WAY TO SUCCESS PUBLICATIONS

12th Std. Mathematics

September 2020 - Public Exam Answer Key

Part - I

1. (2) $(A^T)^2$ Ex. 1.8 (11)
2. (4) $e^{(\Delta_1/\Delta_3)}, e^{(\Delta_2/\Delta_3)}$ Ex. 1.8 (19)
3. (1) $\frac{1}{2}$ Ex. 2.9 (6)
4. (4) Collinear creative (old Book)
5. (4) $|k| \geq 6$ Ex. 3.7 (6)
6. (3) $\pi/10$ Ex. 4.6 (7)
7. (2) $0 \leq x \leq \pi$ Ex. 4.6 (5)
8. (3) 4 creative (old Book)
9. (2) $(-3, 2)$ Ex. 5.6 (25)
10. (1) 0 Ex. 6.10 (8)
11. (4) $(5, -1, 1)$ Ex. 6.10 (4)
12. (3) $\pi/2$ Ex. 7.10 (9)
13. (4) 2 Ex. 7.10 (2)
14. (3) yx^{y-1} Ex. 8.8 (5)
15. (1) 0 creative (old Book)
16. (3) $\cos x$ Ex. 9.10 (7)
17. (4) $-\tan x$ Creative (old Book)
18. (2) $\phi(y/x) = kx$ Ex. 10.9 (17)
19. (1) 4 Creative (old Book)
20. (2) Z Ex. 12.3 (15)

Part - II

21. Creative old book Ex. 3.1 (3)

$$\frac{1+i}{1-i} = \frac{1+i}{1-i} \times \frac{1+i}{1+i}$$

$$= \frac{(1+i)(1+i)}{(1-i)(1+i)}$$

$$= \frac{1+i+i+i^2}{1^2+i^2}$$

$$\frac{1+i}{1-i} = \frac{2i}{2} = i$$

$$\left(\frac{1+i}{1-i}\right)^n = 1$$

$$(i)^n = 1$$

$$i^4 = i^8 = i^{12} = i^{16} = i^{20} = \cdots = 1$$

$\therefore$ The Least Positive integer $n = 4$

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22. $|z+i| = |z-1|$ $\boxed{\text{WTS}}$

Ex. 2.6 – 3 (iii)

$|x+iy+i| = |x+iy-1|$

$|x+i(y+1)| = |(x-1)+iy|$

$\sqrt{x^2+(y+1)^2} = \sqrt{(x-1)^2+y^2}$

Squaring on both sides

$x^2+y^2+2y+1 = x^2-2x+1+y^2$

$x^2+y^2+2y+1-x^2+2x-1-y^2=0$

$2x+2y=0$

$\div 2$ $\quad x+y=0$

The locus of z is $x+y=0$

23. Given $2x^4+5x^3-7x^2+8=0$

Ex. 3.1 – (8)

$a=2, b=5, c=-7, d=0, e=0$

$\Sigma 1 = \alpha+\beta+\gamma+\delta = -\frac{b}{a} = -\frac{5}{2}$

$\Sigma 4 = \alpha\beta\gamma\delta = \frac{e}{a} = \frac{8}{2} = 4$

The new roots are $\alpha+\beta+\gamma+\delta$, $\alpha\beta\gamma\delta$

$\Sigma 1 = \alpha+\beta+\gamma+\delta + \alpha\beta\gamma\delta$

$= -\frac{5}{2}+4$

$= \frac{-5+8}{2} = \frac{3}{2}$

$\Sigma 2 = (\alpha+\beta+\gamma+\delta)(\alpha\beta\gamma\delta)$

$= -\frac{5}{2}(4) = -10$

$\therefore$ The required equation

$x^2 - x(\Sigma 1)+\Sigma 2 = 0$

$x^2 - x\left(\frac{3}{2}\right) - 10 = 0$

$2x^2 - 3x - 20 = 0$

24. Let $\tan^{-1}(\sqrt{3}) = y$

Example 4.8

$\tan y = \sqrt{3}$

Thus $y = \pi/3$

since $\pi/3 \in (-\pi/2, \pi/2)$

$\therefore$ The principal value of $\tan^{-1}(\sqrt{3}) = \pi/3$

25. $\vec{a}\times(\vec{b}\times\vec{c}) = \frac{1}{2}\vec{b}$

Ex. 6.3 – 8

$(\vec{a}\cdot\vec{c})\vec{b} - (\vec{a}\cdot\vec{b})\vec{c} = \frac{1}{2}\vec{b} - (0)\vec{c}$

