hat{k}$, respectively, act on a particle which is displaced from the point with position vector $4\hat{i}-3\hat{j}-2\hat{k}$ to the point with position vector $6\hat{i}+\hat{j}-3\hat{k}$. Find the work done by the forces.

Solution

Let $\vec{F}_1 = 5\sqrt{2}\hat{F}_1 = 5\sqrt{2}\dfrac{3\hat{i}+4\hat{j}+5\hat{k}}{\sqrt{50}} = 3\hat{i}+4\hat{j}+5\hat{k}$

$\vec{F}_2 = 10\sqrt{2}\hat{F}_2 = 10\sqrt{2}\dfrac{10\hat{i}+6\hat{j}-8\hat{k}}{\sqrt{200}} = 10\hat{i}+6\hat{j}-8\hat{k}$

$\therefore$ Resultant of the given forces $\vec{F} = \vec{F}_1 + \vec{F}_2$

$\vec{F} = 13\hat{i}+10\hat{j}-3\hat{k}$

$\therefore$ Displacement vector $\vec{d} = (6\hat{i}+\hat{j}-3\hat{k})-(4\hat{i}-3\hat{j}-2\hat{k})$

$= 2\hat{i}+4\hat{j}-\hat{k}$

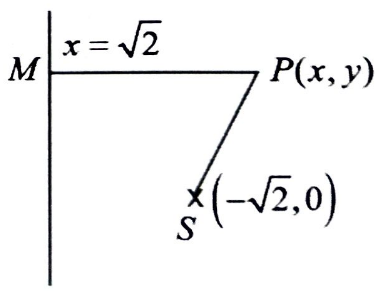
Hence, work done $W = \vec{F}\cdot\vec{d} = (13\hat{i}+10\hat{j}-3\hat{k})\cdot(2\hat{i}+4\hat{j}-\hat{k})$

$W = 69.$

36. Find the equation of the parabola with focus $(-\sqrt{2},0)$ and directrix $x=\sqrt{2}$.

Solution

The fixed point and the fixed line are given



$$\frac{SP}{PM} = e = 1$$

$$SP^2 = PM^2$$

$$\Rightarrow (-\sqrt{2}-x)^2 + y^2 = \left(\frac{x-\sqrt{2}}{\sqrt{1}}\right)^2$$

i.e., $y^2 = -4\sqrt{2}x$

$$a^2 = \frac{625}{16}$$

$$b^2 = 4a = 25.$$

Thus the equation of the ellipse is $\frac{x^2}{\frac{625}{16}} + \frac{y^2}{25} = 1$.

35. Forces of magnitudes $5\sqrt{2}$ and $10\sqrt{2}$ units acting in the directions $3\hat{i}+4\hat{j}+5\hat{k}$ and $10\hat{i}+6\hat{j}-8\hat{k}$, respectively, act on a particle which is displaced from the point with position vector $4\hat{i}-3\hat{j}-2\hat{k}$ to the point with position vector $6\hat{i}+\hat{j}-3\hat{k}$. Find the work done by the forces.

Solution

$$\text{Let } \vec{F_1} = 5\sqrt{2}\hat{F_1} = 5\sqrt{2}\frac{3\hat{i}+4\hat{j}+5\hat{k}}{\sqrt{50}} = 3\hat{i}+4\hat{j}+5\hat{k}$$

$$\vec{F_2} = 10\sqrt{2}\hat{F_2} = 10\sqrt{2}\frac{10\hat{i}+6\hat{j}-8\hat{k}}{\sqrt{200}} = 10\hat{i}+6\hat{j}-8\hat{k}$$

$\therefore$ Resultant of the given forces $\vec{F} = \vec{F_1}+\vec{F_2}$

$$\vec{F} = 13\hat{i}+10\hat{j}-3\hat{k}$$

$\therefore$ Displacement vector $\vec{d} = (6\hat{i}+\hat{j}-3\hat{k})-(4\hat{i}-3\hat{j}-2\hat{k})$

$$= 2\hat{i}+4\hat{j}-\hat{k}$$

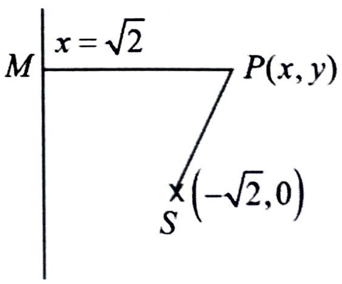
Hence, work done $W = \vec{F}\cdot\vec{d} = \left(13\hat{i}+10\hat{j}-3\hat{k}\right)\cdot\left(2\hat{i}+4\hat{j}-\hat{k}\right)$

$$W = 69.$$

36. Find the equation of the parabola with focus $\left(-\sqrt{2},0\right)$ and directrix $x=\sqrt{2}$.

Solution

The fixed point and the fixed line are given

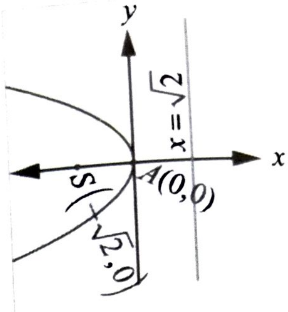


$$\frac{SP}{PM} = e = 1$$

$$SP^2 = PM^2$$

$$\Rightarrow \left(-\sqrt{2}-x\right)^2 + y^2 = \left(\frac{x-\sqrt{2}}{\sqrt{1}}\right)^2$$

i.e., $y^2 = -4\sqrt{2}x$



Another method

Parabola is open left and axis of symmetry as x-axis and vertex $(0,0)$.

Then the equation of the required parabola is

$$(y-0)^2 = -4\sqrt{2}(x-0)$$
$$y^2 = -4\sqrt{2}\,x.$$

37. The time T, taken for a complete oscillation of a single pendulum with length l, is given by the equation $T=2\pi\sqrt{\frac{l}{g}}$, where g is a constant. Find the approximate percentage error in the calculated value of T corresponding to an error of 2 percent in the value of t

Solution

$$T = 2\pi\sqrt{\frac{l}{g}} \;;\; \frac{dl}{l}\times 100 = 2 \quad \text{(given)}$$

$$T = \frac{2\pi}{\sqrt{g}}\sqrt{l}$$

Take log on both sides

$$\log T = \log\frac{2\pi}{\sqrt{g}} + \frac{1}{2}\log l$$

$$\Rightarrow \frac{1}{T}dT = 0 + \frac{1}{2}\cdot\frac{1}{l}dl$$

$$\frac{dT}{T}\times 100 = \frac{1}{2}\left(\frac{dl}{l}\times 100\right)$$

$$\frac{dT}{T}\times 100 = \frac{1}{2}\times 2$$

appropriate percentage error in T is 1%.

