12th - PHYSICS - 2024
English Medium.
UNIT TEST - 1
MODEL EXAM - 1

Total Mark. 5×5=25M

Date:- 14/04/2024. Total Mark:- 25M

1) Calculate the electric field due to dipole on its axial line

2) Calculate the electric field due to dipole on its equatorial line

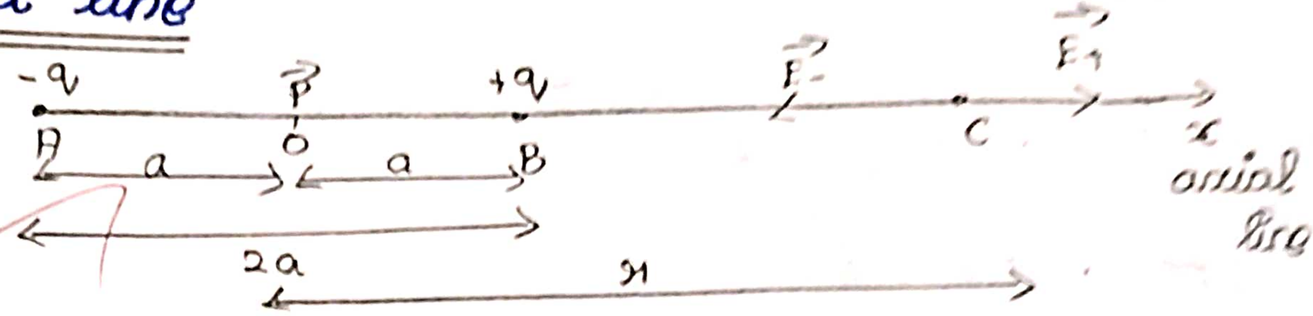
3) Derive an expression for electrostatic Potential due to a point charge

4) Derive the Expression for resultant capacitance when capacitors are connected in series

5) Derive the Expression for resultant capacitance when capacitors are connected in parallel

Answer

Axial line



AB → Two Point

O → centre of the point

a → Distance

r → Distance

2a → very small Distance

+q → positive charge

-q → negative charge

+E → Positive Electric field

E- → negative Electric field

AB = 2a

OA = a

OB = a

OC = r

p → Electric dipole Moment

$P = q \times 2a$

$\vec{p} = q \times 2a \cdot \hat{p}$

Electric field

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

B Point (+q charge)

Electric field

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

$$\vec{E}_+ = \frac{1}{4\pi\epsilon_0}\frac{q}{(r-a)^2}\cdot\hat{p} \rightarrow ①$$

A Point (-q charge)

Electric field

$$\vec{E}_- = \frac{1}{4\pi\epsilon_0}\frac{-q}{(r+a)^2}\cdot\hat{p} \rightarrow ②$$

Total Electric field

$$\vec{E}_{Total} = \vec{E}_+ + \vec{E}_-$$

$$\vec{E}_{Total} = \left[\frac{1}{4\pi\epsilon_0}\frac{q}{(r-a)^2}\cdot\hat{p}\right] + \left[\frac{1}{4\pi\epsilon_0}\frac{-q}{(r+a)^2}\cdot\hat{p}\right]$$

$$\vec{E}_{Total} = \left[\frac{1}{4\pi\epsilon_0}\frac{q}{(r-a)^2}\right] + \left[-\frac{1}{4\pi\epsilon_0}\frac{q}{(r+a)^2}\right]\cdot\hat{p}$$

$$\vec{E}_{Total} = \left[\frac{1}{4\pi\epsilon_0}\frac{q}{(r-a)^2} - \frac{1}{4\pi\epsilon_0}\frac{q}{(r+a)^2}\right]\cdot\hat{p}$$

$$\vec{E}_{Total} = \left[\frac{q}{4\pi\epsilon_0}\frac{1}{(r-a)^2} - \frac{q}{4\pi\epsilon_0}\frac{1}{(r+a)^2}\right]\cdot\hat{p}$$

$$\vec{E}_{Total} = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]\cdot\hat{p}$$

$$\vec{E}_{Total} = \frac{q}{4\pi\epsilon_0}\left[\frac{(r+a)^2 - (r-a)^2}{(r-a)^2(r+a)^2}\right]\cdot\hat{p}$$