Equating like component

$\vec{a}\cdot\vec{c} = 1/2 \Rightarrow ac\cos\theta = 1/2$

$(1)(1)\cos\theta = 1/2$

$\cos\theta = 1/2$

$\cos\theta = \cos\pi/3$

$\theta = \pi/3$

26. If directly substitute $x=0$ we get an ~~indetermined~~ indeterminate form $\frac{0}{0}$ and hence we apply the l'Hopital's rule to evaluate the limit as

Example 7.35

$\lim_{x\to 0}\left(\frac{\sin mx}{x}\right) = \lim_{x\to 0}\left(\frac{m\times\cos mx}{1}\right)$

$= m$

27. $\int_3^4 \frac{dx}{x^2-4}$ $\boxed{WTS}$

$= \left[\frac{1}{2}\log\left|\frac{x-2}{x+2}\right|\right]_3^4$

$\left(\because \int\frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c\right)$

Ex. 9.3 1-(i)

$= \frac{1}{4}\left[\log\left(\frac{4-2}{4+2}\right) - \log\left(\frac{3-2}{3+2}\right)\right]$

$= \frac{1}{4}\left[\log\frac{2}{6} - \log\frac{1}{5}\right]$

$= \frac{1}{4}\left[\log\frac{2}{6}\times\frac{5}{1}\right]$

$= \frac{1}{4}\left[\log\frac{5}{3}\right]$

29. $a*b$ is in the quotient form since, division by 0 is undefined.

Since the division by 0 is undefined, the denominator $b-1$ must be nonzero.

Example 12.1 (ii)

It is clear that $b-1=0$ if $b=1$

As $1\in Q$, $*$ is not a binary operation on the whole of Q.

However it can be found that by omitting 1 from Q, the output $a*b$ exists in $Q\setminus\{1\}$.

Hence, $*$ is a binary operation on $Q/\{1\}$

28.

$y = ax^2+bx+c \rightarrow (1)$

$y' = 2ax+b \rightarrow (2)$

$y'' = 2a$

$\frac{y''}{2} = a \rightarrow (3)$

Creative

From (2), $y' = y''x+b$

$b = y'-y''x$

Sub a, b values in (1)

$y = \frac{y''}{2}x^2 + (y'-y''x)x + c$

$2y = y''x^2 + 2y'x - 2y''x^2 + 2c$

$x^2y'' - 2xy' + 2y - 2c = 0$

30. $x = r\cos\theta$, $y = r\sin\theta$

$x^2 = r^2\cos^2\theta$, $y^2 = r^2\sin^2\theta$

$x^2+y^2 = r^2\cos^2\theta + r^2\sin^2\theta$

$= r^2(\cos^2\theta+\sin^2\theta)$

$x^2+y^2 = r^2(1)$

$x^2+y^2 = r^2$

Creative

Diff. w.r. to 'x'

$2x+0 = 2r\frac{dr}{dx}$

$\frac{dr}{dx} = \frac{x}{r} = \frac{r\cos\theta}{r} = \cos\theta$

Part - III

WTS

31. $A = \begin{bmatrix} 0 & -3 \\ 1 & 4 \end{bmatrix}, B = \begin{bmatrix} -2 & -3 \\ 0 & -1 \end{bmatrix}$

Example 1.9

$AB = \begin{bmatrix} 0 & -3 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} -2 & -3 \\ 0 & -1 \end{bmatrix}$

$= \begin{bmatrix} 0+0 & 0+3 \\ -2+0 & -3-4 \end{bmatrix}$

$= \begin{bmatrix} 0 & 3 \\ -2 & -7 \end{bmatrix}$

$(AB)^{-1} = \frac{1}{(0+6)}\begin{bmatrix} -7 & -3 \\ 2 & 0 \end{bmatrix}$

$= \frac{1}{6}\begin{bmatrix} -7 & -3 \\ 2 & 0 \end{bmatrix} \rightarrow (1)$

$A^{-1} = \frac{1}{(0+3)}\begin{bmatrix} 4 & 3 \\ -1 & 0 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 4 & 3 \\ -1 & 0 \end{bmatrix}$