38. Evaluate : $\int_0^a \frac{f(x)}{f(x)+f(a-x)}dx.$

Solution

$$\text{Let } I = \int_0^a \frac{f(x)}{f(x)+f(a-x)}dx \quad \text{... (1)}$$

Applying the formula $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ in equation (1), we get

$$I = \int_0^a \frac{f(a-x)}{f(a-x)+f(a-(a-x))}dx$$

$$= \int_0^a \frac{f(a-x)}{f(x)+f(a-x)}\,dx. \qquad \ldots (2)$$

Adding equations (1) and (2), we get

$$2I = \int_0^a \frac{f(x)}{f(x)+f(a-x)}\,dx + \int_0^a \frac{f(a-x)}{f(x)+f(a-x)}\,dx$$

$$= \int_0^a \frac{f(x)+f(a-x)}{f(x)+f(a-x)}\,dx$$

$$= \int_0^a dx = a.$$

Hence, we get $I = \frac{a}{2}$.

39. Solve the differential equation $\frac{dy}{dx} = xy\,e^x$

Solution

$$\frac{dy}{y} = xe^x \cdot dx$$

$$\int \frac{dy}{y} = \int xe^x\,dx + c$$

$$\log y = xe^x - e^x + c.$$

40. Find the value of $\tan^{-1}(-1) + \cos^{-1}\left(\frac{1}{2}\right) + \sec^{-1}(-2)$.

Solution

$$\tan^{-1}(-1) + \cos^{-1}\left(\frac{1}{2}\right) + \sec^{-1}(-2) = -\tan^{-1}(1) + \cos^{-1}\left(\frac{1}{2}\right) + [\pi - \sec^{-1}(2)]$$

$$= -\frac{\pi}{4} + \frac{\pi}{3} + \left(\pi - \frac{\pi}{3}\right) = \frac{3\pi}{4}.$$

PART – IV

41. (a) If $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, find the products AB and BA and hence solve the system of equations $x - y + z = 4, x - 2y - 2z = 9, 2x + y + 3z = 1$.

Solution

$$AB = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = 8I$$

$$BA = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = 8I$$

$AB = BA = 8I$. That is, $\left(\frac{1}{8}A\right)B = B\left(\frac{1}{8}A\right) = I$. Hence, $B^{-1} = \frac{1}{8}A$.

Writing the given system of equations in matrix form, we get

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}. \text{ That is, } B\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}.$$

$$\text{So, } \begin{bmatrix} x \\ y \\ z \end{bmatrix} = B^{-1}\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix} = \left(\frac{1}{8}A\right)\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} -16+36+4 \\ -28+9+3 \\ 20-27-1 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$$

Hence, the solution is $x = 3, \quad y = -2, \quad z = -1$.

41. (b) If X is the random variable with probability density function

$$f(x) = \begin{cases} x-1, & 1 \le x < 2 \\ -x+3, & 2 \le x < 3 \\ 0 & \text{otherwise} \end{cases},$$

find the distribution function $F(x)$.

Solution

By definition $F(x) = P(X \le x) = \int_{-\infty}^{x} f(x)\,dx$

When $x < 1$ $\qquad F(x) = P(X \le x) = \int_{-\infty}^{x} 0\,dx = 0$.

When $1 \le x < 2$ $\qquad F(x) = P(X \le x) = \int_{-\infty}^{1} 0\,dx + \int_{1}^{x} (x-1)\,dx$

$$= 0 + \left[\frac{(x-1)^2}{2}\right]_1^x = \frac{(x-1)^2}{2}$$

When $2 \le x < 3$

$$F(x) = P(X \le x) = \int_{-\infty}^{1} 0dx + \int_{1}^{2}(x-1)dx + \int_{2}^{x}(3-x)dx$$

$$= 0 + \left[\frac{(x-1)^2}{2}\right]_1^2 + \left[-\frac{(3-x)^2}{2}\right]_2^x$$

$$= \frac{1^2-0}{2} + \frac{1-(3-x)^2}{2} = 1 - \frac{(3-x)^2}{2}$$

When $x \ge 3$,

$$F(x) = P(X \le x)$$

$$= \int_{-\infty}^{1} 0dx + \int_{1}^{2}(x-1)dx + \int_{2}^{3}(3-x)dx + \int_{3}^{x} 0dx$$

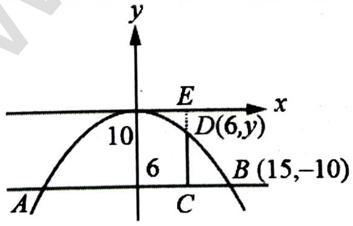
$$= 0 + \left[\frac{(x-1)^2}{2}\right]_1^2 + \left[-\frac{(3-x)^2}{2}\right]_2^3 + 0$$

$$= \frac{1}{2} + \frac{1}{2} = 1.$$

$$F(x) = \begin{cases} 0, & x < 1 \\ \frac{(x-1)^2}{2}, & 1 \le x < 2 \\ 1 - \frac{(3-x)^2}{2}, & 2 \le x < 3 \\ 1 & 3 \le x \end{cases}$$

42. (a) A bridge has a parabolic arch that is 10m height in the centre and 30m wide at the bottom. Find the height of the arch 6m from the centre, on any one side.

Solution



Let the equation of the parabola be $x^2 = -4ay$.

