12th STD

COMMON QUARTERLY EXAMINATION SEPTEMBER - 2019

Reg. No.

Mathematics

TIME ALLOWED : 2.30 Hours MAXIMUM MARKS : 90

Instructions :

1. Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.
2. Use **Blue** or **Black** ink to write and underline and pencil to draw diagrams

PART - I

Note : (i) Answer all the questions. **[20 × 1 = 20]**

(ii) Choose the most appropriate answer from the given four alternatives and write the option code and the corresponding answer.

1. **The rank of the matrix** $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \end{bmatrix}$ **is**

(a) 2 (b) 1 (c) 3 (d) 4

2. **If $0 \leq \theta \leq \pi$ and the system of equations $x + (\sin\theta)y - (\cos\theta)z = 0$, $(\cos\theta)x - y + z = 0$, $(\sin\theta)x + y - z = 0$ has a non-trivial solution then θ is**

(a) $\frac{5\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{2\pi}{3}$ (d) $\frac{3\pi}{4}$

3. **If $|z_1| = 1$, $|z_2| = 2$, $|z_3| = 3$ and $|9z_1z_2 + 4z_1z_3 + z_2z_3| = 12$, then the value of $|z_1 + z_2 + z_3|$ is**

(a) 2 (b) 1 (c) 4 (d) 3

4. **Which one of the points $i, -2+i$, 2 and 3 is farthest from the origin?**

(a) 3 (b) $-2+i$ (c) i (d) 2

5. **If α, β and γ are the zeros of $x^3 + px^2 + qx + r$, then $\sum \frac{1}{\alpha}$ is**

(a) $\frac{q}{r}$ (b) $\frac{-p}{r}$ (c) $\frac{-q}{r}$ (d) $\frac{-q}{p}$

6. **The range of $\sec^{-1} x$ is**

(a) $[-\pi, \pi]\setminus\left\{\frac{\pi}{2}\right\}$ (b) $[0, \pi]\setminus\left\{\frac{\pi}{2}\right\}$

(c) $(0, \pi)\setminus\left\{\frac{\pi}{2}\right\}$ (d) $(-\pi, \pi)\setminus\left\{\frac{\pi}{2}\right\}$

7. **If $P(x, y)$ be any point on $16x^2 + 25y^2 = 400$ with foci $F_1(3,0)$ and $F_2(-3,0)$ then $PF_1 + PF_2$ is**

(a) 6 (b) 8 (c) 12 (d) 10

8. **If the length of the perpendicular from the origin to the plane $2x + 3y + \lambda z = 1$, $\lambda > 0$ is $\frac{1}{5}$, then the value of λ is**

(a) 0 (b) 1 (c) $2\sqrt{3}$ (c) $3\sqrt{2}$

9. **The system of linear equations $x + y + z = 2$, $2x + y - z = 3$, $3x + 2y + kz = 4$ has a unique solution if**

(a) $k \neq 0$ (b) $-1 < k < 1$

(c) $-2 < k < 2$ (d) $k = 0$

10. **If A is an orthogonal matrix, then |A| is**

(a) 1 (b) -1 (c) ± 1 (d) 0

11. **If z is a complex number such that Re (z) = Im(z), then**

(a) Re$(z^2) = 0$ (b) Im$(z^2) = 0$

(c) Re(z^2) = Im (z^2) (d) Re$(z^2) = -$Im(z^2)

12. **If z is any complex number, then the points $z, iz, -z, -iz$**

(a) form a square (b) form a trapezium

(c) are collinear

(d) lie on a circle $|z| = \sqrt{2}$ with centre (0,0) and radius $\sqrt{2}$

13. **If $\sin\alpha$ and $\cos\alpha$ are the roots of $25x^2 + 5x - 12 = 0$ then the value of $\sin 2\alpha$ is**

(a) $\frac{12}{25}$ (b) $\frac{-12}{25}$ (c) $\frac{-24}{25}$ (d) $\frac{4}{5}$

14. **If a and b are odd integers then the roots of the equation $2ax^2 + (2a + b)x + b = 0$ $(a \neq 0)$ are**

(a) rational (b) irrational

(c) non real (d) rational and equal

15. **If $4\cos^{-1} x + \sin^{-1} x = \pi$ then the value of x is**

(a) $\frac{3}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{2}{\sqrt{3}}$

16. **The domain of the function $\cos^{-1}(2x - 1)$ is**

(a) [0,1] (b) [−1, 1]

(c) (−1, 1) (d) $(0, \pi)$

17. **If $5x + 9 = 0$ is the directrix of the hyperbola $16x^2 - 9y^2 = 144$ then its corresponding focus is**

(a) $\left(\frac{-5}{3}, 0\right)$ (b) (5,0) (c) (−5, 0) (d) $\left(\frac{5}{3}, 0\right)$

18. **If $ax^2 + by^2 + (a + b - 4)xy - ax - by - 20 = 0$ represent the circle then its centre is**

(a) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (b) $\left(-\frac{1}{2}, -\frac{1}{2}\right)$

(c) (1, 1) (d) (−1, −1)

19. If $\vec{\alpha} = \vec{a}\times\vec{b}, \vec{\beta} = \vec{b}\times\vec{c}, \vec{\gamma} = \vec{c}\times\vec{a}$ then $\left[\vec{\alpha}\times\vec{\beta}, \vec{\beta}\times\vec{\gamma}, \vec{\gamma}\times\vec{\alpha}\right] =$

(a) $\left[\vec{a}\ \vec{b}\ \vec{c}\right]^4$ (b) $\left[\vec{a}\ \vec{b}\ \vec{c}\right]^2$

(c) $2\left[\vec{a}\ \vec{b}\ \vec{c}\right]^2$ (d) $4\left[\vec{a}\ \vec{b}\ \vec{c}\right]$

20. The foot of the perpendicular from A (1, 0, 0) to the line $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$ is

(a) (3, –4, –2) (b) (5, –8, –4)

(c) (–3, 4, 2) (d) (2, –3, 4)

PART - II

(i) Answer any **SEVEN** questions.

(ii) Question number **30** is compulsory. **7 × 2 = 14**

21. If adj A = $\begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, find A^{-1}.

22. Simplify : $\left(\sin\frac{\pi}{6} + i\cos\frac{\pi}{6}\right)^{18}$.

23. Form a polynomial equation with integer coefficients with $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ as a root.

24. Find the value of $\tan^{-1}\left(\tan\frac{3\pi}{5}\right)$.

25. Find the condition for the line $y = mx + c$ to be a tangent to the circle $x^2 + y^2 = a^2$.

26. If $2\vec{i} - \vec{j} + 3\vec{k}, 3\vec{i} + 2\vec{j} + \vec{k}, \vec{i} + m\vec{j} + 4\vec{k}$ are coplanar, find the value of m.