$B^{-1} = \frac{1}{(2-0)}\begin{bmatrix} -1 & 3 \\ 0 & -2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -1 & 3 \\ 0 & -2 \end{bmatrix}$

$B^{-1}A^{-1} = \frac{1}{2}\begin{bmatrix} -1 & 3 \\ 0 & -2 \end{bmatrix}\frac{1}{3}\begin{bmatrix} 4 & 3 \\ -1 & 0 \end{bmatrix}$

$= \frac{1}{6}\begin{bmatrix} -7 & -3 \\ 2 & 0 \end{bmatrix} \rightarrow (2)$

From (1), (2) $(AB)^{-1} = B^{-1}A^{-1}$

32. $A = \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$

Ex. 1.1 - (6)

$\text{adj } A = \begin{bmatrix} 3 & 4 \\ 5 & 8 \end{bmatrix}$

$A(\text{adj } A) = \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} 3 & 4 \\ 5 & 8 \end{bmatrix}$

$= \begin{bmatrix} 24-20 & 32-30 \\ -15+15 & -20+24 \end{bmatrix}$

$A(\text{adj } A) = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \rightarrow (1)$

$(\text{adj } A)A = \begin{bmatrix} 3 & 4 \\ 5 & 8 \end{bmatrix}\begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$

$= \begin{bmatrix} 24-20 & -12+12 \\ 40-40 & -20+24 \end{bmatrix}$

$(\text{adj } A)A = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \rightarrow (2)$

$|A| = 24-20 = 4$

$|A|I_2 = 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \rightarrow (3)$

From (1), (2), (3) we get

$A(\text{adj } A) = (\text{adj } A)A = |A|I_2$

33. Let the roots be in A.P

Example 3.19

$\therefore$ Roots are $\alpha-d, \alpha, \alpha+d$

Applying the Vieta's formula

$(\alpha-d)+\alpha+(\alpha+d) = -\frac{p}{1} = -p$

$3\alpha = -p$

$\alpha = -p/3$

α is the root of the given equation

$(-p/3)^3 + p(-p/3)^2 + q(-p/3) + r = 0$

$9pq = 2p^3 + 27r$

34. $x+\cancel{y} = 5 \longrightarrow (1)$

Ex. 5.1 (4)

$x-\cancel{y} = 1 \longrightarrow (2)$

$2x = 6$

$\Rightarrow x = 3$

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$3+y=5$ WTS

$y=2$

centre $(h,k)=(3,2)$

Area $=\pi r^2=9\pi$

$r^2=9$

$r=3$

Equation of the circle is

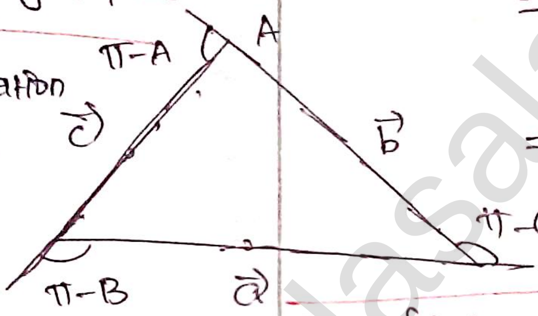
$(x-h)^2+(y-k)^2=r^2$

$(x-3)^2+(y-2)^2=3^2$

$x^2-6x+9+y^2-4y+4=9$

$x^2+y^2-6x-4y+4=0$

35. With usual notation in triangle ABC



Example 6.4

$\vec{BC}=\vec{a}$

$\vec{CA}=\vec{b}$

$\vec{AB}=\vec{c}$

Then $|\vec{BC}|=a$, $|\vec{CA}|=b$ and $|\vec{AB}|=c$

Since in ΔABC, $\vec{BC}+\vec{CA}+\vec{AB}=0$ we have,

$\vec{BC}\times(\vec{BC}+\vec{CA}+\vec{AB})=\vec{0}$

$\vec{BC}\times\vec{CA}=\vec{AB}\times\vec{BC}$ $\rightarrow (1)$

III^{ly} $\vec{BC}+\vec{CA}+\vec{AB}=0$

$\vec{CA}\times(\vec{BC}+\vec{CA}+\vec{AB})=\vec{0}$

$\vec{BC}\times\vec{CA}=\vec{CA}\times\vec{AB}$ $\rightarrow (2)$

From Equation (1), (2)