$B(15,-10)$ is a point on the parabola $x^2 = -4ay$.

$$\therefore 15^2 = -4a(-10)$$

$$\frac{225}{10} = 4a$$

When $2 \le x < 3$

$$F(x) = P(X \le x) = \int_{-\infty}^{1} 0dx + \int_{1}^{2} (x-1)dx + \int_{2}^{x} (3-x)dx$$

$$= 0 + \left[\frac{(x-1)^2}{2}\right]_1^2 + \left[-\frac{(3-x)^2}{2}\right]_2^x$$

$$= \frac{1^2 - 0}{2} + \frac{1-(3-x)^2}{2} = 1 - \frac{(3-x)^2}{2}$$

When $x \ge 3$,

$$F(x) = P(X \le x)$$

$$= \int_{-\infty}^{1} 0dx + \int_{1}^{2} (x-1)dx + \int_{2}^{3} (3-x)dx + \int_{3}^{x} 0dx$$

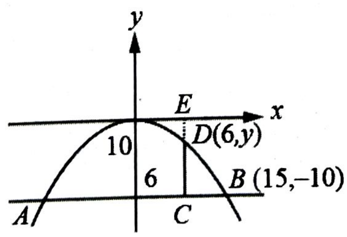
$$= 0 + \left[\frac{(x-1)^2}{2}\right]_1^2 + \left[-\frac{(3-x)^2}{2}\right]_2^3 + 0$$

$$= \frac{1}{2} + \frac{1}{2} = 1.$$

$$F(x) = \begin{cases} 0, & x < 1 \\ \frac{(x-1)^2}{2}, & 1 \le x < 2 \\ 1 - \frac{(3-x)^2}{2}, & 2 \le x < 3 \\ 1 & 3 \le x \end{cases}$$

42. (a) A bridge has a parabolic arch that is 10m height in the centre and 30m wide at the bottom. Find the height of the arch 6m from the centre, on any one side.

Solution



Let the equation of the parabola be $x^2 = -4ay$.

$B(15,-10)$ is a point on the parabola $x^2 = -4ay$.

$$\therefore 15^2 = -4a(-10)$$

$$\frac{225}{10} = 4a$$

$\therefore$ The parabola is $x^2 = \dfrac{-225}{10}y$

Let $DE = y$

$D(6, y)$ lies on the parabola

$$\therefore\ 36 = \frac{-225}{10}y$$

$$y = \frac{-360}{225} = -1.6.$$

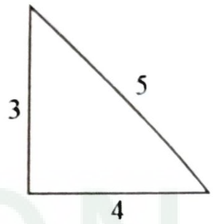
$$DE = 1.6\,\text{m}$$

Height of the arch 6m from the centre is

$$CD = CE - DE = 10 - 1.6 = 8.4\,\text{m}.$$

42. (b) Evaluate $\sin\left[\sin^{-1}\left(\frac{3}{5}\right) + \sec^{-1}\left(\frac{5}{4}\right)\right]$.

Solution



Let $\sec^{-1}\frac{5}{4} = \theta$. Then, $\sec\theta = \frac{5}{4}$ and hence, $\cos\theta = \frac{4}{5}$.

Also, $\sin\theta = \frac{3}{5} \Rightarrow \theta = \sin^{-1}\left(\frac{3}{5}\right) = \sec^{-1}\frac{5}{4}$

Therefore, $\sin^{-1}\frac{3}{5} + \sec^{-1}\left(\frac{5}{4}\right) = 2\sin^{-1}\left(\frac{3}{5}\right)$

We know that $2\sin^{-1}x = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$.

$$2\sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(2\times\frac{3}{5}\sqrt{1-\left(\frac{3}{5}\right)^2}\right) = \sin^{-1}\left(\frac{24}{25}\right).$$

Hence, $\sin\left[\sin^{-1}\left(\frac{3}{5}\right) + \sec^{-1}\left(\frac{5}{4}\right)\right] = \sin\left(\sin^{-1}\left(\frac{24}{25}\right)\right) = \frac{24}{25}$, since $\frac{24}{25} \in [-1, 1]$.

43. (a) Find the equation of the circle passing through the points $(1,1), (2,-1)$, and $(3,2)$.

Solution

Let the general equation of the circle be

$$x^2 + y^2 + 2gx + 2fy + c = 0 \qquad \text{... (1)}$$

It passes through points $(1,1), (2,-1)$ and $(3,2)$.

Therefore $\qquad 2g + 2f + c = -2 \qquad \text{... (2)}$

$$4g-2f+c = -5 \quad \dots(3)$$
$$6g+4f+c = -13. \quad \dots(4)$$

(2) – (3) gives $-2g+4f = 3$ ... (5)

(4) – (3) gives $2g+6f = -8$ ... (6)

(5) + (6) gives $f = -\frac{1}{2}$

Substituting $f = -\frac{1}{2}$ in (6), $g=-\frac{5}{2}$

Substituting $f = -\frac{1}{2}$ and $g=-\frac{5}{2}$ in (2), $c=4$

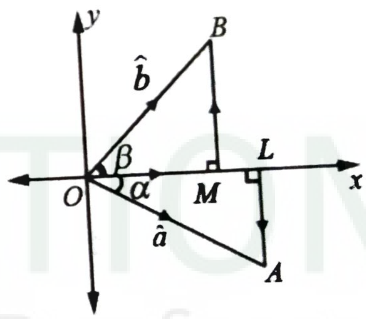
Therefore the required equation of the circle is

$$x^2+y^2+2\left(-\frac{5}{2}\right)x+2\left(-\frac{1}{2}\right)y+4 = 0$$

(or) $x^2+y^2-5x-y+4 = 0$.

43. (b) Prove that $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$, by using vectors.

Solution



Let $\hat{a}=\overrightarrow{OA}$ and $\hat{b}=\overrightarrow{OB}$ be the unit vectors and which make angles α and β, respectively, with positive x-axis, Draw AL and BM perpendicular to the x-axis.

Then A is $(\cos\alpha,\sin\alpha)$, B is $(\cos\beta,\sin\beta)$

So, $\overrightarrow{OL}=|\overrightarrow{OL}|\hat{i} = \cos\alpha\hat{i}, \overrightarrow{LA}=|\overrightarrow{LA}|\sin\alpha(-\hat{j})$.

Therefore, $\hat{a} = \overrightarrow{OA}=\overrightarrow{OL}+\overrightarrow{LA}=\cos\alpha\hat{i}-\sin\alpha\hat{j}$.

Similarly, $\hat{b} = \cos\beta\hat{i}+\sin\beta\hat{j}$

The angle between $\hat{a}$ and $\hat{b}$ is $\alpha+\beta$,

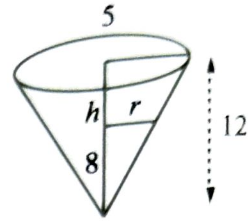
By definition $\hat{a}\cdot\hat{b} = |\hat{a}|\,|\hat{b}|\cos(\alpha+\beta)=\cos(\alpha+\beta)$ ... (1)

By their values $\hat{a}\cdot\hat{b} = (\cos\alpha\hat{i}-\sin\alpha\hat{j})\cdot(\cos\beta\hat{i}+\sin\beta\hat{j})=\cos\alpha\cos\beta-\sin\alpha\sin\beta$. ... (2)

From (1) and (2), we get $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$.