27. Show that $\left(2+i\sqrt{3}\right)^{10} - \left(2-i\sqrt{3}\right)^{10}$ is purely imaginary.

28. Determine the number of positive and negative roots of the equation $x^9 - 5x^8 - 14x^7 = 0$.

29. Find the value of $\cot\left(\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{4}{5}\right)$.

30. If the system of linear equation $x + 2ay + az = 0$, $x + 3by + bz = 0$, $x + 4cy + cz = 0$ has a non-trival solution then show that a, b, c are in H.P.

PART - III

(i) Answer any **SEVEN** questions.

(ii) Question number **40** is compulsory.

7 × 3 = 21

31. Find the inverse of the non-singular matrix $A = \begin{bmatrix} 0 & 5 \\ -1 & 6 \end{bmatrix}$, by Gauss Jordan method.

32. In a competitive examination, one mark is awarded for every correct answer while $\frac{1}{4}$ mark is deducted for every wrong answer. A student answered 100 questions and got 80 marks. How many questions did he answer correctly? (Use Cramer's rule to solve the problem)

33. State and prove triangle inequality.

34. Find the condition that the roots of $ax^3 + bx^2 + cx + d = 0$ are in geometric progression. Assume $a, b, c, d \neq 0$.

35. Find the domain of $\sin^{-1}(2 - 3x^2)$.

36. Find the equation of the parabola with vertex (–1, –2), axis parallel to y - axis and passing through (3, 6).

37. If the normal at the point 't_1' on the parabola $y^2 = 4ax$ meets the parabola again at the point 't_2', then prove that $t_2 = -\left(t_1 + \frac{2}{t_1}\right)$

38. If $\hat{a}, \hat{b}, \hat{c}$ are three unit vectors such that $\hat{b}$ and $\hat{c}$ are non parallel and $\hat{a}\times(\hat{b}\times\hat{c}) = \frac{1}{2}\hat{b}$, find the angle between $\hat{a}$ and $\hat{c}$

39. Determine whether the pair of straight lines $\vec{r} = (2\hat{i} + 6\hat{j} + 3\hat{k}) + t(2\hat{i} + 3\hat{j} + 4\hat{k})$, $\vec{r} = (2\hat{j} - 3\hat{k}) + s(\hat{i} + 2\hat{j} + 3\hat{k})$ are parallel. Find the shortest distance between them.

40. Prove that $\tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right) + \tan\left(\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right) = \frac{2b}{a}$.

PART - IV

Answer *all* questions. **7 × 5 = 35**

41. (a) By using Gaussian elimination method, balance the chemical reaction equation:

$C_5H_8 + O_2 \to CO_2 + H_2O$

(OR)

(b) Solve : $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$

42. (a) Find the value of k for which the equations $kx - 2y + z = 1, x - 2ky + z = -2y. x - 2y + kz = 1$ have

(i) no solution (ii) unique solution

(iii) infinitely many solution

(OR)

(b) Suppose z_1, z_2 and z_3 are the vertices of an equilateral triangle inscribed in the circle $|z| = 2$. If $z_1 = 1 + i\sqrt{3}$ then find z_2 and z_3.

43. (a) If $z = x + iy$ and $\arg\left(\frac{z-1}{z+2}\right) = \frac{\pi}{4}$, then show that $x^2 + y^2 + 3x - 3y + 2 = 0$.

(OR)

(b) If $2 + i$ and $3 - \sqrt{2}$ are the roots of the equation $x^6 - 13x^5 + 62x^4 - 126x^3 + 65x^2 + 127x - 140 = 0$ then find all the roots.

44. (a) i) Find the value of $\tan^{-1}(-1) + \cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(\frac{-1}{2}\right)$

ii) If $\cot^{-1}\left(\frac{1}{7}\right) = \theta$, find the value of $\cos\theta$

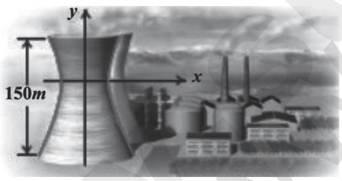
(OR)

(b) For the ellipse $4x^2 + y^2 + 24x - 2y + 21 = 0$, find the centre, vertices, and the foci. Also prove that the length of latus rectum is 2.

45. (a) Find the number of solutions of the equation $\tan^{-1}(x-1) + \tan^{-1}(x) + \tan^{-1}(x+1) = \tan^{-1}(3x)$.

(OR)

(b) Cross section of a Nuclear cooling tower is in the shape of a hyperbola with equation $\frac{x^2}{30^2} - \frac{y^2}{44^2} = 1$. The tower is 150 m tall and the distance from the top of the tower to the centre of the hyperbola is half the distance from the base of the tower to the centre of the hyperbola. Find the diameter of the top and base of the tower.



46. (a) Find parametric form of vector equation and Cartesian equations of the plane passing through the points (2, 2, 1), (1, –2, 3) and parallel to the straight line passing through the points (2, 1, –3) and (–1, 5, –8).

(OR)

(b) Prove by vector method that the perpendiculars (attitudes) from the vertices to the opposite sides of a triangle are concurrent.

47. (a) If $\cos\theta + \cos\phi = \sin\theta + \sin\phi = 0$ then show that

i) $\cos 2\theta + \cos 2\phi = 2\cos(\pi + \theta + \phi)$

ii) $\sin 2\theta + \sin 2\phi = 2\sin(\pi + \theta + \phi)$

(OR)

(b) If a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ makes intercepts h and k on the co-ordinate axes then show that $\frac{a^2}{h^2} + \frac{b^2}{k^2} = 1$.

ANSWERS

PART - I

1.	(b)	1	**11.**	(a)	Re $(z^2) = 0$
2.	(b)	$\frac{\pi}{4}$	**12.**	(a)	form a square
3.	(a)	2	**13.**	(c)	$\frac{-24}{25}$
4.	(a)	3	**14.**	(a)	rational
5.	(c)	$\frac{-q}{r}$	**15.**	(c)	$\frac{\sqrt{3}}{2}$
6.	(b)	$[0, \pi] \setminus \left\{\frac{\pi}{2}\right\}$	**16.**	(a)	[0, 1]
7.	(d)	10	**17.**	(c)	(–5, 0)
8.	(c)	$2\sqrt{3}$	**18.**	(a)	$\left(\frac{1}{2}, \frac{1}{2}\right)$
9.	(a)	$k \neq 0$	**19.**	(a)	$\left[\vec{a}\ \vec{b}\ \vec{c}\right]^4$
10.	(c)	±1	**20.**	(a)	(3, –4, –2)