$\vec{AB}\times\vec{BC}=\vec{CA}\times\vec{AB}=\vec{BC}\times\vec{CA}$

$|\vec{AB}\times\vec{BC}|=|\vec{CA}\times\vec{AB}|=|\vec{BC}\times\vec{CA}|$

$ca\sin(\pi-B)=bc\sin(\pi-A)$

$=ab\sin(\pi-C)$

(ie) $ca\sin B=bc\sin A=ab\sin C$

Dividing by abc,

$$\frac{ca\sin B}{abc}=\frac{bc\sin A}{abc}=\frac{ab\sin C}{abc}$$

$$\frac{\sin B}{b}=\frac{\sin A}{a}=\frac{\sin C}{c}$$

$$\Rightarrow \frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$$

36. $f(x)=x^2-12x+10$

Ex 7.6 1(ii)

$f'(x)=2x-12$

$f'(x)=0$

$2x-12=0 \Rightarrow 2x=12 \Rightarrow x=6$

The critical number $x=6$

$f(6)=6^2-12(6)+10$

$=36-72+10$

$=-26$

$f(1)=1^2-12(1)+10$

$=1-12+10$

$=-1$

$f(7)=7^2-12(7)+10=-25$

Absolute maximum $=-1$

Absolute minimum $=-26$

Creative Old book Example 6.17

37. $\frac{dz}{dt} = \frac{\partial z}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial z}{\partial y}\cdot\frac{dy}{dt}$

$\frac{\partial z}{\partial x} = ye^{x^2}\cdot 2x$

$\frac{\partial z}{\partial y} = e^{x^2}, \quad \frac{dx}{dt} = 2, \quad \frac{dy}{dt} = -1$

$\frac{dz}{dt} = y\,2x\,e^{x^2}(2) + e^{x^2}(-1)$

$= 4xye^{x^2} - e^{x^2} \quad (\because x = 2t,\ y = 1-t)$

$= e^{4t^2}[(8t(1-t)-1)]$

$= e^{4t^2}(8t - 8t^2 - 1)$

Example 11.5

38. $S = \{TT, HT, TH, HH\}$

Let X be the random variable denoting the number of heads.

$X(TT) = 0, \quad X(TH) = 1$

$X(HT) = 1, \quad X(HH) = 2$

Values of R.V	0	1	2	total
Number of elements in inverse image	1	2	1	4

The Probability given by

$f(0) = P(X=0) = \frac{1}{4}$

$f(1) = P(X=1) = \frac{1}{2}$

$f(2) = P(X=2) = \frac{1}{4}$

$\Sigma f(x) = f(0) + f(1) + f(2) = \frac{1}{4} + \frac{1}{2} + \frac{1}{4} = 1$

The Probability mass function is given by

x	0	1	2
$f(x)$	$1/4$	$1/2$	$1/4$

Example 11.21 (i)

39. Mean $= np = 2$

Variance $= npq = 1.5$

$\frac{npq}{np} = \frac{1.5}{2} = \frac{3}{4}$

$q = \frac{3}{4} \Rightarrow p = 1 - q$

$p = 1 - \frac{3}{4}$

$p = \frac{1}{4}$

$np = 2$

$n\left(\frac{1}{4}\right) = 2 \Rightarrow n = 8$

$\therefore X \sim B\left(8, \frac{1}{4}\right)$

The binomial distribution

$P(X = x) = f(x) = \binom{8}{x}\left(\frac{1}{4}\right)^x\left(\frac{3}{4}\right)^{8-x}$

$P(X=0) = f(0)$

$= \binom{8}{0}\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^{8-0}$

$= (1)(1)\left(\frac{3}{4}\right)^8$

$= \left(\frac{3}{4}\right)^8$

Creative Old book Example 9.10 (ii)

40.

p	q	$\neg q$	$(\neg q)\wedge p$	$((\neg q)\wedge p)\wedge q$
T	T	F	F	F
T	F	T	T	F
F	T	F	F	F
F	F	T	F	F

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The Last column contains only F.