Note : (1) The points A and B can be taken in the first and fourth quadrants also.
(2) The result can be proved by drawing a unit circle also.

44. (a) A conical water tank with vertex down of 12 metres height has a radius of 5 metres at the top. If water flows into the tank at a rate 10 cubic m/min, how fast is the depth of the water increases when the water is 8 metres deep?



Solution

$$\frac{dV}{dt} = 10; \text{ find } \frac{dh}{dt} \text{ when } h = 8$$

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{5}{12}\right)^2 h^2 h$$

$$\frac{r}{h} = \frac{5}{12}$$

$$r = \frac{5}{12}h$$

$$V = \frac{25\pi}{3\times144}h^3$$

$$\frac{dV}{dt} = \frac{25\pi}{3\times144}3h^2\frac{dh}{dt}$$

$$10 = \frac{25\pi}{144}\times8^2\times\frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{9}{10\pi} \text{ m/min.}$$

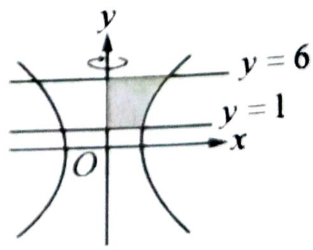
44. (b) Show that $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$.

Solution

p	q	$p \rightarrow q$	$q \rightarrow p$	$p \leftrightarrow q$	$(p \rightarrow q) \wedge (q \rightarrow p)$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

From the table, $p \leftrightarrow q$ and $(p \rightarrow q) \wedge (q \rightarrow p)$ are equivalent.

45. (a) Find the volume of the solid generated by revolving the region bounded by the curve $y = \frac{3}{4}\sqrt{x^2 - 16}$, $x \geq 4$, the y-axis, and the lines $y = 1$ and $y = 6$, about y-axis.



Solution

$y = \frac{3}{4}\sqrt{x^2 - 16} \Rightarrow \frac{x^2}{16} - \frac{y^2}{9} = 1$. So, the given curve is a portion of the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ between the lines $y = 1$ and $y = 6$ and it lies above the $x-$axis.

The volume of the solid generated is

$$V = \pi\int_1^6 x^2 dy = \pi\int_1^6\left(\frac{4}{3}\sqrt{9+y^2}\right)^2 dy = \pi\left(\frac{16}{9}\right)\int_1^6(9+y^2)dy$$

$$= \pi\left(\frac{16}{9}\right)\left(9y + \frac{y^3}{3}\right)_1^6 = \pi\left(\frac{16}{9}\right)(54+72) = 224\pi.$$

45. (b) Solve $(x^2 + y^2)dy = xy\,dx$. Find the value of x_0 when $y(1) = 1$ and $y(x_0) = e$.

Solution

$$\frac{dx}{dy} = \frac{x^2 + y^2}{xy}$$

$$\frac{dx}{dy} = \frac{x}{y} + \frac{y}{x}$$

$$x = vy$$

$$v + y\frac{dv}{dy} = v + \frac{1}{v}$$

$$y\frac{dv}{dy} = \frac{1}{v}$$

$$vdv = \frac{dy}{y}$$

$$\frac{v^2}{2} = \log y + \log c$$

$$yc = e^{\frac{1}{2}\left(\frac{x^2}{y^2}\right)}$$

When $x = 1,\ y = 1 \Rightarrow c = \sqrt{e}$

$$y\sqrt{e} = e^{\frac{1}{2}\left(\frac{x^2}{y^2}\right)}$$

When $x = x_0,\ y = e$

$$e^{\frac{3}{2}} = e^{\frac{1}{2}\left(\frac{x_0^2}{e^2}\right)}$$

$$\frac{3}{2} = \frac{x_0^2}{2e^2} \Rightarrow x_0^2 = 3e^2$$

$$\Rightarrow x_0 = \pm\sqrt{3}e.$$

46 (a) Find all the values of $\left(-\sqrt{3} - i\right)^{\frac{1}{3}}$.

Solution

$$-\sqrt{3} - i = r(\cos\theta + i\sin\theta)$$

$$r = 2$$

$$\theta = -\pi + \frac{\pi}{6} = \frac{-5\pi}{6}$$

$$-\sqrt{3}-i = 2\left[\cos\left(-\frac{5\pi}{6}\right)+i\sin\left(-\frac{5\pi}{6}\right)\right]$$

$$\left(-\sqrt{3}-i\right)^{\frac{1}{3}} = 2^{\frac{1}{3}}\left[\cos\left(\frac{-5\pi}{6}\right)+i\sin\left(\frac{-5\pi}{6}\right)\right]^{\frac{1}{3}}$$

$$= 2^{\frac{1}{3}}\left[\cos\left(2k\pi-\frac{5\pi}{6}\right)+i\sin\left(2k\pi-\frac{5\pi}{6}\right)\right]^{\frac{1}{3}}, k\in\mathbb{Z}$$

$$= 2^{\frac{1}{3}}\left[\cos\left(\frac{12k-5}{6}\right)\pi+i\sin\left(\frac{12k-5}{6}\right)\pi\right], k=0,1,2.$$

46. (b) If $u=\sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{2}\tan u$.

Solution

Note that the function u is not homogeneous.

But $\frac{x+y}{\sqrt{x}+\sqrt{y}}$ is homogeneous i.e., $\sin u$ is homogeneous.

Let $f(x,y)=\frac{x+y}{\sqrt{x}+\sqrt{y}}=\sin u$

$$f(tx,ty) = \frac{tx+ty}{\sqrt{tx}+\sqrt{ty}}=t^{1/2}f(x,y).$$

Thus f is homogeneous with degree $\frac{1}{2}$, and so by Euler's Theorem we have

$$x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}=\frac{1}{2}f.$$

Now substituting $f=\sin u$ in the above equation we obtain

$$x\frac{\partial(\sin u)}{\partial x}+y\frac{\partial(\sin u)}{\partial y} = \frac{1}{2}\sin u$$

$$x\cos u\frac{\partial u}{\partial x}+y\cos u\frac{\partial u}{\partial y} = \frac{1}{2}\sin u \qquad \text{... (19)}$$

Dividing both sides by $\cos u$ we obtain

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u.$$

47. (a) Find the vector and Cartesian equations of the plane passing through the point $(-2,-2,4)$ and perpendicular to the planes $6x+4y-6z=5$ and $10x-8y+2z=1$.