PART - II

21. Solution : We compute |adj A| = 9

So, we get $A^{-1} = \pm\frac{1}{\sqrt{|adj\ A|}}\text{adj}(A)$

$$= \pm\frac{1}{\sqrt{9}}\begin{bmatrix}-1 & 2 & 2\\ 1 & 1 & 2\\ 2 & 2 & 1\end{bmatrix} = \pm\frac{1}{3}\begin{bmatrix}-1 & 2 & 2\\ 1 & 1 & 2\\ 2 & 2 & 1\end{bmatrix}$$

22. Solution :

$$\left(\sin\frac{\pi}{6} - i\cos\frac{\pi}{6}\right)^{18} = \left[i\left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right)\right]^{18}$$

$$= -(\cos 3\pi - i\sin 3\pi) = 1$$

23. Solution : Since $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ is a root, $x - \sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ is a factor. To remove the outermost square root, we take $x + \sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ as another factor and find their product

$$\left(x + \sqrt{\frac{\sqrt{2}}{\sqrt{3}}}\right)\left(x - \sqrt{\frac{\sqrt{2}}{\sqrt{3}}}\right) = x^2 - \sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$$

Still we didn't achieve our goal. So we include another factor $x^2 - \sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ and get the product

$$\left(x^2 - \sqrt{\frac{\sqrt{2}}{\sqrt{3}}}\right)\left(x^2 + \sqrt{\frac{\sqrt{2}}{\sqrt{3}}}\right) = x^4 - \frac{2}{3}$$

So, $3x^2 - 2 = 0$ is a required polynomial equation with the integer coefficients.

Now we identify the nature of roots of the given equation without solving the equation. The idea comes from the negativity, equality to 0, positivity of $= b^2 - 4ac$.

24. Solution : $\tan^{-1}\left(\tan\frac{3\pi}{5}\right) = \tan^{-1}\left(\tan\left(\frac{-2\pi}{5}\right)\right)$

$= \frac{-2\pi}{5} \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

25. Solution : Let the line $y = mx + c$ touch the circle $x^2 + y^2 + a^2$. The centre and radius of the circle $x^2 + y^2 + a^2$ are (0,0) and a respectively.

Condition for a line to be tangent.

$$\left|\frac{0 - m, 0 - c}{\sqrt{1+m^2}}\right| = \frac{|c|}{\sqrt{1+m^2}}$$

This must be equal to radius, Therefore $\frac{|c|}{\sqrt{1+m^2}} = a$

or $c^2 = a^2(1 + m^2)$

Thus the condition for the line $y = mx + c$ to be a tangent to the circle $x^2 + y^2 = a^2$ is $c^2 = a^2(1 + m^2)$.

26. Solution : Since the given three vectors are coplanar,

we have $\begin{vmatrix} 2 & -1 & 3 \\ 3 & 2 & 1 \\ 1 & m & 4 \end{vmatrix} = 0 \Rightarrow m = -3$

27. Solution : Let $z = (2+i\sqrt{3})^{10} - (2-i\sqrt{3})^{10}$

Then, we get $\bar{z} = \overline{(2+i\sqrt{3})^{10} + (2-i\sqrt{3})^{10}}$

$= \overline{(2+i\sqrt{3})^{10}} + \overline{(2-i\sqrt{3})^{10}}$ $(\because \overline{z_1 + z_2} = \bar{z_1} + \bar{z_2})$

$= \left(\overline{2+i\sqrt{3}}\right)^{10} + \left(\overline{2-i\sqrt{3}}\right)^{10}$ $\left(\because \left(\overline{z^n}\right) = \left(\bar{z}\right)^n\right)$

$= (2-i\sqrt{3})^{10} + (2+i\sqrt{3})^{10} = z$

$\bar{z} = -z \Rightarrow z$ is real.

28. Solution : Let $p(x) = x^9 - 5x^2 - 14x^7 = 0$

$p(x)$ has only one sign change

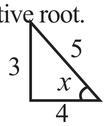
Also $p(-x) = (-x)^9 - 5(-x)^8 - 14(-x)^7 = 0$

$\Rightarrow p(-x) = -x^9 - 5x^8 + 14x^7 = 0$

$p(-x)$ has only one sign change

$\therefore p(-x)$ has at most one positive and one negative root.

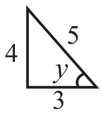
29. Solution : $\cot\left(\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{4}{5}\right)$



$\sin^{-1}\left(\frac{3}{5}\right) = x \Rightarrow \sin x = \frac{3}{5}$

$\therefore \tan x = \frac{\text{opp}}{\text{adj}} = \frac{3}{4}$

Let $\sin^{-1}\left(\frac{4}{5}\right) = y \Rightarrow \sin y = \frac{4}{5}$



$\therefore \tan y = \frac{\text{opp}}{\text{adj}} = \frac{4}{3}$

$\therefore \cot\left(\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{4}{5}\right) = \cot(x + y)$

$$= \frac{1}{\tan(x+y)} = \frac{1}{\frac{\tan x + \tan y}{1 - \tan x \tan y}} = \frac{1 - \tan x \tan y}{\tan x + \tan y}$$

$$= \frac{1 - \frac{3}{\not{4}} \times \frac{\not{4}}{3}}{\frac{3}{4} + \frac{4}{3}} = \frac{1-1}{\frac{3}{4}+\frac{4}{3}} = \frac{0}{\frac{3}{4}+\frac{4}{3}} = 0$$

$\therefore \cot\left(\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{4}{5}\right) = 0$

30. Solution : $\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$

$-bc + 2ac - ab = 0 \Rightarrow \frac{2}{b} = \frac{1}{c} + \frac{1}{a}$

$\therefore a, b, c$ are in H.P

PART - III

31. Solution : Applying Gauss – Jordan method, we get

$$[A|I_2] = \left[\begin{array}{cc|cc} 0 & 5 & 1 & 0 \\ -1 & 6 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_1 \to R_1} \left[\begin{array}{cc|cc} -1 & 6 & 0 & 0 \\ 0 & 5 & 1 & 0 \end{array}\right] \xrightarrow{R_2 \to (-1)R_1} \left[\begin{array}{cc|cc} 1 & -6 & 0 & -1 \\ 0 & 5 & 1 & 0 \end{array}\right]$$

$$\xrightarrow{R_2 \to \frac{1}{5} \times R_2} \left[\begin{array}{cc|cc} 1 & -6 & 0 & -1 \\ 0 & 1 & \left(\frac{1}{5}\right) & 2 \end{array}\right]$$

$$\xrightarrow{R_1 \to R_1 + 6R_2} \left[\begin{array}{cc|cc} 1 & 0 & \left(\frac{6}{5}\right) & -1 \\ 0 & 1 & \left(\frac{1}{5}\right) & 0 \end{array}\right]$$

So, we get $A^{-1} \xrightarrow{R_1 \to R_1 + 6R_2} \begin{bmatrix} \left(\frac{6}{5}\right) & -1 \\ \left(\frac{1}{5}\right) & 0 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 6 & -5 \\ 1 & 0 \end{bmatrix}$

32. Solution : Let x represent the number of question with correct answer and y represent the number of questions with wrong answers.