$\therefore ((\neg q) \wedge p) \wedge q$ is a Contradiction

Part - IV

41. (a) The matrix form of the system is $AX = B$

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & -1 & 1 \\ 3 & 1 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 5 \\ 1 \\ 4 \end{bmatrix}$$

The augmented matrix is

$$[A, B] = \left[\begin{array}{ccc|c} 2 & 2 & 1 & 5 \\ 1 & -1 & 1 & 1 \\ 3 & 1 & 2 & 4 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 2 & 2 & 1 & 5 \\ 3 & 1 & 2 & 4 \end{array}\right] R_1 \leftrightarrow R_2$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 4 & -1 & 3 \\ 0 & 4 & -1 & 1 \end{array}\right] \begin{array}{l} R_2 \rightarrow R_2 - 2R_1 \\ R_3 \rightarrow R_3 - 3R_1 \end{array}$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 4 & -1 & 3 \\ 0 & 0 & 0 & -2 \end{array}\right] R_3 \rightarrow R_3 - R_2$$

Ex. 1.6 1 (iii)

The last equivalent matrix is in row-echelon form.

$\rho[A/B] = 3$, $A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 4 & -1 \\ 0 & 0 & 0 \end{bmatrix}$

It is in row-echelon form.

$\rho(A) = 2$

$\therefore \rho(A) \neq \rho[A|B]$

$\therefore$ The given system is inconsistent and has no solution.

WTS

(or b) Let $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$

$z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$

$|z_1| = r_1 \quad \arg z_1 = \theta_1$

$|z_2| = r_2, \quad \arg z_2 = \theta_2$

$z_1 \cdot z_2 = r_1 r_2 (\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 + i\sin\theta_2)$

$= r_1 r_2 [(\cos\theta_1 \cdot \cos\theta_2 - \sin\theta_1 \sin\theta_2) + i(\sin\theta_1 \cos\theta_2 + \cos\theta_1 \sin\theta_2)]$