$$(a+b)^2 = a^2+b^2+2ab$$

$$(r+a)^2 = r^2+a^2+2ra$$

$$(a-b)^2 = a^2+b^2-2ab$$

$$(r-a)^2 = r^2+a^2-2ra$$

$$\vec{E}_{Total} = \frac{q}{4\pi\varepsilon_0}\left[\frac{(r^2+a^2+2ra)-(r^2+a^2-2ra)}{(r+a)^2(r-a)^2}\right]\cdot\hat{p}$$

$$\vec{E}_{Total} = \frac{q}{4\pi\varepsilon_0}\left[\frac{\cancel{r^2}+\cancel{a^2}+2ra-\cancel{r^2}-\cancel{a^2}+2ra}{(r+a)^2(r-a)^2}\right]\cdot\hat{p}$$

$$\vec{E}_{Total} = \frac{q}{4\pi\varepsilon_0}\left[\frac{2ra+2ra}{(r^2-a^2)^2}\right]\cdot\hat{p}$$

$$\vec{E}_{Total} = \frac{q}{4\pi\varepsilon_0}\left[\frac{4ra}{(r^2-a^2)^2}\right]\cdot\hat{p}$$

$$\vec{E}_{Total} = \frac{q}{4\pi\varepsilon_0}\left[\frac{4ra}{(r^2)^2}\right]\cdot\hat{p}$$

$$r >> a$$

$$\vec{E}_{Total} = \frac{q}{4\pi\varepsilon_0}\left[\frac{4ra}{r^4}\right]\cdot\hat{p}$$

$$p = q \times 2a$$

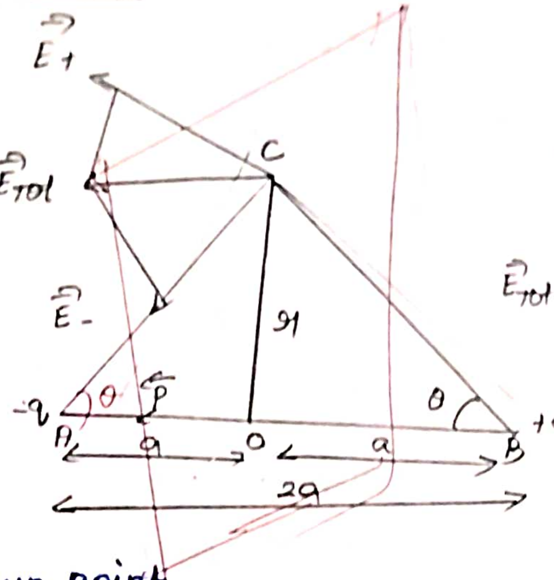
$$2p = q \times 4a$$

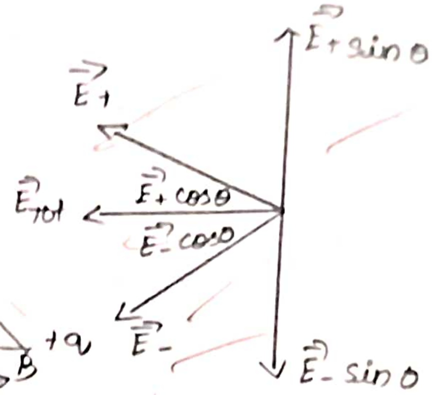
$$\vec{E}_{Total} = \frac{q}{4\pi\varepsilon_0}\left[\frac{4a}{r^3}\right]\cdot\hat{p}$$

$$\vec{E}_{Total} = \frac{1}{4\pi\varepsilon_0}\frac{q\times 4a}{r^3}\cdot\hat{p}$$

$$\vec{E}_{Total} = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}.$$

2) equatorial line





AB → Two point

O → centre of point

θ → Angle

a → Distance

r → Distance

2a → small Distance

+q → Positive charge

−q → Negative charge

$\vec{E}_+$ → Positive Electric field

$\vec{E}_-$ → Negative Electric field

$\vec{E}_{Tot}$ → Total Electric field

ΔABC

AB = 2a

OA = a

OB = a

OC = r

P → Electric dipole Moment

$P = q \times 2a$

$\vec{P} = q \times 2a \cdot \hat{p}$

$\left.\begin{array}{l}\vec{E}_+ \cos\theta \\ \vec{E}_- \cos\theta\end{array}\right\}$ Parallel direction ↓ Horizontal component

$\vec{E_+}\sin\theta$ } Perpendicular direction
$\vec{E_-}\sin\theta$ } vertical component
cancel each other