By the given data, $x + y = 100$ and ... (1)

$1.x - \frac{1}{4}y = 80$

Multiplying by 4 we get, $4x - y = 320$ ... (2)

From (1) and (2)

$\Delta = \begin{vmatrix} 1 & 1 \\ 4 & -1 \end{vmatrix} = -1 - 4 = -5$

$\Delta_1 = \begin{vmatrix} 100 & 1 \\ 320 & -1 \end{vmatrix} = -100 - 320 = -420$

$\Delta_2 = \begin{vmatrix} 1 & 100 \\ 4 & 320 \end{vmatrix} = 320 - 400 = -80$

$\therefore x = \frac{\Delta_1}{\Delta} = \frac{-420}{-5} = +84$ and $y = \frac{\Delta_2}{\Delta} = \frac{-80}{-5} = 16$

Hence, the number of questions with correct answer is 84.

33. Solution : Using $|z_1+z_2|^2 = (z_1+z_2)\overline{(z_1+z_2)}$ $\quad (\because |z|^2 = z\bar{z})$

$\Rightarrow \quad = (z_1+z_2)(\bar{z}_1+\bar{z}_2) \quad (\because \overline{z_1+z_2} = \bar{z}_1+\bar{z}_2)$

$= z_1\bar{z}_1 + (z_1\bar{z}_2 + \bar{z}_1 z_2) + z_2\bar{z}_2 \quad (\because \bar{\bar{z}} = z)$

$= |z_1|^2 + 2\,\text{Re}\,(\bar{z}_1 \bar{z}_2) + |z_2|^2 \quad (\because \text{Re}\,(z) \le |z|)$

$\le |z_2|^2 + 2|z_1||z_2| + |z_2|^2 \quad (\because |z_1 z_2| = |z_1||z_2| \text{ and } |\bar{z}| = |z|)$

$\Rightarrow |z_1+z_2|^2 \le (|z_2|+|z_2|)^2 \Rightarrow |z_1+z_2| \le |z_2|+|z_2|$

34. Solution : Let the roots be in G.P.

Then, we can assume them in the form $\frac{\alpha}{\lambda}$, α, $\alpha\lambda$

$$\Sigma\alpha = \alpha\left(\frac{1}{\lambda}+1+\lambda\right) = -\frac{b}{a}$$

$$\Sigma\alpha\beta = \alpha^2\left(\frac{1}{\lambda}+1+\lambda\right) = \frac{c}{a}$$

$$\Sigma\alpha\beta\Upsilon = \alpha^3 = -\frac{d}{a}$$

Dividing (2) by (1), we get

$$\alpha = -\frac{c}{b}$$

Substituting (4) in (3), we get $\left(-\frac{c}{b}\right)^3 = -\frac{d}{a}$

$$\Rightarrow \quad ac^3 = db^3$$

35. Solution : We know that the domain of $\sin^{-1}(x)$ is $[-1,1]$.

This leads to $1 \le 2 - 3x^2 \le 1$ which implies $-3 \le -3x^2 \le -1$.

Now, $-3 \le -3x^2$, $x^2 \le 1$ and ... (1)

$-3x^2 \le -1$, gives $x^2 \le \frac{1}{3}$ ... (2)

Combining the equations (1) and (2), we get

$\frac{1}{3}x^2 \le 1$ That is $\frac{1}{\sqrt{3}} \; |x| \le 1$, which gives

$x \in \left[-1, -\frac{1}{\sqrt{3}}\right] \cup \left[\frac{1}{\sqrt{3}}, 1\right]$ since $3 \le |x| \le b$ implies

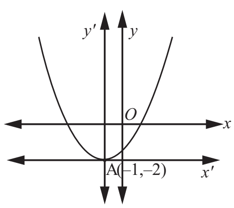
$x \in [-b, -a] \cup [a, b]$

36. Solution : Since axis is parallel to y-axis the required equation of the parabola is

$$(x+1)^2 = 4a\,(y+2).$$

Since this passes through (3,6), we get

$$(3+1)^2 = 4a + (6+2) \Rightarrow a = \frac{1}{2}$$



Then the equation of parabola is $(x+1)^2 = 2\,(y+2)$ which on simplifying yields,

$x^2 + 2x - 2y - 3 = 0.$

37. Solution : Equation of normal at 't_1' to the parabola $y^2 = 4ax$ is $y + xt_1 = at_1^3 + 2at_1$ ...(1)

(1) meets the parabola $y_2 = 4ax$ at 't_2'.

At 't_2', the point on the parabola is $x = at_2^2$, $y = 2at_2$ (2) lies on (1)

$\therefore$ Substituting (2) in (1) we get,

$$2at_2 + (at_2^2)\,t_1 = at_1^3 + 2at_1$$

$$\Rightarrow 2at_2 + at_1t_2^2 = at_1^3 + 2at_1$$

$$\Rightarrow 2at_2 - 2at_1 = at_1^3 - at_1t_2^2$$

$$\Rightarrow 2a(t_2 - t_1) = -at_1\,[t_2^2 - t_1^2]$$

$$\Rightarrow 2a(t_2 - t_1) = -at_1(t_2+t_1)(t_2-t_1)$$

$$\Rightarrow 2 = -t_1(t_2+t_1)$$

$$\Rightarrow \frac{-2}{t_1} = t_2 + t_1 \Rightarrow t_2 = \frac{-2}{t_1} - t_1$$

$$\Rightarrow t_2 = -\left(t_1 + \frac{2}{t_1}\right)$$

Hence proved.

38. Solution : Given $\hat{a}\times(\hat{b}\times\hat{c}) = \frac{1}{2}\hat{b}$

$$\Rightarrow (\hat{a}\cdot\hat{c})\hat{b} - (\hat{a}\cdot\hat{b})\hat{c} = \frac{1}{2}\hat{b}$$

$$\Rightarrow \lambda\hat{b} - \mu\hat{c} = \frac{1}{2}\hat{b} \quad [\because \text{put } \lambda = \hat{a}\cdot\hat{c} \;\&\; \mu = \hat{a}\cdot\hat{b}]$$

$$\Rightarrow \left(\lambda - \frac{1}{2}\right)\hat{b} - \mu\hat{c} = 0.$$

Since $\hat{b}$ and $\hat{c}$ are non-collinear vectors

$$\lambda - \frac{1}{2} = 0 \text{ and } \mu = 0$$

$$\therefore \lambda = \frac{1}{2} \quad \Rightarrow \hat{a}\cdot\hat{c} = \frac{1}{2}$$

$$\Rightarrow |\hat{a}||\hat{c}|\cos\theta = \frac{1}{2}$$

[$\because$ By the definition of scalar product]

$$\Rightarrow (1)\,(1)\cos\theta = \frac{1}{2} \quad [\because |\vec{a}| = |\vec{c}| = 1]$$

$$\Rightarrow \cos\theta = \frac{1}{2} \quad \Rightarrow \theta = 60° = \frac{\pi}{3}$$

Hence angle between $\vec{a}$ and $\vec{c}$ is $\frac{\pi}{3}$.