Solution

$$\vec{a} = -2\hat{i}-2\hat{j}+4\hat{k}$$

$$\vec{u} = 6\hat{i}+4\hat{j}-6\hat{k}$$

$$\vec{v} = 10\hat{i} - 8\hat{j} + 2\hat{k}$$

The plane passes through a point and parallel to two vectors.

Vector equation is

$$\vec{r} = \vec{a} + s\vec{u} + t\vec{v} \quad s,t \in \mathbb{R}$$

$$\vec{r} = \left(-2\hat{i} - 2\hat{j} + 4\hat{k}\right) + s\left(6\hat{i} + 4\hat{j} - 6\hat{k}\right) + t\left(10\hat{i} - 8\hat{j} + 2\hat{k}\right)$$

Cartesian equation is

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

Here $(x_1, y_1, z_1) = (-2, -2, 4)$

$(l_1, m_1, n_1) = (6, 4, -6)$

$(l_2, m_2, n_2) = (10, -8, 2)$

The equation is $\begin{vmatrix} x+2 & y+2 & z-4 \\ 6 & 4 & -6 \\ 10 & -8 & 2 \end{vmatrix} = 0$

$$5x + 9y + 11z - 16 = 0$$

47. (b) Find whether local extrema and points of inflexion exist for the curve $f(x) = \frac{4}{3}x^3 - 8x^2 + 16x + 5$. If so, find them.

Solution

$$f(x) = \frac{4}{3}x^3 - 8x^2 + 16x + 5$$

$$f'(x) = 4x^2 - 16x + 16$$

$$f''(x) = 8x - 16$$

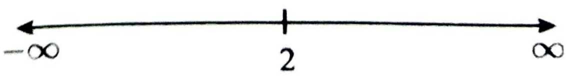
$$f'(x) = 0 \Rightarrow 4x^2 - 16x + 16 = 0$$

$$\Rightarrow x^2 - 4x + 4 = 0 \Rightarrow x = 2$$

When $x = 2$, $f''(x) = 0$

$\therefore$ There is no information regarding local extremum.

$$f''(x) = 0 \Rightarrow 8x - 16 = 0 \Rightarrow x = 2$$



Consider $(-\infty, 2)$

$$\text{Take } x = 0$$
$$f''(x) < 0$$

Consider $(2, \infty)$

$$\text{Take } x = 3$$
$$f''(x) > 0$$

Thus $x=2$ changes the sign of $f''(x)$ from negative to positive.

$\therefore$ $x=2$ gives a point of inflection.

The point of inflection is $(2, f(2)) = \left(2, \frac{47}{3}\right)$.

MARKING SCHEME - MATHEMATICS

GENERAL INSTRUCTIONS

The marking scheme provides general guidelines to reduce subjectivity in the marking. The answer given in the marking scheme are Text Book, Solution Book and COME book bound. If a student has given any answer which is different from one given in the marking scheme, but carries the prescribed content meaning (rigorous), such answers should be given full credit with suitable distribution.

There is no separate mark allotted for formulae. If a particular stage is wrong and if the candidate writes the appropriate formula then award 1 mark for the formula irrespective of stage marks. This mark is attached with that stage. This is done with the aim that a student who did the problem correctly without writing the formula should not be penalised. That is, mark should not be deducted for not writing the formula.

In the case of Part II, Part III and Part IV, if the solution is correct then award full mark directly. The stage mark is essential only if the part of the solution is incorrect.

Only a Rough sketch of the diagram is expected and full credit must be given for such diagrams that form a part and parcel of the solution of a problem.

In questions on Variable Separable, Homogeneous and Linear differential equations, full marks should be given for an equivalent answer and the students should not be penalised for not getting the final answer as mentioned in the marking scheme.

If the method is not mentioned in the question, then one may adopt any method.

Complicated radicals and combinations need not be simplified (for example $\sqrt{7}$, $\sqrt{82}$, $\frac{3}{\sqrt{2}}$, $10c_6$ etc.)

Important Note : In an answer to a question between any two particular stages of marks (greater than one) if a student starts a stage with correct step but reaches the next stage with a wrong result then suitable credits should be given to the related steps instead of denying the entire marks meant for that stage.

PART – II

... (1)

21. $|adjA| = 36$

$$A^{-1} = \pm\frac{1}{\sqrt{(adjA)}}(adjA)$$

$$= \pm\frac{1}{6}\begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}.$$ ... (1)

22. $z = (2+3i)(1-i) = 5+i$ ... (1)

$$z^{-1} = \frac{5}{26} - i\frac{1}{26}.$$ ... (1)

23. Sum of the roots $= \alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = \frac{-3}{4}$.

Product of the roots $= \alpha^2\beta^2 = (\alpha\beta)^2 = \frac{169}{4}$ ... (1)

Quadratic equation is $x^2 + \frac{3}{4}x + \frac{169}{4} = 0$. ... (1)

(OR) $4x^2+3x+169 = 0$

24. $11x^2 - 25y^2 - 44x + 50y - 256 = 0$

$B^2 - 4AC = 0 - 4(11)(-25) > 0$ ... (1)

It represents a hyperbola. ... (1)

25. $$\lim_{x\to 0}\frac{\sin mx}{\sin nx} = \lim_{x\to 0}\frac{\cos mx \cdot m}{\cos nx \cdot n}$$ ... (1)

$$= \frac{m}{n}$$ ... (1)

26. $y = 52\sqrt{x}$

$$dy = 52\cdot\frac{1}{2\sqrt{x}}dx$$ ... (1)

$$\simeq \frac{26}{\sqrt{1}}(0.1) = 2.6 \simeq 3 \text{ words.}$$ ... (1)

PART – II

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$$= \frac{m}{n}$$ ... (1)

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$$dy = 52\cdot\frac{1}{2\sqrt{x}}dx$$ ... (1)

$$\simeq \frac{26}{\sqrt{1}}(0.1) = 2.6 \simeq 3 \text{ words}.$$ ... (1)

27.