Creative

$= r_1 r_2 [\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]$

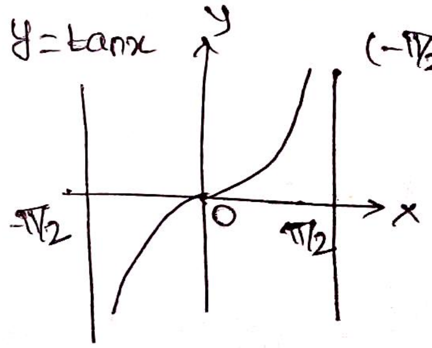
$\therefore |z_1 z_2| = r_1 r_2$

$= |z_1||z_2|$

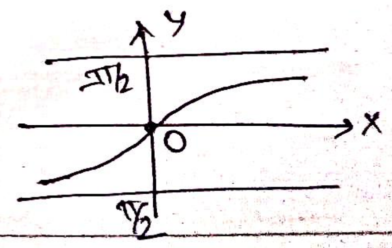
$\arg(z_1 z_2) = \theta_1 + \theta_2$

$= \arg z_1 + \arg z_2$

42. a) $y = \tan x$ $(-\pi/2, \pi/2)$



$y = \tan^{-1} x$ in $(-\infty, \infty)$



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WTP 8

b) $11x^2 - 25y^2 - 44x + 50y - 256 = 0$

$11(x^2 - 4x) - 25(y^2 - 2y) - 256 = 0$

$11(x-2)^2 - 25(y-1)^2 = 256 - 44 + 25$

$11(x-2)^2 - 25(y-1)^2 = 275$

$\frac{(x-2)^2}{25} - \frac{(y-1)^2}{11} = 1$

centre $(2,1)$

$a^2 = 25,\ b^2 = 11$

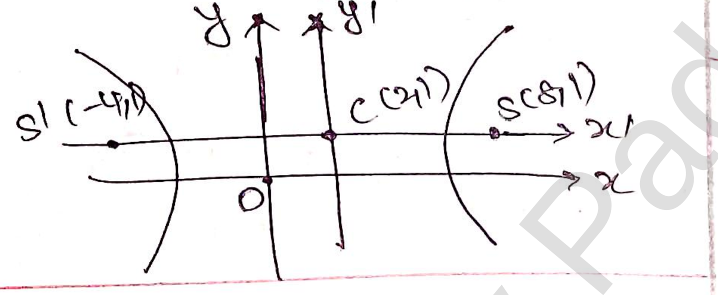
$c^2 = a^2 + b^2 = 25 + 11 = 36$

$c = \pm 6$

$e = \frac{c}{a} = \frac{6}{5}$

foci $(8,1), (-4,1)$

Example 5.26



$\cos^2\theta + \sin^2\theta = 1$

$\left(\frac{x}{0.9}\right)^2 + \left(\frac{y}{0.3}\right)^2 = 1$

$\frac{x^2}{0.81} + \frac{y^2}{0.09} = 1$

$a^2 = 0.81,\ b^2 = 0.09$

$e^2 = \frac{a^2 - b^2}{a^2}$

$= \frac{0.81 - 0.09}{0.81}$

$= \frac{0.72}{0.81}$

$= \frac{8}{9}$

$e^2 = \frac{4 \times 2}{9}$

$e = \frac{2\sqrt{2}}{3}$

43. (a) $P(x,y)$ is the point on the rod

Ex. 5.5 - 11

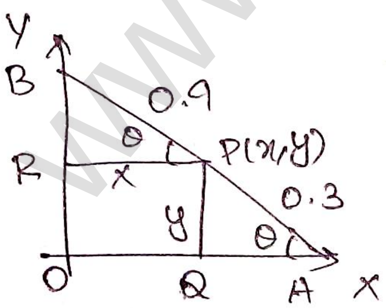
$AB = 1.2$

$AP = 0.3$

$PB = 0.9$

$\angle PAQ = \theta$

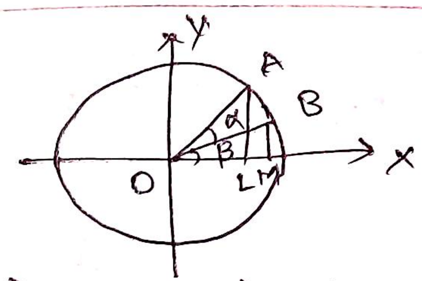
$\angle BPR = \theta$



In right angle Δ, AQP, $\sin\theta = \frac{y}{0.3}$

In right angle Δ BRP, $\cos\theta = \frac{x}{0.9}$

b)



$\vec{OA} = \vec{a},\quad \vec{OB} = \vec{b}$

Ex. 6.1 - 9

Let $\angle AOL = \alpha,\ \angle BOL = \beta$

$\therefore \angle AOB = \alpha - \beta$

$A(\cos\alpha, \sin\alpha),\ B(\cos\beta, \sin\beta)$

$\vec{OA} = \cos\alpha\,\hat{i} + \sin\alpha\,\hat{j}$

$\vec{OB} = \cos\beta\,\hat{i} + \sin\beta\,\hat{j}$

$\vec{OB}\cdot\vec{OA} = |\vec{OB}||\vec{OA}|\cos(\alpha-\beta)$

$= 1 \times 1 \times \cos(\alpha - \beta)$

$= \cos(\alpha-\beta) \rightarrow (1)$

$\vec{OB}\cdot\vec{OA} = (\cos\beta\hat{i}+\sin\beta\hat{j})\cdot(\cos\alpha\hat{i}+\sin\alpha\hat{j})$

$= \cos\alpha\cos\beta + \sin\alpha\sin\beta \rightarrow (2)$

From (1), (2)

$\cos(\alpha-\beta) = \cos\alpha\cos\beta+\sin\alpha\sin\beta$

44. a) The required line passes through $A(1,-2,4)$ and Parallel to $\vec{b}(1,2,-3)$, $\vec{c}=(3,-1,1)$

Ex. 6.7 - (4)

1) Vector Form

$(\vec{r}-\vec{a})\cdot(\vec{b}\times\vec{c})=0$

$\vec{a} = \hat{i}-2\hat{j}+4\hat{k}$

$\vec{b} = \hat{i}+2\hat{j}-3\hat{k}$

$\vec{c} = 3\hat{i}-\hat{j}+\hat{k}$

$$\vec{b}\times\vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 3 & -1 & 1 \end{vmatrix}$$

$= \hat{i}(-1)-\hat{j}(10)+\hat{k}(-7)$

$= -\hat{i}-10\hat{j}-7\hat{k}$

$= (-1)[\hat{i}+10\hat{j}+7\hat{k}]$

$(\vec{r}-\vec{a})\cdot(\vec{b}\times\vec{c})=0$

$(\vec{r}-(\hat{i}-2\hat{j}+4\hat{k}))\cdot(-1)(\hat{i}+10\hat{j}+7\hat{k})=0$

$[\vec{r}-(\hat{i}-2\hat{j}+4\hat{k})]\cdot(\hat{i}+10\hat{j}+7\hat{k})=0 \quad (\div -1)$