To Electric field

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

$$\vec{E_+} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2+a^2)} \rightarrow (1)$$

$$\vec{E_-} = \frac{1}{4\pi\varepsilon_0}\frac{-q}{(r^2+a^2)} \rightarrow (2)$$

Total Electric field

$$\vec{E}_{Total} = -|E_+|\cos\theta\cdot\hat{p} - |E_-|\cos\theta\cdot\hat{p}$$

$$\vec{E}_{Total} = -E\cos\theta\cdot\hat{p} - E\cos\theta\cdot\hat{p}$$

$$\vec{E}_{Total} = -2E\cos\theta\cdot\hat{p}$$

$$\vec{E}_{Total} = -2\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2+a^2)}\right)\cos\theta\cdot\hat{p}$$

$$\vec{E}_{Total} = -\frac{1}{4\pi\varepsilon_0}\frac{2q}{(r^2+a^2)}\cos\theta\cdot\hat{p}$$

$$\cos\theta = \frac{a}{\sqrt{r^2+a^2}}$$

$$\vec{E}_{Total} = \frac{-1}{4\pi\varepsilon_0}\frac{2q}{(r^2+a^2)}\left(\frac{a}{\sqrt{r^2+a^2}}\right)\cdot\hat{p}$$

$$\vec{E}_{Total} = \frac{-1}{4\pi\varepsilon_0}\left(\frac{2qa}{(r^2+a^2)\sqrt{r^2+a^2}}\right)\cdot\hat{p}$$

$$\vec{E}_{Total} = \frac{-1}{4\pi\epsilon_0} \frac{2qa}{(r^2+a^2)(r^2+a^2)^{1/2}} \cdot \vec{p}$$

$$\vec{E}_{Total} = \frac{-1}{4\pi\epsilon_0} \frac{2qa}{(r^2+a^2)^{1+1/2}} \cdot \vec{p}$$

$$\vec{E}_{Total} = \frac{-1}{4\pi\epsilon_0} \frac{2qa}{(r^2+a^2)^{3/2}} \cdot \vec{p}$$

$$\vec{E}_{Total} = -\frac{1}{4\pi\epsilon_0} \frac{2qa \cdot \vec{p}}{(r^2+a^2)^{3/2}}$$

$\boxed{p = q \times 2a}$

$r >> a$

$$\vec{E}_{Total} = -\frac{1}{4\pi\epsilon_0} \frac{\vec{p}}{(r^2)^{3/2}}$$

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$$\vec{E}_{Total} = -\frac{1}{4\pi\epsilon_0} \frac{\vec{p}\, r}{r^3}$$

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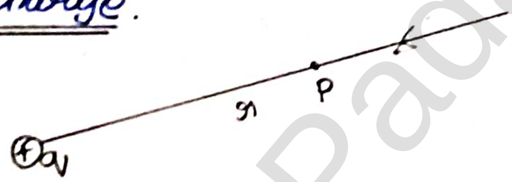
3) Point charge.



Electrostatic Potential

unit = V (volt)

$$V = \int_{\infty}^{r} -\vec{E}\, \vec{dr}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$$

$$V = \int_{\infty}^{r} -\vec{E}\, \vec{dr}$$

$$V = -\int_{\infty}^{r} E\, dr$$

$$V = -\int_{\infty}^{r} \frac{1}{4\pi\varepsilon_0}\,\frac{q}{r^2}\,\vec{dr}$$

$$V = -\int_{\infty}^{r} \frac{q}{4\pi\varepsilon_0}\,\frac{1}{r^2}\,\vec{dr}$$

$$V = -\frac{q}{4\pi\varepsilon_0}\int_{\infty}^{r} r^{-2}\,dr$$

$$V = -\frac{q}{4\pi\varepsilon_0} \qquad \int x\,dx = \frac{x^{n+1}}{n+1} + c$$

$$V = -\frac{q}{4\pi\varepsilon_0}\left[\frac{r^{-2+1}}{-2+1}\right]_{\infty}^{r}$$

$$V = -\frac{q}{4\pi\varepsilon_0}\left[\frac{r^{-1}}{-1}\right]_{\infty}^{r}$$

$$V = -\frac{q}{4\pi\varepsilon_0}\left[-r^{-1}\right]_{\infty}^{r}$$

$$V = \frac{q}{4\pi\varepsilon_0}\left[r^{-1}\right]_{\infty}^{r}$$

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]_{\infty}^{r}$$

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{1}{\infty}\right]$$

$$\frac{1}{\infty} = 0$$

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r} - 0\right]$$

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$$V = \frac{q}{4\pi\varepsilon_0} \frac{1}{r}$$