$$y = mx \qquad \ldots (1)$$

$$\frac{dy}{dx} = m$$

Replace m with $\frac{dy}{dx}$

$$y = x\frac{dy}{dx} \qquad \ldots (1)$$

28.

$$n = 5, p = \frac{3}{4} \text{ and } q = \frac{1}{4} \qquad \ldots (1)$$

$$P(X = 3) = \binom{5}{3}\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right)^2. \qquad \ldots (1)$$

29. When n is a negative integer then m^n is not an integer ... (1)
Closure property is not true. ... (1)

30.

$$\text{Area} = \int_0^{\pi} y\,dx = \int_0^{\pi} \sin x\,dx \qquad \ldots (1)$$

$$= 2 \qquad \ldots (1)$$

PART– III

31.

$$[A \quad B] = \begin{bmatrix} 2 & 2 & 1 & 5 \\ 1 & -1 & 1 & 1 \\ 3 & 1 & 2 & 4 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -1 & 1 & 1 \\ 2 & 2 & 1 & 5 \\ 3 & 1 & 2 & 4 \end{bmatrix} R_1 \leftrightarrow R_2$$

$$\sim \begin{bmatrix} 1 & -1 & 1 & 1 \\ 0 & 4 & -1 & 3 \\ 0 & 4 & -1 & 1 \end{bmatrix} \begin{matrix} \\ R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & -1 & 1 & 1 \\ 0 & 4 & -1 & 3 \\ 0 & 0 & 0 & -2 \end{bmatrix} R_3 \to R_3 - R_1$$

$$\Rightarrow \rho(A) = 2, \ \rho([A \ B]) = 3 \qquad \ldots (2)$$

The system is inconsistent. ... (1)

32.
$$z^2 = \bar{z} \quad \text{... (}$$
$$\Rightarrow |z|^2 = |z|$$
$$\Rightarrow |z|(|z|-1) = 0, \quad \text{... (}$$
$$\Rightarrow |z| = 0, \text{ or } |z|=1.$$
$$|z| = 0 \Rightarrow z=0 \text{ is a solution, } |z|=1 \Rightarrow z\bar{z}=1 \Rightarrow \bar{z}=\frac{1}{z}.$$
$$\text{Given } z^2 = \bar{z} \Rightarrow z^2 = \frac{1}{z} \Rightarrow z^3 = 1. \text{ It has 3 solutions.} \quad \text{... (1}$$

Totally the equation has four solutions.

33. $\sin x = 4$ which is not possible and $\sin x = 1 \Rightarrow x = \frac{\pi}{2}$ ... (2

The general solution is $x = (-1)^n \frac{\pi}{2} + n\pi, \quad n \in Z$ ... (1)

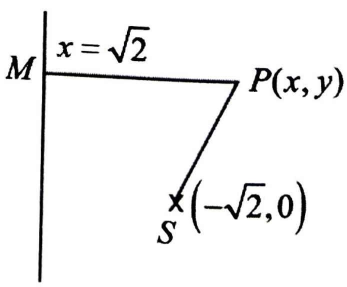
34. The equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. ... (1)

$$a^2 = \frac{625}{16}$$
$$b^2 = 4a = 25.$$

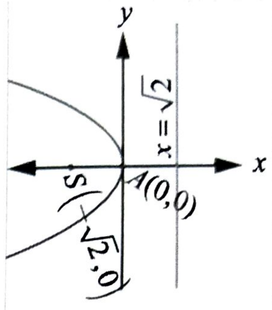
The equation of the ellipse is $\frac{x^2}{\frac{625}{16}} + \frac{y^2}{25} = 1$. ... (2)

35.
$$\vec{F} = 13\hat{i} + 10\hat{j} - 3\hat{k} \quad \text{... (1)}$$
$$\vec{d} = 2\hat{i} + 4\hat{j} - \hat{k} \quad \text{... (1)}$$
$$W = \vec{F}\cdot\vec{d} = \left(13\hat{i} + 10\hat{j} - 3\hat{k}\right)\cdot\left(2\hat{i} + 4\hat{j} - \hat{k}\right)$$
$$W = 69. \quad \text{... (1)}$$

36.
$$\frac{SP}{PM} = e = 1 \quad \text{... (1)}$$
$$SP^2 = PM^2$$
$$\Rightarrow \left((-\sqrt{2} - x)^2 + y^2\right) = \left(\frac{x - \sqrt{2}}{\sqrt{1}}\right)^2 \quad \text{... (1)}$$
$$y^2 = -4\sqrt{2}x \quad \text{... (1)}$$



Another method



Parabola is open left and vertex $(0,0)$. ... (1)

Parabola is

$$(y-0)^2 = -4\sqrt{2}(x-0) \quad \text{... (1)}$$

$$y^2 = -4\sqrt{2}\,x. \quad \text{... (1)}$$

37. $$T = 2\pi\sqrt{\frac{l}{g}}\ ;\ \frac{dl}{l}\times 100 = 2 \quad \text{(given)}$$

$$T = \frac{2\pi}{\sqrt{g}}\sqrt{l}$$

Take log on both sides

$$\log T = \log\frac{2\pi}{\sqrt{g}} + \frac{1}{2}\log l \quad \text{... (1)}$$

$$\frac{dT}{T}\times 100 = \frac{1}{2}\left(\frac{dl}{l}\times 100\right) \quad \text{... (1)}$$

$$\frac{dT}{T}\times 100 = \frac{1}{2}\times 2$$

appropriate percentage error in T is 1%. ... (1)

38. $$I = \int_0^a \frac{f(x)}{f(x)+f(a-x)}dx$$

$$I = \int_0^a \frac{f(a-x)}{f(a-x)+f(a-(a-x))}dx$$

$$= \int_0^a \frac{f(a-x)}{f(x)+f(a-x)}dx. \quad \text{... (1)}$$

$$2I = \int_0^a \frac{f(x)}{f(x)+f(a-x)}dx + \int_0^a \frac{f(a-x)}{f(x)+f(a-x)}dx$$

$$= \int_0^a \frac{f(x)+f(a-x)}{f(x)+f(a-x)}dx \quad \text{... (1)}$$

$$= \int_0^a dx = a.$$

$$I = \frac{a}{2}. \quad \text{... (1)}$$

...