$\vec{r}\cdot(\hat{i}+10\hat{j}+7\hat{k})-(\hat{i}-2\hat{j}+4\hat{k})\cdot(\hat{i}+10\hat{j}+7\hat{k})=0$

$\vec{r}\cdot(\hat{i}+10\hat{j}+7\hat{k})-(1-20+28)=0$

$\vec{r}\cdot(\hat{i}+10\hat{j}+7\hat{k})-9=0$

$\vec{r}\cdot(\hat{i}+10\hat{j}+7\hat{k})=9$

2) Cartesian Form

Put $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$

$(x\hat{i}+y\hat{j}+z\hat{k})\cdot(\hat{i}+10\hat{j}+7\hat{k})=9$

$x+10y+7z=9$

b) Let length of the rectangle $=x$

breadth of the rectangle $=y$

Area of rectangle $=xy$

Ex. 7.8 - (8)

$P=$ Perimeter of rectangle $=2(x+y)$

$P=2x+2y$

$y=\frac{1}{2}(P-2x) \rightarrow (1)$

$\therefore A = x\left(\frac{1}{2}(P-2x)\right)$

$A=\frac{Px}{2}-x^2$

$\frac{dA}{dx}=\frac{P}{2}-2x$

$\frac{d^2A}{dx^2}=-2$

$\frac{dA}{dx}=0$

$\Rightarrow \frac{P}{2}-2x=0 \Rightarrow \frac{P}{2}=2x$

$x=P/4$

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WTO 10

At $x = P/4$, $\frac{d^2A}{dx^2} = -2 = -ve$

$\therefore$ A is maximum when $x = P/4$

$① \Rightarrow y = \frac{1}{2}\left(P - \frac{2P}{4}\right)$

$= \frac{1}{2}\left(\frac{P}{2}\right) = \left(\frac{P}{4}\right)$

Length = Breadth = $P/4$

$\therefore$ The rectangle becomes a square when it has maximum area.

45. a)

$I = \int_0^1 (\tan^{-1}x + \tan^{-1}(1-x))dx$

$= \int_0^1 \tan^{-1}x\,dx + \int_0^1 \tan^{-1}(1-x)\,dx$

$= \int_0^1 \tan^{-1}x\,dx + \int_0^1 \tan^{-1}(1-(1-x))\,dx$

$\left(\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$

$= \int_0^1 \tan^{-1}x\,dx + \int_0^1 \tan^{-1}x\,dx$

$= 2\int_0^1 \tan^{-1}x\,dx$

$= \left[2\int u\,dv\right]_0^1$, $u = \tan^{-1}x$, $dv = dx$

$= 2\left[uv - \int v\,du\right]_0^1$

$= 2\left[x\tan^{-1}x - \int x \frac{dx}{1+x^2}\right]_0^1$

$= 2\left[x\tan^{-1}x - \frac{1}{2}\log(1+x^2)\right]_0^1$

$= \pi/2 - \log 2$

Example 9.28

b) Let T be the temperature of boiling water at time t.

S be the room temperature

$\frac{dT}{dt} \propto T - S$

$\frac{dT}{dt} = k(T-S)$

$T - S = ce^{kt}$

When $t=0$, $T=100$

$100 - S = ce^0$

$c = 100 - S$

$T - S = (100-S)e^{kt} \rightarrow ①$

When $t=5$, $T=80$

$① \Rightarrow 80 - S = (100-S)e^{5k}$

$e^{5k} = \frac{80-S}{100-S}$

When $t=10$, $T=65$

$① \Rightarrow 65 - S = (100-S)e^{10k}$

$= (100-S)(e^{5k})^2$

$65 - S = \cancel{(100-S)}\frac{(80-S)(80-S)}{\cancel{(100-S)}(100-S)}$

$(65-S)(100-S) = (80-S)^2$

$6500 - 165S + S^2 = 6400 - 160S + S^2$

$6500 - 6400 = 165S - 160S$

$100 = 5S$

$S = 100/5 = 20°C$

Kitchen Temperature $= 20°C$

Ex. 10.8 - 9

46. a) $\frac{dy}{dx} = e^{x+y} + x^3 e^y$

Ex. 10.5 - 4 (iv)

