Chapter 10 Ex: 10.8 and Example Problems. (Ordinary Differential Equations). (Key Points only)

Problem Name	Given	Equation	Initial Condition	Second condition	Final condition	Answer
Bacteria	$\frac{dA}{dt} \propto A$	$A = c e^{kt}$	$t=0$ & $A = A_0$ $\therefore c = A_0$ $A = A_0 e^{kt}$	when $t=5$ $\therefore e^{5k} = 3$	when $t=10$, To find $A = ?$	$A = 9A_0$
Population (City)	$\frac{dA}{dt} \propto A$	$A = Ce^{kt}$	$t=0$ & $A = 3,00000$ $\therefore c = 3,00,000$ $A = 3,00,000 e^{kt}$	when $t = 40$ & $A = 4,00,000$ $e^{40k} = \frac{4}{3}$	At any time t, To find $A = ?$	$A = 3,00,000 \left(\frac{4}{3}\right)^{\frac{1}{40}t}$
Population (Colony)	$\frac{dA}{dt} \propto A$	$A = ce^{kt}$	$t=0$ & $A = A_0$ $\therefore c = A_0$ $A = A_0 e^{kt}$.	when $t = 50$, $A = 2A_0$ $k = \frac{1}{50} \log 2$	when $A = 3A_0$ To find $t = ?$	$t = 50\left(\frac{\log 3}{\log 2}\right)$
Boat	$\frac{dv}{dt} = -v$	$V = Ce^{-t}$	$t=0$ & $V=10$ $\therefore C = 10$ $V = 10 e^{-t}$	–	when $t=2$, To find $v = ?$	$V = 10e^{-2}$
Radioactive Nuclei decay	$\frac{dA}{dt} \propto A$	$A = ce^{kt}$	$t=0$ & $A = A_0$ $\therefore c = A_0$ $A = A_0 e^{kt}$	when $t=100$ & $A = 9/10\, A_0$ $e^{100k} = \frac{9}{10}$	when $t=1000$, To find $A =$	$A = A_0 \frac{9^{10}}{10^{10}}$
Radioactive Isotopes	$\frac{dA}{dt} = -kA$	$A = ce^{-kt}$	$t=0$ & $A=200$ $\therefore c = 200$ $A = 200 e^{-kt}$	$t=2$ & $A=150$ $\therefore k = \frac{1}{2}\log\left(\frac{4}{3}\right)$	After 't' years To find mass $A = ?$	$A = 200 e^{\frac{t}{2}\log\frac{4}{3}}$.
Bank deposit	$\frac{dA}{dt} \propto A$ & $k = 5\%$	$A = c e^{0.05t}$	$t=0$ & $A = 10,000$ $\therefore c = 1,0000$ $A = 10000 e^{0.05t}$	–	$t = 18$ months $t = 3/2$ years, To find $A = ?$	$A = 10000 e^{0.075}$
Salt (100g) in water tank (1000 l)	$\frac{dA}{dt}$ = in flow − out flow $\frac{dA}{dt} = 50 - \frac{10}{1000}A$	$A - 5000 = Ce^{-0.01t}$	$t=0$ & $A=100$ $\therefore c = -4900$	–	At any time 't', Amount of salt $= ?$	$A = 5000 - 4900e^{-0.01t}$
Salt (2g) in water tank (50 l)	$\frac{dA}{dt}$ = in flow − out flow $\frac{dA}{dt} = 6 - \frac{3}{50}A$	$A - 100 = Ce^{-3/50\, t}$	$t=0$ & $A=0$ $\therefore C = -100$	–	At any time 't', Amount of salt $= ?$	$A = 100(1 - e^{-3/50\, t})$

Problem Name	Given data	Equation	Initial Condition	II condition	Final Condition	Answer
Water (Boiling)	$\frac{dT}{dt}\propto(T-S)$ $S=25^\circ$	$T-S=ce^{kt}$ $T-25=ce^{kt}$	$t=0,\ T=100^\circ C$ $\therefore c=75^\circ$ $T-25=75e^{kt}$	(i) $t=10$ & $T=80^\circ C$ $e^{10k}=11/15$ (ii) $T=40^\circ C$ To find $t=?$	$t=20$, To find $T=?$ (ii) $T=40^\circ C$ To find $t=?$	$T=65.33^\circ C$ $t=51.89$ minuts
Instant coffee.	$\frac{dT}{dt}\propto(T-S)$ $S=70^\circ$	$T-S=ce^{kt}$ $T-70=ce^{kt}$	$t=0;\ T=180^\circ$ $\therefore C=110$ $T-70=110e^{kt}$	$t=10,\ T=160^\circ$ $e^{10k}=9/11$	(i) $t=15$ To find $T=?$ (ii) when $T=130^\circ F \Rightarrow t_1=30.216$ when $T=140^\circ F \Rightarrow t_2=22.56$	$T=151.4^\circ F$ Between 10.22 AM to 10.30 AM.
Pot of Boiling water	$\frac{dT}{dt}\propto T-S$	$T-S=ce^{kt}$	$t=0$ & $T=100^\circ C$ $\therefore c=100-S$ $\therefore T-S=(100-S)e^{kt}$	$t=5$ & $T=80^\circ$ $\therefore e^{5k}=\frac{80-S}{100-S}$	when $t=10$ & $T=65^\circ$ To find $S=?$	Surrounding Temperature $S=20^\circ C$
Murder Investigation	$\frac{dT}{dt}\propto T-S$ $S=50^\circ F$	$T-50=ce^{kt}$	$t=0$ & $T=70^\circ F$ $\therefore c=-20$	$t=2,\ T=60^\circ F$ $k=\frac{1}{2}\log\left(\frac{1}{2}\right)$ $\therefore T=50+20\left(\frac{1}{2}\right)^{t/2}$	$T=98.6^\circ F$ To find $t=?$ $\therefore t=-2.56$ hours	App. (death) Murder time 5:30 PM
Current $E=Ri+L\frac{di}{dt}$ (solve by linear diff. equation method)	$\frac{di}{dt}+\frac{R}{L}i=\frac{E}{L}$	It is linear in 'i' $P=R/L$ $Q=E/L$	$P=R/L$ $I.F=e^{\int P\,dx}$ $=e^{R/L\,t}$	General solution is $i\,(I.F)=\int Q\,(I.F)\,dt+C$ $i\,e^{R/L\,t}=\frac{E}{R}e^{R/L\,t}+c$	when $E=0$, To find current $i=?$	$i=c\,e^{-\frac{Rt}{L}}$

Y. ARUL KUMAR., M.Sc., B.Ed., ST. JOSEPH'S MHSS
KKI - DT.