39. Solution : Comparing the given two equations with

$$\Rightarrow \vec{r} = \vec{a} + s\vec{b} \text{ and } \vec{c} + s\vec{d}$$

we have

$$\vec{a} = 2\hat{i} + 6\hat{j} + 3\hat{k},$$

$$\vec{b} = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

$$\vec{c} = 2\hat{j} - 3\hat{k}$$

$$\vec{d} = \hat{i} + 2\hat{j} - 3\hat{k}$$

Clearly, $\vec{b}$ is not a scalar multiple of $\vec{d}$. So, the two vectors are not parallel and hence the two lines are not parallel.

The shortest distance between the two straight lines is given by

$$\delta = \frac{\left|(\vec{c}-\vec{a})\cdot(\vec{b}-\vec{d})\right|}{|\vec{b}\times\vec{d}|}$$

$$\text{Now, } \vec{b}\times\vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 1 & 2 & 3 \end{vmatrix} = \hat{i} - 2\hat{j} + \hat{k}$$

So, $(\vec{c}-\vec{a})\cdot(\vec{b}-\vec{d}) = (-2\hat{i} - 4\hat{j} - 6\hat{k}).(\hat{i} - 2\hat{j} + \hat{k}) = 0$

Therefore, the distance between the two given straight lines is zero. Thus, the given lines intersect each other.

40. **Solution :** Let $\frac{1}{2}\cos^{-1}\frac{a}{b} = \theta$

$$\Rightarrow \quad \frac{a}{b} = \cos 2\theta \Rightarrow \sec 2\theta = \frac{b}{a}$$

$$\text{LHS} = \tan^{-1}\left(\frac{\pi}{4}+\theta\right) + \tan^{-1}\left(\frac{\pi}{4}-\theta\right)$$

$$= \frac{1+\tan\theta}{1-\tan\theta} + \frac{1-\tan\theta}{1+\tan\theta}$$

$$= \frac{2\sec^2\theta}{\left(\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta\theta}\right)} = \frac{2}{\cos 2\theta} = \frac{2b}{a}$$

PART - IV

41. (a) **Solution :** We are searching for positive integers x_1, x_2, x_3 and x_4 such that

$$x_1 C_5H_8 + x_2O_2 = x_3CO_2 + x_2H_2O$$

The number of carbon atoms on the left-hand side of (1) should be equal to the number of carbon atoms on the right-hand side of (1). So we get a linear homogeneous equation

$$5x_1 = x_3$$

$$\Rightarrow \quad 5x_1 - x_3 = 0$$

Similarly, considering hydrogen and oxygen atoms, we get respectively,

$$8x_1 = 2x_4$$

$$\Rightarrow \quad 4x_1 - x_4 = 0,$$

$$2x_1 = 2x_3 + x_4$$

$$\Rightarrow \quad 2x_2 - 2x_3 - x_3 = 0.$$

Equations (2), (3), and (4) constitute a homogeneous system of linear equations in four unknowns.

The augmented matrix is [A|B]

$$= \left[\begin{array}{cccc|c} 5 & 0 & -1 & 0 & 0 \\ 4 & 0 & 0 & -1 & 0 \\ 0 & 2 & -2 & -1 & 0 \end{array}\right]$$

By Gaussian elimination method, we get

$$[A|B] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{cccc|c} 4 & 0 & 0 & -1 & 0 \\ 5 & 0 & -1 & 0 & 0 \\ 0 & 2 & -2 & -1 & 0 \end{array}\right]$$

$$\xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{cccc|c} 4 & 0 & 0 & -1 & 0 \\ 0 & 2 & -2 & -1 & 0 \\ 5 & 0 & -1 & 0 & 0 \end{array}\right]$$

$$\xrightarrow{R_3 \to 4R_3 - 5R_1} \left[\begin{array}{cccc|c} 4 & 0 & 0 & -1 & 0 \\ 0 & 2 & -2 & -1 & 0 \\ 0 & 0 & -4 & 5 & 0 \end{array}\right]$$

Therefore, $\rho(A) = \rho([A|B]) = 3 < 4$ = Number of unknowns.

The system is consistent and has infinite number of solutions.

Writing the equations using the echelon form, we get $4x_1 - x_4 = 0, 2x_2 - 2x_3 - x_4 = 0, -4x + 5x_4 = 0$.

So, one of the unknowns should be chosen arbitrarily as a non-zero real number.

Let us choose $x_4 = t$, t 0. Then, by back substitution, we $x_3 = \frac{5t}{4}, x_2 = \frac{7t}{4}, x_1 = \frac{t}{4}$

Since x_1, x_2, x_3 and x_4 are positive integers, let us choose $t = 4$.

Then, we get $x_1 = 1, x_2 = 7, x_3 = 5$ and $x_4 = 4$.

So, the balanced equation is

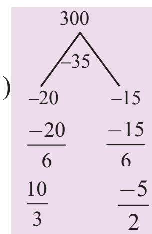
$C_5H_8 + 7O_2 \rightarrow 5CO_2 + 4H_2O$.

(OR)

(b) **Solution :**

This equation is type I even degree reciprocal equation.

Hence, it can be rewritten as

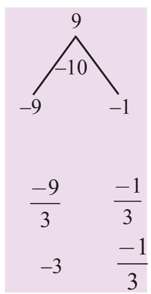


$$6\left(x^2 + \frac{1}{x}\right) - 35\left(x + \frac{1}{x}\right) + 62 = 0 \quad ...(1)$$

putting $x + \frac{1}{x} = y$

$$\Rightarrow \quad x^2 + \frac{1}{x^2} + 2 = y^2$$

$$\Rightarrow \quad x^2 + \frac{1}{x^2} = y^2 - 2$$



∴ (1) becomes as,

$$\Rightarrow \quad 6(y^2 - 2) - 35y + 62 = 0$$

$$\Rightarrow \quad 6y^2 - 12 - 35y + 62 = 0$$

$$\Rightarrow \quad 6y^2 - 35y + 50 = 0$$

$$\Rightarrow \quad (3y - 10)(2y - 5) = 0$$

$$\Rightarrow \quad y = \frac{10}{3}, \frac{5}{2}$$

Case (i) When $y = \frac{10}{3}, x + \frac{1}{x} = \frac{10}{3}$

$\Rightarrow \quad \frac{x^2+1}{x} = \frac{10}{3} \Rightarrow 3x^2 + 3 = 10x$

$\Rightarrow \quad 3x^2 - 10x + 3 = 0$

$\Rightarrow \quad (x-3)(3x-1) = 0$

$\Rightarrow \quad x = 3, \frac{1}{3}$

Case (ii)