$$V = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}$$

+q potential decreases → distance increases

−q potential increases → distance increases

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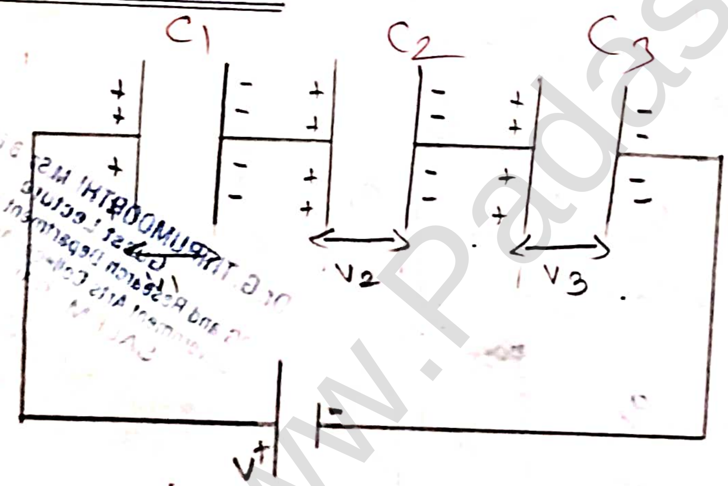
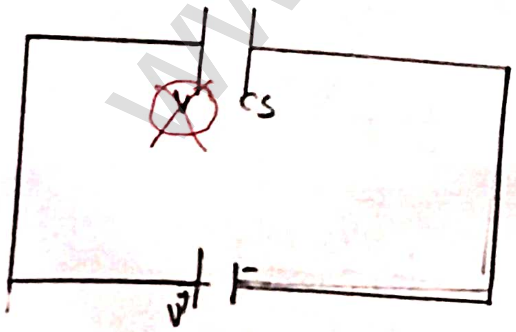
$r = \infty$ $V = 0$

4) <u>Series connection</u>

a)



b)



$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$V = V_1 + V_2 + V_3$$

$V \Rightarrow$ Potential difference

$$C = \frac{Q}{V}$$

$$V = \frac{Q}{C}$$

$$V_1 = \frac{Q}{C_1} \rightarrow (1)$$

$$V_2 = \frac{Q}{C_2} \rightarrow (2)$$

$$V_3 = \frac{Q}{C_3} \rightarrow (3)$$

<u>Total potential difference</u>

$$V = V_1 + V_2 + V_3$$

$$V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$V = \frac{Q}{C}$$

$$C = C_S$$

$$\frac{Q}{C_S} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

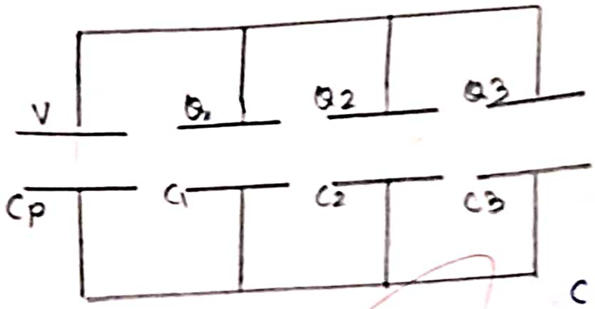
$$\frac{Q}{C_S} = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

$$\boxed{\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

This is equivalent capacitance series is less than the smallest individual capacitance in the series

5) Parallel connection

a)



$$C_p = C_1 + C_2 + C_3$$

b)

$$\boxed{Q = Q_1 + Q_2 + Q_3}$$

$$C = \frac{Q}{V}$$

$$Q = CV$$

$$Q_1 = C_1 V \rightarrow (1)$$

$$Q_2 = C_2 V \rightarrow (2)$$

$$Q_3 = C_3 V \rightarrow (3)$$

$$CV = C_1 V + C_2 V + C_3 V$$

$$C = C_p$$

$$C_p V = C_1 V + C_2 V + C_3 V$$

$$C_p V = V(C_1 + C_2 + C_3)$$

$$\boxed{C_p = C_1 + C_2 + C_3}$$

In a parallel connection is equivalent of each area of add to give more elastic area such the total parallel connection increases

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