39. $$\frac{dy}{y} = xe^x \cdot dx$$ ... (

$$\log y = xe^x - e^x + c.$$ ... (

40. $$\tan^{-1}(-1)+\cos^{-1}\left(\frac{1}{2}\right)+\sec^{-1}(-2) = -\tan^{-1}(1)+\cos^{-1}\left(\frac{1}{2}\right)+[\pi-\sec^{-1}(2)]$$ ... (2

$$= -\frac{\pi}{4}+\frac{\pi}{3}+\left(\pi-\frac{\pi}{3}\right)=\frac{3\pi}{4}.$$

PART – IV

41. (a) $AB = 8I$

$BA = 8I$ ... (2)

$AB = BA = 8I.$ ... (1)

$$B^{-1} = \frac{1}{8}A$$

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}. \text{ That is, } B\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}.$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = B^{-1}\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix} = \frac{1}{8}A\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

$x=3,\ y=-2,\ z=-1.$ ... (2)

41. (b)

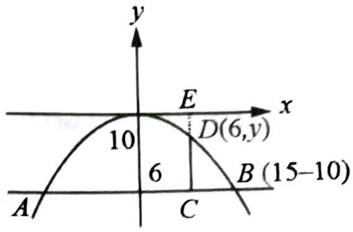
When $x<1$ $\quad F(x) = P(X \le x) = \int_{-\infty}^{x} 0dx = 0.$ ... (1)

When $1 \le x < 2$ $\quad F(x) = \frac{(x-1)^2}{2}$ ... (1)

When $2 \le x < 3$ $\quad F(x) = 1-\frac{(3-x)^2}{2}$ ... (1)

When $x \ge 3$, $\quad F(x) = 1$ ... (2)

42. (a) Rough diagram ... (1)



The parabola is $x^2 = \frac{-225}{10} y$ ... (2)

$DE = y$

$D(6, y)$ lies on the parabola

$DE = 1.6\text{ m}$ ... (1)

Height of the arch

$CD = CE - DE = 10 - 1.6 = 8.4\text{ m}.$ ... (1)

42. (b) $\sec^{-1}\frac{5}{4} = \sin^{-1}\left(\frac{3}{5}\right)$ ... (1)

$\sin^{-1}\frac{3}{5} + \sec^{-1}\left(\frac{5}{4}\right) = 2\sin^{-1}\left(\frac{3}{5}\right)$ ... (1)

$2\sin^{-1}\left(\frac{3}{5}\right) = \sin^{-1}\left(2\times\frac{3}{5}\sqrt{1-\left(\frac{3}{5}\right)^2}\right) = \sin^{-1}\left(\frac{24}{25}\right).$ ... (2)

$\sin\left[\sin^{-1}\left(\frac{3}{5}\right) + \sec^{-1}\left(\frac{5}{4}\right)\right] = \sin\left(\sin^{-1}\left(\frac{24}{25}\right)\right) = \frac{24}{25}.$ ... (1)

43. (a)

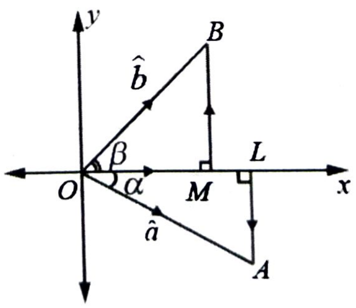
Let the general equation of the circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$ ... (1)

$f = -\frac{1}{2}, \; g = -\frac{5}{2}, c = 4$ ... (3)

$x^2 + y^2 - 5x - y + 4 = 0.$ ... (1)

43. (b) Rough diagram ... (1)



$$\hat{a} = \cos\alpha\hat{i} - \sin\alpha\hat{j}.$$ ... (1)

$$\hat{b} = \cos\beta\hat{i} + \sin\beta\hat{j}$$ ... (1)

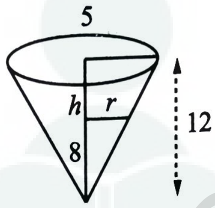
$$\hat{a}\cdot\hat{b} = |\hat{a}|\,|\hat{b}|\cos(\alpha+\beta) = \cos(\alpha+\beta)$$ ... (1)

$$\hat{a}\cdot\hat{b} = (\cos\alpha\hat{i} - \sin\alpha\hat{j})\cdot(\cos\beta\hat{i} + \sin\beta\hat{j}) = \cos\alpha\cos\beta - \sin\alpha\sin\beta.$$ ... (1)

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

Note : (1) The points A and B can be taken in the first and fourth quadrants also.
(2) The result can be proved by drawing a unique circle also. ... (1)

44. (a) Rough diagram



$$V = \frac{25\pi}{3\times144}h^3$$ ... (1)

$$\frac{dV}{dt} = \frac{25\pi}{3\times144}3h^2\frac{dh}{dt}$$ ... (1)

$$\frac{dh}{dt} = \frac{9}{10\pi} \text{ m/min.}$$ ... (2)

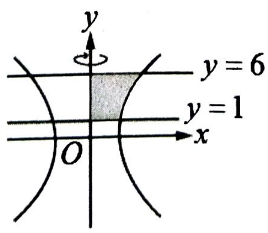
44. (b)

p	q	$p\to q$	$q\to p$	$p\leftrightarrow q$	$(p\to q)\wedge(q\to p)$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	T	F	F	F
F	F	T	T	T	T

Third, fourth, fifth and sixth column – each one mark ... (4)

They are equivalent. ... (1)

45. (a) Rough diagram ... (1)



$$V = \pi\int_1^6 x^2 dy = \pi\int_1^6\left(\frac{4}{3}\sqrt{9+y^2}\right)^2 dy = \pi\left(\frac{16}{9}\right)\int_1^6 (9+y^2)dy \quad \text{... (2)}$$

$$= \pi\left(\frac{16}{9}\right)\left(9y+\frac{y^3}{3}\right)_1^6 = \pi\left(\frac{16}{9}\right)(54+72) = 224\pi. \quad \text{... (2)}$$