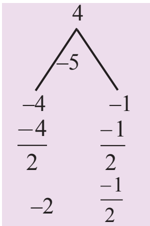
When $y = \frac{5}{2}, x + \frac{1}{x} = \frac{5}{2} \Rightarrow \frac{x^2+1}{x} = \frac{5}{2}$

$\Rightarrow \quad 2x^2 + 2 = 5x \Rightarrow 2x^2 - 5x + 2 = 0$

$\Rightarrow (x-2)(2x-1) = 0$

$\Rightarrow \quad x = 2, \frac{1}{2}$

Hence the roots are $2, \frac{1}{2}, 3, \frac{1}{3}$



42. (a) **Solution :**

$kx - 2y + z = 1, x - 2ky + z = -2, x - 2y + k = 1$

The matrix form of the system is AX = B

Where $A = \begin{bmatrix} k & -2 & 1 \\ 1 & -2k & 1 \\ 1 & -2 & k \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

Applying elementary row operation on the augment matrix [A|B] we get,

$$[A/B] = \left[\begin{array}{ccc|c} k & -2 & 1 & 1 \\ 1 & -2k & 1 & -2 \\ 1 & -2 & k & 1 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & -2 & k & 1 \\ 1 & -2k & 1 & -2 \\ k & -2 & 1 & 1 \end{array}\right]$$

$$\xrightarrow[R_3 \to R_3 - kR_1]{R_2 \to R_2 - R_1} \left[\begin{array}{ccc|c} 1 & -2 & k & 1 \\ 1 & -2k+2 & k & -3 \\ 0 & -2+2k & 1-k^2 & 1-k \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 + R_2} \left[\begin{array}{cccc} 1 & -2 & k & 1 \\ 0 & -2k+2 & 1-k & -3 \\ 0 & 0 & 1-k^2+1-k & -k-2 \end{array}\right]$$

$$\to \left[\begin{array}{cccc} 1 & -2 & k & 1 \\ 0 & -2k+2 & 1-k & -3 \\ 0 & 0 & -k^2-k+2 & -k-2 \end{array}\right]$$

$$\to \left[\begin{array}{cccc} 1 & -2 & k & 1 \\ 0 & -2k+2 & 1-k & -3 \\ 0 & 0 & (k+2)(1-k) & -k-2 \end{array}\right] \quad ...(1)$$

Case (i) when $k = 1$

$$[A|B] \to \left[\begin{array}{cccc} 1 & -2 & 1 & 1 \\ 0 & 0 & 0 & -3 \\ 0 & 0 & 0 & -3 \end{array}\right] \xrightarrow{R_3 \to R_3 - R_2} \left[\begin{array}{cccc} 1 & -2 & 1 & 1 \\ 0 & 0 & 0 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Here $\rho(A) = 1$ and $\rho[A|B] = 2$

So, $\rho(A) \neq \rho[A|B] \Rightarrow$ The system has no solution.

Case (ii) When $k \neq 1, k \neq -2$

$$[A|B] \to \left[\begin{array}{ccc|c} 1 & -2 & k & 1 \\ 0 & -2k+2 & 1-k & -3 \\ 0 & 0 & \text{not zero} & \text{not zero} \end{array}\right]$$

$\Rightarrow \rho(A) = 3$ and $\rho[A|B] = 3$

so, $\rho(A) = \rho[A|B] = 3$ = the number of unknowns

Hence, the system has unique solution.

Case (iii) when $k = -2$

$$\rho[A|B] \to \left[\begin{array}{cccc} 1 & -2 & -2 & 1 \\ 1 & 6 & 3 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Here $\rho(A) = 2$ and $\rho[A|B] = 2$

$\therefore \rho(A) = \rho[A|B] = 2 < 3$, the number of unknowns so the system is consistent with infinitely many solutions

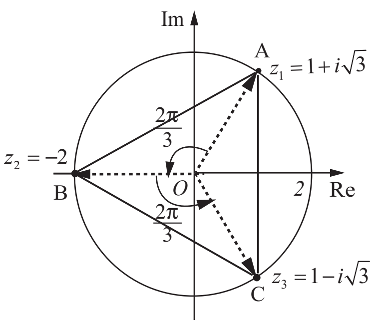
(OR)

(b) **Solution :**

$|z| = 2$ represents the circle with centre (0,0) and radius 2.

Let A, B, and C be the vertices of the given triangle. Since the vertices z_1, z_2, and z_2 form an equilateral triangle inscribed in the circle $|z| = 2$, the sides of this triangle AB, BC, and CA subtend $\frac{2\pi}{3}$ radians (120 degree) at the origin (circumcenter of the triangle).

(The complex number $ze^{i\theta}$ is a rotation of z by θ radians in the counter clockwise direction about the origin.)



Therefore, we can obtain z_2 and z_3 by the rotation of z_1 by $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$ perspectively.

Given that $\overline{OA} = z_1 = 1\,i\sqrt{3}$;

$\overline{OB} = z_i e^{i\frac{2\pi}{3}} = (1+i\sqrt{3})e^{i\frac{2\pi}{3}}$

$= (1+i\sqrt{3})\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) = 2$;

$\overrightarrow{OC} = z_i e^{i\frac{4\pi}{3}} = z_i e^{i\frac{2\pi}{3}} = -2e^{i\frac{2\pi}{3}}$

$= -2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

$= -2\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = 1 - i\sqrt{3}$

Therefore, $z_2 = -2$, and $z_3 = 1 - i\sqrt{3}$

43. (a) **Solution :**

Given $z = x + iy$ and $\arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$

$\Rightarrow \arg(z-i) - \arg(z+2) = \frac{\pi}{4}$

$\Rightarrow \arg(x+iy-i) - \arg(x+iy+2) = \frac{\pi}{4}$

$\Rightarrow \arg(x+i(y-1)) - \arg((x+2)+iy) = \frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\frac{y-1}{x}\right) - \tan^{-1}\left(\frac{y}{x+2}\right) = \frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left(\frac{\frac{y-1}{x} - \frac{y}{x+2}}{1 + \frac{y-1}{x}\cdot\frac{y}{x+2}}\right) = \frac{\pi}{4}$

$\left[\because \tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)\right]$

$\Rightarrow \frac{\left(\frac{(x+2)(y-1)-xy}{x(x+2)}\right)}{\left(\frac{x(x+2)+y(y-1)}{x(x+2)}\right)} = \tan\frac{\pi}{4} = 1$

$\Rightarrow \frac{(x+2)(y-1)-xy}{x(x+2)+y(y-1)} = 1$

$\Rightarrow -x + 2y - 2 = x^2 + 2x + y^2 - y$

$\Rightarrow x^2 + 2x + y^2 - y + x - 2y + 2 = 0$

$\Rightarrow x^2 + y^2 + 3x - 3y + 2 = 0$

Hence proved.