$\frac{dy}{dx} = e^x e^y + x^3 e^y$

$\frac{dy}{dx} = e^y (e^x + x^3)$

$e^{-y} \frac{dy}{dx} = (e^x + x^3)\, dx$

Integrating on both sides

$\int e^{-y}\, dy = \int e^x dx + \int x^3 dx$

$\frac{e^{-y}}{-1} = e^x + \frac{x^4}{4} + c_1$

$-e^{-y} = e^x + \frac{x^4}{4} + c_1$

$-c_1 = e^x + e^{-y} + \frac{x^4}{4}$

$e^x + e^{-y} + \frac{x^4}{4} = c \quad (\because c = -c_1)$

b) $n = 6$

Ex. 11.5 - (8)

$4\, P(X=4) = P(X=2)$

$4\,(6C_4\, p^4 q^{6-4})$

$= 6C_2\, p^2 (q)^{6-2}$

$4 \times 6C_2\, p^4 q^2 = 6C_2\, p^2 q^4$

$\frac{4p^4}{p^2} = \frac{q^4}{q^2}$

$4p^2 = q^2$

Taking square root

$2p = q$

$2p = 1 - p$

$3p = 1$

$p = 1/3$

$q = 1 - p = 1 - 1/3$

$q = 2/3$

The distribution is

$P(X=x) = nC_x\, p^x q^{n-x}$

$= 6C_x (1/3)^x (2/3)^{6-x} \quad x = 0, 1, 2 \ldots 6$

Mean $= np = 6 \times 1/3 = 2$

Variance $= npq = 6 \times 1/3 \times 2/3$

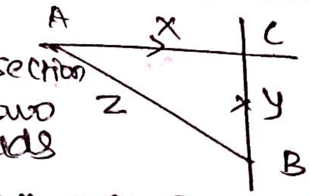
$= 4/3$

S.D $= \sqrt{npq} = \sqrt{4/3} = 2/\sqrt{3}$

47. a)

(Creative) - old book Example 5.8

C - Intersection of two roads



x - Distance from A to C

y → Distance from B to C

z - Distance from A to B

$\frac{dx}{dt} = -50\, km/hr, \quad dy/dt = -60\, km/$

$z^2 = x^2 + y^2$

Diff. w.r.to 't'

$2z \cdot \frac{dz}{dt} = 2x \frac{dx}{dt} + 2y \frac{dy}{dt}$

$\frac{dz}{dt} = \frac{1}{z}\left(x \frac{dx}{dt} + y \frac{dy}{dt}\right)$

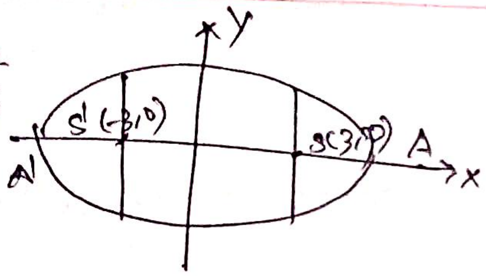
when $x=0.3$, $y=0.4$ we get $z=0.5$

$$\frac{dz}{dt}=\frac{1}{0.5}\left[0.3(-50)+0.4(-60)\right]$$

$$=-78 \text{ km/hr}$$

∴ The cars are approaching each other at a rate of 78 km/hr

b)



$$\frac{x^2}{25}+\frac{y^2}{16}=1$$

$a^2=25$
$b^2=16$
$c^2=a^2-b^2$
$c^2=25-16$
$c^2=9$
$c=3$

The eqn of latus rectum is $x=\pm 3$

The required Area

$= 4 \times$ Area of Ist quadrant

$$=4\int_0^3 y\,dx$$

$$=4\int_0^3 \frac{4}{5}\sqrt{25-x^2}\,dx$$

$$=\frac{16}{5}\int_0^3 \sqrt{5^2-x^2}\,dx$$

$$=\frac{16}{5}\left[\frac{x}{2}\sqrt{25-x^2}+\frac{25}{2}\sin^{-1}\left(\frac{x}{5}\right)\right]_0^3$$

$$=\frac{16}{5}\left[\frac{3}{2}\sqrt{25-3^2}+\frac{25}{2}\sin^{-1}\left(\frac{3}{5}\right)-0\right]$$

$$=\frac{8}{5}\left[12+25\sin^{-1}\left(\frac{3}{5}\right)\right]$$

$$=\frac{96}{5}+40\sin^{-1}\left(\frac{3}{5}\right)$$

Creative

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