45. (b)

$$\frac{dx}{dy} = \frac{x^2+y^2}{xy}$$

$$\frac{dx}{dy} = \frac{x}{y}+\frac{y}{x} \quad \text{... (1)}$$

$$x = vy$$

$$vdv = \frac{dy}{y} \quad \text{... (1)}$$

$$\frac{v^2}{2} = \log y + \log c$$

$$yc = e^{\frac{1}{2}\left(\frac{x^2}{y^2}\right)} \quad \text{... (1)}$$

When $x = 1, \; y = 1 \Rightarrow c = \sqrt{e}$

$$y\sqrt{e} = e^{\frac{1}{2}\left(\frac{x^2}{y^2}\right)} \quad \text{... (1)}$$

When $x = x_0, \; y = e$

$$e^{\frac{3}{2}} = e^{\frac{1}{2}\left(\frac{x_0^2}{e^2}\right)}$$

$$\frac{3}{2} = \frac{x_0^{\,2}}{2e^2} \Rightarrow x_0^{\,2} = 3e^2$$

$$\Rightarrow x_0 = \pm\sqrt{3}e. \quad \text{... (1)}$$

46. (a)

$$-\sqrt{3}-i = 2\left[\cos\left(-\frac{5\pi}{6}\right)+i\sin\left(-\frac{5\pi}{6}\right)\right] \quad \text{... (2)}$$

$$\left(-\sqrt{3}-i\right)^{\frac{1}{3}} = 2^{\frac{1}{3}}\left[\cos\left(\frac{-5\pi}{6}\right)+i\sin\left(\frac{-5\pi}{6}\right)\right]^{\frac{1}{3}} \quad \text{... (1)}$$

$$= 2^{\frac{1}{3}}\left[\cos\left(\frac{12k-5}{6}\right)\pi+i\sin\left(\frac{12k-5}{6}\right)\pi\right], \; k=0,1,2. \quad \text{... (2)}$$

46. (b)

$$f(x, y) = \frac{x+y}{\sqrt{x}+\sqrt{y}} = \sin u$$ ... (1)

Thus f is homogeneous with degree $\frac{1}{2}$ ... (1)

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = \frac{1}{2} f.$$ ... (1)

$$x\frac{\partial(\sin u)}{\partial x} + y\frac{\partial(\sin u)}{\partial y} = \frac{1}{2}\sin u$$

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u.$$ ... (2)

47. (a)

$$\vec{a} = -2\hat{i} - 2\hat{j} + 4\hat{k}$$
$$\vec{u} = 6\hat{i} + 4\hat{j} - 6\hat{k}$$
$$\vec{v} = 10\hat{i} - 8\hat{j} + 2\hat{k}$$

Vector equation is

$$\vec{r} = \left(-2\hat{i} - 2\hat{j} + 4\hat{k}\right) + s\left(6\hat{i} + 4\hat{j} - 6\hat{k}\right) + t\left(10\hat{i} - 8\hat{j} + 2\hat{k}\right)$$ ... (2)

Cartesian equation :

Here $(x_1, y_1, z_1) = (-2, -2, 4)$

$(l_1, m_1, n_1) = (6, 4, -6)$

$(l_2, m_2, n_2) = (10, -8, 2)$

The equation is $\begin{vmatrix} x+2 & y+2 & z-4 \\ 6 & 4 & -6 \\ 10 & -8 & 2 \end{vmatrix} = 0$ ... (2)

$$5x + 9y + 11z - 16 = 0$$ ... (1)

47. (b)

$$f(x) = \frac{4}{3}x^3 - 8x^2 + 16x + 5$$
$$f'(x) = 4x^2 - 16x + 16$$
$$f''(x) = 8x - 16$$
$$f'(x) = 0 \Rightarrow 4x^2 - 16x + 16 = 0$$
$$\Rightarrow x^2 - 4x + 4 = 0 \Rightarrow x = 2$$

When $x = 2$, $f''(x) = 0$

There is no information regarding local extremum.

$$f''(x) = 0 \Rightarrow 8x - 16 = 0 \Rightarrow x = 2$$ ... (2)

$x = 2$ gives a point of inflection. ... (1)

The point of inflection is $(2, f(2)) = \left(2, \frac{47}{3}\right)$.

... (2)

8. Two tangents are drawn to the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$ parallel to the straight line $2x-y=1$. One of the points of contact of tangents on the hyperbola is

(1) $\left(\frac{9}{2\sqrt{2}},\frac{-1}{\sqrt{2}}\right)$ (2) $\left(\frac{-9}{2\sqrt{2}},\frac{1}{\sqrt{2}}\right)$ (3) $\left(\frac{9}{2\sqrt{2}},\frac{1}{\sqrt{2}}\right)$ (4) $\left(3\sqrt{3},-2\sqrt{2}\right)$

9. The angle between the line $\vec{r}=\left(\hat{i}+2\hat{j}-3\hat{k}\right)+t\left(2\hat{i}+\hat{j}-2\hat{k}\right)$ and the plane $\vec{r}\cdot\left(\hat{i}+\hat{j}\right)+4=0$ is

(1) 0° (2) 30° (3) 45° (4) 90°

10. Which of the following statement is incorrect?

(1) if two lines are coplanar then their direction ratios must be same

(2) two coplanar lines must lie in a plane

(3) skew lines are neither parallel nor intersecting

(4) if two lines are parallel or intersecting then they are coplanar

11. The number given by the Mean value theorem for the function $\frac{1}{x}$, $x\in[1,9]$ is

(1) 2 (2) 2.5 (3) 3 (4) 3.5

12. The slope of the tangent to the curve $f(x)=2\cos 4x$ at $x=\frac{\pi}{12}$ is

(1) $-4\sqrt{3}$ (2) -4 (3) $\frac{\sqrt{3}}{12}$ (4) $4\sqrt{3}$

13. $\int_a^b f(a+b-x)dx=$

(1) $f(a)-f(b)$ (2) $\int_b^a f(x)dx$ (3) 0 (4) $\int_a^b f(x)dx$

14. The value of $\int_0^1 x(1-x)^{99}dx$ is

(1) $\frac{1}{11000}$ (2) $\frac{1}{10100}$ (3) $\frac{1}{10010}$ (4) $\frac{1}{10001}$

15. The order and degree of $\frac{d^2y}{dx^2}=xy+\cos\left(\frac{dy}{dx}\right)$ are

(1) 2, does not exist (2) 2,1 (3) 1,2 (4) 2,2

16. For a certain substance, the rate of change of vapor pressure P with respect to temperature T is proportional to the vapor pressure and inversely proportional to the square of the temperature. The corresponding differential equation is (k is constant of proportionality)

(1) $\frac{dP}{dT}=k\cdot\frac{T^2}{P}$ (2) $\frac{dT}{dP}=k\cdot T^2$ (3) $\frac{dP}{dT}=k\cdot\frac{P}{T^2}$ (4) $\frac{dP}{dT}=k\cdot P$