(OR)

(b) **Solution :**

Since the coefficient of the equations are all rational numbers, and $2 + i$ and $3 -$ are roots, we get $2 - i$ and $3 + \sqrt{2}$ are also roots of the given equation. Thus $(x - (2+i))$, $(x - (2-i))$, $(x - (3+\sqrt{2}))$ and $(x - (3+\sqrt{2}))$ are factors. Thus their product $(x-(2+i))(x-(2-i))(x-(3+\sqrt{2}))(x-(3+\sqrt{2}))$ is a factor of the given polynomial equation. That is, $(x^2 - 4x + 5)(x^2 - 6x + 7)$ is a factor. Dividing the given polynomial equation by this factor, we get the other factor as $(x^2 - 3x - 4)$ which implies that 4 and -1 are the other two roots. Thus

$2+i,\ 2-i,\ 3+\sqrt{2},\ 3-\sqrt{2},\ -1$ and 4 are the roots of the given polynomial equation.

44. (a) (i) **Solution :** Let $\tan^{-1}(-1) = y$. Then,

$\tan y = -1 = -\tan\frac{\pi}{4} = \tan\left(-\frac{\pi}{4}\right)$

As $-\frac{\pi}{4} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ $\tan^{-1}(-1) = -\frac{\pi}{4}$

Now, $\cos^{-1} = \left(\frac{1}{2}\right) y$ implies $\cos y = \frac{1}{2}$ $\cos = \frac{\pi}{3}$

As $-\frac{\pi}{3} \in [0, \pi]$, $\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

Now, $\sin^{-1} = \left(-\frac{1}{2}\right) y$ implies $\sin y = \frac{1}{2}$ $\sin\left(-\frac{\pi}{6}\right)$

As $-\frac{\pi}{6} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, $\sin^{-1}\left(-\frac{1}{2}\right) = \frac{\pi}{6}$

Therefore, $\tan^{-1}(-1) + \cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$

$= \frac{\pi}{4} + \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{12}$

(ii) **Solution :**

By definition, $\cot^{-1} x \in (0, \pi)$

Therefore, $\cot^{-1} x = \theta$ implies $\theta \in (0, \pi)$

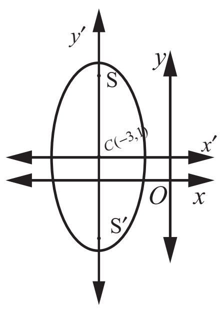
But $\cot^{-1}\left(\frac{1}{7}\right) = \theta$ implies $\theta = \frac{1}{7}$ and hence $\tan\theta = 7$ and θ is acute.

Using $\tan\theta = \frac{7}{1}$, we construct a right triangle as shown, Then, we have $\cos\theta = \frac{1}{5\sqrt{2}}$.

(OR)

(b) **Solution :**

Rearranging the terms, the equation of ellipse is



$$4x^2 + 24x + y^2 - 2y + 21 = 0$$

That is, $4(x^2 + 6x + 9 - 9) + (y^2 - 2y + 1 - 1) + 21 = 0,$

$$4(x+3)^2 - 36 + (y-1)^2 - 1 + 21 = 0,$$

$$4(x+3)^2 + (y-1)^2 = 16,$$

$$\frac{(x+3)^2}{4} + \frac{(y-1)^2}{16} = 1$$

Centre is (–3,1) $a = 4$, $b = 2$, and the major axis is parallel to y-axis

$$c^2 = 16 - 4 = 12$$

$$c = \pm 2\sqrt{3}$$

Therefore, the foci are $(-3, 2\sqrt{3} + 1)$ and $(-3, -2\sqrt{3} + 1)$.

Vertices are $(1, \pm 4 + 1)$. That is the vertices are (1,5) and (1, –3), and the length of Latus rectum $= \frac{2b^2}{a} = 2$ units.

45. (a) **Solution :** Consider $\tan^{-1}(x-1) + \tan^{-1}(x+1)$

$$= \tan^{-1}\left(\frac{x-1+x+1}{1-(x-1)(x+1)}\right) = \tan^{-1}\left(\frac{2x}{1-(x^2-1)}\right)$$

$$= \tan^{-1}\left(\frac{2x}{1-x^2+1}\right) = \tan^{-1}\left(\frac{2x}{2-x^2}\right)$$

$$\therefore \tan^{-1}\left(\frac{2x}{2-x^2}\right) + \tan^{-1}(x) = \tan^{-1}(3x)$$

$$\Rightarrow \tan^{-1}\left(\frac{2x}{2-x^2}\right) = \tan^{-1}(3x) - \tan^{-1}(x)$$

$$\Rightarrow \tan^{-1}\left(\frac{2x}{2-x^2}\right) = \tan^{-1}\left(\frac{3x-x}{1+3x^2}\right)$$

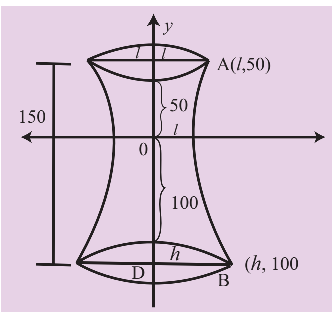
$$\Rightarrow \frac{2x}{2-x^2} = \frac{2x}{1+3x^2}$$

Cross multiply, we get a cubic equations.

Hence, there are 3 solutions for the given equation.

(OR)

(b) **Solution :** The cross section of a nuclear cooling tower is in the shape of a hyperbola.



Given OC $= \frac{1}{2}$ OD and CD = 150 m

Its equation is OC = 50 m & OD = 100 m

$$\frac{x^2}{30^2} - \frac{y^2}{44^2} = 1 \quad \text{...(1)}$$

Let l be the radius of the top of the tower.

$\therefore$ A(l, 50) is a point on the hyperbola

$$\therefore \frac{l^2}{30^2} - \frac{50^2}{44^2} = 1$$

$$\Rightarrow \frac{l^2}{30^2} = 1 + \frac{50^2}{44^2} = \frac{44^2 + 50^2}{44^2}$$

$$\Rightarrow l^2 = \frac{30^2}{44^2}(1936 + 2500)$$

$$\Rightarrow l = \frac{30}{44}\sqrt{4436} = \frac{30}{44}(66.60)$$

$$= \frac{1998}{44} = 45.40 \text{ m}$$

Radius of the top of the tower is 45.40 m.

Let h be the radius of the base of the tower.

$\therefore$ B(h, 100) is a point on the hyperbola

$\therefore$ (1) becomes

$$\frac{h^2}{30^2} - \frac{100^2}{44^2} = 1 \Rightarrow \frac{h^2}{30^2} = 1 + \frac{100^2}{44^2} = \frac{44^2 + 100^2}{44^2}$$

$$\Rightarrow h^2 = \frac{30^2}{44^2}(1936 + 10000)$$

$$\Rightarrow h^2 = \frac{30}{44}\sqrt{11936} = \frac{30}{44}(109.25)$$

$$\Rightarrow h = \frac{3277.5}{44} = 74.48 \text{ m}$$

Radius of the base of the tower is 74.48 m.

46. (a) **Solution :**

The plane passes through two points $\vec{a} = 2\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$.

The straight line passing through the points (2, 1, –3) and (–1, 5, –8) is

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}.$$

$$\Rightarrow \frac{x-2}{-1-2} = \frac{y-1}{5-1} = \frac{z+3}{-8+3}$$

$$\Rightarrow \frac{x-2}{-3} = \frac{y-1}{4} = \frac{z+3}{-5}$$

Hence the required plane is parallel to the vector $\vec{c} = -3\hat{i}+4\hat{j}-5\hat{k}$.

The parametric form of vector equation of the plane passing through two points $\vec{a},\vec{b}$ and parallel to a vector $\vec{c}$ is $\vec{r} = \vec{a} + s(\vec{b}-\vec{a})+t\vec{c}$, $s, t \in \mathbb{R}$.

$$\vec{r} = 2\hat{i}+2\hat{j}+\hat{k}+s(-3\hat{i}-4\hat{j}+2\hat{k}) + (-3\hat{i}+4\hat{j}-5\hat{k}), s, t \in \mathbb{R}$$

Cartesian form of the plane passing through two points and parallel to a vector is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$$

[$\because$ (x_1, y_1, z_1) is (2, 2, 1), (x_2, y_2, z_2) is (–1, –2, 3) & c_1, c_2, c_3 is –3, 4 –5]

$$\Rightarrow \begin{vmatrix} x-2 & y-2 & z-1 \\ -1 & -4 & 2 \\ -3 & 4 & -5 \end{vmatrix} = 0$$

$\Rightarrow (x-2)(20-8)-(y-2)(5+6)+(z-1)(-4-12) = 0$

$\Rightarrow (x-2)(12)-(y-2)(11)+(z-1)(-16) = 0$

$\Rightarrow 12x-24-11y+22-16z+16 = 0$

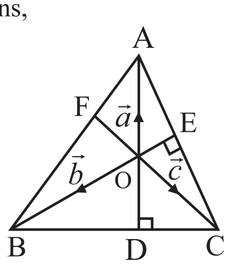
$\Rightarrow 12x-11y-16z+14 = 0$

(OR)

(b) **Solution :** Consider a triangle ABC in which the two altitudes AD and BE intersect at O. Let CO be produced to meet AB at F. We take O as the origin and let $\overrightarrow{OA} = \vec{a}, \overrightarrow{OB} = \vec{b}$ and $\overrightarrow{OC} = \vec{c}$, which means Since $\overrightarrow{AD}$ is perpendicular to $\overrightarrow{BC}$, we have $\overrightarrow{OA}$ is perpendicular to $\overrightarrow{BC}$, and hence we get $\overrightarrow{OA}\cdot\overrightarrow{BC}$ = 0. That is, $\vec{a}.(\vec{c}-\vec{b}) = 0$, which means

$$\vec{a}.\vec{c}-\vec{a}.\vec{b} = 0$$

Similarly, since $\overrightarrow{BE}$ is perpendicular to, $\overrightarrow{CA}$ we have $\overrightarrow{OB}$ is perpendicular to $\overrightarrow{CA}$, and hence we get $\overrightarrow{OC}\cdot\overrightarrow{BA}$ =0. That is, $\vec{b}.(\vec{a}-\vec{c})$, which means,



$$\vec{a}.\vec{b}-\vec{b}.\vec{c} = 0$$

Adding equations (1) and (2), gives $\vec{a}.\vec{b}-\vec{b}.\vec{c}$.

That is, $\vec{c}.(\vec{a}-\vec{b}) = 0$.

That is, $\overrightarrow{OC}, \overrightarrow{BA}$ = 0. Therefore, $\overrightarrow{BA}$ is perpendicular to $\overrightarrow{OC}$ which implies that $\overrightarrow{CF}$ is perpendicular to $\overrightarrow{AB}$. Hence, the perpendicular drawn form C to the side AB passes through O. Thus, the altitudes are concurrent.

47.(a) **Solution :**

Given cos θ + cos ϕ = sin θ + sin ϕ = 0

Let a = cos θ + i sin θ, b = cos ϕ + sin ϕ

$a + b$ (cos θ + cos ϕ) + i (sin θ + sin ϕ) = 0

Since $a + b = 0$ then $(a + b)^2 = 0$

$a^2 + b^2 + 2ab = 0 \Rightarrow a^2 + b^2 = 2(-1)ab$

$\therefore$ (cos θ + i sin θ)^2 + (cos ϕ + sin ϕ)^2 = 2

(cos π + i sin π) (cos θ + i sin θ)(cos ϕ + sin ϕ)

(cos 2θ + cos 2ϕ) + i (sin 2θ + sin 2ϕ) = 2

(cos (π + i sin π)(cos θ + i sin θ)(cos ϕ + i sin ϕ)

(cos 2θ + cos 2ϕ) + i (sin 2θ + sin 2ϕ) = 2

[cos (π + θ + ϕ) + i sin (π + θ + ϕ)]

Equating real and imaginary parts

cos 2θ + cos 2ϕ = 2 cos (π + θ + ϕ)

sin 2θ + sin 2ϕ = 2 sin (π + θ + ϕ)

(OR)

(b) **Solution :** Tangent at (x_1, y_1) is $\frac{xx_1}{a^2}+\frac{yy_1}{b^2} = 1$

$$\Rightarrow \frac{x}{\left(\frac{a^2}{x_1}\right)}+\frac{y}{\left(\frac{b^2}{y_1}\right)} = 1 \qquad \therefore h = \frac{a^2}{x_1}$$

$$\Rightarrow x_1 = \frac{a^2}{h} \text{ and}$$

$$k = \frac{b^2}{y_1}$$

$$\Rightarrow y_1 = \frac{b^2}{k}$$

$(x_1, y_1) = \left(\frac{a^2}{k}, \frac{b^2}{k}\right)$ is a point on the ellipse $\frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}$

$$= 1 \Rightarrow \frac{(a^2)^2}{h^2(a^2)}+\frac{(b^2)^2}{k^2(b^2)} = 1$$

$$\therefore \frac{a^2}{h^2}+\frac{b^2}{k^2} = 1$$

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